Lecture 2: Return Performance

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FINM 36700: Portfolio Management

Outline

Measuring Performance

Evaluating Performance

Hedging and Tracking

Log returns

Let r denote the log-return:

$$\mathrm{r}_t \equiv \log(1 + r_t)$$

Log returns are particularly useful when dealing with compounding returns across different time horizons.

$$\mathrm{r}_{t,t+h} \equiv \log(1 + r_{t,t+h}) = \sum_{i=1}^{h} \mathrm{r}_{t+i}$$

Thus, the cumulative return from t to $t+h$ is just the sum of one-period returns.

Annualizing Returns

- Suppose that log returns are iid (independent, and identically distributed).
- Then the mean and variance are linear in the cumulative return horizon, h.

$$\boxed{\begin{aligned}\mathbb{E}\left[\mathrm{r}_{t,t+h}\right] &= h\left(\mathbb{E}\left[\mathrm{r}_t\right]\right)\\ \mathrm{var}\left[\mathrm{r}_{t,t+h}\right] &= h\left(\mathrm{var}\left[\mathrm{r}_t\right]\right)\end{aligned}}$$

vol = $\sqrt{h}$ std(rt)

Example: Suppose the *monthly* return of a security has unconditional mean, μ. Then the *annual* return of the security is 12μ.

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Interview question

Question

- Daily excess returns on the U.S. stock index from July 1963 to June 2012 have a volatility of 0.0010 (0.1%).
- Assume there are 21 business days per month.

What is the volatility of monthly returns on the U.S. stock index over this time?

252 business days in a year

Interview question

Answer

- Calculate $\sqrt{21} = 4.58$.
- According to iid cumulation formula, monthly volatility would be 0.458%.
- Empirically, monthly excess return volatility is 4.55 times as large as the daily volatility.

Source: CRSP index. July 1963 - June 2012.

The iid assumption

These scalings are widely used in quoting stats.

- Theoretically requires iid assumption, but this is often ignored.
- Return series have low autocorrelation, so assumption of independence may not be too bad.
- However, returns are often distributed in clearly non-identical ways, as illustrated by the following figure.

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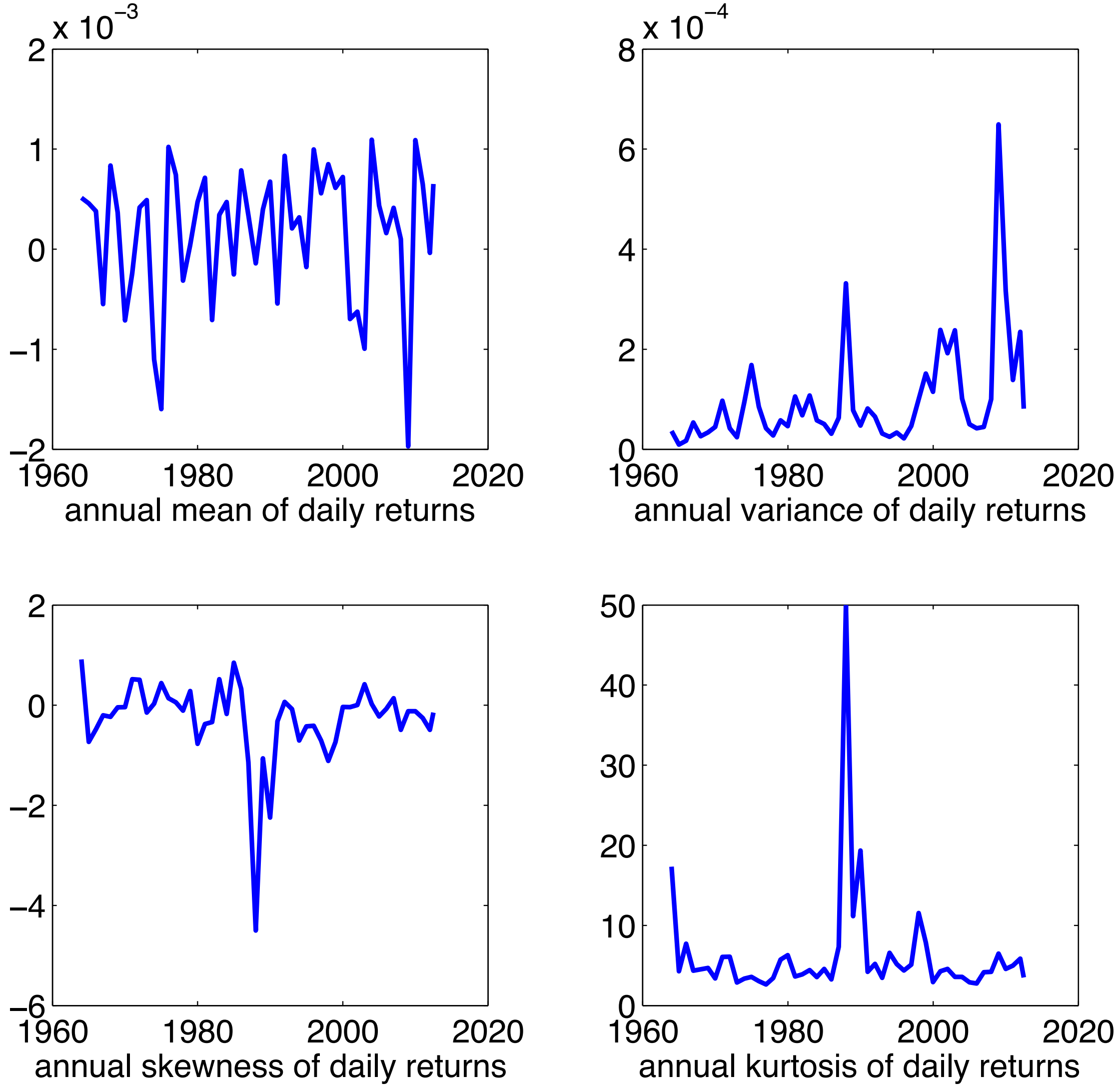

Figure: Sub-sample estimates of first four moments of excess log-returns on equity index. Source: CRSP U.S. stock index. July 1963 to June 2012.

Higher moments

Consider a random variable x, with

$$\mu = \mathbb{E}\left[x\right]$$
$$\sigma^2 = \mathbb{E}\left[(x-\mu)^2\right]$$

We are not assuming x is normally distributed.

Skewness is defined as the (scaled) third centralized moment of the distribution:

$$\varsigma = \frac{\mathbb{E}\left[(x-\mu)^3\right]}{\sigma^3}$$

Kurtosis is defined as

$$\kappa = \frac{\mathbb{E}\left[(x-\mu)^4\right]}{\sigma^4}$$

A normal distribution has kurtosis equal to 3, so *excess kurtosis* refers to $\kappa - 3$.

Scaling higher moments

Unlike quotes of return mean and volatility,

- Skewness and kurtosis of returns are not typically scaled with cumulative horizon.
- The formulas for how skewness and kurtosis compound for $\mathrm{r}_{t,t+h}$ are messier, especially without the iid assumption.

Example: higher moment market returns

Consider excess returns from the U.S. equity market.

- Skewness of **daily** returns: -0.8150.
  Skewness of **monthly** returns: -0.8023.
- Kurtosis of **daily** returns: 22.1085.
  Kurtosis of **monthly** returns: 5.8157.

Source: CRSP Index . July 1963 to June 2012

Estimating higher moments

Given samples of size T, the sample mean, variance, skewness, and kurtosis are given by

$$
\begin{aligned}
\bar{\mu}_x =& \frac{1}{T}\sum_{i=1}^{T} x_i \\
\bar{\sigma}_x^2 =& \frac{1}{T-1}\sum_{i=1}^{T}(x_i-\bar{\mu}_x)^2 \\
\bar{\varsigma}_x =& \left(\frac{1}{\bar{\sigma}_x^3}\right)\frac{1}{T-1}\sum_{i=1}^{T}(x_i-\bar{\mu}_x)^3 \\
\bar{\kappa}_x =& \left(\frac{1}{\bar{\sigma}_x^4}\right)\frac{1}{T-1}\sum_{i=1}^{T}(x_i-\bar{\mu}_x)^4
\end{aligned}
$$

Value at Risk

how bad can things get

The τ-day, π% Value at Risk (VaR) of a portfolio is defined as $\text{VaR}_{\pi,\tau}$ such that

- there is a π% chance
- that over a horizon of τ days
- the portfolio will lose an amount greater than VaR.

Assume Normal Distribution

$r^{VaR} = \mu_t + z_q \sigma_t$ 可以直接得到 VaR

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VaR as a Return

eg last 50 points with VaR 0.01 ⇒ 根本找不到. 但如果 assume normal distribution → 可以解决这个问题, Also, limits data has no statistical power, hard to estimate

尽管 assumption N not correct, but has statistical power

The VaR expressed as a return rate is then,

$$r^{\text{VaR}}_{\pi,\tau} = F^{r\,(-1)}_{\tau}(\pi)$$

where F^r_τ is the cdf of the return distribution.

The VaR in terms of returns is simply the π quantile of the observed returns.

assume

0

pdf of distribution

$$r^{cVaR} = \mu_{\tau,t} - \frac{\phi_z(z_q)}{q}\sigma_{\tau,t}$$

quantile you choose

$$\boxed{r^{VaR_{q,\tau}} = \sqrt{\tau}\, r^{VaR}}$$

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Expected Shortfall

Expected Shortfall (ES) refers to the expected loss conditional on a loss greater than $\text{VaR}_{\pi,\tau}$ occurring.

- ES is the expected horizon-τ loss, conditional on a loss in the $(1-\pi)\,\%$ tail of the loss cdf occurring.
- That is, ES looks at expected loss given that a loss of at least $\text{VaR}_{\pi,\tau}$ has occurred.

Thus, ES is simply the mean of all returns smaller than the π quantile.

ES is also sometimes known as Conditional Value-at-Risk (CVaR).

calculate mean loss for VaR range

Maximum Drawdown more on history

最大回撤

The maximum drawdown (MDD) of a return series is the maximum **cumulative** loss suffered during the time period.

- Visually, this is the largest peak-to-trough during the sample.
- This is a path-dependent statistic, and it is much less precise to estimate for the future.
- It is widely cited in performance evaluation to understand how badly the investment might perform.

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Outline

$r^M_{2000-2029} < 30\, r_s$

$r^M_{2000-2023} + r^M_{2024-2029} < 30\, r_s$

$r^M_{2024-2029} < 30 r_s - 24 \cdot r^M_{mean}$

$\frac{1}{6}$ ____ $< 5 r_s - 4 r^m$

Measuring Performance

Evaluating Performance

Hedging and Tracking

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Risk-adjusted performance

- Asset may have impressive $\mathbb{E}[r]$, but we want to know how this compares to risk.
- Otherwise, a fund manager might obtain high returns by loading excessively on risk.

Mean-Variance

Mean-variance analysis is adjusting for one type of risk: variance.

- Optimize mean per variance (or equivalently, volatility.)
- The Sharpe Ratio is a measure of this risk and return tradeoff.
- Did not consider any other measure of return risk.

Factor decomposition of return variation

A Linear Factor Decomposition (LFD) of $\tilde{r}^i$ onto the factor $\mathbf{x}_t$ is given by the regression,

$$\tilde{r}^i_t = \alpha + \boldsymbol{\beta}^{i,\mathbf{x}} \mathbf{x}_t + \epsilon_t$$

- The variation in returns is decomposed into the variation explained by the benchmark, x_t and by the residual, ϵ_t.
- These factors, $\mathbf{x}$, in the LFD should give a high R-squared in the regression if they really explain the variation of returns well.

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Elements of the regression

Consider the following elements of the regression:

- Alpha. Expected return beyond what can be explained by the factor.
- Beta. Risk related to the factor. If x moves, how much will our return move?
- Residual. The risk of the return uncorrelated to the factor.

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Interpreting alpha

Using alpha as a measure of performance is sensitive to which factors are used in the regression.

- High α will always lead to the question of whether the performance is good, or whether we used a bad model.
- Perhaps alpha is really just some missing beta from the model.
- Still, if this missing beta is not widely known or understood, it may make sense for investors to pay fees to get access to this beta knowledge, even though the fund is passively tracking a model.

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Luck or skill

Estimating alpha is statistically imprecise.

- It would be easy to get a large α due to in-sample luck.
- So when faced with a large α, it may be a sign of high performance, or it may just be luck in that sample.

Treynor's Ratio

Treynor's measure is an alternative measure of the risk-reward tradeoff. For the return of asset, i,

$$\text{Treynor Ratio} = \frac{\mathbb{E}\left[\tilde{r}^i\right]}{\beta^{i,m}}$$

Information ratio

unexplained factor

The **information ratio** refers to the Sharpe Ratio of the non-factor component of the return: $\alpha + \epsilon_t$.

$$\text{IR} = \frac{\alpha}{\sigma_\epsilon}$$

not explained by sharpe ratio

where σ_ϵ and α come from

$$\tilde{r}^i_t = \alpha + \beta^{i,j}\tilde{r}^j_t + \epsilon_t$$

- α measures the excess return beyond what is explained by the factor, j.
- σ_ϵ measures the non-factor volatility.

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Manipulating the benchmark

Investment managers (of hedge funds, mutual funds, etc.) might prefer to be evaluated against a benchmark which does not capture all risks.

- Hedge funds may want to be evaluated against a simple equity benchmark.
- But these funds are likely achieving excess returns by taking on other forms of risk which do not show up in the regression.

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Net exposure

Investor is long \$1 of i and hedges by selling h\$ of j.

- Time t net exposure is,

$$\epsilon_t = r_t^i - h\, r_t^j$$

- This net exposure after hedging is known as **basis**.
- A position in i is perfectly hedged over horizon t if $\epsilon_t = 0$ with probability one.

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Why basis?

Why cross-hedging rather than perfectly hedging with i through futures, options, etc.

- Maybe asset i is a non-tradable exposure.

  eg. private real estate (interest rate risk)

- Maybe asset i is a traded security, but it has no market in futures and shorting i is costly.

Instead, the investor must hedge using asset j.

Basis risk

$$\epsilon_t = r_t^i - h\, r_t^j$$

Basis risk refers to volatility in ϵ_t, denoted as σ_ϵ.

$$\sigma_\epsilon^2 = \sigma_i^2 + h^2\sigma_j^2 - 2h\,\sigma_i\sigma_j\,\rho_{i,j}$$

- Denoted as σ_ϵ^2 because basis is the error in the hedge.
- For $\rho_{i,j} = \pm 1$, the basis risk can be eliminated.

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Optimal hedge ratio

The **optimal hedge ratio**, h^*, minimizes basis risk.

$$\begin{aligned} h^* &= \arg\min_h \sigma_\epsilon^2 \\ &= \arg\min_h \left\{\sigma_i^2 + h^2\sigma_j^2 - 2h\,\sigma_i\sigma_j\,\rho_{i,j}\right\} \end{aligned}$$

Solve by taking derivative,

如果 σ_i 比 σ_j 大很多, 则需要更多的 σ_j 来hedge

$$\boxed{h^* = \frac{\sigma_i}{\sigma_j}\,\rho_{i,j}}$$

- Higher correlation implies larger hedge ratio, h.
- High relative volatility of i implies larger hedge ratio.
- With negative correlation, must go long the hedging security.

Basis as a regression residual

From the previous slide, we can write

$$r_t^i = \beta^{i,j} r_t^j + \epsilon_t$$

where

$$\boxed{\beta^{i,j} = h^*}$$

- The optimal hedge ratio, h^*, is simply a regression beta!
- Optimized basis risk is simply the regression residual variance.
- (Thus the notation of using ϵ to denote basis.)

Hedging returns

需要 update 因为 β 会变

These results also apply to hedging with multiple assets:

$$r_t^i = \beta^{i,1} r_t^1 + \beta^{i,2} r_t^2 + \ldots + \beta^{i,k} r_t^k + \epsilon_t$$

- Basis is then the net return exposure.
- Optimal hedge ratios are given by the betas in the return regression above.

Hedging excess returns

These results also apply to hedging excess returns.

$$\tilde{r}^i_t = \beta^{i,1}\tilde{r}^1_t + \beta^{i,2}\tilde{r}^2_t + \ldots + \beta^{i,k}\tilde{r}^k_t + \epsilon_t$$

- Optimal hedge ratios are given by the regression beta.
- Intercept is portion of mean returns which can not be replicated by the hedge strategy.

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Include an intercept?

$\alpha + \beta^1 r_1 + \beta^2 r_2 + \cdots + \varepsilon_t$

In regression for optimal hedge ratio, should we include a constant, (alpha?) Depends on our purpose...

- Do we want to explain the total return (including the mean) or simply the excess-mean return?
- In short samples, mean returns may be estimated inaccurately, (whether in r^i or $\tilde{r}^i$,) so we may want to include α (eliminate means) to focus on explaining variation.

α并不是 risk free rate (而是和 hedging assets 比较 constant)

没有α：更接近实际情况 (因为没有 instrument 是 no risk & constant)

有α： just want to maximize correlation，不 care mean return same

(mean return not accurately estimated, variance is accurate
则不希望 hedge ratio based on mean return)

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Investment with hedging a factor

- Suppose a hedge fund wants to trade on information regarding a certain asset return, r^i.
- But does not want the trade to be subject to the overall market factor, r^m.

don't want to bet on S&P500, hedge out S&P500

- More generally, imagine anyone that wants to trade on the performance of return r^i **relative** to another factor r^j.

This idea of trading on specific information while hedging out broader market movements is the origination of the term, hedge funds.

Building the market-hedged position

A hedge fund would first run the regression

$$\tilde{r}^i_t = \alpha + \beta^{i,m}\tilde{r}^m_t + \epsilon_t$$

- Then the hedge-fund can go long $\tilde{r}^i$, while shorting $\beta^{i,m}$ times the overall market.
- The fund is then holding

$$\tilde{r}^i_t - \beta^{i,m}\tilde{r}^m_t = \alpha + \epsilon_t$$

Properties of the market-hedged position

$$\tilde{r}^i_t - \beta^{i,m}\tilde{r}^m_t = \alpha + \epsilon_t$$

- This hedged position has mean excess return α; volatility σ_ϵ.
- Compared to simply going long $\tilde{r}^i$, the strategy is no longer subject to the volatility coming from $\beta^{i,m}\tilde{r}^m_t$.
- This allows the hedge fund to minimize the variance of the hedged position.

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Hedging vs Tracking

- We have considered the case where an investor wants to completely hedge out some factor, r^j.

- This optimal hedging allows the investor to just trade on the portion of r^i uncorrelated with r^j: $\alpha + \epsilon$.

- Now consider a **tracking portfolio**, r^i, which tracks a factor, r^j, rather than hedging it out.

(tracking)
如果 mimic：希望 $\alpha = 0$ 因为这样 easily captured   buy r^j

如果 hedge：希望 α 越大越好，   short r^j

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Tracking portfolios

Regress

$$\tilde{r}_t^i = \alpha + \beta \tilde{r}_t^j + \epsilon_t$$

- ϵ is known as the **tracking error** of $\tilde{r}^i$ relative to $\tilde{r}^j$.
- R-squared measures how well j tracks i. want high R^2
- The Information Ratio, α/σ_ϵ, measures the tradeoff between obtaining extra mean return α at the cost of taking on tracking error ϵ from the target portfolio.

Of course, this is just another way of looking at the hedging problem.

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Tracking funds and hedged funds

=tracking fund

For broad market factors, mutual funds are often tracking some factor while hedge funds are trying to hedge it out.

$$\tilde{r}_t^i = \underbrace{\beta \tilde{r}_t^j}_{\text{mutual fund position}} + \underbrace{\alpha + \epsilon_t}_{\text{hedge fund position}}$$

for factors such as the overall market index, industry indexes, value/growth indexes, etc.

- This is not exactly true.
- Hedge funds retain some factor exposure, β, while mutual funds deviate from their benchmark.

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Mutual funds benchmarks

Most mutual funds explicitly state that they track some type of benchmark. r^j may equal...

- Market index
- Value index
- Growth index
- Small stock index
- Large stock index
- Foreign equities
- AAA Corporate bonds

Along with many others...

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