

# Time Variant Causality Model Applied in Brain Connectivity Network Based on Event Related Potential

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**Abstract.** Granger causality model mostly used to find the interaction between different time series are more and more applied to natural neural network at present. Brain connectivity network that could imply interaction and coordination between different brain regions is a focused research of brain function. Usually synchronization and correlation are used to reveal the connectivity network based on event-related potential (ERP) signals. However, these methods lack the further information such as direction of the connectivity network. In this paper, we performed an approach to detect the direction by Granger causality model. Considering the non-stationary of ERP data, we used traditional recursive least square (RLS) algorithm to calculate time variant Granger causality. In particular, we extended the method on the significance of causality measures in order to make results more reasonable. These approaches were applied to the classic Stroop cognitive experiment to establish the causality network related to attention process mechanism.

## 1 Introduction

The study on the localization of brain functions and the coordination mechanism among the different neuronal structures is important to neuroscience. Electroencephalogram (EEG) is a kind of biological spontaneous potential that records electric temporal signal from the scalp position. Event related potential (ERP) also called evoked potential is a kind of cognitive EEG. The potential evoked by a single stimulus is often so weak that it has to be enhanced by repeating the stimulus many times and averaging across every EEG trials caused by the same stimulus. In other words, ERP signal is the average result across a series of trials each of which related to the same stimulus. ERP directly reflects the electric activities of neuron assembly at different scalp electrode during cognitive task. Traditional analysis of ERP signals usually focus on the specific ERP components within some specific electrodes, and the interactivities between different electrodes are usually analyzed by synchronization or correlation at present. However, these approaches do not reveal anything about how different brain regions communicate with each other and what causality exist. Characterizing brain activity requires causal model, by which regions and connections of interest are specified [1].

The best approach of the causal relation, which is so-called Granger causality, was introduced by Wiener (1956) and formulated by Granger (1969) in the form of linear autoregressive (AR) model [2][3]. The original concept refers to the improvement in

predictability of a time series when the knowledge of the past of another time series is considered. That is, if a time series  $X$  causes a time series  $Y$ , the variance of the prediction error of both the two time series will be less than the prediction of only  $Y$ . With Granger causality applied in neuroscience, cortical networks are engaged. Many researchers used such causality conception to study the cortical connectivity networks based on EEG or ERP data by means of kinds of measurements of Granger causality, which were given by Geweke (1982) or others [4][5][6]. However, these methods require stationary time series. Usually, in cognitive experiment, EEG or ERP signals are not stationary. To solve this problem, Ding (2000) and Liang (2000) developed adaptive vector autoregressive models using short-time windows for multiple trials [7][8]. Moller (2001) and Hesse (2003) introduced generalized recursive least square (RLS) approach [9][10]. Another problem is statistic significant test that could estimate the significance of causality at last. In above papers, the statistical significant test was performed by the construction of surrogate data. Their results may be better convinced if they had considered the temporal construction within the original time series.

For the aim of both finding transient direction of connectivity networks and overcoming non-stationary of EEG time series, in the present study, we used Granger causality to analyze ERP data by means of traditional RLS algorithm. We also improved the statistical significant test by using value of correlation coefficient of surrogate data. In addition, this approach was applied to analyze ERP data recorded from the classic psychological Stroop experiment in order to investigate the causal connectivity network since the event related potential technique could increase the signal to noise ratio.

## 2 Method

### 2.1 Time Variant Granger Causality Model

Let  $X = \{x(t)\}$  and  $Y = \{y(t)\}$  be the time series. The univariate AR models of  $X$  and  $Y$  are:

$$\begin{aligned} x(t) &= \sum_{i=1}^p a_1(i)x(t-i) + \varepsilon_1(t) \\ y(t) &= \sum_{i=1}^p a_2(i)y(t-i) + \varepsilon_2(t) \end{aligned} \quad (1)$$

Here, the parameters  $a_1(i)$  and  $a_2(i)$  are the time variant model coefficients,  $\varepsilon_1(t)$  and  $\varepsilon_2(t)$  are their time variant prediction errors. The variance of  $\varepsilon_1(t)$  and  $\varepsilon_2(t)$  are  $\Sigma_{X|X^-}(t)$  and  $\Sigma_{Y|Y^-}(t)$  respectively. The bivariate AR models of  $X$  and  $Y$  are:

$$\begin{aligned} x(t) &= \sum_{i=1}^p a_{11}(i)x(t-i) + \sum_{i=1}^p a_{12}(i)y(t-i) + \eta_1(t) \\ y(t) &= \sum_{i=1}^p a_{21}(i)y(t-i) + \sum_{i=1}^p a_{22}(i)x(t-i) + \eta_2(t) \end{aligned} \quad (2)$$

Where the parameters  $a_{jl}(i)$ ,  $j, l=1, 2$ , are the time variant model coefficients,  $\eta_1(t)$  and  $\eta_2(t)$  are their time variant prediction errors. The variance of  $\eta_1(t)$  and  $\eta_2(t)$  are  $\Sigma_{X|X^-,Y^-}(t)$  and  $\Sigma_{Y|Y^-,X^-}(t)$  respectively.

When  $X$  causes  $Y$ , the variance of the prediction error  $\Sigma_{Y|Y^-,X^-}(t)$  will be less than  $\Sigma_{Y|Y^-}(t)$ . The measure of Granger causality from  $X$  to  $Y$  is defined as:

$$F_{X \rightarrow Y}(t) = \ln \frac{\Sigma_{Y|Y^-}(t)}{\Sigma_{Y|Y^-,X^-}(t)}. \quad (3)$$

Symmetrically, the measure of Granger causality from  $Y$  to  $X$  is defined as:

$$F_{Y \rightarrow X}(t) = \ln \frac{\Sigma_{X|X^-}(t)}{\Sigma_{X|X^-,Y^-}(t)}. \quad (4)$$

The recursive least square (RLS) algorithm can be used to estimating the time-variant Granger causality. In this paper, we used the traditional RLS algorithm with forgetting factor to calculate the Equation (3) and (4). According to Moller (2001), the recursive computation is:

$$\Sigma(t) = (1 - \lambda)\Sigma(t) + \lambda z(t). \quad (5)$$

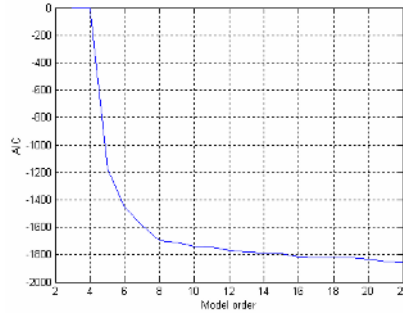
with  $0 < \lambda < 1$ ,  $z(t)$  denotes the prediction errors and  $\Sigma(t)$  denotes the variance of the prediction. In the case of ERP data analysis, the model was fitted with  $\lambda = 0.025$ .

## 2.2 Order Determination of AR Model

In order to determining the optimal model order  $p$  in equation (2), we use the Akaike information criterion (AIC) defined as:

$$AIC(i) = N \ln(\det(\Sigma_i)) + 2iL^2. \quad (6)$$

where  $L$  is the number of variables,  $N$  is the length of data and  $\Sigma_i$  is the variance of the prediction of the  $i$ th order model. For ERP data, AR model orders were calculated by least square estimation, and then, an optimal order can be used in RLS algorithm. In fact, the optimal model order should be selected for which the AIC reaches the minimum. However, in most cases, the AIC curve decreased when the model order increased. Fig. 1 shows an example of the AIC determination for electrode pair P3 and CP3. It is suitable for choosing the order  $p=12$ , since there is little change beyond that value and the similar order appeared in other studies for EEG data [6].



**Fig. 1.** The AIC curve of the bivariate AR model for electrode pair P3 and CP3 according to equation (6)

### 2.3 Statistical Test of Significance

Because the distribution of Granger causality measures as shown in Equation (3) and (4) are not easily established, we used surrogate data to construct an empirical distribution [11]. According to this method, the time series was shuffled without replacement. Thus, one surrogate data was established, and then we calculated the value of Granger causality between the surrogate data and another time series. After this procedure was carrying out 100 times, we created the empirical distribution of Granger causality and constructed the threshold with 5% statistic significance. We confirmed that there was causality if the value of equation (3) or (4) was larger than threshold.

In addition, since the brain does not remain the same state in the whole experiment, the ERP time series must have its temporal construction. Although the shuffle procedure can conserve the distributional properties of the time series, it cannot conserve the temporal construction. For avoiding destroying the original temporal construction, we calculated the correlation coefficient of surrogate data and the time series, by assuming the correlation coefficient of surrogate data had uniform distribution on  $[-1, 1]$ . We only considered the surrogate data whose correlation coefficient was between  $[-1, -0.05]$  and  $[0.05, 1]$  for 5% significance.

## 3 Experiment and Results

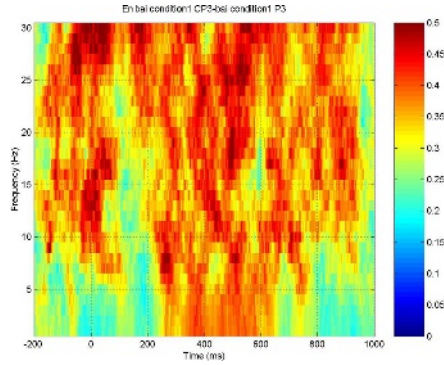
### 3.1 Cognitive Experiment and Data Acquisition

Cognitive experiment was designed the classic psychological Stroop task. When a subject is asked to identify the display color of a color meaning word, his (her) reaction will be affected by the word's meaning. Stimuli were 4 colorful Chinese characters (red, yellow, blue and green) in different colors. Subjects were asked to identify the display color of the Chinese character using keystroke. There were six blocks in the experiment and 96 stimuli in each block with color-meaning consistent word and color-meaning inconsistent word appeared randomly. The continuous EEG was recorded from 32 electrodes using ESI 128 channel workshop (NeuroScan, USA) with two referenced electrodes to two mastoids (band-pass 0.05~30 Hz, sampling rate 500 Hz). The analysis time course was at about 1000 ms post stimulus onset with

baseline at 200 ms pre-stimulus. The ERP signals were obtained by averaging across EEG trials that are associated with a correct response [12].

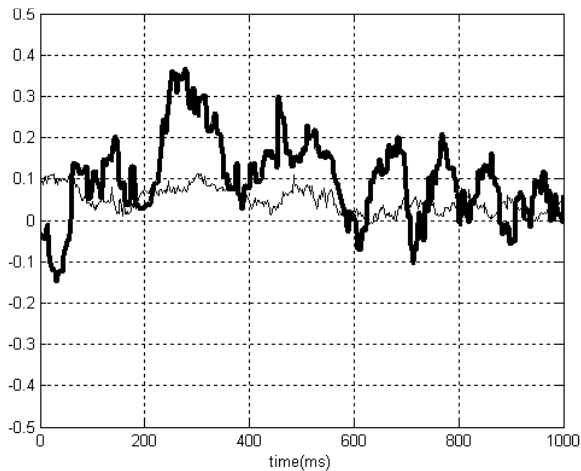
### 3.2 Data Analysis

We analyzed the phase synchronization between all electrodes firstly, and found there was strong synchronization among P3, CP3, P7, T7, P4, and CP4 [13]. Fig. 2 showed the phase synchrony index distribution between P3 and CP3.



**Fig. 2.** The time-frequency representation of phase synchrony index between P3-CP3, the red color means the strong synchronization

According to the above synchronization results, we analyzed the causality among these electrodes. The time-variant Granger causality analysis from P3 to CP3 was shown in Fig. 3. It showed that the value of causality was larger than the threshold after the time 160 ms and the causality occurs mostly from 160 ms to 862 ms.

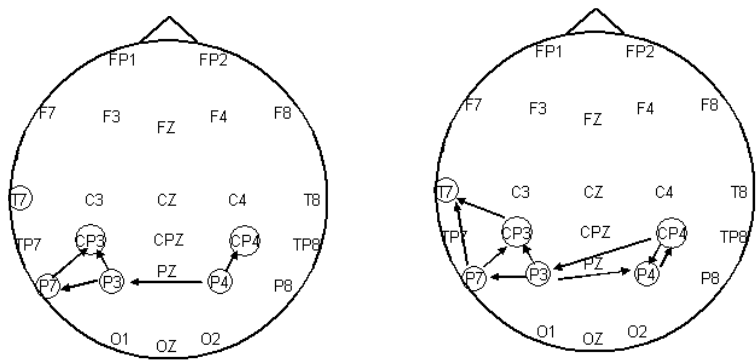


**Fig. 3.** Time variant Granger causality from P3 to CP3. The bold line represents the values of Granger causality and another line represents the values of threshold.

We found that there were three connectivity networks: P3-CP3-P7, P3-P4-CP4 and T7-CP3-P7. The details of the causality were displayed in Table 1. We list the time interval in which causality mostly occurs between electrode pair. Fig. 4 showed the spatial map of causal connectivity networks on the whole scalp.

**Table 1.** The time interval of the causality occurring mostly between electrode pair

Electrode pair	Time Interval (ms)	
P3→P7	184-712	
CP3→P7	496-598	
P7→CP3	236-416	
P3→CP3	160-862	
CP3→P3	116-272	362-684
P3→P4	424-470	570-632
P4→P3	104-272	
CP4→P3	728-836	
P4→CP4	226-276	368-612
CP4→P4	278-352	494-574
CP3→T7	272-442	460-500
P7→T7	256-268	296-414



**Fig. 4.** The brain map of Granger causality before 250 ms (left) and after 250 ms (right)

4 Discussion

Applying time variant Granger causality model to ERP data is a research way that tries to explore the interactivity of brain function. There were two aspects to discuss from our results. One was related to non-stationary of signal. Although RLS algorithm for Granger causality has high time resolution for non-stationary data, there was fluctuation in the causality results (Fig. 3). For example, the causality between some electrode pairs occurred at time 0 ms and negative values might appear sometimes. That may be for the following reasons. (1) Because the RLS algorithm results in iterative computation around the real values, the values of Granger causality might be

fluctuated at the beginning, but the time of fluctuation did not last long. (2) Signal noise is a significant influential factor. The noise can increase the value of the prediction error.

The statistical test of significance is another important issue when the statistical validity of causality is considered. Because statistical properties of Granger causality are unknown, in this paper, by the use of surrogate, the threshold is provided. The main idea, on one hand, is shuffling the series and establishing an empirical distribution of Granger causality for significant test. On the other hand, to avoiding destroying temporal construction within the time series of ERP signals, we use the value of correlation coefficient to choose the surrogate data. The surrogate data with very small value of correlation coefficient is not considered.

Color word Stroop experiment was classic paradigm of studying attentional network. The current related research mainly depended on traditional method [14][15]. In our work, it was clearly that there were three causality networks: P3-CP3-P7, P3-P4-CP4 and T7-CP3-P7 (Fig. 4 and Table 1). And the primary causality of them mostly took place around 300 ms. The time and position of the causality was consistent with that of P300 component in this experiment [12]. In the network of P3-CP3-P7, the causality from P3 to P7 and from P3 to CP3 lasted a long time and remained strong. P3 also caused P4 in the right hemisphere in network of P3-P4-CP4. It implied that the brain region around P3 mainly affected other regions. Furthermore, there was also evidential causality between two hemispheres in network of P3-P4-CP4, and the causality from P4 to P3 occurred very early. That may be related to the attentional function between two hemispheres. Further work will focus on the nonlinear model and advanced statistic test, and the cognitive significance of causality network needs to be discussed deeply.

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