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Vehicle-to-vehicle connectivity on two parallel roadways with a general headway distribution

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A B S T R A C T

Vehicle-to-Vehicle (V2V) connectivity via a multihop connectivity process underlies vehicular ad hoc networks that enable vehicles to disseminate traffic-related information through short-range wireless communication. In this paper, we propose analytical models for the vehicle connectivity on two parallel roadways, assuming general distributions for vehicle headways. Specifically, we derive models for the expectation, variance and probability distribution of information propagation distance. Closed form approximation to the expectation is developed and is numerically shown to agree well with the exact models. Monte Carlo simulation results further validate the proposed models. Through simulations, the developed models are also shown to have overcome the deficiencies associated with the commonly used one-roadway models and models with the Poisson assumption for vehicle distribution.

1. Introduction

Vehicle-to-Vehicle (V2V) wireless communication is a new thrust in Intelligent Transportation Systems (ITS) (Hartenstein and Laberteaux, 2010). In contrast to the conventional centralized traffic information systems, the V2V system supports the development of ad hoc vehicular networks (VANETs) and provides a distributed, stand-alone platform suitable for managing transient traffic information. This new V2V platform provides a great opportunity for the public–private partnerships. As a result, it has attracted much attention from transportation agencies and automobile industries alike in recent years (Ford Sync Technology, 2012; BMW ConnectedDrive, 2012). Seven channels in the 5.9 GHZ band for Dedicated Short-Range Communication (DSRC) have been allocated and regulated by the U.S. Federal Communications Commissions (FCC) to support V2V applications (DSRC, 2012). The U.S. Department of Transportation (DOT) has initiated a Connected Vehicle program aiming at multimodal V2V and Vehicle-to-Infrastructure (V2I) systems at a national level to support emergency response and incident management, and to improve the general road traffic operations and control (Connected Vehicle Research, 2012).

Vehicles in a VANET send and receive messages between each other through wireless communication. Specific standards for VANET such as IEEE 802.11p enable vehicles to connect to each other by a so-called multihop connectivity process (also referred to as information relay). This multihop process allows two vehicles beyond a transmission range to communicate via intermediate ones. VANETs must address many technical issues in such areas as routing protocol (Saleet et al., 2011; Hartenstein and Laberteaux, 2008), network capacity (Andrews et al., 2010; Gupta and Kumar, 2000) and message delivery latency (Pereira et al., 2012). Successful solutions to these issues depend on the understanding of VANETs evolution, which to a great extent depends on the understanding of the multihop connectivity process between vehicles (Jin and Recker, 2010). Generally, a larger transmission range, a higher traffic density, and a higher proportion of equipped vehicles (referred to

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as market penetration rate) make more vehicles connected and information propagated further. Analytical models to quantify this relationship are therefore of critical importance and remain to be an important undertaking.

V2V systems may also be considered as a special mobile ad hoc networks (MANETs) (Tsugawa, 2002), in which the multihop connectivity has been studied extensively such as in Xue and Kumar (2004) and Bettstetter, 2004, for examples. However, in contrary to most studies in MANETs that assume the nodes follow a spatial Poisson or uniform distribution on a line or in a continuous plane (Wang, 2007; Gilbert, 1964), the V2V system demonstrates distinct traffic flow characteristics due to the unique vehicular traffic behavior and the roadway network topology (Jin and Recker, 2010; Ng and Waller, 2010; Yousefi et al., 2006). For example, at a high vehicular traffic density, especially around highway ramps, platoons often tend to be present, and the vehicle headway (or, vehicle spacing) is claimed to follow a compound distribution (Cowan, 1975). In another study at a low market penetration rate, the vehicle headway is said to obey a general distribution (Jin and Recker, 2010). Traffic with this non-Poisson process partially explains why traditional routing protocols that are based on Poisson assumptions of vehicles show a poor performance in VANETs (Saleet et al., 2011; Naumov and Gross, 2007). Therefore, it is meaningful to consider a general distribution of vehicles in a network, a distribution not restricted to Poisson. Assumption of a general distribution of vehicle headway on a traffic network allows application of the subsequent results to situations of any particular vehicular traffic distribution in field.

In addition, most studies on connectivity of V2V systems are based on one-dimensional models, focusing on one lane highway (Jin and Recker, 2010; Wang, 2007), or treating multiple lanes and roadways as one lane (Kesting et al., 2010). This simplification into one lane traffic may work well when parallel roadways are close enough relative to the transmission range. However, recent experiments find significant errors from those simplified methods when the road separation is large compared with the transmission range (Bai et al., 2010; Wang et al., 2012). It is necessary to explicitly consider vehicle interactions between parallel, separated roadways (Wang et al., 2012). This paper may be considered as a continued effort from our earlier work (Wang et al., 2010), which also adopts a general vehicle headway studying information propagation, but on one single road. The models in this paper are significantly different from those in Wang et al. (2010) and are more challenging.

In this paper, we study vehicle connectivity or equivalently, information propagation, in a V2V system assuming the vehicle headway follows *general* distributions on two parallel roads. We endeavor to answer the following question: *How far can information propagate along two parallel roadways*? We further assume that a packet can be transmitted from a sender to its receiver *almost* instantaneously relative to vehicle speed. The instantaneity assumption helps develop analytical results and may provide a basis for further studying information propagation latency. We present analytical models to characterize the propagation distance in terms of expectation, variance and probability. This study follows our earlier work (Wang et al., 2012) by extending the vehicle headway distribution into a general one. As shown in numerical tests later, the propagation distance can be dramatically different between Poisson and general vehicle distributions. Our work allows explicit consideration of any actual (from field traffic data) vehicle headway distribution for information propagation.

The remainder of this paper is organized as follows. In Section 2, we review related literature. Section 3 presents recursive models for the expectation, variance and probability of propagation distance. We also propose an approximate solution for the expected distance. In Section 4, we conduct numerical tests before we conclude this paper in Section 5.

2. Relevant literature

Connectivity in MANETs including the one-road V2V system has been well studied. The research on connectivity of networks traces back to the work of Gilbert (1964), in which two random points in a network are considered to be connected if the distance between them becomes less than a certain threshold. Gilbert (1964) also studies the asymptotic probability for a full connectivity. Gilbert's models, which leads to what are now known as random graphs (Haenggi et al., 2009), along with stochastic geometry, have become basic tools to study connectivity of wireless networks with such special considerations as multihop strategy (Cheng and Robertazzi, 1989; Piret, 1991; Gupta and Kumar, 1998), interference impact (Dousse et al., 2003), throughput scaling (Dousse et al., 2005), information propagation speed (Jacquet et al., 2010; Jacquet, 2011; Baccelli et al., 2012), and protocol (Baccelli and Baszczyszyn, 2010). Most of the studies in network connectivity explore asymptotic properties largely because the network settings are too broad to develop exact models.

In contrast, exact models in a V2V system are often possible because of the special features about the roadway vehicle system. Specifically, many analytical models have been proposed for the V2V connectivity on one single road. Wang (2007) investigates the instantaneous information propagation through traffic on a single roadway, by assuming a Poisson distributed traffic stream. Similarly, Dousse et al. (2002) obtain a formula for the probability of multihop connectivity of two nodes on a line. Yousefi et al. (2008) consider a model in which cars with the same speed passed a point on the highway are separated by exponentially distributed durations, and they obtain a compound Poisson distribution for the vehicle spatial distribution. These current studies are all restricted to traffic with vehicles of either (compound) Poisson point processes (Ukkusuri and Du, 2008) or uniformly distributed nodes (Wu et al., 2009) on one line. As mentioned earlier, Poisson distribution or uniform distribution of vehicles on roadways is a strong assumption that greatly limits applicability of the derived results in practice.

Attempts are made to relax the Poisson or uniform assumptions for V2V connectivity. Jin and Recker (2010) investigate the case of non-Poisson distributed traffic and derive a recursive model for multihop probabilities, but still on one line of traffic. Thanks to Chen et al. (2010), the lane configuration is found through simulation to affect connectivity by means of channel interference. In addition, the authors in Kesting et al. (2010) and Schönhof et al. (2006) study longitudinal hopping within one lane and transversal hopping across lanes of bi-directional traffic when adopting a store-and-forward V2V communication strategy. However, none of these connectivity models consider the effect of distance between roadways. Our work is different from those in the current literature in two respects. First, we explicitly consider the distance between two roadways in our analytical models. This makes an interesting step forward from a single road towards a network. Second, we assume a general distribution of vehicle headway on the roadway, enabling our models to apply to a broad class of traffic conditions and eliminate the limitations of Poisson assumption. Numerical results in Section 4 show significant differences in propagation distance between exponential and other distributions of vehicle headway and between treating two roads as one and treating them as two roads explicitly.

3. Modeling vehicle connectivity with a general headway distribution

3.1. The research problem

Consider two parallel roadways R_1 and R_2 that are d distance apart, and the mean densities of communicating vehicles on these roadways are λ_1 and λ_2, respectively. Vehicles form ad hoc networks through a transmission range L. Two vehicles are considered connected if the distance between them is less than or equal to L. Information propagates in one direction along the roads among connected vehicles. If no additional vehicle is present within the range, information propagation terminates. The propagation distance measures from the initial transmitter vehicle (also called sender) to the furthest receiving vehicle (also referred to as receiver). The objective is to characterize the propagation distance in relation to traffic density, road separation and transmission range.

Communications can only take place among equipped vehicles. Therefore, the words ‘vehicle’ and ‘equipped vehicle’ are interchangeable from now on in this paper. The headway between two vehicles is assumed to be independent and identically distributed with a general density function $f_1(x)$ and $f_2(x)$ on R_1 and R_2, respectively. In addition, signal interference and signal fading are not considered. In another word, the connectivity is a graph connectivity, which implies the instantaneity assumption of transmission mentioned earlier. The VANET in this study can also be viewed as a random graph with vertices on two parallel lines and with edges connecting two vertices within distance L. We further restrict the study to the case where $0 \leqslant d \leqslant \frac{\sqrt{3}L}{2}$ due to a technical reason, which will be explained later.

3.2. Transmission regions and transitions

We first introduce a concept of transmission region that is associated with each transmitter vehicle, after which we will establish a transitional relationship between transmission regions. Detailed information to a similar transmission region is also available in Wang et al. (2012) and Yin (2010). Transitions between transmission regions will establish recursive analytical models later in this paper. In Fig. 1, assuming rightward propagation, the transmitter vehicle A on R_2 has an effective region for forward propagation as characterized by a parallelogram $ABCD$, where the length of segment $|AB| = |CD| = |AC| = L$. Clearly, if there is no additional vehicle in the region $ABCD$, vehicle A is the last receiving vehicle in this transitional process, and the information ‘dies’ out at vehicle A. The parallelogram $ABCD$ is defined as the transmission region associated with vehicle A.

Under the condition of $0 \leqslant d \leqslant \frac{\sqrt{3}L}{2}$, the entire parallelogram $ABCD$ falls within the transmission range of vehicle A. Any vehicle within this transmission region under the assumption of $0 \leqslant d \leqslant \frac{\sqrt{3}L}{2}$ is connected to vehicle A and can potentially further the propagation. There are two types of transmission regions. If the transmitter is on road R_2, the corresponding

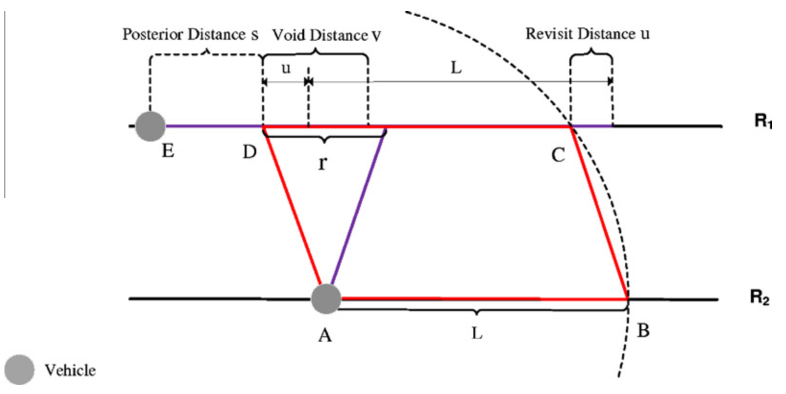


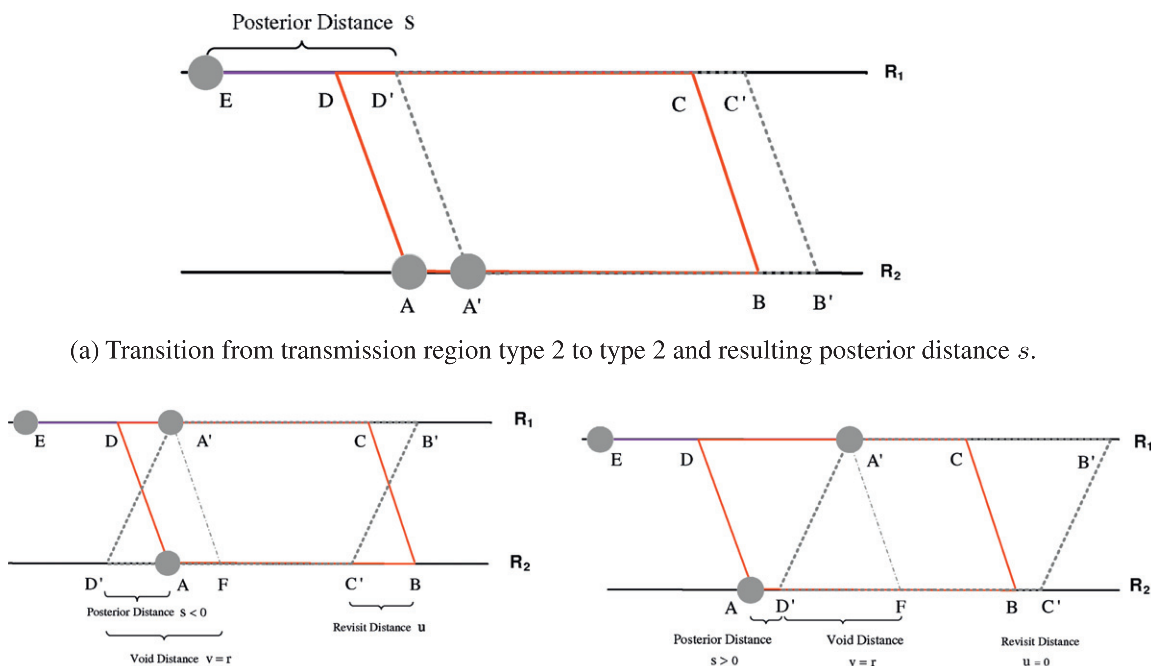
Fig. 1. Transmission region of equipped vehicle A on R_2 with length $|AB| = |CD| = |AC| = L$, and associated three parameters, assuming rightward information propagation. Note that $s > 0$ implies $u = 0$ from analysis in context.

transmission region is defined as type 2. In contrast, transmission regions associated with vehicles on road R_1 are defined as type 1. At $d > \frac{\sqrt{3}L}{2}$, no parallelogram *ABCD* is completely contained within the range, which is not studied because it demands different modeling methods that are currently beyond our grasp.

Next, we will introduce the associated parameters and transitions between transmission regions. The process of propagation is one in which one transmission region transits into another, each associated with a unique vehicle. The key in this transition is the immediate next vehicle within each transmission region, which we explain through an example. Suppose that transmitter *A* takes place on R_2 with a transmission region *ABCD* as in Fig. 2a. If we move the edge *AD* rightward and *AD* encounters the first vehicle A' that happens to be on R_2, A' becomes the next transmitter with a new transmission region $A'B'C'D'$. This implies a transition from *ABCD* to $A'B'C'D'$, accompanied by a forward distance moved. The propagation distance from *A* can be determined with distance $|AA'|$ plus the potential propagation distance from A'. If the probability of transition is known, a recursive process may be analytically expressed. Note that this probability of transiting to A' is for no vehicle presence in DD' and only one vehicle at A' in AA'. Fig. 2a is a transition from type 2 to type 2 while Fig. 2b and c are both for transitions from type 2 to 1. There are also transitions from type 1 to type 2. A notion is that this transitional process between transmission regions equivalently determines the propagation distance.

We next introduce the first parameter associated with a transmission region. We still use Fig. 2 as an example. Note that a general headway distribution does not allow to calculate the probability of no vehicle presence on DD' without knowledge of the closest prior vehicle location such as node *E* left of *D* in Fig. 2a. Therefore, we introduce the parameter *posterior distance s* to associate with a transmission region. As illustrated in Fig. 2a, a posterior distance *s* measures from the nearest prior vehicle location to the left most point of a transmission region. A posterior distance is always on the opposite road of the current transmitter vehicle. Be aware that the section of length *s* has been proven to be void of vehicle. We allow *s* to take negative values when the node *E* locates within the section *CD* in Fig. 2a, i.e., when *E* is to the right of node *D*.

We now introduce the second and third parameters associated with a transmission region through an example transition from a type 2 to a type 1 region. Suppose that the first encountered vehicle locates at A' on R_1 when moving the line *AD* rightward (so that FA' is parallel to *AD*), as shown in Fig. 2b. The new vehicle A' has a transmission region $A'B'C'D'$, which is of type 1. In Fig. 2b, if the length of $A'D$ is shorter than a threshold $r = 2(L - \sqrt{L^2 - d^2})$, the vertex D' would be located left of point *A*, in which case *s* becomes negative for the transmission region $A'B'C'D'$ of vehicle A'. In this transition, the section $D'F$ has been proven to be void of vehicle. Therefore, the length of $D'F$ is referred to as *void distance* denoted by *v*. In addition, in the case of transition from *A* to A' as in Fig. 2b, the vehicles in the region $C'B$ have been able to get connected to *A* and potentially contribute to further propagation. The section $C'B$ covered before shall not be dropped from consideration when the new transmission region for the transmitter A' is considered. The ability of further propagation from A' is complete only if $C'B$ is considered together with the new transmission region $A'B'C'D'$. The length of $C'B$ is referred to as *revisit distance* denoted by *u*. By now, we have introduced all three parameters associated with a transmission region. As in Fig. 2b, the length of



(a) Transition from transmission region type 2 to type 2 and resulting posterior distance s.

(b) Transition from type 2 to type 1, $s < 0, u > 0$.

(c) Transition from type 2 to type 1, $s > 0, u = 0$.

Fig. 2. Transition between transmission regions and parameters of resulting transmission region.

$D'A$ is a posterior distance s, $D'F$ void distance v, and $C'B$ revisit distance u. Note that, in light of Fig. 2b, we have $u = -s$ when $s < 0$ because $|D'A| = |C'B|$. Fig. 2c illustrates another case in which $s > 0$ and $u = 0$ during a transition from vehicle A to A', a transition from a type 2 to a type 1 region, which occurs when $|A'D|$ is larger than r. To summarize, we have the following proposition.

Proposition 1. *When $s < 0, u = -s$; when $s \geqslant 0$, $u = 0$.*

Proposition 1 may be equivalently presented as $u = \max\{-s, 0\}$. Three notions may help understand the presentation here. (I) The void distance v for the transmission region $A'B'C'D'$ is equal to r in Fig. 2b and c. And v becomes a smaller positive in the transition in Fig. 2a. (II) When moving the edge AD rightward, the probability of simultaneously encountering two vehicles on R_1 and R_2 is *almost* zero. Therefore, encountering two vehicles with edge AD is ignored in the derivation. (III) In spite of the relationship between u and s, we choose to keep all the three parameters within a transmission region for the sake of presentation. Next, we define the transmission state.

Definition 1. A transmission state of a vehicle on road R_i, $i = 1$ or 2, is denoted by $S_i(v, u, s)$ for a transmission region of type i, where v, u and s are the three parameters: void distance, revisit distance, and posterior distance, respectively.

A transmission state is simply referred to as a state later. The process of propagation is essentially a transitional process of states. Fig. 1 illustrates a state $S_2(v, u, s)$ according to vehicle A on R_2. Also note that the void and revisit distances are always on the opposite road of the transmitting vehicle. In light of the above analysis in Fig. 2, the following proposition becomes obvious.

Proposition 2. *In the case $d \leqslant \frac{\sqrt{3}}{2}L$, for any transmission state $S_i(v, u, s)$, $i \in \{1, 2\}$, there holds $u \leqslant v \leqslant r$, where $u = \max\{-s, 0\}$, as in Proposition 1.*

In the following, We define the notations.

r	A characteristic constant, $r = 2(L - \sqrt{L^2 - d^2})$.
$D_i(v, u, s)$	Stochastic propagation distance from a starting state $S_i(v, u, s)$.
$d_i(v, u, s)$	Expectation of $D_i(v, u, s)$.
$V_i(v, u, s)$	Variance of $D_i(v, u, s)$.
$P_i(x, v, u, s)$	The probability of propagation beyond distance x when the initial node has a state. $S_i(v, u, s)$, where x is the horizontal distance propagated.
H_i	Vehicle headway on road R_i.
$\mathbb{P}(H_i \leqslant x)$	Probability of H_i less than x. $\mathbb{P}(H_i \leqslant x) = \int_0^x f_i(t)dt$.

In the above notations, $i = 1$ or 2.

3.3. A recursive model for expected propagation distance

We explain the recursive models through examples for simplicity. Start from vehicle A on road R_2 with a state $S_2(v, u, s)$ as in Fig. 3. In light of the earlier analysis, $D_2(v, u, s)$ can be recursively expressed conditional on the first encountered transmitter vehicle along the direction of propagation. In particular, three cases are identified.

Case 1: The first vehicle is encountered on road R_2 when moving the edge AD rightward as in Fig. 3. Suppose that the first vehicle is located at a distance t from A. Three intervals on R_2 for t are identified. (I) $t \in [0, u]$ as shown by A_1 in Fig. 3. The resulting transmission region and state associated with A_1 are respectively $A_1B_1C_1D_1$ and $S_2(v - t, u - t, s + t)$. Note that the propagation distance from A is t plus the additional potential propagation distance from A_1. (II) $t \in (u, v]$ as A_2 in Fig. 3, the resulting state is $S_2(v - t, 0, s + t)$ corresponding to a transmission region $A_2B_2C_2D_2$. (III) $t \in (v, L]$ as A_3 in Fig. 3. The

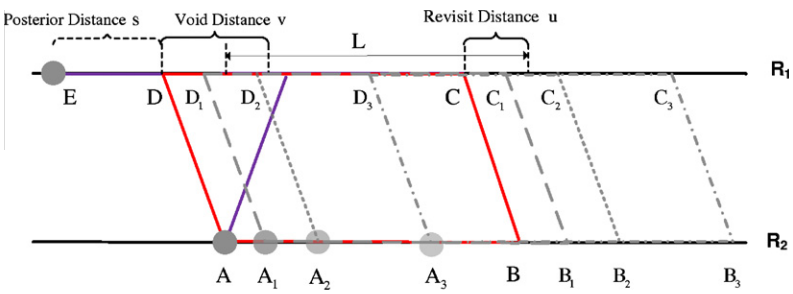


Fig. 3. Transition between regions of type 2. Note that $u = \max\{-s, 0\}$.

resulting state is $S_2(0, 0, s + t)$ with a transmission region $A_3B_3C_3D_3$. Note that cases (II) and (III) both result in a revisit distance zero and that the void distance in (III) also becomes zero.

The propagation distance from A has the following terms corresponding to Case 1:

$$\int_0^u f_2(t)\big(t + D_2'(v - t, u - t, s + t)\big)\,dt + \int_u^v f_2(t)(t + D_2'(v - t, 0, s + t))\,dt + \int_v^L \frac{\mathbb{P}(H_1 \geqslant s + t)}{\mathbb{P}(H_1 \geqslant s + v)} f_2(t)\big(t + D_2'(0, 0, s + t)\big)\,dt, \tag{1}$$

where D_2' represents additional propagation distance from the first vehicle on R_2. $D_i'(v, u, s)$ is independent and identical distribution (*i.i.d.*) with $D_i(v, u, s)$, i = 1 or 2. In the last term of Eq. (1), $\mathbb{P}(H_1 \geqslant s + v)$ denotes the probability that the vehicle headway H_1 on R_1 is larger than $s + v$. Hence, $\frac{\mathbb{P}(H_1 \geqslant s+t)}{\mathbb{P}(H_1 \geqslant s+v)}$ means the probability for event $\{H_1 \geqslant s + v + t\}$ conditional on the event $\{H_1 \geqslant s + v\}$.

Case 2: The first vehicle is encountered on road R_1 by moving the edge AD rightward as in Fig. 4. Again we use t for the horizontal displacement of AD. Similarly there are three intervals for t. (I) $t \in [v, r]$ as shown by A_1 in Fig. 4. The resulting state is $S_1(r, r - t, t - r)$. One may also consult Fig. 2b for additional help for a better understanding. Be aware that the horizontal distance of A and D is $\frac{r}{2}$. When information is transmitted from A to A_1, the propagation distance from A can be represented by $t - \frac{r}{2}$ plus the additional potential distance from A_1. (II) $t \in [r, L]$ as A_2 in Fig. 4. The state of A_2 is $S_1(r, 0, t - r)$. (III) $t \in [L, L + u]$, where the first vehicle as A_3 in Fig. 4 is within the revisit distance. The state of A_3 is $S_1(r - (t - L), 0, t - r)$. Similar to Eq. (1), the propagation distance from A has the following terms corresponding to Case 2:

$$\int_v^r \frac{f_1(s + t)}{\mathbb{P}(H_1 \geqslant s + v)} \mathbb{P}(H_2 \geqslant t)\Big(t - \frac{r}{2} + D_1'(r, r - t, t - r)\Big)dt + \int_r^L \frac{f_1(s + t)}{\mathbb{P}(H_1 \geqslant s + v)} \mathbb{P}(H_2 \geqslant t)\Big(t - \frac{r}{2} + D_1'(r, 0, t - r)\Big)dt + \int_L^{L+u} \frac{f_1(s + t)}{\mathbb{P}(H_1 \geqslant s + v)} \mathbb{P}(H_2 \geqslant L)\Big(t - \frac{r}{2} + D_1'(r - (t - L), 0, t - r)\Big)dt. \tag{2}$$

Case 3: No vehicle is present in the transmission region and its revisit distance. The propagation terminates at A. However, A might not be horizontally (rightward) as far as the previous vehicle on R_1 that has caused the revisit distance u in the state $S_2(v, u, s)$ for vehicle A. In this case, an adjustment to the propagation distance is needed, by comparing the horizontal locations of A and the prior vehicle on R_1. Note that the prior vehicle leads to the current state for A. Therefore, the location of that prior vehicle on R_1 can be determined by the revisit distance in $S_2(v, u, s)$ for vehicle A. We use Fig. 1 as an example to illustrate this idea, where there is no new vehicle in the transmission region of vehicle A. If $u \geqslant \frac{r}{2}$, we have $s = -u < -\frac{r}{2}$, which indicates there is a prior transmitter horizontally right of A. This transmitter instead of A is horizontally the furthest vehicle receiving information, although information dies out at A. In this case, $D_2(v,u,s)$ is adjusted by adding the following quantity:

$$\mathbb{P}(H_2 \geqslant L)\frac{\mathbb{P}(H_1 \geqslant L + u + s)}{\mathbb{P}(H_1 \geqslant v + s)}\left[u - \frac{r}{2}\right]_+, \tag{3}$$

where $[x]_+$ equals to x if $x \geqslant 0$, and 0 if $x < 0$. The term $\mathbb{P}(H_2 \geqslant L)\frac{\mathbb{P}(H_1 \geqslant L+u+s)}{\mathbb{P}(H_1 \geqslant v+s)}$ is the probability of propagation termination.

The recursive propagation distance analyzed above considers a starting transmitter on R_2. If the starting transmitter is on R_1, a recursive distance can be expressed in a similar way. The following theorem summarizes the discussions above.

Theorem 1. *The propagation distance along both roads are described by the following recursive models. Replacing each* $D_i(\cdot, \cdot, \cdot)$ *and* $D_i'(\cdot, \cdot, \cdot)$ *with* $d_i(\cdot, \cdot, \cdot)$ *gives the expected distances.*

$$D_j(v, u, s) = \int_0^L \frac{\mathbb{P}(H_i \geqslant s + [t - v]_+ + v)}{\mathbb{P}(H_i \geqslant s + v)} f_j(t)\Big(t + D_j'([v - t]_+, [u - t]_+, s + t)\Big)dt + \int_v^{L+u} \frac{f_i(s + t)}{\mathbb{P}(H_i \geqslant s + v)} \mathbb{P}(H_j \geqslant L - [L - t]_+)\Big(t - \frac{r}{2} + D_i'(r - [t - L]_+, [r - t]_+, t - r)\Big)\,dt + \mathbb{P}(H_j \geqslant L)\frac{\mathbb{P}(H_i \geqslant L + u + s)}{\mathbb{P}(H_i \geqslant v + s)}\left[u - \frac{r}{2}\right]_+, \tag{4}$$

where (i, j) = (1, 2) or (2, 1).

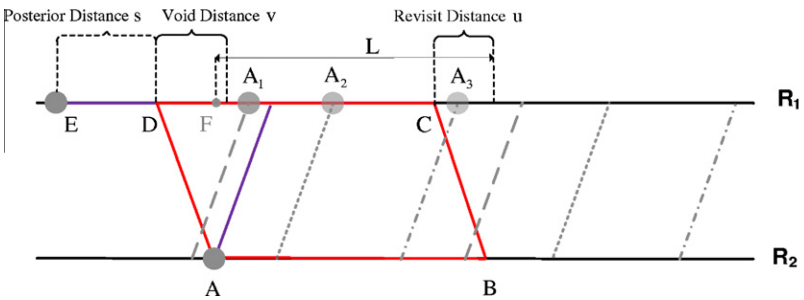


Fig. 4. Transition from type 2 region to type 1.

Eq. (4) allows numerical calculation of the expected propagation distance from an initial state $S_i(u, v, s)$, $i \in \{1, 2\}$. Discretization of integration gives an array of linear equations, which can be easily solved. In the numerical test, the relationship $u = \max\{-s, 0\}$ according to Proposition 1 reduces the combinations of u and s. In addition, one can verify that when $d = 0$ and when the vehicle headway is exponentially distributed, Eq. (4) and the subsequent Eq. (7) reduce to those for a single road as in Wang (2007).

3.4. Variance of information propagation distance

The variance of the propagation distance is similarly derived by conditioning on the location of the first vehicle, denoted by τ, when applying the law of total variance. That is,

$$\begin{aligned} Var(D_i(v,u,s)) &= E(Var(D_i(v,u,s)|\tau)) + Var(E(D_i(v,u,s)|\tau)) \\ &= E(Var(D_i(v,u,s)|\tau)) + E\Big([E(D_i(v,u,s)|\tau) - E(D_i(v,u,s))]^2\Big) \end{aligned} \tag{5}$$

where $i \in \{1, 2\}$. We explain the variance of $D_1(v, u, s)$ as an example by starting with a transmitter vehicle on R_1. If the next vehicle takes place on R_1 and falls in the range $[0, u]$, i.e., $\tau \in [0, u]$, the resulting state is $S_1(v - \tau, u - \tau, s + \tau)$, as in the analysis of Case 1 in Section 3.3. The corresponding terms of variance are expressed as follows:

$$\begin{aligned} E(Var(D_1(v,u,s)|\tau)) + Var(E(D_1(v,u,s)|\tau)) &= \int_0^u f_1(t)V_1(v-t,u-t,s+t)dt + \int_0^u f_1(t)(t + d_1(v-t,u-t,s+t) \\ &\quad - d_1(v,u,s))^2\,dt \\ &= \int_0^u f_1(t)[V_1(v-t,u-t,s+t) + (t + d_1(v-t,u-t,s+t) \\ &\quad - d_1(v,u,s))^2]\,dt. \end{aligned} \tag{6}$$

Results can be obtained similarly according to Case 2 and Case 3 in Section 3.3. Worth mentioning is Case 3, which holds $E(Var(D_1(v, u, s)|\tau)) = 0$ and $Var(E(D_1(v, u, s)|\tau))$ is not zero. Since $E(D_1(v,u,s)|\tau) = \left[u - \frac{r}{2}\right]_+$ and $E(D_1(v, u, s))) = d_1(v, u, s)$, the corresponding variance term in this case is given as $Var(E(D_1(v,u,s)|\tau)) = \mathbb{P}(H_1 \geqslant L)\frac{\mathbb{P}(H_2 \geqslant L+u+s)}{\mathbb{P}(H_2 \geqslant v+s)}\Big(\left[u - \frac{r}{2}\right]_+ - d_i(v,u,s)\Big)^2$.

The following summarizes the above analysis.

Theorem 2. *The variance of $D_i(v, u, s)$ can be described as follows.*

$$\begin{aligned} V_i(v,u,s) &= \int_0^L \frac{\mathbb{P}(H_j \geqslant s + [t-v]_+ + v)}{\mathbb{P}(H_j \geqslant s+v)} f_i(t)\cdot[V_i([v-t]_+,[u-t]_+,s+t) + (t + d_i([v-t]_+,[u-t]_+,s+t) \\ &\quad - d_i(v,u,s))^2]\,dt + \int_v^{L+u} \frac{f_j(s+t)}{\mathbb{P}(H_j \geqslant s+v)}\mathbb{P}(H_i \\ &\quad \geqslant L - [L-t]_+)\cdot\left[\left(t - \frac{r}{2} + d_j(r-[t-L]_+,[r-t]_+,t-r) - d_i(v,u,s)\right)^2 + V_j(r-[t-L]_+,[r-t]_+,t-r)\right]dt \\ &\quad + \mathbb{P}(H_i \\ &\quad \geqslant L)\frac{\mathbb{P}(H_j \geqslant L+u+s)}{\mathbb{P}(H_j \geqslant v+s)}(\left[u - \frac{r}{2}\right]_+ - d_i(v,u,s))^2, \end{aligned} \tag{7}$$

where (i, j) = (1, 2) or (2, 1).

3.5. Probability distribution of propagation distance

$P_i(x, v, u, s)$ denotes the probability of propagation beyond a horizontal distance x starting from a state $S_i(v, u, s)$, $i \in \{1, 2\}$. We model $P_i(x, v, u, s)$, $i \in \{1, 2\}$, also by conditioning on the location of the first vehicle in a transmission region. The following is obvious.

$$P_i(x,v,u,s) = 1, \quad \text{if } x \leqslant max\Big\{0, u - \frac{r}{2}\Big\}, \quad i \in \{1,2\}. \tag{8}$$

A negative value of x refers to a location horizontally to the left of the transmitter. The transmitter itself proves a successful propagation beyond that negative point. In addition, at the horizontal location $u - \frac{r}{2}$ from the transmitter vehicle, where $u \geqslant \frac{r}{2}$, a prior vehicle has received the information on the road opposite of the current transmitter (refer to Section 3.3). It is that prior vehicle that has covered the revisit distance associated with the current transmission region. That prior vehicle itself also proves a successful propagation beyond location x, where $x \leqslant u - \frac{r}{2}$. Accordingly we have the following results.

Theorem 3. *If $x \leqslant \max\{0, u - \frac{r}{2}\}$, the propagation probability $P_i(x, v, u, s)$ satisfies Eq. (8). Otherwise, $P_i(x, v, u, s)$ satisfies the following with (i, j) = (1, 2) or (2, 1).*

$$P_i(x, v, u, s) = \int_0^L \frac{\mathbb{P}(H_j \geqslant s + [t - v]_+ + v)}{\mathbb{P}(H_j \geqslant s + v)} f_i(t) P_i(x - t, [v - t]_+, [u - t]_+, s + t)\, dt + \int_v^{L+u} \frac{f_j(s + t)\mathbb{P}(H_i \geqslant L - [L - t]_+)}{\mathbb{P}(H_j \geqslant s + v)} P_j\left(x - t + \frac{r}{2}, r - [t - L]_+, [r - t]_+, t - r\right) dt. \tag{9}$$

3.6. Approximate solution

Eq. (4), although solvable through numerical methods, poses great practical inconvenience because special commercial solvers are needed for the solution. Therefore, we propose a closed-form approximation to Eq. (4), but only for the state $S_i(0, 0, s)$, $s > 0$, $i \in \{1, 2\}$. Note that $S_i(0, 0, s)$, $s > 0$, $i \in \{1, 2\}$, is the most desired measure of propagation distance. Particularly, consider using the following:

$$d_j(0, 0, s) = \int_0^L \frac{\mathbb{P}(H_i \geqslant s + t)}{\mathbb{P}(H_i \geqslant s)} f_j(t)(t + d_j(0, 0, s + t))\, dt + \int_0^r \frac{f_i(s + t)}{\mathbb{P}(H_i \geqslant s)} \mathbb{P}(H_j \geqslant t)\left(t - \frac{r}{2} + d_i(r, r - t, t - r)\right) dt + \int_r^L \frac{f_i(s + t)}{\mathbb{P}(H_i \geqslant s)} \mathbb{P}(H_j \geqslant t)\left(t - \frac{r}{2} + d_i(r, 0, t - r)\right) dt, \tag{10}$$

where $(i, j) = (1, 2)$ or $(2, 1)$. The approximation is developed based on an assumption that the expectation $d_i(v, u, s)$ is continuous and differentiable except at some finite points. We approximately let $d_i(r, r - t, t - r) \cong d_i(0, 0, 0) + O(r)$ and $s = 0$, then:

$$d_j(0, 0, 0) \cong \int_0^L \mathbb{P}(H_i \geqslant t) f_j(t)(t + d_j(0, 0, 0))dt + \int_0^L f_i(t)\mathbb{P}(H_j \geqslant t)\left(t - \frac{r}{2} + d_i(0, 0, 0)\right) dt. \tag{11}$$

Eq. (11) is much simpler than Eq. (4) as people are generally interested in $d_i(0, 0, 0)$ instead of $d_i(v, u, s)$. This approximation uses the fact that $d_i(v, u, s) = d_i(0, 0, 0) + (v\partial_v + u\partial_u + s\partial_s)d_i(0, 0, 0) + O(v^2)$. We have the following result.

Corollary 1. *The approximate solution to Eq. (4) can be simplified as Eq. (12). The error of approximation is of the order $O(rL)$, and the solution follows Eq. (11) when $1 > A_{i,j} + A_{j,i}$.*

$$d_i(0, 0, 0) = \frac{B_{i,j} + B_{j,i} - \frac{r}{2}A_{j,i}}{1 - A_{i,j} - A_{j,i}}, \tag{12}$$

where $A_{i,j} = \int_0^L f_i(t)\mathbb{P}(H_j \geqslant t)\, dt$, $B_{i,j} = \int_0^L t f_i(t)\mathbb{P}(H_j \geqslant t)\, dt$, where $(i, j) = (1, 2)$ or $(2, 1)$.

The derivation above indicates close proximity of the approximation to the original solution at small density λ_i and small separation distance d, as supported in later numerical tests. In a special case in which headways on both roads are i.i.d., $d_1(0, 0, 0) = d_2(0, 0, 0) = d(0, 0, 0)$, Eq. (12) can be further simplified into the following:

$$d(0, 0, 0) \cong \frac{\int_0^L f(t)\mathbb{P}(H \geqslant t)\left(2t - \frac{r}{2}\right) dt}{1 - 2\int_0^L f(t)\mathbb{P}(H \geqslant t)\, dt}. \tag{13}$$

The headway requirement for the further simplification from Eqs. (12) and (13) is $\int_0^L f(t)\mathbb{P}(H \geqslant t)dt \leqslant 0.5$. This requirement is satisfied by headway of exponential distributions.

4. Numerical results

In this section, we numerically compare the analytic models with Monte Carlo simulation using varying road separation distance, non-Poisson distributions, and vehicle traffic densities.

Eqs. (4) and (7) can be solved by discretizing the integral of the equations, i.e., $\int_a^b f(t)dt \approx \sum_{i=k}^m f(ih)h$ or by the Simpson's rule, and by solving the resulting arrays of linear equations. In our numerical solutions, $[0, r]$ is partitioned into n subintervals each having a width $h = r/n$. The parameters u and v take values of integer number of subintervals. Because $s \geqslant -v$, we subjectively restrict s to the interval $[-r, L]$. The computation was conducted with Matlab on a personal computer with a 2.66 GHz/2.66 GHz Intel Core 2 Quad CPU on a Microsoft Windows platform and a 4.00 GB RAM.

Each simulation generates a snapshot of vehicles on the roadway according to a particular vehicle headway distribution. The instantaneous propagation distance is measured from the sender to the vehicle that first fails to further propagate the information. In the numerical test, we scale the transmission range to be a standard unit 1.0 for simplicity; and all other measures including the vehicle density are scaled accordingly. The study on packet delivery ratio in the V2V communication Bai et al. (2010) indicates that an appropriate transmission range might be around 100*m*, which is used as one unit length here. In setting up vehicle density, Yousif (2010) indicates that an average density for freeway at level of service A (a stable free flow state) is about 5.5 vehicles per mile per lane (vpmpl) and is 22 vpmpl at level of service C. Because of the market penetration rate, a much lower density for equipped vehicles may result. Luttinen (1996) concludes that Gamma distribution may be appropriate for light traffic. Gamma distribution as a generalized Erlangian distribution is often adopted in modeling traffic flow. There are also other studies about the vehicle headway distribution. For example, Ha et al. (2012) finds lognormal distribution for the intermediate density traffic. If market penetration rate is considered, density of equipped vehicles shall be very modest. In our numerical tests, we will mainly use Gamma distribution supplemented by a Lognormal normal distribution. Note that our general models apply to traffic with any headway distributions, which allows test for the effect of headway distribution on information propagation.

4.1. Effects of road separation and vehicle density

We let R_2 be the road with larger traffic density. The density λ_2 for road R_2 is set to be 2.0 and 1.5 sequentially, each corresponding to a set of lower densities on road R_1, denoted by λ_1. Gamma distributions of vehicle headway are used. Note that the mean headway is λ_i^{-1} on R_i, $i \in \{1, 2\}$. The parameters in each Gamma distribution are set according to the coefficient of variation (*CV*). (*CV* for the exponential distribution is 1.0). In this test, *CV* is set to be 0.81. The results are tabulated in Table 1. In each cell, the results from Eqs. (4) and (7) and simulation are paired. The expectation and variance are for $d_i(0, 0, 0)$ and $V_i(0, 0, 0)$, respectively, which are of most interest. The simulation includes a plus and minus standard deviation. The standard deviation is for the means over 20 runs, each run conducting 2000 simulations. This standard deviation, usually small, gives high confidence of an interval containing the true propagation distance.

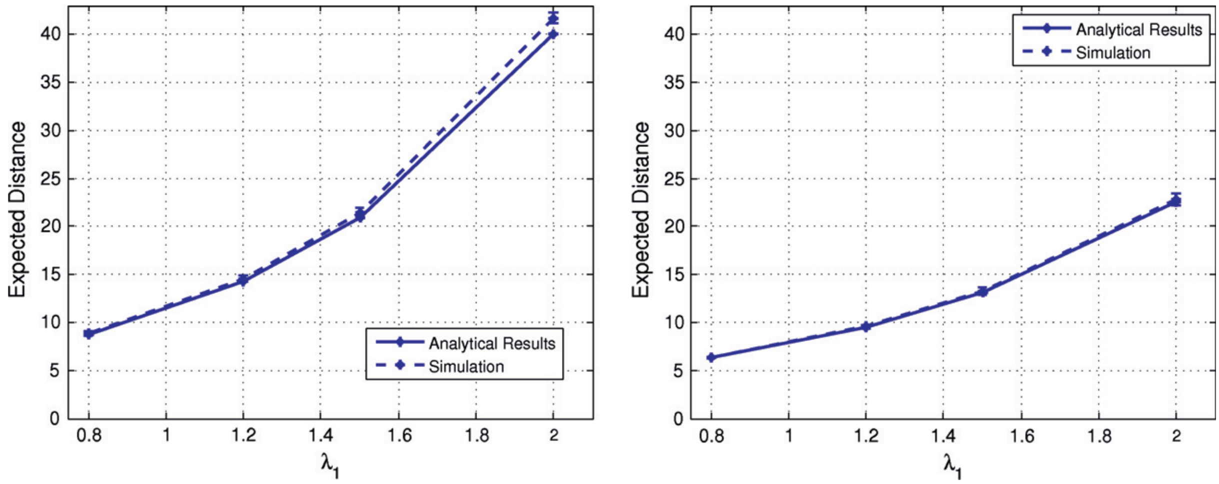
Table 1 shows that Eqs. (4) and (7) are generally agreeable with simulation, especially in terms of the expected distance. The propagation distance decreases with road separation—consistent with commonsense. For example, when λ_1 decreases from 2.0 to 1.5 with a fixed λ_2, the expected propagation distance decreases by 42 percent at $d = 0.3$ and by 37 percent at $d = 0.8$. In addition, the propagation distance appears volatile because of the large variance relative to expectation. In almost all the cases in Table 1, the *CV* of propagation distance is fairly close to 1.0, an indicator of exponential propagation distance in the limiting case at large traffic densities, which is conjectured in Wang (2007) for one road and Wang et al. (2012) for two roads.

4.2. Effect of vehicle distribution

How do different vehicular headway distributions affect the connectivity of V2V networks? We use Gamma distribution to illustrate. Fig. 5a and b compares the analytical and simulation results for the expected propagation distance between two different Gamma distributions of vehicle headway, one having a $CV = 0.7071$ and the other having a $CV = 0.8165$. In both cases, the mean density λ_1 varies from 0.8 to 2.0 at $\lambda_2 = 2.0$ and $d = 0.3$. Comparison of Fig. 5a and b illustrates that a smaller *CV* significantly increases the expected propagation distance. This shows a major advantage of Eq. (4) for explicitly considering any specific vehicle headway distribution.

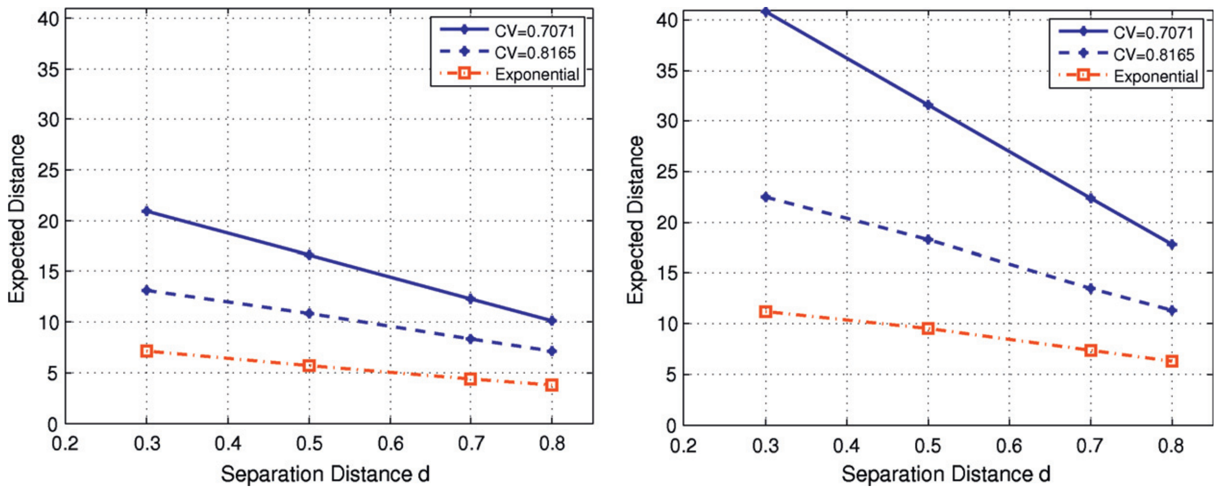
Table 1
Numerical results for traffic with gamma headway distributions (CV = 0.8165).

d	$\lambda_2 = 2.0$			
	0.3	0.5	0.7	0.8
$\lambda_1 = 2.0$				
Expectation	22.47	18.14	13.17	11.31
	(22.54 ± 0.62)	(18.52 ± 0.43)	(14.14 ± 0.42)	(12.10 ± 0.32)
Variance	513.75	331.92	180.01	126.15
	(522.76 ± 34.16)	(356.87 ± 22.82)	(209.74 ± 15.58)	(149.27 ± 7.00)
$\lambda_1 = 1.5$				
Expectation	13.08	10.82	8.29	7.07
	(13.30 ± 0.36)	(11.01 ± 0.26)	(8.40 ± 0.25)	(7.22 ± 0.18)
Variance	177.94	121.23	70.49	51.04
	(180.57 ± 12.77)	(131.10 ± 7.63)	(76.22 ± 6.38)	(58.99 ± 4.16)
	$\lambda_2 = 1.5$			
$\lambda_1 = 1.5$				
Expectation	7.53	6.34	4.97	4.30
	(7.64 ± 0.15)	(6.35 ± 0.16)	(4.97 ± 0.11)	(4.36 ± 0.12)
Variance	61.17	43.48	26.83	20.18
	(62.76 ± 3.18)	(43.88 ± 2.63)	(28.62 ± 1.72)	(22.34 ± 1.68)
$\lambda_1 = 1.2$				
Expectation	5.43	4.64	3.70	3.25
	(5.47 ± 0.133)	(4.56 ± 0.14)	(3.68 ± 0.09)	(3.13 ± 0.08)
Variance	32.82	24.04	15.45	11.93
	(33.28 ± 2.15)	(23.64 ± 1.62)	(16.53 ± 1.21)	(12.19 ± 0.91)



(a) $CV = 0.7071$, $d = 0.3$ and $\lambda_2 = 2.0$. (b) $CV = 0.8165$, $d = 0.3$ and $\lambda_2 = 2.0$.

Fig. 5. Propagation distance with Gamma distribution of vehicle headway.



(a) $\lambda_1 = 1.5$, $\lambda_2 = 2.0$. (b) $\lambda_1 = 2.0$, $\lambda_2 = 2.0$.

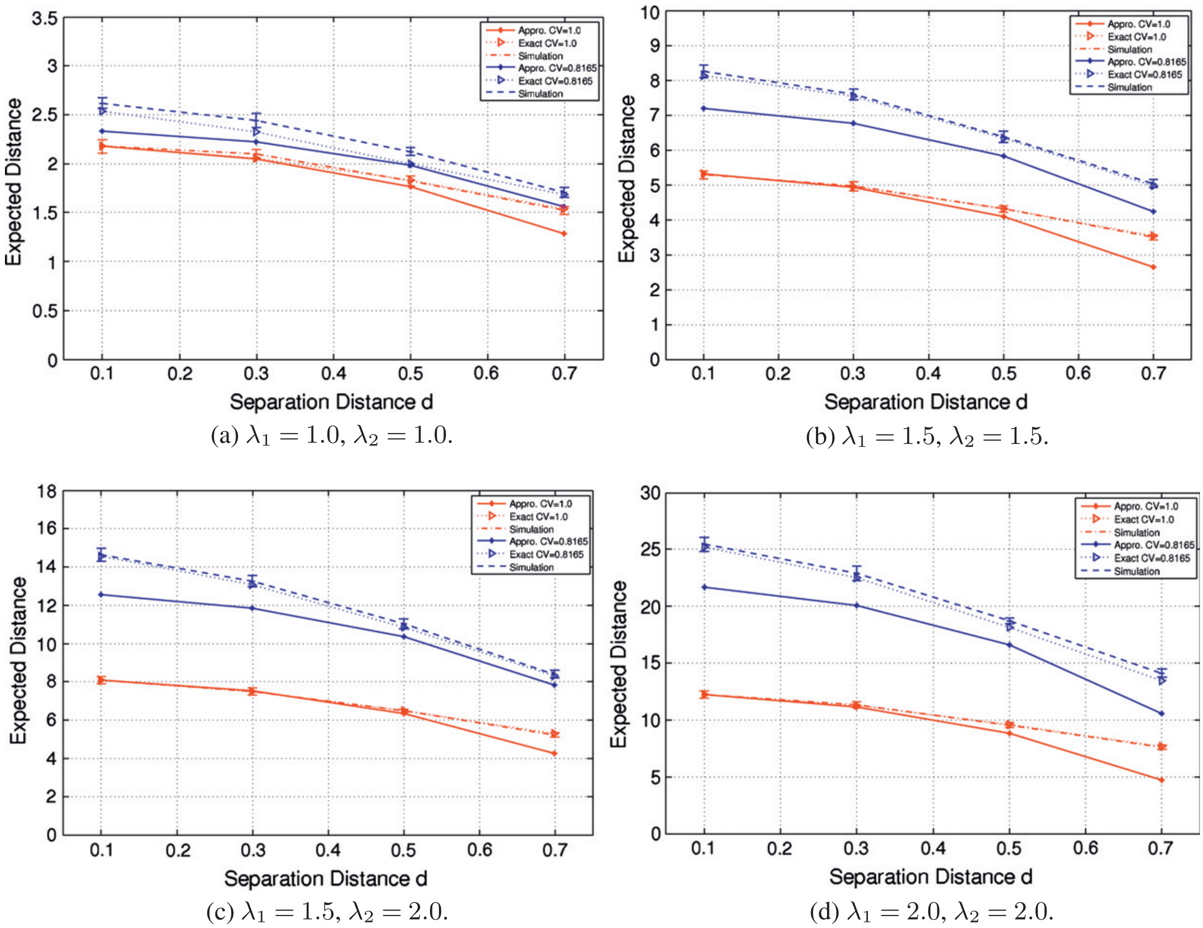
Fig. 6. Comparison with Gamma distribution of vehicle headway with different *CV* values.

4.3. Effect of Poisson assumption

Research often tends to use Poisson assumption for vehicle spatial distribution because of technical tractability. Here we illustrates errors due to the Poisson assumptions compared with our models with general headway distribution. We consider Gamma headway distribution of *CV* = 1.0, 0.7061 and 0.8165, respectively. Fig. 6a shows a comparison between $\lambda_1 = 1.5$ and $\lambda_2 = 2.0$, while Fig. 6b shows a comparison between $\lambda_1 = \lambda_2 = 2.0$. In all these cases, the propagation distance with exponentially distributed headway is shorter. When the two parallel roads are close to each other, the effects of non-Poisson distribution become more significant. This result demonstrates the necessity of extending (Wang, 2007; Yousefi et al., 2008; Schönhof et al. (2006)), which assume exponential headway distributions, to traffic with a general vehicle headway distribution as modeled in this paper.

4.4. Assessment of the approximation

The approximation Eq. (12) is expected to work well at low traffic density. To assess accuracy of the approximation, a Gamma distribution with *CV* = 0.8165 and an exponential distribution of vehicle headway are used to compare the expected



(a) $\lambda_1 = 1.0$, $\lambda_2 = 1.0$.

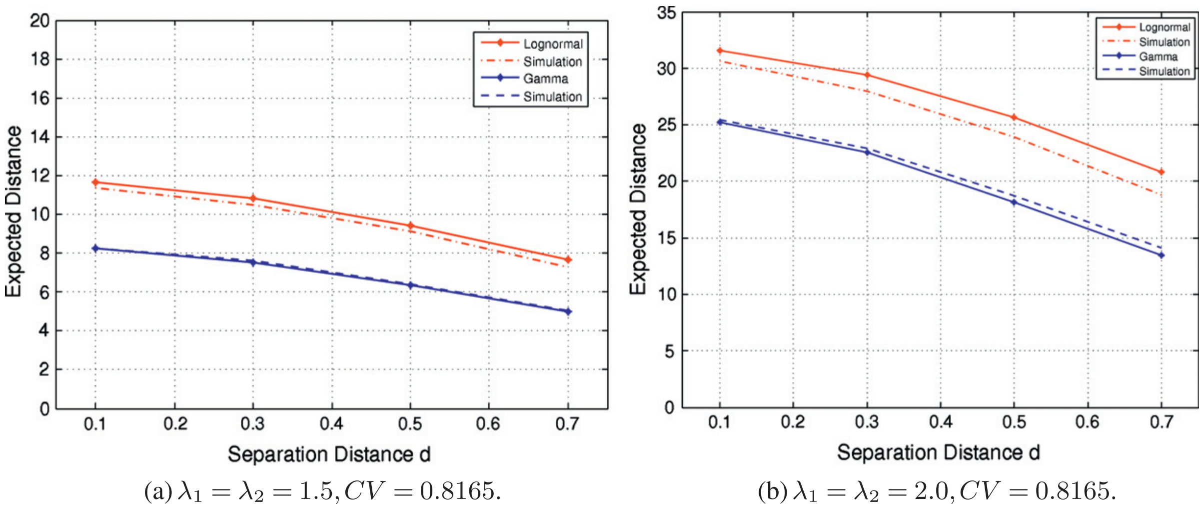
(b) $\lambda_1 = 1.5$, $\lambda_2 = 1.5$.

(c) $\lambda_1 = 1.5$, $\lambda_2 = 2.0$.

(d) $\lambda_1 = 2.0$, $\lambda_2 = 2.0$.

Fig. 7. Approximation, exact solution, versus simulation.

propagation distance between the exact, approximation, and simulation methods at varying *d* and vehicle densities as in Fig. 7a–d. The exact solutions in these figures are from Eq. (4). Since the exact solutions are very close to the simulations, the two lines are hardly differentiable in Fig. 7. The approximations are close to the exact solutions at low λ_1 and λ_2. The



(a) $\lambda_1 = \lambda_2 = 1.5, CV = 0.8165$.

(b) $\lambda_1 = \lambda_2 = 2.0, CV = 0.8165$.

Fig. 8. Propagations with Gamma and lognormal distributions.

results indicate that the approximation works particularly well for the exponential distribution of vehicle headway, in which case, the gap between approximation and simulation solutions increases with road separation, which is consistent with Corollary 1. Numerically, it appears that Eq. (12) works fine largely because it incorporates the effect from a general vehicle distribution and from a road separation distance.

4.5. Case of lognormal distribution

While the Gamma distribution is found for light traffic (Luttinen (1996)), lognormal distribution is identified for the intermediate density traffic Ha et al. (2012), Yan and Olariu (2011). Fig. 8a and b compare the expected propagation distance from Gamma and lognormal distributions respectively. The two distributions are generated to have the same mean and variance. The analytical numbers are also verified by simulation. Clearly, Fig. 8a and b indicate a system difference in propagation distance between the two distributions. Our proposed general model therefore can be applied to test the effect of varying headway distributions, which is important for robust design.

5. Conclusion

In this paper, we develop recursive models for the expectation, variance and probability of propagation distance in a V2V system for vehicular traffic on two parallel roads. The models explicitly consider road separation and general distributions of vehicle headway. Clearly, the interaction of vehicles between the two roads is significant to vehicle connectivity as evidenced in the numerical tests. We further validate the models by Monte Carlo simulation, and the models prove to work well. In addition, in order to facilitate applications, we develop a closed form approximation for the expected distance, which is easier to use than the exact, recursive model. The approximation numerically shows reasonable accuracy at low traffic densities. Overall, the developed models overcome the limitations of the existing one-road models and models that assume spatial Poisson distribution of vehicles. Our models reveal significant errors from the traditional models that approximate the headways with exponential distributions, and from those that approximate two roads as one by summing up the two road densities. This work may provide a modeling framework to further consider signal fading and other practical issues in the V2V communication. It serves as a first step in an effort to study propagation on general, realistic roadway networks.

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