In [111…

```
# 1.
## 1.1
```

In [112…

```
# import libraries
import os
import sys
import numpy as np
import matplotlib.pyplot as plt
import scipy.stats as stats
#from scipy.stats import skew, kurtosis

# Set the current working directory
os.chdir(sys.path[0])
```

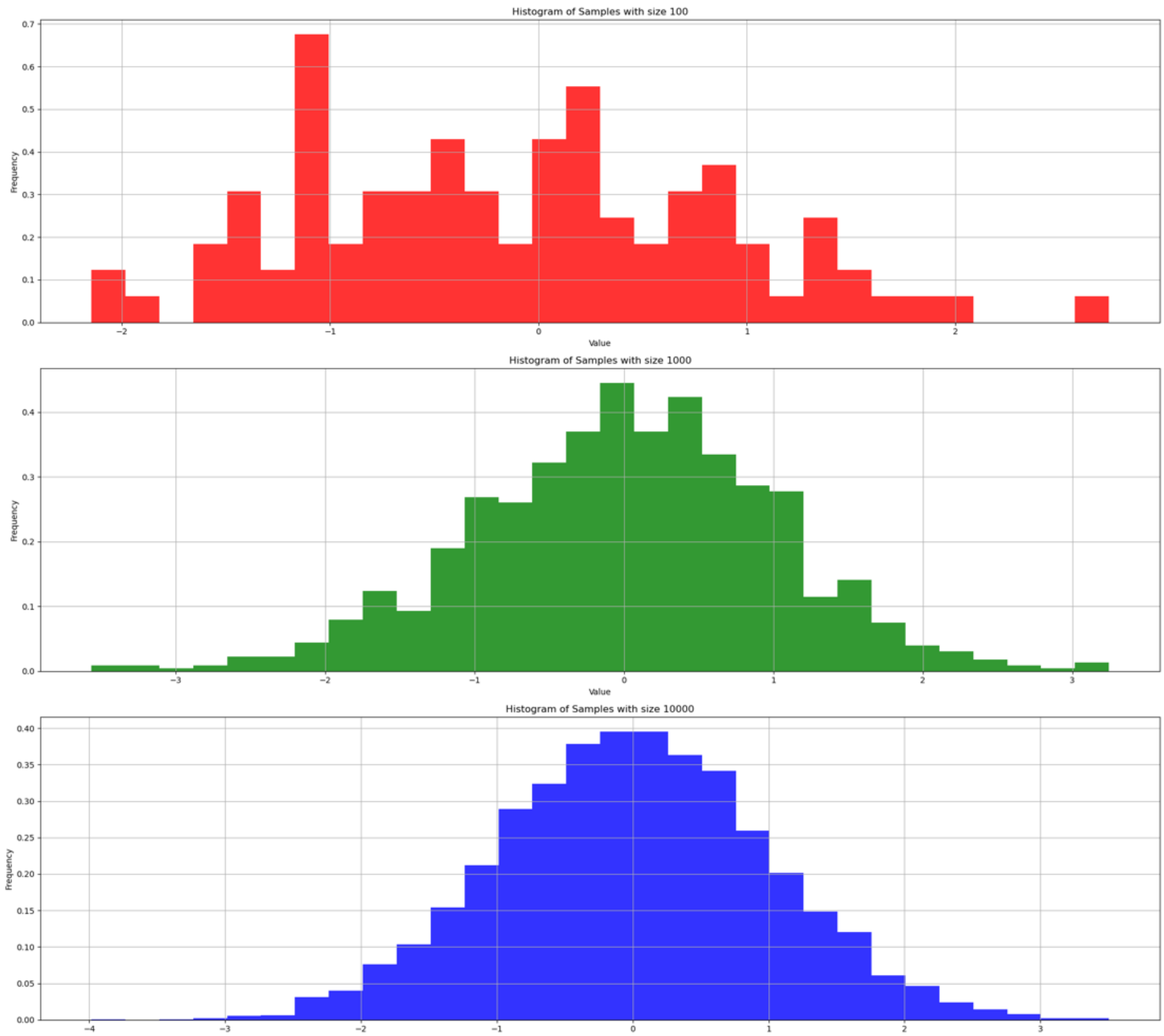
In [113…

```
# define sample sizes
sizes = [100, 1000, 10000]
#kurtosis_values = []
```

In [114…

```
# create a figure for histograms
plt.figure(figsize = (20,18))
# sample size 100
samples_100 = np.random.normal(0, 1, 100)
kurtosis_100 = stats.kurtosis(samples_100)

# sample size 1000
samples_1000 = np.random.normal(0, 1, 1000)
kurtosis_1000 = stats.kurtosis(samples_1000)

# sample size 10000
samples_10000 = np.random.normal(0, 1, 10000)
kurtosis_10000 = stats.kurtosis(samples_10000)

# plot histogram for sample size 100
plt.subplot(3, 1, 1)
plt.hist(samples_100, bins=30, density=True, alpha=0.8, color='r')
plt.title(f'Histogram of Samples with size 100')
plt.xlabel('Value')
plt.ylabel('Frequency')
plt.grid(True)

# plot histogram for sample size 1000
plt.subplot(3, 1, 2)
plt.hist(samples_1000, bins=30, density=True, alpha=0.8, color='g')
plt.title(f'Histogram of Samples with size 1000')
plt.xlabel('Value')
plt.ylabel('Frequency')
plt.grid(True)
```

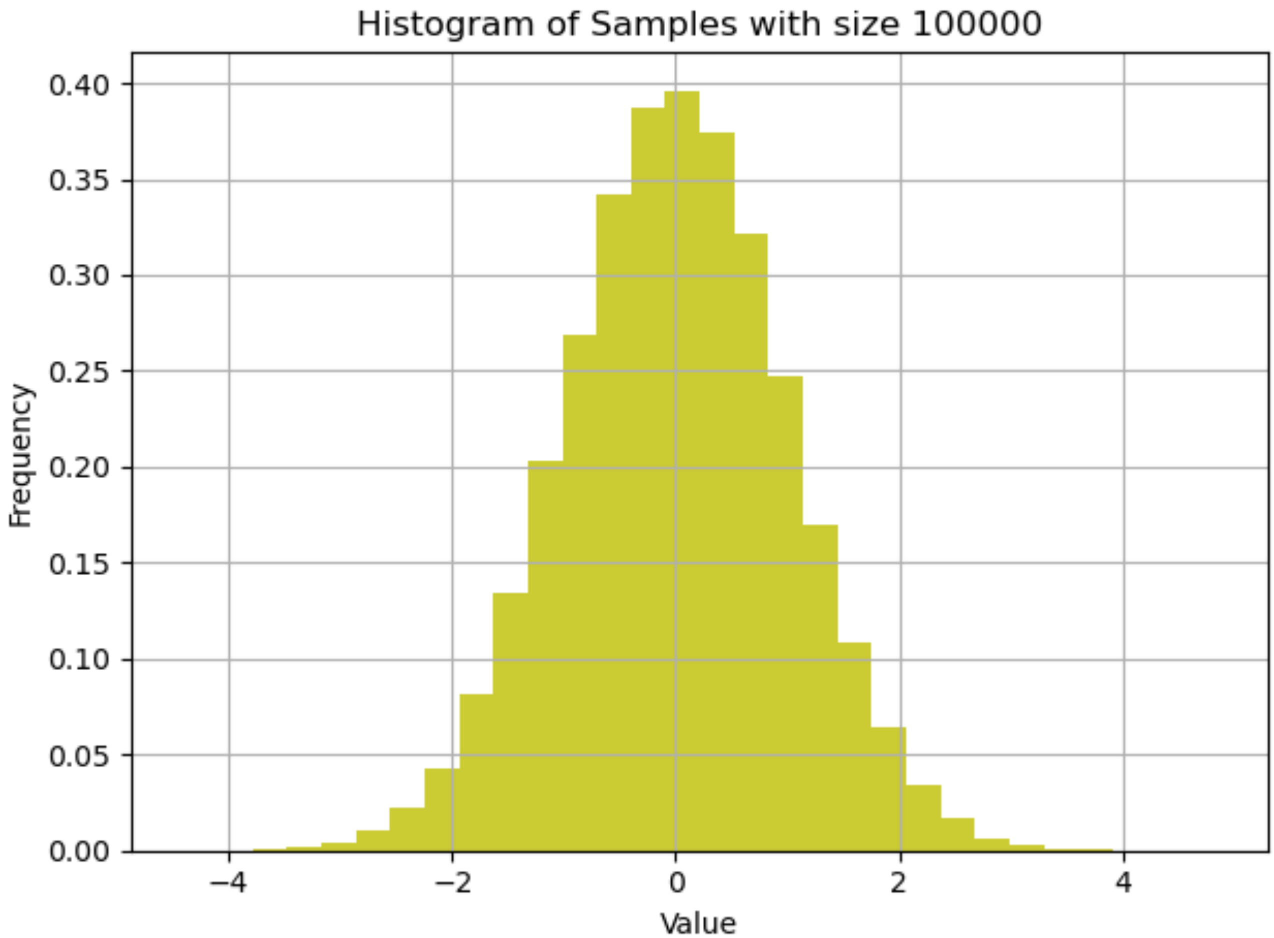
```
# plot histogram for sample size 10000
plt.subplot(3, 1, 3)
plt.hist(samples_10000, bins=30, density=True, alpha=0.8, color='b')
plt.title(f'Histogram of Samples with size 10000')
plt.xlabel('Value')
plt.ylabel('Frequency')
plt.grid(True)

# show layout and plot
plt.tight_layout()
plt.show()

# print Kurtosis Values
print(f"Sample size: 100, Kurtosis: {kurtosis_100}")
print(f"Sample size: 1000, Kurtosis: {kurtosis_1000}")
print(f"Sample size: 10000, Kurtosis: {kurtosis_10000}")
```



```
Sample size: 100, Kurtosis: -0.33766638750736755
Sample size: 1000, Kurtosis: 0.3356225774933672
Sample size: 10000, Kurtosis: -0.05024741281744616
```

In [41]:
```
## 1.2
```

In [115…
```
# sample size 100000
samples_100000 = np.random.normal(0, 1, 100000)
kurtosis_100000 = stats.kurtosis(samples_100000)

# plot histogram for sample size 100
#plt.figure(figsize = (8, 6)
plt.hist(samples_100000, bins=30, density=True, alpha=0.8, color='y')
plt.title(f'Histogram of Samples with size 100000')
plt.xlabel('Value')
plt.ylabel('Frequency')
plt.grid(True)

# show layout and plot
plt.tight_layout()
plt.show()

# print Kurtosis Values
print(f"Sample size: 100000, Kurtosis: {kurtosis_100000}")
```




```
Sample size: 100000, Kurtosis: 0.009374276827441186
```

In [116…

```
# Observations:
# 1. As the increasing of sample size, the histogram will be close to normal
# distribution and the kurtosis values should approach the theoretical value
# 2. For samller sample size, the kuritosis values will show more variablity
# due to the samll number of samples.
# 3. For bigger sample size, the kuritosis values should be close to 0.

# Consistency with Theoretical Understanding:
# 1. The theoretical kurtosis of a standard normal distribution is 0, as
# normal distribution doesn't have excess kurtosis. As the sample size
# increase, the sample kurtosis values should converge to the theoretical
# value.
# 2. Due to the variability of sampling, the kurtosis of small samples
# varies more.
```

In [117…

```
## 1.3
# When sample size is 100 or 1000, the kurtosis be returned is excess
# kurtosis. Also, when sample size is 10000 or more, the kurtosis be
# returned is kurtosis. This is because kurtosis is used to measures
# the "tailedness" of the distribution. For a standard normal distribution,
# the kurtosis is 0, and the normal distribution haven't the excess kurtosis
```

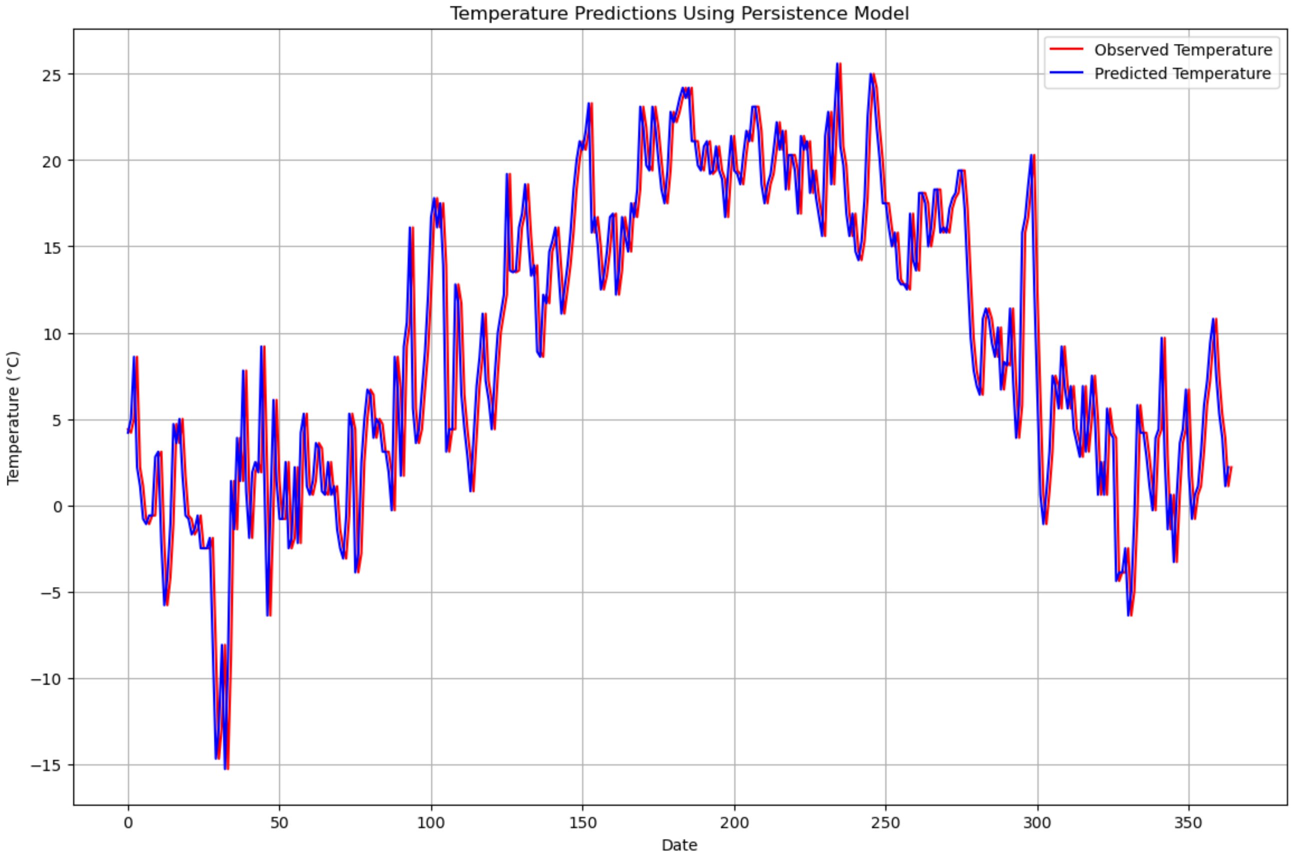
In [118…

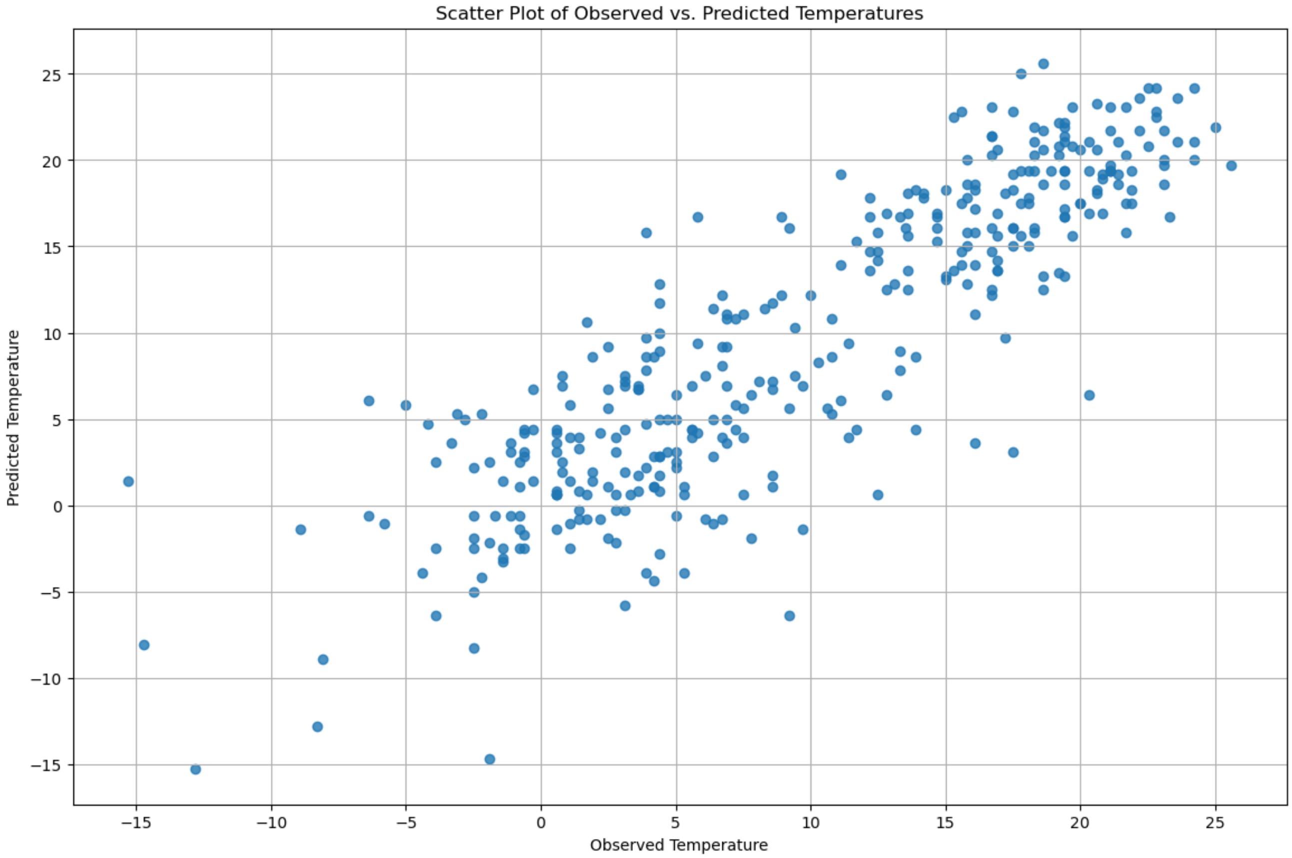
```
# 2.
## 2.1
```

In [161…

```
# import libraries
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
from sklearn.metrics import confusion_matrix, accuracy_score

# load the dataset
dt = pd.read_csv('Ann-Arbor-Temp.csv')
# display the first few rows of the dt
print(dt.head())

temps = dt['T'].values

# predictions for next day
pred = np.roll(temps, shift = -1)
# last day has no pred
pred[-1] = np.nan

plt.figure(figsize = (14, 9))

# plot for pred
plt.plot(dt.index, temps, label = 'Observed Temperature', color = 'r',
         linestyle = '-')
plt.plot(dt.index, pred, label='Predicted Temperature', color='b',
         linestyle='-')
plt.title('Temperature Predictions Using Persistence Model')
plt.xlabel('Date')
plt.ylabel('Temperature (°C)')
plt.legend()
plt.grid(True)

plt.show()


# Scatter plot
plt.figure(figsize=(14, 9))
plt.scatter(temps[:-1], pred[1:], alpha=0.8)
plt.xlabel('Observed Temperature')
plt.ylabel('Predicted Temperature')
plt.title('Scatter Plot of Observed vs. Predicted Temperatures')
plt.grid(True)
plt.show()
```

	STATION	NAME	DATE	T
0	USW00094889	ANN ARBOR MUNICIPAL AIRPORT, MI US	1/1/23	4.4
1	USW00094889	ANN ARBOR MUNICIPAL AIRPORT, MI US	1/2/23	4.2
2	USW00094889	ANN ARBOR MUNICIPAL AIRPORT, MI US	1/3/23	5.0
3	USW00094889	ANN ARBOR MUNICIPAL AIRPORT, MI US	1/4/23	8.6
4	USW00094889	ANN ARBOR MUNICIPAL AIRPORT, MI US	1/5/23	2.2





In [120…

```
## 2.2
```

In [162…

```
# drop the last value and remove NaN values
r_temps = temps[:-1]
#r_pred = pred[1:]
r_pred = pred[:-1]
# compute metrics
mse = np.mean((r_temps - r_pred) ** 2)
rmse = np.sqrt(mse)
mae = np.mean(abs(r_temps - r_pred))
bias = np.mean(r_temps - r_pred)
relative_bias = bias / np.mean(r_temps)
corr_coeff = np.corrcoef(r_temps, r_pred)[0, 1]

# check for NaN vlaues in r_temps ans r_pred
print("Number of NaNs in observed temperatures:", np.isnan(r_temps).sum())
print("Number of NaNs in predicted temperatures:", np.isnan(r_pred).sum())

#remove NaN values

# Print the results
print(f"MSE: {mse}")
print(f"RMSE: {rmse}")
print(f"MAE: {mae}")
print(f"Bias: {bias}")
```

```python
print(f"Relative Bias: {relative_bias}")
print(f"Correlation Coefficient: {corr_coeff}")
```

```
Number of NaNs in observed temperatures: 0
Number of NaNs in predicted temperatures: 0
MSE: 9.889615384615386
RMSE: 3.1447758878202094
MAE: 2.364285714285714
Bias: 0.006043956043956042
Relative Bias: 0.000621644532353772
Correlation Coefficient: 0.9330884725991082
```

In [122…

```python
## 2.3
```

In [123…

```python
# According to the result, I think the persistence model is neither
# underestimating nor overestimating for the temperature. This is
# because the 'Bias' is 0.006 which is very close to 0. And for
# 'Relative Bias', it is also very samll ('0.062%'), which means
# the persistence model haven't a significant tendency to overestimate or
# underestimate the tempreture.
```

In [124…

```python
## 2.4
```

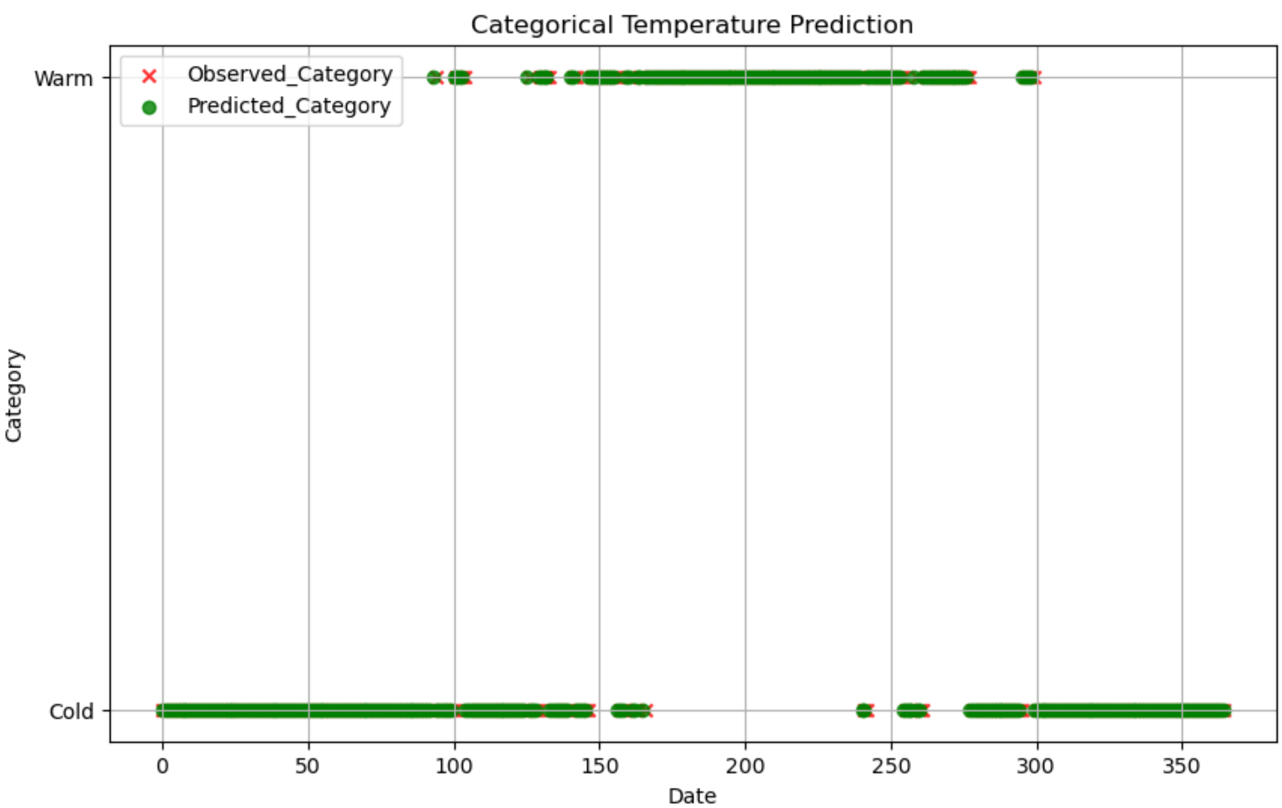
In [163…

```python
# transforming temp to categorical variables
def t_categories(temp_series, threshold=15):
    return np.where(temp_series >= threshold, 'Warm', 'Cold')

# transform
obs_categories = t_categories(temps)
# drop the last NaN value and use 'Cold' to fill up ?
pred_categories = t_categories(pred[:-1])

# adding the categories to the dataset
dt['Observed_Category'] = obs_categories
# use 'Cold' to fill up ?
dt['Predicted_Category'] = np.append(pred_categories, 'Cold')

print(dt.head())
```

```
       STATION                               NAME    DATE    T  \
0  USW00094889  ANN ARBOR MUNICIPAL AIRPORT, MI US  1/1/23  4.4
1  USW00094889  ANN ARBOR MUNICIPAL AIRPORT, MI US  1/2/23  4.2
2  USW00094889  ANN ARBOR MUNICIPAL AIRPORT, MI US  1/3/23  5.0
3  USW00094889  ANN ARBOR MUNICIPAL AIRPORT, MI US  1/4/23  8.6
4  USW00094889  ANN ARBOR MUNICIPAL AIRPORT, MI US  1/5/23  2.2

  Observed_Category Predicted_Category
0              Cold               Cold
1              Cold               Cold
2              Cold               Cold
3              Cold               Cold
4              Cold               Cold
```

In [126…

```
plt.figure(figsize = (10, 6))
# plot observed and predicted categories using scatter plot
plt.scatter(dt.index, dt['Observed_Category'], label = 'Observed_Category',
            alpha = 0.8, color = 'r', marker = 'x')
plt.scatter(dt.index, dt['Predicted_Category'], label = 'Predicted_Category'
            alpha = 0.8, color = 'g', marker = 'o')
plt.title('Categorical Temperature Prediction')
plt.xlabel('Date')
plt.ylabel('Category')
plt.legend()
plt.grid(True)
plt.show()
```



In [127…

```
## 2.5
```

In [164…

```
from sklearn.metrics import confusion_matrix, accuracy_score

# Calculate the confusion matrix and accuracy
conf_matrix = confusion_matrix(dt['Observed_Category'],
                               dt['Predicted_Category'],
                               labels = ['Warm', 'Cold'])
accuracy = accuracy_score(dt['Observed_Category'], dt['Predicted_Category'])

# Print the confusion matrix and accuracy
print("Confusion Matrix:")
print(conf_matrix)
print(f"Accuracy of the persistence model: {accuracy:.2f}")
```

```
Confusion Matrix:
[[120  13]
 [ 13 219]]
Accuracy of the persistence model: 0.93
```

In [129…

```
## 2.6
```

In [130…

```
# I think the persistence model is a very good and useful model. However,
# its performance depends the dataset we used. For example, in our homework,
# we want to predicate tomorrow tempreture based on today tempreture.
# According the result,  for 2.5, accuracy of the persistence model is 0.93,
# which means 93% of the days were correctly categorized as "Warm" or "Cold"
# For the last question, I think the ML will beat the persistence model,
# this is because ML could learn from hug data to notice some reasons be
# ignored such as seasonlity and get a better model.
```

In [131…

```
# 3.
## 3.1
```

In [165…

```
import pandas as pd

# it's a wrong way to solve this problem,
#because the result just shows 1 or 0.
# Create the transition matrix
#t_counts = pd. crosstab (index = dt['Observed_Category'][:-1],
                         #columns = dt['Observed_Category'][1:],
                         #normalize = 'index')

# display the transition probability matrix
#print("Transition Probability Matrix:")
#print(t_counts)
```

In [166…

```
dt['Observed_Category']
```

Out[166…

```
0      Cold
1      Cold
2      Cold
3      Cold
4      Cold
       ...
360    Cold
361    Cold
362    Cold
363    Cold
364    Cold
Name: Observed_Category, Length: 365, dtype: object
```

In [167…

```
# there are two ways to get probability matrix.
# Create a transition DataFrame
transitions = pd.DataFrame(0, index = dt['Observed_Category'].unique(),
                           columns = dt['Observed_Category'].unique())

# Fill in the transition counts
for (i, j) in zip(dt['Observed_Category'][:-1], dt['Observed_Category'][1:]):
    transitions.loc[i, j] += 1

#transition_counts = pd. crosstab (index = dt['Observed_Category'][:-1],
                        # columns = dt['Observed_Category'][1:],
                         #normalize = 'index')

# Convert counts to probabilities
transition_probabilities = transitions.div(transitions.sum(axis=1), axis=0)

# display the transition probability matrix
print("Transition Probability Matrix:")
print(transition_probabilities)
```

```
Transition Probability Matrix:
          Cold      Warm
Cold  0.943723  0.056277
Warm  0.097744  0.902256
```

In [168…

```
x = dt['Observed_Category'].values
n_warm_warm = np.sum((x[:-1]=='Warm') & (x[1:]=='Warm'))
n_warm_cold = np.sum((x[:-1]=='Warm') & (x[1:]=='Cold'))
n_cold_cold = np.sum((x[:-1]=='Cold') & (x[1:]=='Cold'))
n_cold_warm = np.sum((x[:-1]=='Cold') & (x[1:]=='Warm'))

# Convert counts to probabilities
#transition_probabilities = transitions.div(n_warm_warm, n_warm_cold,
# n_cold_cold, n_cold_warm)

# display the transition probability matrix
print("Transition Probability Matrix:")
```

```
print(transition_probabilities)
```

```
Transition Probability Matrix:
          Cold      Warm
Cold  0.943723  0.056277
Warm  0.097744  0.902256
```

In [136…

```
np.sum((x[:-1]=='Warm') & (x[1:]=='Cold'))
```

Out[136… 13

In [137…

```
np.sum((x[:-1]=='Cold') & (x[1:]=='Cold'))
```

Out[137… 218

In [138…

```
np.sum((x[:-1]=='Cold') & (x[1:]=='Warm'))
```

Out[138… 13

In [139…

```
## 3.2
```

In [169…

```
def markov_chain(initial_state, n, P):

   # define state
    states = ['Cold', 'Warm']
   # create sate and map them to indices
    state_index =  {state: idx for idx, state in enumerate(states)}

   # get the initial state to its index
    current_state_idx = state_index[initial_state]

   # initialize the sequence with the initial state
    sequence = [states[current_state_idx]]

   # get the sequence
    for _ in range(n - 1):
        # Get the transition probabilities for the current state
        transition_probs = P[current_state_idx]

        # Choose the next state based on the transition probabilities
        next_state_idx = np.random.choice([0, 1], p = transition_probs)
        # choose the next state based on current state
        #next_state_idx = np.random.choice([0, 1], p = P[current_state_idx])

        # append the next state to the sequence
        sequence.append(states[next_state_idx])

       # update the current state index
        current_state_idx = next_state_idx
```

```python
    return sequence
```

In [170…

```python
# get transition_probs from 3.1
transition_probs = np.array([
    [0.943723, 0.056277],
    [0.097744, 0.902256]
])
# Generate five simulation sequences of length 30 starting from the initial
# state 'Cold'
sim_1 = markov_chain('Cold', 30, transition_probs)
sim_2 = markov_chain('Cold', 30, transition_probs)
sim_3 = markov_chain('Cold', 30, transition_probs)
sim_4 = markov_chain('Cold', 30, transition_probs)
sim_5 = markov_chain('Cold', 30, transition_probs)

# create dataframe to show
df = pd.DataFrame({'sim_1': sim_1, 'sim_2': sim_2, 'sim_3': sim_3, 'sim_4':
```

In [142…

```python
df
```

Out[142…

	sim_1	sim_2	sim_3	sim_4	sim_5
0	Cold	Cold	Cold	Cold	Cold
1	Cold	Cold	Cold	Cold	Cold
2	Cold	Cold	Cold	Cold	Cold
3	Cold	Cold	Cold	Cold	Warm
4	Cold	Cold	Cold	Cold	Warm
5	Cold	Cold	Cold	Cold	Warm
6	Cold	Cold	Cold	Cold	Warm
7	Cold	Cold	Cold	Cold	Warm
8	Warm	Cold	Cold	Cold	Warm
9	Warm	Cold	Cold	Cold	Warm
10	Cold	Cold	Cold	Cold	Warm
11	Cold	Cold	Cold	Cold	Warm
12	Cold	Cold	Cold	Cold	Warm
13	Cold	Cold	Cold	Cold	Warm
14	Cold	Cold	Warm	Cold	Warm
15	Cold	Cold	Warm	Cold	Cold

16	Cold	Cold	Cold	Cold	Cold
17	Cold	Cold	Cold	Cold	Cold
18	Cold	Cold	Cold	Cold	Cold
19	Cold	Cold	Cold	Cold	Cold
20	Cold	Cold	Warm	Cold	Cold
21	Cold	Cold	Warm	Cold	Cold
22	Cold	Cold	Warm	Cold	Warm
23	Cold	Cold	Warm	Cold	Warm
24	Cold	Cold	Warm	Cold	Warm
25	Cold	Warm	Warm	Cold	Warm
26	Cold	Warm	Warm	Cold	Warm
27	Cold	Warm	Warm	Cold	Warm
28	Cold	Warm	Warm	Warm	Warm
29	Cold	Warm	Warm	Warm	Warm

In [143…

```
# As the result, we could observe all simulations are not similar to one
# another. Also, they are identical to observations. For reason, I think
# this is because the randomness of the Markov Chain will lead to different
# simulation sequences, even if the same transition probability matrix is
# used, different results will be obtained.
```

In [144…

```
## 3.3
```

In [173…

```
# divide the date
march_to_august = dt.loc['3/1/23':'8/31/23', 'Observed_Category']
august_to_december = dt.loc['8/1/23':'12/31/23', 'Observed_Category']

# get two time quantum transition matrix
transition_matrix_march_to_august = compute_transition_matrix(march_to_augus
transition_matrix_august_to_december = compute_transition_matrix(august_to_d

# print
print("Transition Matrix (March to August):")
print(transition_matrix_march_to_august)
print("\nTransition Matrix (August to December):")
print(transition_matrix_august_to_december)
```

```
Transition Matrix (March to August):
          Cold      Warm
Cold  0.891566  0.108434
Warm  0.090000  0.910000

Transition Matrix (August to December):
          Cold      Warm
Cold  0.955556  0.044444
Warm  0.080645  0.919355
```

In [180…

```
# According to the result, we could observe the transition matrix of Aug to
# Dec('Cold' to 'Cold') is higher and ('Cold'
# to 'Warm') is lower than Mar to Aug, I think maybe this is because the
# season change and the sample is not enough.
```

In [181…

```
# 4.
# 4.1
# I think this question has different answer based on different outliers.
# 1. If the more the outlier deviates from the mean,  the mean is less
# robust than other.
# 2. If the outlier impact the distribution on both sides of the
# median, the median will be significanty impacted.
# 3. The outlier change the number of mode, the mode will be
# significantly impacted.
```

In [182…

```
# 4.2
# Both can be influenced by outlier in the data. Because they measure
# systematic error and a few extreme value or outliers can distort
# the estimation process.
```

In [183…

```
# 4.3
# Mean Absolute Error (MAE) is more robust to outliers compared to
# Root Mean Squared Error (RMSE).  This is because MAE treats all
# errors equally without amplifying the impact of large deviations,
# while RMSE disproportionately penalizes larger errors due to squaring.
```

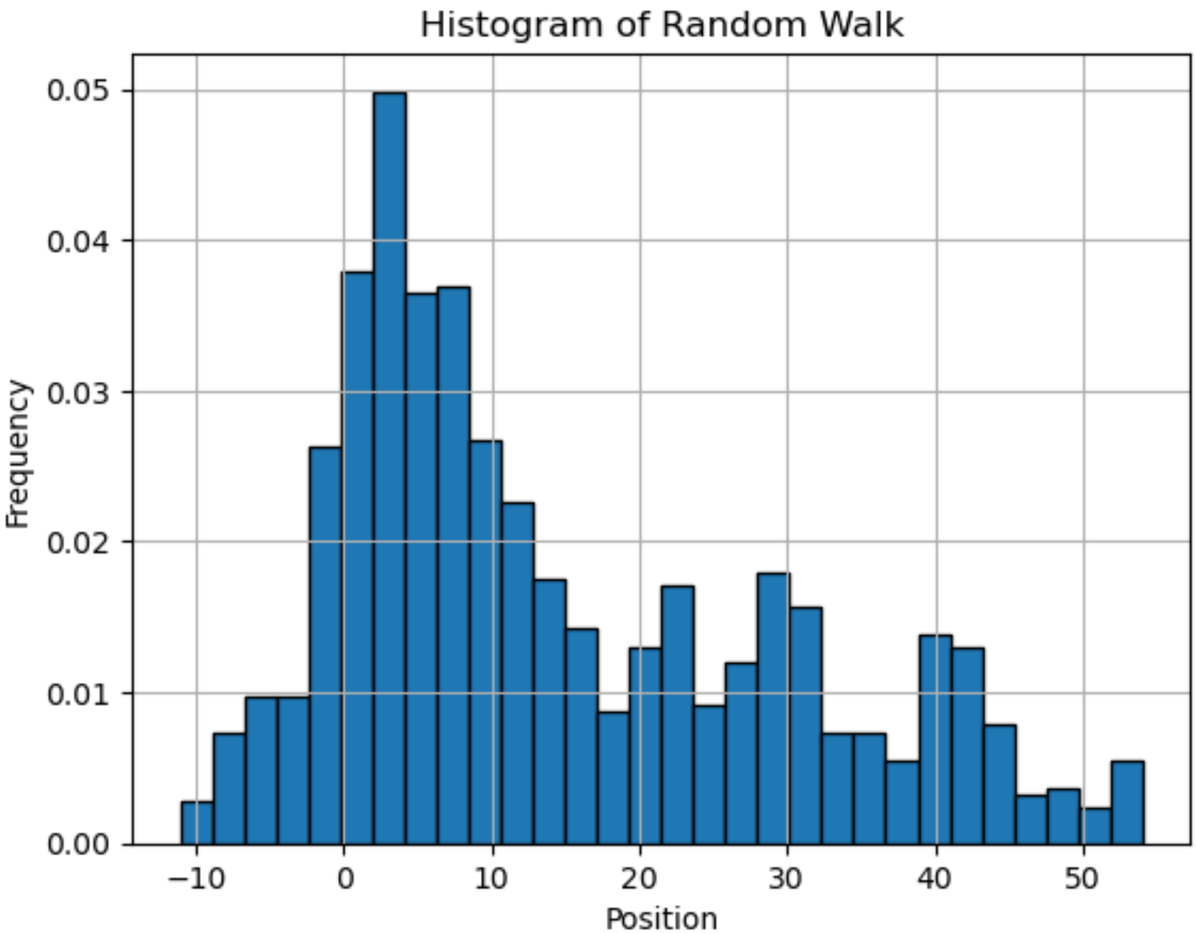
In [75]:

```
# Extra Credit
```

In [179…

```
def random_walk(n):
    # initialize the first loction
    cur_loc = 0
    # Array to store the positions
    positions = np.zeros(n)

    # Generate steps: +1 or -1 with equal probability
    steps = np.random.choice([-1, 1], size = n)

    # Perform the random walk
    for i in range(n):
```

```
        cur_loc += steps[i]
        positions[i] = cur_loc

    return positions

# Generate a sequence of 1,000 values
n = 1000
walk_sequence = random_walk(n)

# Plot the histogram of the data
plt.hist(walk_sequence, bins= 30, edgecolor='black', density=True)
plt.title('Histogram of Random Walk')
plt.xlabel('Position')
plt.ylabel('Frequency')
plt.grid(True)
plt.show()
```



In [184…

```
# Properties of Distribution
# 1. When I try to run this model more times, I could know the distribution
# is symmetric around 0 because the random walk has equal probabilities of
# moving +1 or -1 at each step.
# 2. As the more steps, the distribution of histogram will close to normal
# distribution.
```

```
# 3. Most of it is still concentrated near zero, with no significant
# deviation.
```