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Vehicle-to-vehicle connectivity on parallel roadways with large road separation

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This paper studies vehicle multihop connectivity of vehicular ad hoc networks (VANETs) on two parallel roads. It complements our earlier studies by considering a road separation distance larger than $\frac{\sqrt{3}}{2}L$ and considering signal blockage between roads, where L represents the transmission range. Assuming vehicles follow Poisson processes, we specifically derive exact formulas for the expectation, variance, and probability distribution of the information propagation distance. We further develop a closed form approximation for the expected distance. The analytical results are verified and compared with those from other headway distributions of varying coefficient of variation through Monte Carlo simulation.

1. Introduction

The last decade has witnessed fast development of research on vehicular ad hoc networks (VANETs) largely because VANETs herald a significant potential to improve roadway traffic safety and efficiency (Ford Sync; BMW ConnectedDrive). The U.S. Department of Transportation (USDOT) has initiated the Connected Vehicles Program among other efforts in VANETs (Connected Vehicle Research). Vehcile connectivity is critical to VANETs efficiency, which depends to a great extent on roadway topological and geometric characteristics such as network connectivity and road spacing. Uncertain traffic flow, random vehicle location and mobility all add to the variations in vehicle connectivity on roadways.

Early studies on vehicle connectivity of VANETs generally resort to simulations (Yang and Recker, 2005; Tsugawa, 2002). Most analytical literature on vehicle connectivity so far only considers one road for technical maneuverability (Wang, 2007; Jin and Recker, 2010; Dousse et al., 2002; Ukkusuri and Du, 2008; Yousefi et al., 2008). The means of using one road to approximate the case of multiple roads or a network is coarse (Wang, 2007; Chen et al., 2010; Yin et al., 2013). Therefore, the recent years have seen research efforts to explicitly consider information propagation on parallel roadways. Wang et al. (2012) specifically study information propagation (an equivalent measure of vehicle connectivity) along parallel roadways when the roads are separated by a distance no larger than $\frac{\sqrt{3}L}{2}$, where L is the transmission range of a vehicle. These works show that it may lead to significant errors to approximate two parallel roads with a single one by virtually consolidating traffic onto one road. Following the same line of efforts, this paper complements Wang et al. (2012) by considering a road separation distance larger than $\frac{\sqrt{3}L}{2}$. It completes a full range of road separation distance. Clearly, the type of transmission region defined in the case in which the road separation is no larger than $\frac{\sqrt{3}}{2}L$ (see in Wang et al. (2012)) does not apply to

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Nomenclature

L	transmission range of vehicles
d	separation distance of road R_1 and R_2, satisfying $L \geqslant d \geqslant \frac{\sqrt{3}}{2}L$
λ_i	equipped vehicles densities on $R_i,\ i = 1,\ 2$
v	void distance
u	revisit distance
$d_i(v,u)$	expected propagation distance on $R_i,\ i = 1,\ 2$
$V_i(v,u)$	variance of propagation distance on $R_i,\ i = 1,\ 2$
h	equal to $2\sqrt{L^2 - d^2}$, a characteristic measure of road topology
$P_i(x, v, u)$	probability of propagation beyond a horizontal distance x starting with a vehicle
p_i	success probability of transmission from a vehicle on road R_i to the other road R_j, where $i \neq j$, when blockage between roads is present when both vehicles are within range of communication. $(1 - p_i)$ is the blockage probability in this case

our case of large road separation here because no parallelogram can be identified to be contained within a transmission range of a vehicle in our study problem. To be seen later, the transmission region in this paper, a critical concept for the modeling, is defined differently. Compared with the parallelogram defined as the transmission region in Wang et al. (2012), we identify an irregular shape inherent to the process of transmission to be defined as the transmission region. A unique feature in this case but not in the case of a shorter road separation distance is that vehicles within the propagation distance may be left out of communication because of 'holes' in the propagation distance. We propose formulas to estimate the number of vehicles left in the 'holes' and that outside of the 'holes'.

In this paper, we study connectivity by measuring instantaneous information propagation distance, i.e., the maximum horizontal distance to which a piece of information can propagate in a VANET on two parallel roadways. Instantaneity of information propagation is first argued in Jin and Recker (2010), and is later used in Wang et al. (2012). Analytically, this paper completes a full range of road separation distance together with earlier efforts. Practically, the roadway separation in this paper indicates a relatively sparse roadway network: roads are barely reachable within a transmission range, which takes place in rural or some urban areas. This study could serve as a springboard toward studying connectivity on grid networks. The layout of this paper follows that of Wang et al. (2012) by presenting exact models for the expectation, variance and probability of information propagation distance. An approximated closed-form expression for the expectation is also provided. A special feature of this paper is that it explicitly considers transmission failure due to signal blockage when transmitted between roads particularly because of obstructions within the large separation distance.

In what follows, we first define the study problem and transmission regions, and describe the transition process between regions in Section 2. Models for information propagation distance are developed in Section 3. We further give an approximation solution in Section 4. And numerical simulation is conducted in Section 5. We summarize the major findings in Section 6.

2. Definitions and problem statement

Consider two roadways R_1 and R_2 with a distance d apart where $\sqrt{3}L/2 \leqslant d \leqslant L$. Traffic densities on R_1 and R_2 are assumed to be λ_1 and λ_2, respectively. And vehicles on both roads follow Poisson processes. Vehicles have wireless communication with each other within an effective range L. We specifically consider instantaneous information propagation between vehicles as long as two vehicles are within L of each other. As a result, information is instantly propagated from vehicle A to vehicle B if they both communicate with vehicle C even if A and B are not within L of each other, so called multihop connectivity. When two vehicles are connected, they are considered to be in the same VANET. The objective of this paper is to characterize the longitudinal size of a VANET in a snapshot from a vehicle forward on the two roads.

To make the study general, we assume a failure probability, denoted by $1 - p_i$, where $i = 1, 2$, for within-range transmissions between the two roads due to buildings and other obstructions. As an example, a vehicle on road R_1 transmits a signal to a vehicle on R_2 within range L at a success probability p_1. In contrast, we assume no failure for within-range transmissions on the same road.

Note that the longitudinal size of a VANET from a particular vehicle forward also represents the maximum horizontal distance that information may be propagated in a direction each time. Because of randomness of vehicle presence on the roadway and because of vehicular mobility, VANETs may be considered as random graphs whose nodes are confined to locations on the roads. This paper is particularly interested in the random size of this graph. Note that the assumption of instantaneity implies no channel conflict and information latency, which appears to be a strong assumption. However, if one is able to characterize the probability of lost connectivity due to channel conflict, terrain changes, and other factors, this probability of loss may be multiplied to the vehicle presence probability to get an *adjusted* vehicle presence probability to use in the subsequent models of this paper.

As in Wang et al. (2012) and Yin et al. (2013), the process of information propagation is one of transitions between states, which requires a definition of transmission region and associated parameters. As in Fig. 1, we define the region *ABCD* as a

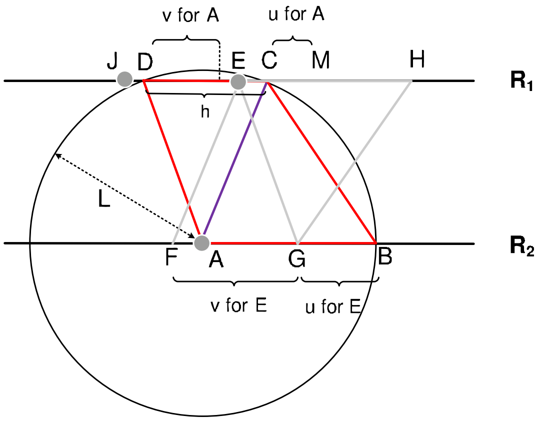


Fig. 1. Transition from A to the first vehicle E on R_1; Or equivalently, transmission region ABCD transitioning to EFGH (solid dots are for vehicles). $|JM| = L$.

transmission region associated with vehicle A on road R_2, where $|AD| = |AC| = |AB| = L$. Vehicles within a transmission region, for instance, vehicle E within transmission region $ABCD$, are able to receive and further propagate the information. There are two types of transmission regions: type 2 if the associated vehicle is on R_2, and type 1 if the vehicle is on R_1. In Fig. 1, EFGH is a type 1 region associated with vehicle E while ABCD is a type 2 region associated with vehicle A. In addition, we define a vehicle within a transmission range as the '*first vehicle*' if it is a vehicle within the transmission region and if it is horizontally closest to the left side of the region. In Fig. 1, for example, vehicle E is referred to as the *first* vehicle within the region $ABCD$ if there are no vehicles on the segments ED and AG, where EG and AD are parallel. The 'first' vehicle is identified by horizontally moving section AD rightwards until the first vehicle E is 'hit'. The concept of *first* vehicle will remain similar and be used to recursively establish our connectivity models later. Note that if the *first* vehicle takes place on the different road than the transmitting vehicle, its presence probability shall be multiplied by the according success probability p_i.

Similar to Wang et al. (2012), each transmission region is associated with two parameters: *void distance* v and *revisit distance* u, which we illustrate through the example in Fig. 1. Consider transmitting vehicle A (assuming $v = u = 0$) and the *first* vehicle E within the transmission region $ABCD$. Information propagates from A to E with a new transmission region $EFGH$ with parameters $v = |FG|$ and $u = |GB|$. Again, we shall only consider vehicles within the revisit distance and within transmission region lest the void distance. Note that transition from A to E proves void of vehicles on the segment AG according to the definition of '*first*' vehicle after A. And any vehicle within FA shall have already been accounted for before transitioning to A. Therefore, when we continue to identify the next 'first' vehicle within region $EFGH$, vehicles, if any, in the segment FG shall be dropped from consideration. FG is called *void distance*, denoted by v, of the transmission region $EFGH$. Furthermore, any vehicle on segment GB, although not connected to E, has been able to connect to A and been able to further the propagation. GB in this case is called revisit distance, denoted by u. So vehicles in the *revisit distance* GB associated with region $EFGH$ should be considered for additional propagation distance in order not to compromise the total propagation distance. The process of information propagation from one vehicle to another is one that transits between transmission regions with the two parameters updated accordingly.

3. Models on two parallel roadways

3.1. A transitional process of states

The information propagation is equivalent to a process of transitions among different transmission regions with parameters v and u. We define a transmission region with associated parameters u and v as a state $S_i(v,u)$, where i is the road index. Each state corresponds to a finite propagation distance. The process of transition between states defines the propagation performance. Transiting from vehicle A to E in Fig. 1 indicates that the total propagation distance from A with its associated state equals the horizontal distance forward till E plus an additional propagation distance from the new state associated with vehicle E. The following is useful for later derivations.

Proposition 1. *We have* $v \leqslant h$ *and* $u \leqslant L$. *Furthermore, when* $v \geqslant 0$, *there exists the relation* $u \in [0, L + v - h]$.

3.2. Expectation and variance of propagation distance

Theorem 1. *The expected propagation distance and the variance from a state* $S_i(v,u)$ *are given as follows, where* $(i,j) = (1,2)$ *or* $(2,1)$, $w = L \wedge (h+u) \equiv \min\{L, h+u\}$ *and* $m = L \vee (h+u) \equiv \max\{L, h+u\}$.

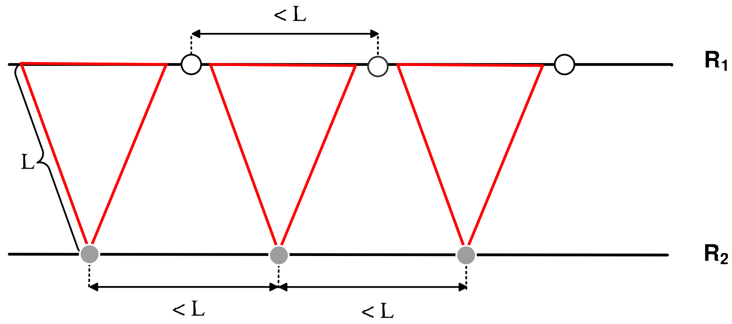


Fig. 2. An illustrative example of skipped vehicles (empty dots on Road R1 skipped and solid dots on Road R2 connected).

$$\begin{aligned} d_i(v,u) = & \int_v^w \lambda_j p_i e^{-(\lambda_j p_i (x-v)+\lambda_i x)} \left(x - \frac{h}{2} + d_j(h, L-x)\right) dx + \int_L^m \lambda_j p_i e^{-(\lambda_j p_i (x-v)+\lambda_i L)} \left(x - \frac{h}{2} + d_j(h-x+L, 0)\right) dx \\ & + \int_0^v \lambda_i e^{-\lambda_i x}(x + d_i(v-x, [u-x]_+))dx + \int_v^w \lambda_i e^{-(\lambda_j p_i (x-v)+\lambda_i x)}(x + d_i(0, [u-x]_+))dx \\ & + \int_w^L \lambda_i e^{-(\lambda_j p_i (u+h-v)+\lambda_i x)}(x + d_i(0,0))dx e^{-(\lambda_j p_i (u+h-v)+\lambda_i L)} \left[\frac{h}{2} - L + u\right]_+, \end{aligned} \tag{1}$$

and

$$\begin{aligned} V_i(v,u) = & \int_v^w \lambda_j p_i e^{-(\lambda_j p_i (x-v)+\lambda_i x)} \left[\left(x - \frac{h}{2} + d_j(h, L-x) - d_i(v,u)\right)^2 + V_j(h, L-x)\right] dx \\ & + \int_L^m \lambda_j p_i e^{-(\lambda_j p_i (x-v)+\lambda_i L)} \left[\left(x - \frac{h}{2} + d_j(h-x+L, 0) - d_i(v,u)\right)^2 + V_j(h-x+L, 0)\right] dx \\ & + \int_0^v \lambda_i e^{-\lambda_i x}[(x + d_i(v-x, [u-x]_+) - d_i(v,u))^2 + V_i(v-x, [u-x]_+)]dx + \int_v^w \lambda_i e^{-(\lambda_j p_i (x-v)+\lambda_i x)} \\ & \cdot [(x + d_i(0, [u-x]_+) - d_i(v,u))^2 + V_i(0, [u-x]_+)]dx + \int_w^L \lambda_i e^{-(\lambda_j p_i (u+h-v)+\lambda_i x)}[(x + d_i(0,0) - d_i(v,u))^2 \\ & + V_i(0,0)]dx + e^{-(\lambda_j p_i (u+h-v)+\lambda_i L)}([h/2 - L + u]_+ - d_i(v,u))^2. \end{aligned} \tag{2}$$

Proof. We show by example. Suppose the information is initiated and propagated rightward from vehicle A on road R_2 in Fig. 1. The terms are explained as follows. Note that each transition has a probability accompanied by a propagation distance forward.

Case 1: The first two terms in Eq. (1) account for the *first* vehicle located on the revisit distance and the segment DC lest v on road R_1. Whether the value $h+u$ is larger than L defines two situations, as represented by the first two terms. The first term accounts for the situation in which the first vehicle falls on R_1 within a distance between v and L right of AD, the probability of which is represented by $\lambda_j p_i e^{-(\lambda_j p_i (x-v)+\lambda_i x)} dx$. The *first* location corresponds to a rightward propagation of $x - \frac{h}{2}$ plus a potential propagation distance $d_j(h, L-x)$. The second term in Eq. (1) is similar to the first term, but accounts for the case in which the *first* vehicle in the revisit distance on R_1 falls a distance larger than L right of AD. Both the first and second terms have a success probability p_i (e.g. a blockage probability $1-p_i$) for cross road transmissions, which makes an equivalent case in which vehicle presence density on road R_2 is $\lambda_j p_i$ but without cross road transmission failure.
Case 2: The terms three to five account for the probability of the *first* vehicle on segment AB of road R_2, dependent on the value of $h+u$ as opposed to L. Term three represents a situation in which the *first* vehicle falls within distance v right of vehicle A. Note that the same distance on road R_1 is void of vehicles by definition of v. The term four is for the situation in which the first vehicle takes place within a distance of (v, w) on road R_2, the probability for which is $\int_v^w \lambda_i e^{-(\lambda_j p_i (x-v)+\lambda_i x)} dx$. Term five is for the case in which w falls short of value L.
Case 3: The last term accounts for the absence of vehicle in the transmission region and re-visit distance, in which case, the prior transmitting vehicle on the road of revisit distance may be horizontally to the right of the current transmitting vehicle (Note that they are on opposite roads.). In this case, the propagation distance is adjusted in the last term.
The variance of propagation distance results in light of the Law of Total Variance. The terms represent situations as discussed for the equations of expectation. A similar explanation to Eq. (2) is also available in Wang et al. (2012). □

3.3. Skipped vehicles

In this study problem, where $d \geqslant \frac{\sqrt{3}}{2}L$, there exist situations in which some vehicles may not be able to receive the information even if they are within the defined propagation distance. Consider a special example as in Fig. 2, in which information is propagated forward along road R_2. The two vehicles on road R_1 do not receive the information, and appear to fall into 'holes' within the propagation distance. Existence of 'holes' makes this study unique compared with earlier ones such as Wang et al. (2012).

We denote respectively the expected number of unconnected and connected vehicles by $N_i^{uc}(v,u)$ and $N_i^c(v,u),\ i \in \{1,2\}$ on a VANET starting from a state $S_i(v,u)$. Again, by assuming Poisson processes of vehicles.

$$\begin{aligned} N_i^c(v,u) = 1 &+ \int_v^w \lambda_j p_i e^{-(\lambda_j p_i (x-v)+\lambda_i x)} N_j^c(h, L-x)dx + \int_L^m \lambda_j p_i e^{-(\lambda_j p_i (x-v)+\lambda_i L)} N_j^c(h-x+L, 0)dx \\ &+ \int_0^v \lambda_i e^{-\lambda_i x} N_i^c(v-x, [u-x]_+)dx + \int_v^w \lambda_i e^{-(\lambda_j p_i (x-v)+\lambda_i x)} N_i^c(0, [u-x]_+)dx \\ &+ \int_w^L \lambda_i e^{-(\lambda_j p_i (u+h-v)+\lambda_i x)} N_i^c(0,0)dx \end{aligned} \tag{3}$$

$$\begin{aligned} N_i^{uc}(v,u) = &\int_v^w \lambda_j p_i e^{-(\lambda_j p_i (x-v)+\lambda_i x)} N_j^{uc}(h, L-x)dx + \int_L^m \lambda_j p_i e^{-(\lambda_j p_i (x-v)+\lambda_i L)} N_j^{uc}(h-x+L, 0)dx \\ &+ \int_0^v \lambda_i e^{-\lambda_i x} N_i^{uc}(v-x, [u-x]_+)dx + \int_v^w \lambda_i e^{-(\lambda_j p_i (x-v)+\lambda_i x)} N_i^{uc}(0, [u-x]_+)dx \\ &+ \int_w^L \lambda_i e^{-(\lambda_j p_i (u+h-v)+\lambda_i x)} \left(N_i^{uc}(0,0) + \lambda_j p_i (x-h-u)\right)dx \end{aligned} \tag{4}$$

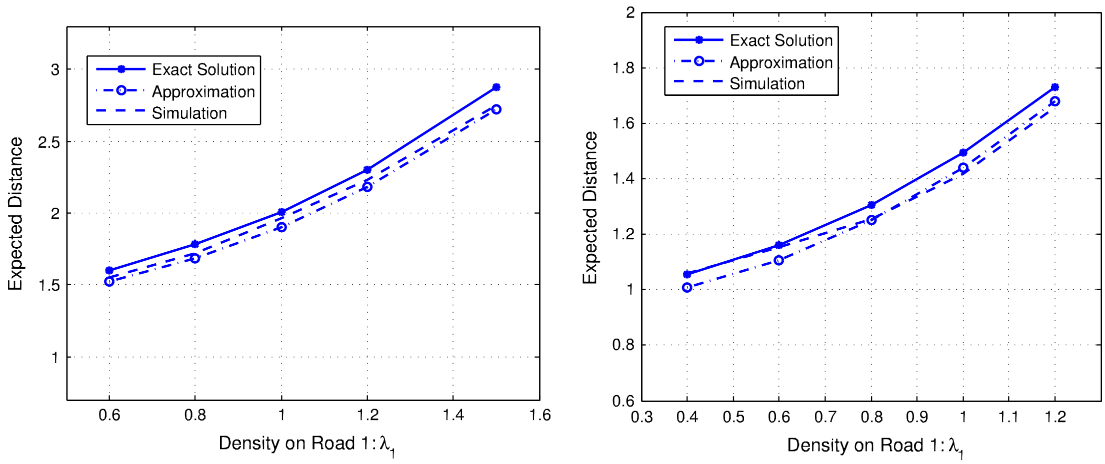
Terms in Eq. (3) correspond to the locations of *first* vehicle as explained for Eq. (1). In each situation, the term $N_i^c(\cdot)$ or $N_j^c(\cdot)$ represents the potential connected number beyond the *first* vehicle. The explanation to $N_i^{uc}(\cdot)$ is similar with an exception in the last term $\lambda_j p_i (x-h-u)$, which represents a length of road in the 'hole' on road R_1.

We have calculated and simulated by setting vehicle density on road R_1 to be 1.0 and 1.5 respectively and by varying vehicle density on road R_2 from 1.0 to 3.0, both assuming Poisson processes. In all the cases, the skipped vehicles within propagation distances account for less than 0.1% of all the vehicles within transmission range. This percentage turns out to be ignorable.

3.4. Propagation distance probability

The propagation distance is random. The probability of propagation distance could be the most important characteristic about vehicle connectivity of the VANETs. We denote by $P_i(x, v, u)$ the probability of propagation beyond a horizontal distance x starting from a vehicle with a state $S_i(v,u)$.

Theorem 2. *The propagation probability* $P_i(x, v, u) = 1$, *if* $x \leqslant \max\{0, \frac{h}{2} - L + u\}$; *otherwise, it satisfies the following equation, where* $(i,j) = (1,2)$ *or* $(2,1)$, $w = L \wedge (h+u)$ *and* $m = L \vee (h+u)$.



(a) $d = 0.90, \lambda_2 = 1.5$ with varying λ_1 (b) $d = 0.92, \lambda_2 = 1.2$ with varying λ_1

Fig. 3. Comparison of exact solution, approximation (using $\alpha = 1.15$) and simulation.

$$P_i(x,v,u) = \int_v^w \lambda_j p_i e^{-(\lambda_j p_i (t-v)+\lambda_i t)} P_j\left(x-t+\frac{h}{2}, h, L-t\right)dt + \int_L^m \lambda_j p_i e^{-(\lambda_j p_i (t-v)+\lambda_i L)} P_j\left(x-t+\frac{h}{2}, h-t+L, 0\right)dt$$
$$+ \int_0^v \lambda_i e^{-\lambda_i t} P_i(x-t, v-t, [u-t]_+)dt + \int_v^w \lambda_i e^{-(\lambda_j p_i (t-v)+\lambda_i t)} P_i(x-t, 0, [u-t]_+)dt$$
$$+ \int_w^L \lambda_i e^{-(\lambda_j p_i (u+h-v)+\lambda_i t)} P_i(x-t, 0, 0)dx. \tag{5}$$

The terms in Eq. (5) correspond to possible locations of the first vehicle on both roads, as explained for Eq. (1). We only take the first term in Eq. (5) as an example to interpret. $P_j(x-t+\frac{h}{2}, h, L-t)$ represents the probability of further propagation beyond a distance $x-t$. The first term is therefore for the probability of propagation beyond a distance x via the *first* vehicle within (v,w) on road R_1. There are other alternative ways to propagate beyond x, each according to a case as explained for Eq. (1).

4. Approximation

Specifically, we approximate Eq. (1) at $v = u = 0$. That is,

$$d_i(0,0) = \int_0^h \lambda_j p_i e^{-(\lambda_j p_i x+\lambda_i x)} \left(x - \frac{h}{2} + d_j(h, L-x)\right)dx + \int_0^h \lambda_i e^{-(\lambda_j p_i x+\lambda_i x)}(x + d_i(0,0))dx$$
$$+ \int_h^L \lambda_i e^{-(\lambda_j p_i h+\lambda_i x)}(x + d_i(0,0))dx. \tag{6}$$

Parameters v and u being continuous implies that $d_j(v,u)$ is also continuous. If we further assume that $d_j(v,u)$ be differentiable in v and u except a finite number of points, we have $d_j(h, L-x) \simeq d_j(0,0) + O(h)$ for the relevant terms in Eq. (6). $d_j(h, L-x)$, where $x \in [0,h]$, indicates that the largest revisit distance reaches $\frac{h}{2}$ further than the distance reached by the associated vehicle. Therefore, we have $d_j(h, L-x) \simeq d_j(0,0) + \left[\frac{h}{2} - x\right]_+$. Our numerical experiences show that the approximation is significantly improved by changing into $d_j(h, L-x) \simeq d_j(0,0) + \left[\frac{h}{2} - x^\alpha\right]_+$, where α is chosen between 1.0 and 2.0. Generally, a larger distance d justifies a larger α to reflect a larger effect from a larger road separation. In our test above, we set $\alpha = 1.15$ for $d = 0.90$ and $\alpha = 1.9$ for $d = 0.94$. Note that setting α to specific values is an empirical decision.

We have found that this approximation retains better accuracy at lower vehicle density. After plugging it into Eq. (1), the closed form for $d_1(0,0)$ and $d_2(0,0)$ reads:

$$\begin{pmatrix} d_1(0,0) \\ d_2(0,0) \end{pmatrix} = \begin{pmatrix} 1 - A_1 - A_{1h} & -A_2 \\ -A_1 & 1 - A_2 - A_{2h} \end{pmatrix}^{-1} \begin{pmatrix} C_1 \\ C_2 \end{pmatrix}. \tag{7}$$

where $i,\ j \in \{1,2\},\ i \neq j$, and

$$A_i = \int_0^h \lambda_i e^{-(\lambda_j p_i x+\lambda_i x)}\,dx, \quad A_{ih} = \int_h^L \lambda_i e^{-(\lambda_j p_i h+\lambda_i x)}\,dx,$$
$$C_i = \int_0^h (\lambda_j p_i [x - h/2]_+ + \lambda_i x) e^{-(\lambda_j p_i x+\lambda_i x)}\,dx + \int_h^L \lambda_i x e^{-(\lambda_j p_i h+\lambda_i x)}\,dx.$$

5. Numerical results

This section numerically compares analytical solutions with Monte Carlo simulations, following a similar process as in Wang et al. (2012). In the simulation, we scale the transmission range to be a standard distance unit 1.0. The simulation each time generates a snapshot of vehicles on the roadway. In Section 5.1, we first examine the case of exponential headway. In

Table 1
Exact solution vs. simulation with varying d.

d	0.00	0.30	0.50	0.88	0.90	0.92	0.94	0.97
$\lambda_1 = \lambda_2 = 1.0$								
Exact	2.1945	2.0295	1.7630	1.2613	1.2644	1.2348	1.1997	1.0001
Simulation	2.1556	2.0468	1.8216	1.2586	1.2349	1.1853	1.1396	1.0524
$\lambda_1 = \lambda_2 = 1.2$								
Exact	3.1763	2.9700	2.6766	1.7929	1.7877	1.7292	1.6643	1.5052
Simulation	3.1552	2.9224	2.6502	1.7620	1.7126	1.6563	1.5982	1.4662
$\lambda_1 = \lambda_2 = 1.5$								
Exact	5.3618	4.8046	4.0421	2.9465	2.8728	2.7777	2.6386	2.3450
Simulation	5.3690	4.9357	4.4329	2.8281	2.7472	2.6490	2.5570	2.3490

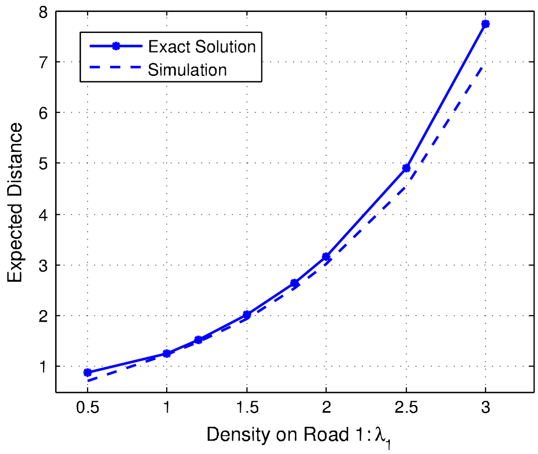


Fig. 4. Example propagation distance with vehicle density λ_1 on Road R1$(\lambda_2 = 1.0, d = 0.9)$.

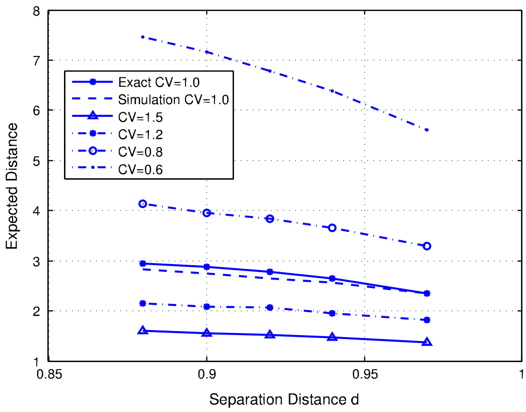


Fig. 5. Propagation distance $(\lambda_1 = \lambda_2 = 1.5)$.

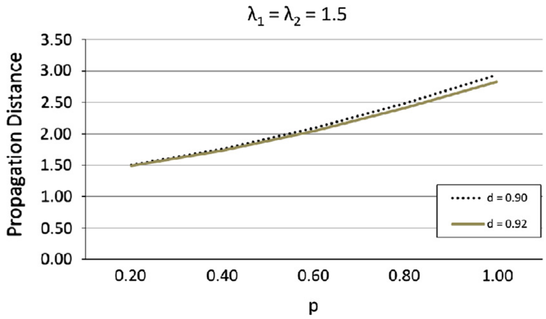
Section 5.2, we compare the exponential headway distribution with other distributions to see the effect of the exponential assumption. In each test, the average distance is obtained over 2000 simulations. The numerical tests in this section assume $p_1 = p_2 = 1.0$. Section 6 will specially test on the effect of p_i when $p_i < 1.0, \ \forall i = 1, \ 2$.

Solution to Eq. (1), called exact solution, is obtained by simple linear discretization, and by solving the resulting array of linear equations. For this method to work, we propose to use the average value of the term $e^{-(\lambda_j p_i (x-v)+\lambda_i x)}$ in Eq. (1) during an interval. The average is obtained by dividing an area within this interval with the interval size. Our numerical experiences here showed that computational error may be up to 30% if not a right value is used for $e^{-(\lambda_j p_i (x-v)+\lambda_i x)}$. An observation is that the computational capability limits the discretization interval from being very small because the number of equations increases dramatically when the partitioning interval decreases. Therefore, handling the term $e^{-(\lambda_j p_i (x-v)+\lambda_i x)}$ appropriately becomes necessary. Another observation is that using the end point of each interval for $e^{-(\lambda_j p_i (x-v)+\lambda_i x)}$ leads to numerical results slightly closer to the computer simulation, which we do not easily identify justifications except for a guess that it overcomes some possible computational errors. A program coded in Matlab™ is provided in the online supplemental materials to facilitate numerical tests by interested readers.

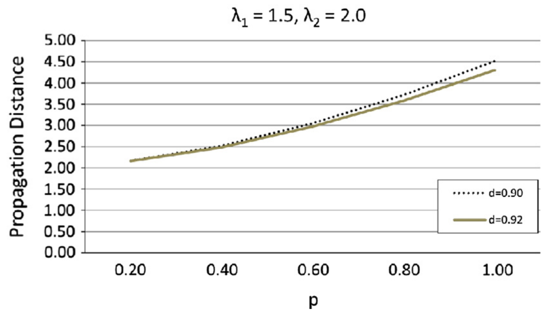
5.1. Information propagation with Poisson vehicle processes

Figs. 3(a) and (b) show exact, approximate, and simulated distances. The three values are all within 5% of each other. In addition, Table 1 demonstrates the effect of d, where $d = 0$ refers to the case of treating two traffic streams as one. Obviously, treating two traffic streams as one in modeling by ignoring the actual separation distance causes significant errors, especially when road separation becomes large. In Table 1, the results with smaller road separation distances are from methods in Wang et al. (2012).

Fig. 4 examines propagation distance at higher vehicle densities. As one may find, if there are 3–4 vehicles in one transmission range as the vehicle density, the propagation distance is already sufficiently far for meaningful technical implementation.



(a) $d = 0.90$ and $0.92, \lambda_1 = \lambda_2 = 1.5$ with varying $p_1 = p_2 = p$.



(b) $d = 0.90$ and $0.92, \lambda_1 = 1.5, \lambda_2 = 2.0$ with varying $p_1 = p_2 = p$.

Fig. 6. Effects of road blockage probability on propagation distance.

Table 2
Poisson (exact solution) v.s. Gamma distributions (simulation) with varying *CV*.

Exponential				
d	0.88	0.90	0.92	0.94
$\lambda_1 = \lambda_2 = 1.0$ Exact	1.2613	1.2644	1.2348	1.1997
Gamma $CV = 1.2$ Simulation	1.0936	1.0747	1.0447	1.0119
Gamma $CV = 1.5$ Simulation	0.9377	0.9100	0.8998	0.8749
$\lambda_1 = \lambda_2 = 1.5$ Exact	2.9465	2.8728	2.7777	2.6386
Gamma $CV = 1.2$ Simulation	2.1576	2.0838	2.0624	1.9548
Gamma $CV = 1.5$ Simulation	1.6014	1.5567	1.5216	1.4758

5.2. Comparison with other headway distributions of vehicular traffic

The assumption on Poisson vehicle distribution on the roadway is a strong one. Traffic are often found to defy it. We have numerically checked the errors of using our formulas to cases of vehicle headway having Gumma distributions with varying correlation of variation (CV). We have found that the errors may be significant when CV significantly deviates from 1.0. In Fig. 5, the propagation distances, when $CV \neq 1.0$, are each from 2000 times simulation. Fig. 5 illustrates the systemic effect of CV on propagation distance, which indicates an overestimate of propagation distance at $CV > 1.0$ from using formulas in this paper and an underestimate at $CV < 1.0$. Our additional observation is that at CV close to 1.0 such as 0.95 and 1.05, the formulas in this paper can be borrowed with relatively small errors, the results of which are not included in Fig. 5 for graphic clarity. According to Jang et al. (2011), the CV of vehicular headway on freeways changes from 0.82 to 0.92 when traffic increases from light at 60–300 vph to heavy at 1260–1500 vph per lane. Fig. 5 might lend light as to how to make empirical adjustment to the computational results from the developed equations.

6. Effect of signal blockage between roads

The success rate p_i for cross road transmission considers a special situation that might arise in urban areas: signals may be blocked by buildings or special terrain situations between the two roads. This blockage effect becomes more obvious when road separation becomes larger as in this study case.

Corresponding to Fig. 3(a) and (b) regarding the parameters of road separation d and vehicle densities, we have obtained exact propagation distances with varying success rate of cross road transmission as in Fig. 6(a) and (b). As an example, at $p = 0.6$ in Fig. 6(a), the propagation distance reduces by about 30% as compared to $p = 1.0$. A point in Fig. 6(a) and (b) may be compared to an according point in Fig. 3(a) and (b) respectively that have the same λ_1 but λ_2(Fig. 3) = $p\lambda_2$(Fig. 6), where λ_2(Fig. 3) is the vehicle density number referenced in Fig. 3 and λ_2(Fig. 6) is that for Fig. 6. This comparison reveals that the propagation distance in Fig. 6 is slightly larger than that from discounting one road vehicle density with the cross road success rate and assuming no signal blockage between roads (see Table 2).

7. Conclusions

Along with the authors' earlier papers (Wang et al., 2012; Yin et al., 2013), this paper completes a full range of roadway separation in an effort to characterize information propagation, or in other words, vehicle connectivity in VANETs, on parallel roadways. The large road separation distance makes modeling more challenging compared to the case $d < \frac{\sqrt{3}L}{2}$ (Wang et al., 2012). Specifically, we develop recursive models for the expectation, variance and probability of information propagation distance on two parallel roads with a large separation distance $d \geqslant \frac{\sqrt{3}}{2}L$ while explicitly considering success rate of cross road transmission. Noteworthy, the closed form approximation numerically shows relatively high accuracy at low traffic density.

The developed models all assume Poisson vehicle processes. Our numerical tests show that the Poisson assumption is a strong one when vehicle headway distributions have a CV significantly deviated from the value 1.0. The assumption about Poisson vehicle processes necessitates adequate adjustment to results from the equations, as indicated in Fig. 3. Additionally, we have tests to show that the blockage has a significant effect on propagation distance. However, by assuming λ_1 and $\lambda_2 p$ as the new road densities while assuming no cross-road blockage as a way of approximation, one can get to a propagation distance lower than the exact distance by a minor amount. A significant signal blockage between roads might make it trivial to explicitly consider two-road propagation as for the case studied in this paper.

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