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An approximate Bernoulli process for information propagation along two parallel roads

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ABSTRACT

This research studies information propagation via inter-vehicle communication along two parallel roads. By identifying an inherent Bernoulli process, we are able to derive the mean and variance of propagation distance. A road separation distance of $\frac{\sqrt{3}}{2}$ times the transmission range distinguishes two cases for approximating the success probability in the Bernoulli process. In addition, our results take the single road as a special case. The numerical test shows that the developed formulas are highly accurate. We also explore the idea of approximating the probability distribution of propagation distance with the Gamma distribution.

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1. Background

There is a fast growing research on mobile vehicular network (VANET) built on short range wireless communication. This ad hoc grid network opens a potential for decentralized traffic information systems in contrast to the currently popular centralized ones. The VANET potentially makes traffic of isolated individuals into a collection of collaborative drivers, integrates vehicles with infrastructure (VII) (Dong et al., 2006), and revolutionizes the network traffic operations and control. A great interest in information propagation via inter-vehicle communication is born on VANET. The information propagation process holds a key to many other research such as communication protocol development, routing algorithms, mobile ad hoc grid network computing, all being important to VANET. Worth mentioning is a large body of literature on multihop connectivity in the wireless communication area, which models essentially the same process as information propagation on a single road (see, for examples, Bettstetter, 2002; Cheng and Robertazzi, 1989; Piret, 1991; Philips et al., 1989).

This paper extends the scope of information propagation from a single road in literature into the case of two parallel roads. There is an extensive literature on information propagation on a single road (Jin and Recker, 2006, 2009, 2010; Yang and Recker, 2005; Ukkusuri and Du, 2008; Wang, 2007; Cheng and Robertazzi, 1989, as examples). Study in cases as simple as a single road gains insights into the general characteristics of the propagation process. For example, Wang (2007) found that the probability of successful propagation along a line of traffic generally follows a Gamma type. Jin and Recker (2009) characterize the process in the case of a uniform vehicle presence with a closed form formula that is said to be more precise than indicated in Wang (2007). Interestingly, when instantaneous connectivity is concerned, the problem becomes a random geometric graph one (see for example, Penrose, 2003).

The extension of study into network of two or more roads is significant. Although studies on single road propagation have appeared successful in deriving analytical results and gaining insights (Jin and Recker, 2009; Wang et al., 2010a, for examples), the ultimate goal is results applicable to the general network. Usually places with dense traffic network and congestion

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are where inter-vehicle communication sees a need for application. The network topology with two or more roads poses challenges to the applicability of results from the single road case. However, the complexity of network with multiple roads is immense. In this paper, as a small step, we have made progress in the case of a special network with two parallel roads. Although there are studies on problems in two dimensional wireless communication (see Mukherjee and Avidor, 2008, for example), they all focus on a continuous two dimensions instead of the discrete case as in this paper.

In a network with two parallel roads, the complexity in modeling mainly arises from the interaction between the two roads: information could possibly take a detour through the other road bypassing a vehicle headway larger than the transmission range on one road. In this paper, we have identified a Bernoulli process to approximate the process of propagation. With this process, we are able to characterize the stochastic propagation distance in terms of its mean and variance. By means of simulation, we are able to assess the accuracy of approximation. Numerical tests show that the approximation is fairly accurate especially when the road separation is less than $\frac{\sqrt{3}}{2}$ times the transmission range.

The paper is organized as follows: Section 2 defines the study problem. Section 3 introduces a Bernoulli process to approximate the propagation process. With introduction of a Bernoulli region, we derive the mean and variance of the propagation distance. In Section 4, numerical simulations are conducted to asses accuracy of the results from using the Bernoulli process, and to show how distance between the two parallel roads impacts the propagation distance. Section 5 describes the probability distribution of the propagation distance. Section 6 concludes the paper.

2. Problem definition

Consider two parallel straight lines (roadways) of traffic with a distance d apart from each other. Traffic densities on the lines are λ_1 and λ_2 respectively. We assume vehicles follow Poisson processes on both roads. Starting from a node, A, on one line, information is propagated forward in one direction of interest. Suppose the transmission range is L. Vehicles on both roads within a transmission range are able to receive and instantly further transmit the information forward. When there is no vehicle present within the range, the propagation process terminates at the last receiving vehicle. The propagation distance measures from the initiating vehicle to the last receiving one on the same road. The objective is to find out the distance of information propagation in terms of its expected value, variance, and probability distribution.

Fig. 1 illustrates this process.

We denote the two roadways by R_1 and R_2 respectively. Consider a location A on road R_2 where the first transmitting vehicle is located as in Fig. 1. With a transmission range L, this vehicle is able to reach the furthest point G on road R_1. Consider such a special situation: if there is no vehicle present on road R_2 within the transmission range L, node A is not able to directly reach node C. However, with help from node G' on road R_2, information is able to propagate to node C. Clearly, information propagation is enhanced by vehicles on the other road. Therefore, there is an effect of distance d on information propagation on both roads. If $d > L$, it reduces to the single road case. In this paper, we only consider the case where $0 < d < L$.

Note that the point assumption of vehicles is implied in this paper. For this reason, we use the terms *gap*, *headway* and *vehicle spacing* interchangeably. In addition, only instantaneous propagation is considered. Therefore, there is no need to distinguish lanes on a road. Each road may be considered a combination of multilanes.

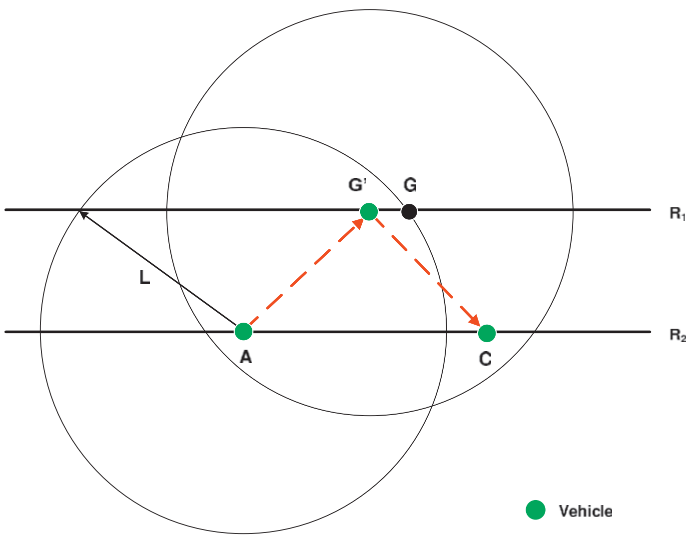

Fig. 1. An illustrative process of propagation with a transmission range L.

3. Modeling the propagation with a bernoulli process

There could be multiple vehicles present within a transmission range each time. In studying connectivity of vehicles or propagation distance, some literature (for examples, Jin and Recker, 2010; Wang, 2007) only allows the furthest vehicle within range to further the propagation using a concept called The Most Forwarded Within Range (MFWR) to construct an equivalent propagation process. This paper follows a method similar to Wang et al. (2010a) which only allows the next vehicle within the transmission range along the same road to receive and further propagate information. If there is no vehicle on the same road within range, the rightmost vehicle on the other road within range receives and continues the propagation process. We refer it as an *equivalent* process. This equivalent process clearly attains the same propagation distance as the original. The problem is therefore a random geometric graph measured by horizontal distance in this special application.

Imagine information being relayed along a road, say R_2, between adjacent vehicles. There are three outcomes about the relay. When the vehicle headway is smaller than L, the relay succeeds; when the headway is larger than L but the information successfully detours on the other road R_1, the relay succeeds; otherwise, the relay fails. With a headway distribution, the success and failure have a *deterministic* probability. If we assume the relays on road R_2 have a probability independent of each other, the propagation resembles a Bernoulli process.

In what follows, we are going to examine the probability of successful relay in this equivalent propagation process. As will be explained later, exact calculation of the Bernoulli probability is complex. Although we adopt an approximation method for the Bernoulli probability, the test results later show satisfactory accuracy from the formulas.

We are interested in the propagation distance on road R_2 without significant loss of generality. Denote by D_2 the random propagation distance to the last communicating vehicle on road R_2, and $E[D_2]$ and $V[D_2]$ its mean and variance respectively. We also assume that road R_2 has higher vehicle density in order for better approximation, the reason for which will be explained later.

There are two cases based on road separation relative to the transmission range, both bringing challenges to modeling the Bernoulli process. We explain each of the two below.

3.1. Case I

Suppose on road R_2, vehicle A is the transmitting vehicle, and the immediate next gap with vehicle C is larger than L as in Fig. 2. The key here is the probability of propagating information to vehicle C on road R_2 by taking a detour on road R_1. Suppose G and H are the furthest points directly reachable by vehicle A on both roads. $ABGH$ is the according parallelogram. Fig. 3 shows the two cases: the left of it has point B within the transmission range of vehicle A. This represents Case I. If point B of the parallelogram is outside vehicle A's transmission region, one gets Case II as in the right of Fig. 3. In Case II, it is not possible to have a parallelogram $ABGH$ within the transmission range of A, and point B will be out of reach by the vehicle at point A, as in Fig. 3. For convenience in the later part for Case II, we define point B as the leftmost reachable point on road R_1 by A, in which case $ABGH$ will not be a parallelogram.

Case I has the following relationship: $d \leqslant \frac{\sqrt{3}}{2}L$. Similarly, Case II corresponds to the situation in which $d > \frac{\sqrt{3}}{2}L$.

Here $d = \frac{\sqrt{3}}{2}L$ is the critical point separating Case I and Case II. At this value, the parallelogram $ABGH$ has exactly three points BGH on the circle of radius L about point A. Increasing the separation distance d makes point B fall outside the circle, implying Case II, while decreasing the distance d draws point B to within the circle, implying Case I.

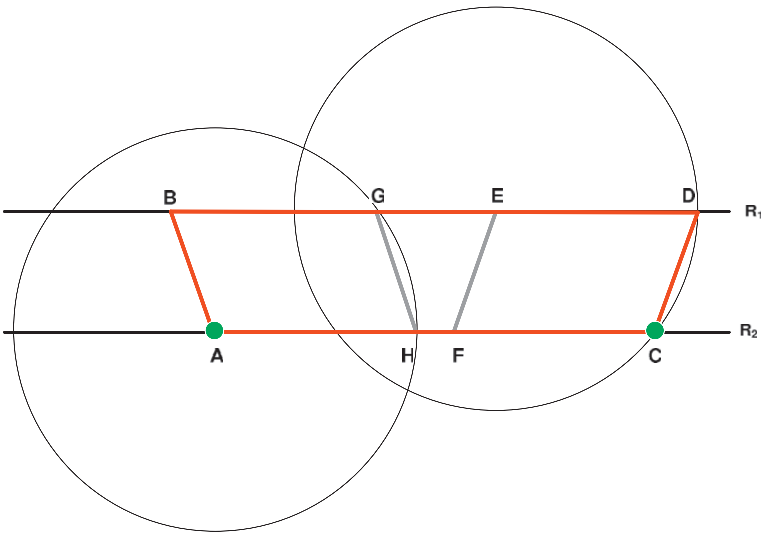

Fig. 2. A Bernoulli region $ABDC$ with $AG = AH = ED = EC = L$.

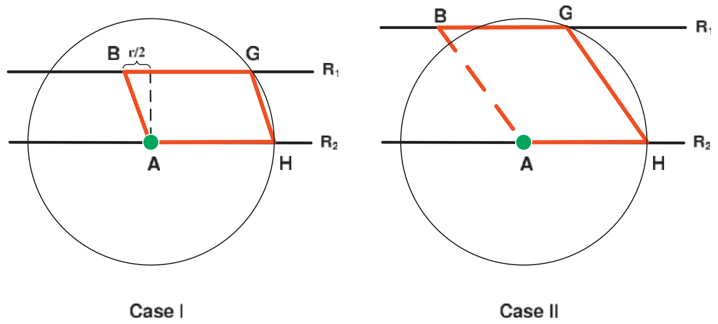

Fig. 3. Two cases of point B on road R_1 with $AG = AH = L$.

We first discuss Case I.

As in Fig. 2, we assume that E is the leftmost point on road R_1 that can reach vehicle C on road R_2. Let $ED = EC = FC = L$, and $CDEF$ is the according parallelogram. We further define $r = 2(L - \sqrt{L^2 - d^2})$, representing two times the horizontal *shift* of the two horizontal sides of each parallelogram. Note that r is an important parameter in this study as it indicates the extent to which vehicular interaction takes place between the two roads. In addition, r has other implications. For example, r represents the size of overlap between two consecutive Bernoulli regions as in Fig. 4, which will be explained later.

It is clear that vehicles left of the parallelogram $ABGH$ do not matter in further propagating information rightward from vehicle A: any vehicle left of point B on road R_1 would have to resort to vehicles within BG for further propagation. In addition, we have identified two types of *transmission regions*: Type I such as $ABGH$ and Type II such as $EFCD$ as in Fig. 2. Note that $EFCD$ corresponds to a transmitting vehicle at location E. Similarly, we call $ABGH$ the transmission region of vehicle A. The immediate implication of a transmission region of each type is that information does not propagate further if no vehicle is present in the region of an according vehicle. Developing exact propagation method will have to resort to the transmission regions, which is explained in details in Wang et al. (2010b). This paper, however, develops approximate methods using this concept of transmission region. An advantage of approximation is its ease of use.

In Fig. 2, $ABDC$ accords to a vehicle gap AC on road R_2, and is defined by two transmission regions: a type II region of vehicle A and a type I region that has vehicle C as its rightmost point on road R_2. We call the region $ABDC$ the *Bernoulli region* of vehicle A. Be aware that BD has the length of $AC + r$ as indicated in Fig. 4. Any vehicle on road R_2 has a Bernoulli region with according points B and D defined above.

The following becomes obvious.

Proposition 1. *Information propagation along road R_2 is gapped out at a vehicle location A if and only if the associated Bernoulli region has a vehicle gap larger than L on both roads.*

We provide a brief proof. If the vehicle gap following vehicle A on road R_2 is not larger than L, the propagation does not terminate. Consider the situation on road R_1 given that the gap AC on road R_2 is larger than L, as in Fig. 2. If and only if there is a gap larger than L within BD can information from A be prevented from a detour through R_1 to vehicle C. □

Clearly, information propagation gaps out if and only if the process fails to get through a Bernoulli region. Each vehicle on road R_2 has an according Bernoulli region with a probability of success. The vehicle gaps on each road are *i.i.d*. Therefore, one may roughly treat each Bernoulli region as having an identical probability of success. Propagating through vehicles on road R_2 can therefore be considered as a Bernoulli process whose 'trials' are the Bernoulli regions. The success means successful propagation through the Bernoulli region and failure means otherwise. The gapout probability associated with a Bernoulli region is denoted by p_r. In this paper, we assess the value of p_r by assuming independent Bernoulli regions. This assessment of p_r could be fairly accurate but not exact, as explained below. Therefore, we call the Bernoulli process here an *approximate* Bernoulli process.

This treatment using the Bernoulli process is not accurate because the probability of gapout at a 'trial' could be slightly correlated to the probability of no gapout at its proceeding trial. The Bernoulli regions associated with two consecutive vehi-

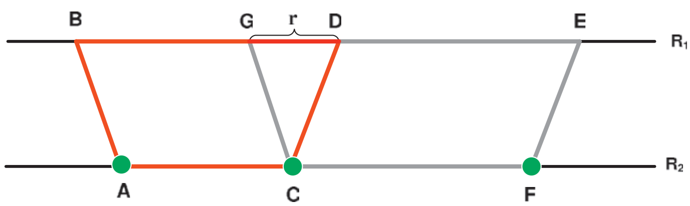

Fig. 4. Overlap between two consecutive Bernoulli regions.

cles on road R_2 have an overlap of length r on road R_1 as in Fig. 4. When two consecutive gaps larger than L are present on road R_2, not gapping out in the first Bernoulli region might indicate, to a certain degree, vehicle presence in the overlap section on road R_1.

Therefore, the gapout probability at a Bernoulli region might not be independent of the proceeding region. We believe this correlation can be technically measured and explicitly considered in modeling the process. However, we believe this correlation is small enough. Our later numerical tests strongly support the conviction of this negligible weak correlation in practically meaningful cases. And the approximation based on the independence assumption generally works well. Choosing the larger density on road R_2 helps reduce the likelihood of having two consecutive larger-than-L gaps on road R_2, and therefore reduces the effect from this correlation.

According to the Bernoulli process, the random number of gaps on R_2 (or, Bernoulli regions) until the first gapout follows a Geometric distribution, whose mean is $\frac{1}{p_r}$. In other words, if the number of gaps before a gapout is denoted by N where $N \geqslant 0$, $N + 1$ has a Geometric distribution. Therefore $E[N] = \frac{1}{p_r} - 1$. In order to get the expected propagation distance on road R_2, we need to calculate the expected length of the gap before a gapout. Note that there could have been gaps larger than L on road R_2 but that the according Bernoulli regions do not fail.

As vehicle presence on both roads are independent of each other, the probability of gapout, p_r, is calculated as the product of two probabilities: one for a gap g larger than L on road R_2 and the other for a gap larger than L on road R_1 within the range $g + r$, where g is the vehicle gap on road R_2. Be aware that the Bernoulli region has a section on road R_1 larger than on road R_2 by r as indicated in Fig. 4. The probability of a gap larger than L on road R_1 within a range y, denoted by $p(y)$, can be calculated recursively as follows:

$$p(y) = \begin{cases} \int_0^L f_1(t)p(y-t)dt + 1 - F_1(L), & \text{when } y \geqslant L, \\ 0, & \text{when } y < L, \end{cases} \tag{1}$$

where $f_i(\cdot)$ is the probability density function of the headway on road i, whose cumulative function is $F_i(\cdot)$. For the gapout probability of a distance y on road R_1, Eq. (1) means that if no vehicle is present in $[0,L]$ when measured from the left, whose probability is $1 - F_1(L)$, there is a gapout; if a vehicle is present at distance t in $[0,L]$, the failure of detour on R_1 then depends on gapout over the remaining length $y - t$, the probability for which is therefore $f_1(t)p(y - t)dt$. Here t is the location of the first vehicle from the beginning of the Bernoulli region on road R_1. Fig. 5 illustrates this recursive relation.

Eq. (1) is accurate for exponential vehicle headway distributions, but is approximate for other distributions. As a result, the approximation errors are different in the subsequent numerical tests. Clearly, the failure (gapout) probability of a Bernoulli region is $p_r = \int_L^\infty f_2(t)p(t+r)dt$.

We denote the expected length of the *gaps* propagated on road R_2 by $E[g—S]$, where S represents a success event of a Bernoulli region. It can be calculated as follows:

$$E[g|S] = \frac{\int_0^L tf_2(t)dt + \int_L^\infty tf_2(t)(1 - p(t+r))dt}{1 - \int_L^\infty f_2(t)p(t+r)dt}. \tag{2}$$

The denominator is the success probability of a Bernoulli region. The probability density function $f_2(\cdot)$ in the numerator divided by the denominator leads to the conditional probability density function on no gapout. The first term in the numerator corresponds to the case of a gap smaller than L on R_2; and the second term refers to the case of a larger than L gap on road R_2 but of a success of the according Bernoulli region.

If we denote by $E[g]$ the expected headway on road R_2, Eq. (2) can be simplified as follows:

$$E[g|S] = \frac{E[g] - \int_L^\infty tf_2(t)p(t+r)dt}{1 - \int_L^\infty f_2(t)p(t+r)dt}. \tag{3}$$

Therefore, the expected propagation distance can be approximated by the following:

$$E[D_2] = E[g|S] \times \left(\frac{1}{p_r} - 1\right) = \frac{E[g|S](1-p_r)}{p_r} = \frac{E[g] - \int_L^\infty tf_2(t)p(t+r)dt}{p_r}. \tag{4}$$

In addition, by using the Bernoulli process, we are able to find the variance of propagation distance D_2, $V(D_2)$, as follows.

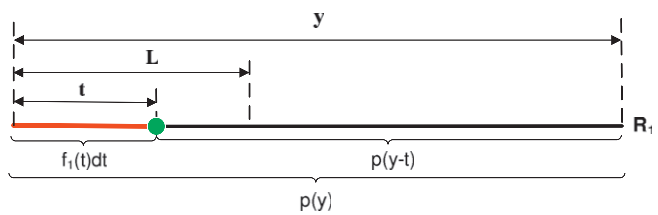

Fig. 5. Illustrate Eq. (1): conditional on next vehicle.

Use X_i for the ith vehicle gap indexed from the initial transmitting vehicle, and N for the number of vehicle gaps on road R_2 before gapout. Then we have

$$D_2 = \sum_{i=1}^{N} X_i. \tag{5}$$

All the X_i are assumed *i.i.d.* distributed with X.

Using Law of Total Variance, the variance of propagation distance may be expressed as in the following.

Proposition 2. *The mean and variance of information propagation distance on road R_2 may be expressed as follows.*

$$E[D_2] = \frac{E[g] - \int_L^\infty t f_2(t) p(t+r) dt}{\int_L^\infty f_2(t) p(t+r) dt},$$

and,

$$V(D_2) = V\left[E\left[\sum_{i=1}^{N} X_i | N\right]\right] + E\left[V\left(\sum_{i=1}^{N} X_i | N\right)\right] = V(NE[X]) + E[NV(X)] = E^2(X)V(N) + E[N]V(X), \tag{6}$$

where X has a probability density function as follows:

$$f(t) = \begin{cases} f_2(t)/(1-p_r), & \textit{for } t \leqslant L, \\ f_2(t)(1-p(t+r))/(1-p_r), & \textit{for } t > L. \end{cases} \tag{7}$$

With formula (7), we are able to numerically get $E[X]$ and $V(X)$. As $N + 1$, the number of Bernoulli trials, follows the Bernoulli process, we can easily find $E[N]$ and $V(N)$ respectively with properties of the Bernoulli process. Therefore, Eq. (6) can be evaluated with numerical method easily.

Two road case reduces to one road case

In the case $\lambda_1 = 0.0$, one can show that Eqs. (4) and (6) leads to the following results.

Proposition 3. *Information propagation along a single road has an expected propagation distance and variance as follows:*

$$E[D] = \frac{\int_0^L t f(t) dt}{1 - F(L)}, \text{ and}$$
$$V(D) = \frac{\int_0^L t^2 f(t) dt}{1 - F(L)} + (E[D])^2, \tag{8}$$

where f(·) and F(·) are the respective density and cumulative functions of headway with two roads combined, and D is the propagation distance.

Proof. We always have $p(t + r) = 1.0$ when $\lambda_1 = 0$. Eq. (4) gives $E[D_2] = \frac{\int_0^L t f_2(t) dt}{1 - F_2(L)}$. Here $D = D_2$. Note that $N + 1$ has a Geometric distribution. Therefore

$$E[N] = \frac{F(L)}{1 - F(L)}, \quad \text{and} \quad V(N) = \frac{FL}{(1 - F(L))^2}.$$

In addition, according to Eq. (6), we have

$$V(D) = E^2[X]V(N) + E[N]V[X] = E[X^2]E[N] + E^2[X](V(N) - E[N]) = \frac{\int_0^L t^2 f(t) dt}{F(L)} \cdot \frac{F(L)}{1 - F(L)} + \left(\frac{\int_0^L t f(t) dt}{F(L)}\right)^2 \cdot \frac{F^2(L)}{(1 - F(L))^2}$$
$$= \frac{\int_0^L t^2 f(t) dt}{1 - F(L)} + (E[D])^2.$$

This finishes the proof. □

Proposition 3 was developed in Wang et al. (2010a). Here by a Bernoulli process, we have provided an alternative proof.

As another special case, when the two road separation is zero, e.g. $d = 0.0$, the Bernoulli process presents an accurate method for information propagation. This implies that the vehicles on one single road may be divided into two groups, a virtual case of two roads, with one group helping the other. Regardless of ways of traffic division, the final result on propagation distance shall be identical.

3.2. Case II

As said earlier, Case II satisfies such a condition: $d > \frac{\sqrt{3}}{2} L$.

In Fig. 6, the gap between two consecutive vehicles A and C is larger than L on road R_2 as we are interested in the gapout probability. B is the left most point on road R_1 that the vehicle at point A on road R_2 can reach within a transmission range L, i.e., $AB = L$. E is the furthest point on road R_1 horizontally left of point C that is able to directly reach point C, i.e., $EC = L$. D is the furthest reach to the right directly from point E on road R_1, i.e., $ED = L$. In addition, G is the rightmost point on road R_1 directly reachable by vehicle A, and clearly $BG = 2\sqrt{L^2 - d^2} \neq r$. F is the leftmost point reachable on road R_2 by a vehicle at point E with $CC_1 = BG$. E_1 is the leftmost point on road R_1 to the right of vehicle C horizontally that can reach vehicle C. C_1 is the rightmost point reachable on R_2 by point E_1. C_2 on road R_2 corresponds to point D on road R_1. We have $CC_2 = L$. Obviously, $BE = AC$ and $BD = AC + L$.

We still refer to $ABDC$ as a Bernoulli region, although with slight notational abuse about points B and D defined earlier. Note that the Bernoulli region in Case I has $BD = AC + r$, which is different from Case II due to different situations. In order to take a detour on road R_1 to C on R_2, there are two necessary conditions. First, there must be vehicle presence within the range $[B,G]$ on road R_1 if otherwise, no vehicle within the section of a length $L–BG$ left of B is capable of propagating further till beyond G. This first condition implies ignorance of information coming from left of point B on road R_1. Second, vehicles cannot gapout from the first vehicle in the section $[B,G]$ till point D in order to assist the propagation. Note that vehicles have probabilistic presence on road R_1, given a vehicle gap AC on road R_2.

In other words, a vehicle within the segment $[B,G]$ has to propagate information to beyond point E, which has two cases.

- Information is propagated to a vehicle within the range $[E,E_1]$. In this case, information is able to reach vehicle C on road R_1.
- There is no vehicle within the range $[E,E_1]$. But information is propagated to a vehicle within the range $[E_1,D]$. we consider this case as not gapping out on road R_2 (approximately). Note that in this case, the vehicle at location C may actually be skipped.

Note that ED has a length L. The above discussions conclude that *there should be no gap of L or larger over the distance BD and section BG must have vehicle presence in order for information to detour to the vehicle at point C*.

As a matter of fact, information getting through BD could have come from the vehicle at point A or from vehicles prior to location B on road R_1. For manoeuvrability, we simplify the process by ignoring the information that could have come from vehicles on road R_1 left of point B. The effect of this ignorance is minimized by choosing the larger vehicle density for road R_2.

The above discussions simplify the process by assuming that information getting through to a vehicle at point within E_1D successfully propagates to the vehicle at point C at a probability 1.00. This assumption could partially compensate for the underestimate of information getting through BG by ignoring that coming from left of point B, as in Fig. 6.

Therefore, the probability of getting through R_1 from vehicle A to C is approximated as follows by slightly modifying Eq. (1), using the fact that $BG = 2\sqrt{L^2 - d^2}$ and $BD = AC + L$.

$$p(y) = \begin{cases} \int_0^{2\sqrt{L^2-d^2}} f_1(t)p(y-t)dt + 1 - F_1(2\sqrt{L^2-d^2}), & \text{for } y \geqslant L, \\ 0, \text{for } y < L. \end{cases} \tag{9}$$

In calculation for Case II, the according new formulas in Proposition 2 are as follows:

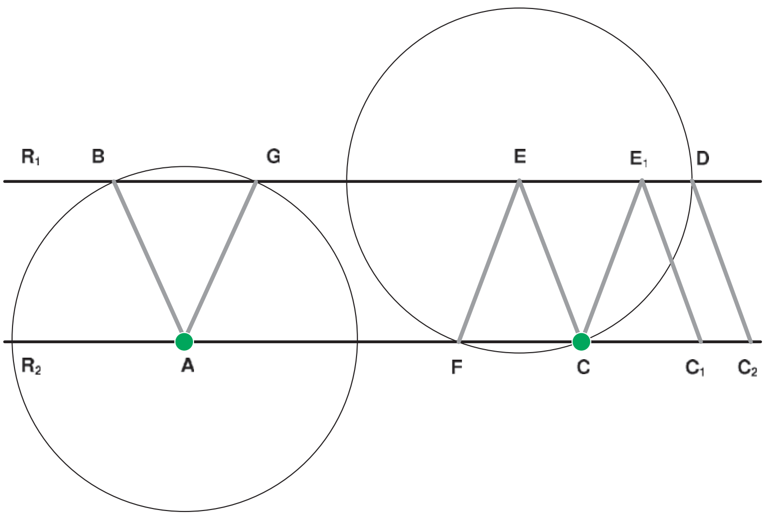

Fig. 6. An *approximate* Bernoulli region in Case II.

$$E[D_2] = \frac{\int_0^L tf_2(t)dt + \int_L^\infty tf_2(t)(1 - p(t+L))dt}{\int_L^\infty f_2(t)p(t+L)dt}.$$

and X has a probability density function as follows:

$$f(t) = \begin{cases} f_2(t)/(1-p_r), & \text{for } t \leqslant L, \\ f_2(t)(1-p(t+L))/(1-p_r), & \text{for } t \geqslant L. \end{cases}$$

and $p_r = \int_L^\infty f_2(t)p(t+L)dt$.

4. Numerical tests

The numerical tests are designed to show the propagation distance in relation to the system parameters such as transmission range, vehicle density and road separation distance.

A discrete numerical method was applied to the equations for the expectation and variance of successful propagation distance. A small step-length h is used for discretization to estimate the integrals of function $f(x)$, i.e., the integral $\int_0^a f(x)dx$ is approximated by $\sum_{i=0}^n f(ih)h$. When a is sufficiently large, for example, when a includes a vehicle headway at a probability of *almost* 1.0, one gets the integration $\int_0^\infty f(x)dx$. In our case, we take $a = 30L$. The $p(t)$, seen in Eqs. (1)–(8), can be estimated recursively by discretizing the integral and using the boundary value $p(0) = 0$.

An observation is worthy of a note. When a test instance is constructed, it only reflects the relativity of distance and vehicle density. The test instances constructed shall cover a range of the relative magnitude of parameters. We set the transmission range to be a standard unit 1. All the other lengths are measured against this unit, including the vehicle density.

We have tested both the case of Poisson vehicle distribution and other independent vehicle headway distributions using the formulas developed. We conduct 4000 times simulation for each instance to benchmark results from the analytical formulas.

4.1. Poisson distribution of vehicles on the roads

In testing the formulas for Case I, the density for road R_2 is set to be 0.2, 0.6, 1.0, 1.5 and 2.0 respectively, each corresponding to a set of lower densities on road R_1. Each pair of densities for both roads has a set of varying road separation distances of 0.1, 0.5 and 0.8 respectively. Except for a particular case with a road R_2 density of 0.2, the expectation and variance of propagation distance are highly accurate in all instances of Case I.

The test result for Case II shows high accuracy, but slightly poorer than in Case I. For each case in Case I, by increasing the road separation distance to 0.9, 0.94 and 0.98, we get corresponding instances in Case II. All the results are provided in Tables 3–7 in the Appendices.

The analytical approximation in case II is generally very close to the simulation results if the vehicle density on road 1 is smaller than that on road 2. However, we have a few cases in which both roads have high densities, which gives rise to large errors in the calculated numbers compared with simulation results, a distinct example of which is seen at $\lambda_1 = \lambda_2 = 2.0$ and $D = 0.98$. See Table 3 for details. This is most likely attributed to the violation of our assumptions in the development of the formulas for Case II. In Case II, the derivation implies that road 2 has a longer propagation distance. Based on this assumption, no vehicle left of point B on road 1 in Fig. 6 is able to transmit the information to vehicles beyond point B. Magnitude of errors in Case II may be interpreted as a result of the extent to which this assumption is violated. For example, when road separation distance gets closer to the critical value $\frac{\sqrt{3}}{2}$, the approximation errors in Case II become smaller because in this instance, the formulas for Case II depend less on the assumption. The reason for the less dependence is due to *ABDC* being closer to the Bernoulli region defined in Fig. 2.

4.2. Gamma headway distribution

We also test a select number of cases in which the vehicle headway follows Gamma distributions on both roads. We set μ_2 to be 2.0, and μ_1 being 1.2, 1.0. and 0.8 respectively, where μ_i is the mean headway on road R_i, $i = 1,2$. The road separation is 0.01, 0.1, 0.3, 0.5, 0.7, 0.8 respectively. In these cases, we set the variance of vehicle headway on road 2 is 0.5 times its mean, and road 1 has a variance of vehicle headway equal to its mean. Note that Gamma distribution becomes exponential when the mean and variance are equal. The test results are seen in Tables 8 and 9 in Appendix A.2. It appears that formulas in Case I generally provide satisfactory approximations. As a minor note, analytical variance tends to be slightly larger than from simulation. The difference between them increases with road separation.

4.3. Truncated Gaussian headway distribution

In addition, we test on traffic with headways following the truncated Gaussian distribution, whose density function is as follows:

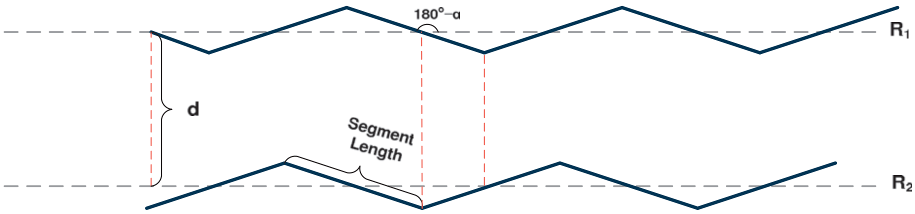

Fig. 7. Propagation along zigzag roads.

$$f(x;a) = \frac{\phi\left(\frac{x-\mu}{\sigma}\right)}{1-\Phi\left(\frac{a-\mu}{\sigma}\right)}, \ x \geqslant a,$$

where $\phi(\cdot)$ is Gaussian density function, $\Phi(\cdot)$ Gaussian cumulative function, a the truncation point, μ the mean and σ the standard deviation of Gaussian distribution. Here a can be set up to control the minimum headway. Generally, the smaller the μ reaches, the higher the density becomes. As earlier, L is set to 1.0. The coefficient of variation, ratio between standard deviation and mean, of the distribution is set to 0.8. The results of each instance are provided with analytical values followed by simulation values, as in Table 10 of Appendix A.3.

4.4. Information propagation along two zigzag roads

In this section, we apply our developed formulas to two parallel zigzag roads as illustrated in Fig. 7 to test the robustness of our roadway configuration.

The center lines of the two zigzag roads are apart from each other by a distance d. The actual zigzag roads have an angle α with the center line. We test a series of instances with α varying from 5° to 20°. Each zigzag section is set to be 0.8L, where L is the length of transmission range standardized to be 1.0. The headways on both roads follow Poisson distributions and the successful information propagation distance is measured horizontally between the initiating vehicle and last receiving one. In the Tables 11–14, λ_1 and λ_2 denote the densities on roads R_1 and R_2 (referred to as curve densities), respectively. The projected density is calculated on each road, called horizontal density, using the curve density divided by $cos(\alpha)$. In the tables, the first number in each cell is from our formulas using the horizontal densities followed by the second number from simulation.

The tests show that the model works well in estimation of both the expectation and variance, especially in Case I. We also tested using the curve densities in our formulas. It appears that if the curve densities instead of horizontal densities are used in our formulas as λ_1 and λ_2, the approximation errors of propagation distance are slightly larger, the details of which are not included here.

5. Probability distribution of successful propagation

An important measure is the probability distribution of the propagation distance. In light of the result in Wang (2007), we suspect that the probability distribution of propagation distance follows a Gamma type. Therefore, we construct for each case a Gamma distribution by setting the Gamma parameters in such a way that the resulting Gamma distribution has the same mean and variance as calculated with our formulas.

The simulated frequency distribution and the constructed Gamma distribution show good fit in general. Two examples are presented. In the first, the headway follows an exponential distribution; in the second, the headway follows a Gamma distribution.

Case of exponential headway

Table 1
Parameters for Gamma approximation to exponential headway.

Parameters for Poisson headway distribution	
λ_1 = 1.2	d = 0.5
λ_2 = 1.5	L = 1.0
Moments of successful propagation distance	
Theoretical mean = 3.1859	Simulation mean = 3.2927
Theoretical variance = 12.9437	Simulation variance = 13.8804
Parameters for Gamma approximation	
Mean of Gamma distribution = theoretical mean of propagation distance	
Variance of Gamma distribution = theoretical variance of propagation distance	

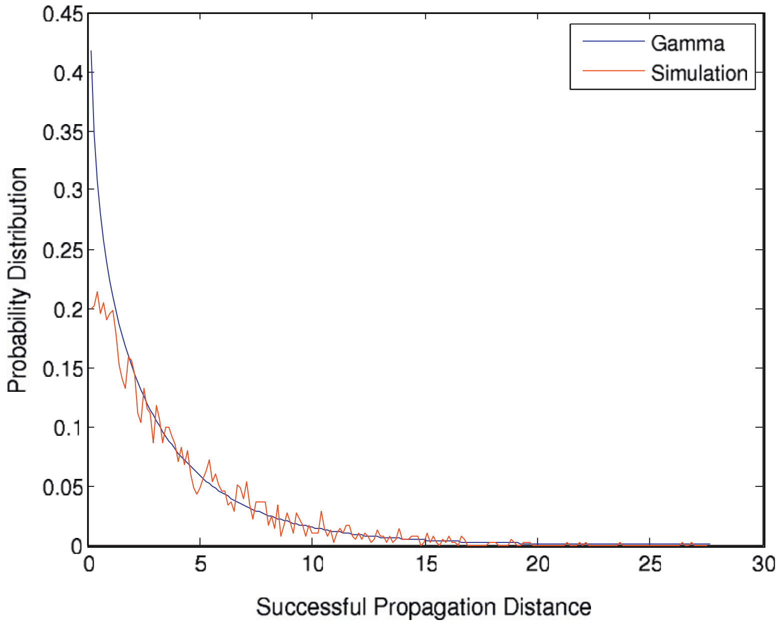

Fig. 8. Gamma approximation with exponential headway.

Table 2
Parameters for Gamma approximation.

Parameters for Gamma headway distribution	
Mean headway on $R_1 = 1.0$	$d = 1.0$
Mean headway on $R_2 = 1.0$	$L = 1.0$
Moments of successful propagation distance	
Theoretical mean = 1.9108	Simulation mean = 1.8816
Theoretical variance = 5.5412	Simulation variance = 5.3965
Parameters for Gamma approximation	
Mean of Gamma distribution = theoretical mean of propagation distance	
Variance of Gamma distribution = theoretical variance of propagation distance	

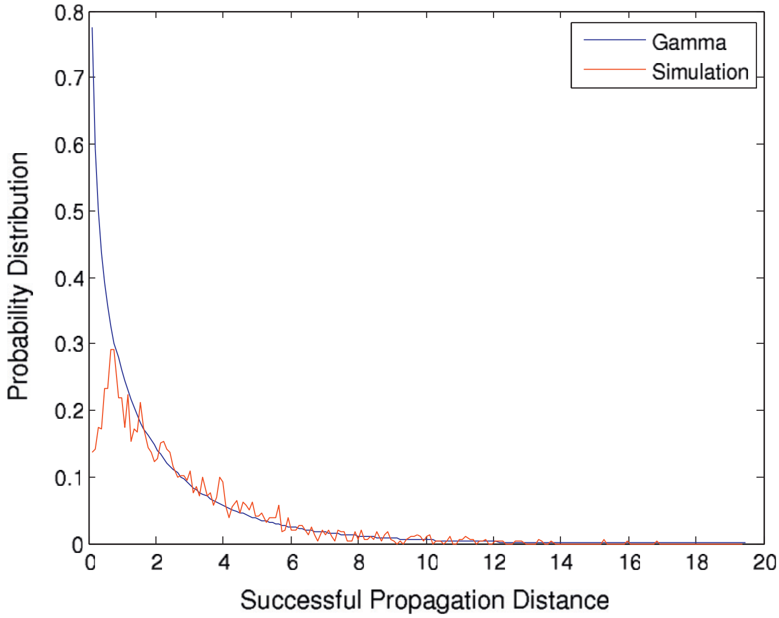

Fig. 9. Gamma approximation when headway has a gamma distribution.

Table 3
Instances of exponential headway at $\lambda_2 = 2.0$.

d		Case I			Case II		
		0.1	0.5	0.8	0.9	0.94	0.98
$\lambda_2 = 2.0$							
$\lambda_1 = 0.2$	E	2.6163	2.4660	2.2576	2.2136	2.2009	2.2576
		2.6291	2.4815	2.1707	2.1509	2.3129	2.2393
	V	8.3862	7.4931	6.3406	6.1096	6.0429	6.0349
		8.3771	7.6324	6.2323	5.8723	6.9114	6.2274
$\lambda_1 = 0.4$	E	3.1113	2.8018	2.3877	2.2633	2.2134	2.2009
		3.0062	2.6694	2.4599	2.2434	2.2808	2.1916
	V	11.6432	9.5446	7.0530	6.3727	6.1081	6.0438
		10.8335	9.1231	7.2428	6.4460	6.1823	6.5348
$\lambda_1 = 0.6$	E	3.6960	3.2129	2.5862	2.3517	2.2398	2.2029
		3.7140	3.1669	2.6131	2.4177	2.3431	2.2929
	V	16.1372	12.3752	8.2113	6.8543	6.2467	6.0544
		17.1476	11.8683	8.1146	7.8916	6.4712	6.2594
$\lambda_1 = 0.8$	E	4.3863	3.7093	2.8551	2.4821	2.2834	2.2054
		4.3901	3.7136	2.8347	2.5968	2.4435	2.3348
	V	22.3380	16.2562	9.9120	7.5969	6.4797	6.0681
		22.4843	16.0576	9.7043	8.3828	7.4897	7.3776
$\lambda_1 = 1.0$	E	5.2010	4.3036	3.1991	2.6579	2.3468	2.2090
		5.1607	4.2211	3.1708	2.7975	2.6367	2.5097
	V	30.8972	21.5632	12.3064	8.6551	6.8265	6.0867
		30.9821	19.6625	12.2637	9.6766	8.8439	7.9201
$\lambda_1 = 1.2$	E	6.1623	5.0113	3.6258	2.8827	2.4323	2.2138
		6.1833	4.9362	3.6066	3.1974	3.0978	2.6989
	V	42.7192	28.8155	15.6130	10.1024	7.3078	6.1126
		39.9055	29.6488	15.7733	12.8741	11.9535	9.1578
$\lambda_1 = 1.4$	E	7.2966	5.8511	4.1455	3.1609	2.5418	2.2206
		7.0643	5.6454	4.1856	3.4504	3.2973	2.9280
	V	59.0621	38.7316	20.1396	12.0378	7.9472	6.1480
		58.6426	38.3815	20.6432	14.7272	14.1258	11.3427
$\lambda_1 = 1.6$	E	8.6356	6.8452	4.7712	3.4979	2.6772	2.2296
		8.6578	6.8334	4.6095	4.1060	3.7288	3.3272
	V	81.6785	52.3054	26.3137	14.5944	8.7735	6.1959
		86.7528	52.5455	26.1259	20.6484	18.1198	15.8284
$\lambda_1 = 1.8$	E	10.2167	8.0206	5.5190	3.9003	2.8407	2.2415
		10.0141	8.0166	5.5345	4.6526	4.3466	3.6814
	V	113.0139	70.9145	34.7279	17.9493	9.8216	6.2589
		105.0044	69.7134	34.2559	26.4321	22.4226	17.2950
$\lambda_1 = 2.0$	E	12.0844	9.4093	6.4087	4.3760	3.0344	2.2567
		11.9310	9.4989	6.4212	5.2115	4.9744	4.1341
	V	156.4857	96.4719	46.2018	22.3378	11.1355	6.3401
		148.8270	94.0844	48.9465	33.7764	30.0973	22.5572

In this case, the headway on each road is assumed to follow an exponential distribution. The parameters are shown in Table 1. And the results are shown in Fig. 8 where the approximation Gamma function is denoted by the blue (smooth) line,[1] and the simulation result frequency distribution by the red (zigzag) line.

Case of headway with Gamma distribution

The headway on each road is assumed to follow a Gamma distribution to make a more general case, and the variance of headway on road R_2 equals 0.5 times its mean, and the variance of headway on road R_1 equals its mean. The values of parameters can be found in Table 2. The results are shown in Fig. 9 where the Gamma distribution is denoted by blue line and the simulation by red line.

6. Conclusion

Information propagation along traffic streams through inter-vehicle communication is an important process in the VANETs. We study a special case in which a network of two parallel highways is present as a step towards addressing information propagation on a discrete network.

[1] For interpretation of color in Figs. 1–9, the reader is referred to the web version of this article.

In this paper, we develop an approximate method based on the Bernoulli process to characterize the process of information propagation in terms of their expected value and variance. We use simulation to evaluate the quality of this approxi-

Table 4
Instances of exponential headway at $\lambda_2 = 1.5$.

d		Case I			Case II		
		0.1	0.5	0.8	0.9	0.94	0.98
$\lambda_2 = 1.5$							
$\lambda_1 = 0.2$	E	1.5909	1.4914	1.3601	1.3330	1.3248	1.3241
		1.5857	1.4734	1.3512	1.3444	1.2504	1.3641
	V	3.5498	3.1431	2.6463	2.5492	2.5197	2.5177
		3.5836	3.0704	2.6561	2.4956	2.3066	2.6697
$\lambda_1 = 0.4$	E	1.9102	1.7079	1.4448	1.3650	1.3326	1.3254
		1.8504	1.7163	1.4866	1.3631	1.3638	1.3455
	V	4.9905	4.0507	2.9668	2.6661	2.5475	2.5229
		4.9656	4.0167	3.0003	2.4637	2.7275	2.5846
$\lambda_1 = 0.6$	E	2.2898	1.9777	1.5776	1.4231	1.3493	1.3269
		2.2175	1.9910	1.6233	1.4202	1.4391	1.3648
	V	6.9873	5.3281	3.5051	2.8860	2.6080	2.5289
		6.7457	5.4012	3.7979	2.8366	2.9953	2.7851
$\lambda_1 = 0.8$	E	2.7401	2.3078	1.7609	1.5108	1.3775	1.3288
		2.6796	2.2873	1.7607	1.5999	1.5299	1.4206
	V	9.7504	7.1047	4.3145	3.2334	2.7119	2.5362
		9.6620	6.8648	4.5104	3.5000	3.3558	2.8838
$\lambda_1 = 1.0$	E	3.2738	2.7070	1.9990	1.6308	1.4192	1.3313
		3.3438	2.6597	2.0260	1.7561	1.6595	1.4942
	V	13.5699	9.8603	5.4757	3.7386	2.8696	2.5457
		14.6653	9.3438	5.8894	4.7211	4.0426	3.1665
$\lambda_1 = 1.2$	E	3.9053	3.1859	2.2977	1.7864	1.4767	1.3347
		3.7922	3.2927	2.3335	2.0241	1.8102	1.6269
	V	18.8475	12.9437	7.1038	4.4418	3.0924	2.5583
		17.5766	13.8804	7.7589	5.7811	5.0603	3.9668
$\lambda_1 = 1.4$	E	4.6522	3.7593	2.6647	1.9812	1.5500	1.3392
		4.5355	3.6857	2.6114	2.3179	2.0486	1.8385
	V	26.1395	17.5985	9.3606	5.3964	3.3931	2.5752
		24.7830	16.5469	8.9771	7.9980	6.1810	5.4124

Table 5
Instances of exponential headway at $\lambda_2 = 1.0$.

d		Case I			Case II		
		0.1	0.5	0.8	0.9	0.94	0.98
$\lambda_2 = 1.0$							
$\lambda_1 = 0.2$	E	0.8789	0.8174	0.7407	0.7253	0.7203	0.7203
		0.8809	0.8219	0.7308	0.7316	0.6976	0.7084
	V	1.3878	1.2109	1.0085	0.9707	0.9575	0.9583
		1.4658	1.2166	0.9456	0.9976	0.9541	0.9878
$\lambda_1 = 0.4$	E	1.0723	0.9488	0.7929	0.7448	0.7249	0.7214
		1.0714	0.9657	0.8019	0.7526	0.7396	0.7266
	V	2.0005	1.5995	1.1496	1.0219	0.9690	0.9623
		1.9719	1.7040	1.1348	1.0275	1.0360	1.0293
$\lambda_1 = 0.6$	E	1.3057	1.1175	0.8781	0.7814	0.7348	0.7227
		1.2590	1.1373	0.8873	0.8314	0.8244	0.7585
	V	2.8618	2.1661	1.3994	1.1218	0.9947	0.9667
		2.6795	2.2642	1.4475	1.3145	1.2957	1.0434
$\lambda_1 = 0.8$	E	1.5863	1.3288	0.9993	0.8380	0.7521	0.7243
		1.5416	1.3634	0.9877	0.9298	0.8625	0.7557
	V	4.0670	2.9756	1.7909	1.2856	1.0402	0.9718
		3.8586	3.1107	1.7473	1.6350	1.3956	1.1332
$\lambda_1 = 1.0$	E	1.9226	1.5890	1.1609	0.9177	0.7781	0.7262
		1.9858	1.5901	1.0878	0.9856	0.9231	0.8432
	V	5.7473	4.1179	2.3717	1.5320	1.1112	0.9778
		6.0198	4.1090	2.2898	1.8237	1.6483	1.3757

Table 6
Instances of exponential headway at $\lambda_2 = 0.6$.

d		Case I			Case II		
		0.1	0.5	0.8	0.9	0.94	0.98
$\lambda_2 = 0.6$							
$\lambda_1 = 0.2$	E	0.4616	0.4256	0.3826	0.3746	0.3714	0.3718
		0.4625	0.4349	0.3895	0.3808	0.3999	0.3742
	V	0.5626	0.4822	0.3952	0.3817	0.3736	0.3768
		0.5507	0.5260	0.4185	0.3912	0.4303	0.3642
$\lambda_1 = 0.4$	E	0.5746	0.5027	0.4139	0.3864	0.3740	0.3732
		0.5752	0.4833	0.4115	0.3907	0.3795	0.3908
	V	0.8412	0.6615	0.4620	0.4084	0.3795	0.3821
		0.5266	0.6441	0.5065	0.4147	0.4103	0.3934
$\lambda_1 = 0.6$	E	0.7142	0.6056	0.4673	0.4091	0.3799	0.3746
		0.7300	0.5955	0.4662	0.4205	0.4057	0.3754
	V	1.2436	0.9366	0.5912	0.4605	0.3921	0.3877
		1.2706	0.8762	0.6109	0.5146	0.4454	0.3619

Table 7
Instances of exponential headway at $\lambda_2 = 0.2$.

d		Case I			Case II		
		0.1	0.5	0.8	0.9	0.94	0.98
$\lambda_2 = 0.2$							
$\lambda_1 = 0.2$	E	0.2462	0.2380	0.2338	0.2370	0.2344	0.2362
		0.1366	0.1201	0.1125	0.1097	0.1113	0.1041
	V	0.1337	0.1118	0.0894	0.1053	0.0872	0.1058
		0.1369	0.1007	0.0874	0.0827	0.0829	0.0789

Table 8
Configuration of Gamma distribution.

On road 1	Mean of Gamma distribution: μ_1 Variance of Gamma distribution: μ_1
On road 2	Mean of Gamma distribution: μ_2 Variance of Gamma distribution: $0.5\mu_2$

Table 9
Analytical and simulation results.

d		0.01	0.1	0.3	0.5	0.7	0.8
$\mu_2 = 1.0$							
$\mu_1 = 1.2$	E	2.0091	1.9950	1.8726	1.6380	1.3361	1.1807
		2.0073	1.9935	1.8924	1.6563	1.3195	1.1819
	V	6.0299	5.9556	5.3284	4.2145	2.9533	2.3791
		6.3135	5.7853	5.6661	4.1474	3.0711	2.3346
$\mu_1 = 1.5$	E	1.6163	1.6072	1.5260	1.3611	1.1380	1.0235
		1.7259	1.6868	1.6146	1.3781	1.1591	1.0294
	V	4.1071	4.0663	3.7099	3.0324	2.2139	1.8365
		4.8706	4.6104	4.4811	3.2178	2.3368	1.8945
$\mu_1 = 1.8$	E	1.3556	1.3496	1.2956	1.1803	1.0154	0.9303
		1.4862	1.4275	1.4163	1.2126	0.9954	0.9105
	V	3.0120	2.9883	2.7773	2.3519	1.8012	1.5426
		3.8272	3.4366	3.3335	2.5664	1.8260	1.6399

mation. According to simulation, our developed formulas are highly accurate in almost all the cases. The developed formulas are also robust, especially in Case I with a road separation distance below $\frac{\sqrt{3}}{2}$ times the transmission range, e.g. $d < \frac{\sqrt{3}}{2}L$. The derivation in Case I is accurate for Poisson vehicle distribution, except for dependence on a weak assumption of independent Bernoulli regions. This weak assumption is *roughly* satisfied in almost all the cases if the higher density of the two roads is on road R_2. The numerical tests indicate that the correlation between Bernoulli trials is normally negligible. Furthermore, when other vehicle headway distributions are applied, the formulas show great robustness and still yield results of high accuracy. An interesting result is that the formulas proposed in Case I take one road as a special case.

The proposed formulas for Case II in which $d > \frac{\sqrt{3}}{2}L$ also perform well except for a very small number of instances which have two comparable high densities for both roads and the road separation distance is large.

Our numerical test also shows an encouraging fit of Gamma distribution to the propagation distance distribution. The Gamma curve is defined by the calculated mean and variance through our proposed formulas. Another interesting observation is that the ratio between the mean and standard deviation of propagation distance is close to 1.0 in cases of large vehicle densities, an evidence of the two road case to support a conjecture in Wang (2007) that an exponential distribution is the limiting distribution for propagation distance.

Note that the presence probability of vehicles may be taken as the final probability after considering hopping failure, market penetration, signal conflict and other factors. Users need to calibrate the probability according to practice. In addition, when significant transmission delay takes place, explicit consideration of multiple lanes on each road would be of interest.

Table 10
Instances of truncated Gaussian headway.

d		Case I			Case II		
		0.2	0.5	0.8	0.9	0.94	0.98
μ_2 = 0.5, a = 0.01							
μ_1 = 4.0	E	4.0664	3.9808	3.7870	3.5697	3.6107	3.6720
		4.3995	4.2121	3.8474	3.7832	3.7621	3.6763
	V	19.1593	18.4042	16.7506	14.9849	15.3117	15.8053
		23.8971	23.0550	17.1167	15.4219	16.3400	16.0258
μ_1 = 3.0	E	4.2193	4.0975	3.8167	3.4963	3.5556	3.6446
		4.5921	4.4444	3.7313	3.6975	3.6791	3.7320
	V	20.5435	19.4358	16.9992	14.4097	14.8738	15.5836
		23.9911	23.0739	16.0244	15.9636	16.6468	16.4426
μ_2 = 1.0, a = 0.01							
μ_1 = 3.0	E	0.5395	0.5046	0.4535	0.3362	0.3638	0.4020
		0.5732	0.5185	0.4518	0.4178	0.4334	0.4471
	V	0.7169	0.6353	0.5268	0.2559	0.3186	0.4072
		0.8505	0.6933	0.5448	0.4534	0.4773	0.5318
μ_1 = 2.0	E	0.6247	0.5634	0.4707	0.2595	0.3088	0.3763
		0.6858	0.5716	0.4660	0.4461	0.4360	0.4557
	V	0.9238	0.7701	0.5633	0.0896	0.1961	0.3473
		1.2108	0.8508	0.5561	0.5358	0.5116	0.5521

Table 11
Instances of exponential headway on zigzag roads for λ_1 = 0.5.

d		Case I			Case II
		0.3	0.5	0.7	0.9
λ_1 = 0.5, λ_2 = 2.0					
α = 5	E	3.2825	3.0227	2.6762	2.3201
		3.3259	3.0508	2.6980	2.3797
	V	12.8769	11.0139	8.7469	6.6739
		12.8059	11.2535	8.9341	6.7402
α = 10	E	3.3689	3.1007	2.7431	2.3743
		3.3664	3.0594	2.7132	2.3175
	V	13.5001	11.5346	9.1449	6.9555
		13.7026	11.6453	9.1897	6.5161
α = 15	E	3.5213	3.2382	2.8608	2.4696
		3.3509	3.1932	2.8206	2.5264
	V	14.6347	12.4809	9.8666	7.4638
		13.4040	12.1114	9.3242	7.8947
α = 20	E	3.7539	3.4479	3.0401	2.6141
		3.6461	3.3592	2.9912	2.6634
	V	16.4545	13.9953	11.0177	8.2688
		16.3080	13.5451	10.5167	8.3137

Appendix A

A.1. Poisson distribution of vehicles

In this section, the headway distribution is assumed exponential with parameters λ_1 and λ_2 on road R_1 and road R_2, respectively. Clearly, the two parameters reflect the traffic densities on the two roads, respectively. In the following tables, both analytical results based on the equations and simulation results are provided. In the tables that follow, d denotes the distance between the two roads, E the expected propagation distance, and V variance of the propagation distance. Furthermore, the numbers for E and V in each instance below start with the analytical value followed by its according simulation counterpart (see Tables 3–7).

A.2. Gamma distribution of the vehicle headway

In this section, the headway on each road is assumed to follow a Gamma distribution with the parameters shown in Table 8. The theoretical and simulation results are shown in Table 9.

A.3. Truncated Gaussian distribution of the vehicle headway

Table 10.

Table 12
Instances of exponential headway on zigzag roads for λ_1 = 0.7.

d		Case I			Case II
		0.3	0.5	0.7	0.9
λ_1 = 0.7, λ_2 = 2.0					
α = 5	*E*	3.8584	3.4806	2.9883	2.4306
		3.8205	3.4192	2.9260	2.5679
	V	17.5000	14.3955	10.7881	7.2928
		16.7599	14.0251	10.3331	8.1068
α = 10	*E*	3.9664	3.5758	3.0673	2.4898
		3.9288	3.5479	3.0228	2.5199
	V	18.4081	15.1225	11.3109	7.6142
		18.0274	15.2847	11.3196	8.5763
α = 15	*E*	4.1572	3.7440	3.2068	2.5938
		4.0655	3.5859	3.1493	2.5967
	V	20.0693	16.4498	12.2629	8.1962
		19.5763	15.0090	12.5503	8.3987
α = 20	*E*	4.4496	4.0015	3.4199	2.7521
		4.3199	3.8764	3.2776	2.9147
	V	22.7546	18.5891	13.7913	9.1223
		21.7084	17.3916	13.4926	9.7362

Table 13
Instances of exponential headway on zigzag roads for λ_1 = 1.0.

d		Case I			Case II
		0.3	0.5	0.7	0.9
λ_1 = 1.0, λ_2 = 2.0					
α = 5	*E*	4.9280	4.3455	3.6086	2.6806
		4.8781	4.3954	3.5772	2.8290
	V	27.8731	21.9508	15.4408	8.7889
		27.2648	23.4424	14.8826	9.3860
α = 10	*E*	5.0783	4.4749	3.7125	2.7508
		4.9275	4.4419	3.7132	2.9614
	V	29.4708	23.1713	16.2638	9.2081
		26.7850	22.0993	16.4029	11.0786
α = 15	*E*	5.3451	4.7042	3.8966	2.8745
		5.2089	4.6258	3.7804	3.1169
	V	32.4157	25.4158	17.7728	9.9713
		30.5780	24.7879	16.7966	11.8781
α = 20	*E*	5.7563	5.0573	4.1793	3.0636
		5.6613	4.9242	3.9747	3.1638
	V	37.2323	29.0738	20.2211	11.1960
		35.5937	27.7942	17.8093	11.2393

Table 14
Instances of exponential headway on zigzag roads for $\lambda_1 = 1.5$.

d		Case I			Case II
		0.3	0.5	0.7	0.9
$\lambda_1 = 1.5$, $\lambda_2 = 2.0$					
$\alpha = 5$	E	7.4322	6.3973	5.1373	3.3540
		7.3142	6.2269	5.1114	3.8299
	V	61.1840	45.9234	30.2431	13.4636
		63.5820	43.6087	28.4120	17.7021
$\alpha = 10$	E	7.6912	6.6145	5.3065	3.4539
		7.6225	6.3632	5.3606	3.8998
	V	65.2747	48.8942	32.1199	14.2038
		63.4701	46.7651	33.3066	18.7436
$\alpha = 15$	E	8.1539	7.0022	5.6079	3.6311
		7.9230	6.9048	5.5446	4.1859
	V	72.9145	54.4267	35.6033	15.5652
		66.3784	52.8162	33.9887	20.7906
$\alpha = 20$	E	8.8751	7.6049	6.0754	3.9042
		8.8628	7.3733	5.9583	4.4887
	V	85.6677	63.6214	41.3629	17.7853
		88.2312	60.8332	42.0374	22.6167

A.4. Information propagation for two zigzag roads

Tables 11–14.

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