PRIORITIZED-MVBA: A NEW APPROACH TO DESIGN AN OPTIMAL ASYNCHRONOUS BYZANTINE AGREEMENT PROTOCOL

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ABSTRACT

The multi-valued byzantine agreement protocol (MVBA) in the authenticated setting has been widely used as a core to design atomic broadcast and fault-tolerant state machine replication protocols in asynchronous networks. Originating from the seminal work of Cachin et al. [13], subsequent research endeavors have sought to optimize protocol efficiency in terms of communication complexity. Notable advancements following Cachin's contributions include: i) VABA [23], requiring multiple protocol instances to achieve agreement on a party's request, and ii) Dumbo-MVBA [29], employing a cryptographic asynchronous dispersal and recovery methods to manage communication complexity alongside additional computational and communication rounds overheads.

Our objective is to devise an MVBA protocol that achieves agreement in each instance without extra computation and communication rounds while maintaining the optimal metrics. Through meticulous research, we analyze the network/adversary's message distribution patterns and observe how a party's message reception varies with the number of request senders. We ascertain that opting for a subset of parties (f+1, ensuring the inclusion of at least one honest party, at most f number of parties can be faulty) rather than the entire set (n) can facilitate the design of a protocol meeting the aforementioned criteria.

Central to our design approach is the introduction of the committee in the classic MVBA protocol, wherein a randomly selected subset of (f+1), where n=3f+1 parties get selected and simultaneously broadcast their requests (transactions) to gather verifiable proofs. Successive distributions of these proofs afford us the necessary properties to employ the asynchronous binary Byzantine agreement (ABBA) protocol for reaching an agreement on a selected party's requests. By integrating the committee and ABBA protocols, we devise the optimal MVBA protocol, termed pMVBA (Prioritized-MVBA). This protocol exhibits resilience to tolerate up to $\lfloor \frac{n}{3} \rfloor$ Byzantine failures, with an expected runtime of O(1), optimal message complexity of $O(n^2)$, and optimal communication complexity $O((l+\lambda)n^2)$ (where n represents the number of parties, l denotes the bit length of the input, and λ signifies the security parameter).

Keywords Distributed Systems · Blockchain · Consensus Protocols

1 Introduction

Byzantine Agreement (BA) protocols are fundamental to implementing decentralized infrastructures, particularly in the context of blockchain and decentralized applications. These protocols ensure agreement among parties, even in the presence of malicious actors, making them crucial for maintaining the integrity and functionality of distributed systems. The renewed interest in BA protocols has been driven by the success of Bitcoin [35] and other decentralized applications [44], which rely heavily on these protocols for achieving agreement [14]. Despite the existence of practical BA protocols [1, 33, 5, 37, 41, 31], their reliance on time parameters for ensuring liveness poses challenges in designing agreement

protocols. The FLP impossibility result [32] demonstrates that deterministic BA protocols cannot guarantee agreement in asynchronous networks. This necessitates the development of efficient asynchronous Byzantine Agreement protocols [23, 42, 11, 5, 29, 10, 39, 45, 22] that can overcome the limitations of existing approaches, particularly in terms of communication complexity.

Achieving agreement in asynchronous networks efficiently remains a significant challenge. Classic Multi-Valued Byzantine Agreement (MVBA) protocol like Cachin's [13], while effective, suffer from high communication complexity, often involving terms like $O(n^3)$. This high complexity is impractical for large-scale systems, necessitating new approaches to reduce communication overhead while ensuring optimal resilience and correctness. Cachin et al. [13] introduced the concept of MVBA, where nodes agree on a value from a large input space rather than a binary decision. Cachin's MVBA protocol provides polynomial-time complexity but suffers from high communication overhead, with a complexity of $O(n^3)$. VABA [23] introduced a view-based approach to eliminate the $O(n^3)$ term, while Dumbo-MVBA [29] used erasure codes to handle large input sizes more efficiently, reducing the communication complexity to $O(ln + \lambda n^2)$. However, these optimizations often introduced additional rounds of communication and computation.

In this context, we propose a novel approach to optimizing MVBA protocols by introducing the Prioritized Multi-Valued Byzantine Agreement (pMVBA) protocol. Our goal is to achieve agreement in each instance without the need for extra computation and communication rounds, maintaining optimal performance metrics. The key innovation of our approach is the integration of a committee-based method within the classical MVBA framework, where a randomly selected subset of parties, rather than the entire set of parties, broadcast proposals.

The pMVBA protocol leverages the concept of a committee, wherein a subset of f+1 parties is chosen to broadcast their proposals. This approach ensures that at least one honest party is always included in the subset, enhancing the protocol's resilience. By dynamically selecting parties for each protocol instance, we mitigate the risk of adversarial attacks targeting specific nodes. This method significantly reduces the number of broadcast messages and the associated communication overhead. Central to our protocol is the integration of the Asynchronous Binary Byzantine Agreement (ABBA) protocol, which facilitates reaching an agreement on one of the proposals broadcast by the committee members. The ABBA protocol ensures that the selected parties' proposals are agreed upon with probability 1, maintaining the integrity and functionality of the distributed systems.

Our pMVBA protocol achieves several key improvements over existing MVBA protocols while exhibiting optimal resilience against Byzantine failures, with an expected runtime of O(1), optimal message complexity of $O(n^2)$, and optimal communication complexity of $O((l+\lambda)n^2)$, where n is the number of parties, l is the bit length of the input, and λ is the security parameter. The key improvements include removing the need for multiple instances of the protocol or extra rounds of messages and cryptographic computation to reach an agreement on a party's request while keeping the optimal communication complexity. These enhancements make the pMVBA protocol suitable for large-scale decentralized applications, addressing the scalability challenges faced by traditional MVBA protocols.

The main contributions of this paper are as follows:

- We introduce a novel committee selection protocol that ensures the presence of at least one honest party in the selected subset, enhancing the protocol's resilience against Byzantine failures.
- We propose the pMVBA protocol, which reduces the communication complexity from $O(n^3)$ to $O((l + \lambda)n^2)$ by dynamically selecting a subset of parties to broadcast their proposals.
- Our protocol integrates the ABBA protocol to achieve agreement with probability 1 on the proposals broadcast by committee members, ensuring robust agreement mechanisms.
- We provide a thorough theoretical analysis demonstrating that the pMVBA protocol achieves an expected constant asynchronous round count while maintaining optimal resilience and correctness.
- We present extensive case studies and performance evaluations, showing that the pMVBA protocol is effective
 and scalable for large-scale decentralized applications, outperforming traditional MVBA protocols in terms of
 communication overhead and efficiency.

2 Related Work

This section provides an overview of the Byzantine Agreement protocols and the key developments in the Byzantine Agreement protocols, focusing on four main areas: Byzantine Agreement Protocols in partially-synchronous network, MVBA Protocols in asynchronous network, Committee-Based Protocols, and Optimistic Fastlane Mechanisms to combine both the partially-synchronous network and the asynchronous network.

Byzantine Agreement Protocols. Byzantine Agreement (BA) protocols have long been a cornerstone of fault-tolerant distributed computing. The Byzantine Generals Problem, introduced by Lamport et al. [26], laid the theoretical

foundation for BA protocols, emphasizing the challenges of achieving an agreement in the presence of faulty or malicious nodes. Early BA protocols, such as those by Pease et al. [38], focused on synchronous networks where communication occurs in fixed rounds. However, these protocols often relied on time assumptions for ensuring liveness.

Practical Byzantine Fault Tolerance (PBFT), proposed by Castro and Liskov [15], was a significant advancement in making BA protocols practical for real-world applications. PBFT introduced an efficient consensus protocol for partially-synchronous networks, ensuring safety and liveness even with up to $f < \frac{n}{3}$ Byzantine faults. Several modern BA protocols build upon the principles of PBFT, including HotStuff [46] and Tendermint [12], optimizing various aspects such as communication complexity and responsiveness.

The FLP impossibility result [18] demonstrated that no deterministic BA protocol could guarantee an agreement in an asynchronous network with a single faulty node. This led to the exploration of probabilistic and randomized approaches for ABA. Ben-Or's protocol [8] was one of the earliest randomized BA protocols, using coin-flipping to break symmetry among nodes. Recent protocols like HoneyBadgerBFT [34] and Dumbo [28] have focused on optimizing ABA for high throughput and low latency.

Multi-Valued Byzantine Agreement Protocols. Cachin et al. [13] introduced the concept of MVBA, where nodes agree on a value from a large input space rather than a binary decision. Cachin's MVBA protocol provides polynomial-time complexity but suffers from high communication overhead, with a complexity of $O(\ln^2 + \lambda n^2 + n^3)$.

Subsequent works, such as VABA [23] and Dumbo-MVBA [29], aimed to reduce the communication complexity of MVBA. VABA introduced a view-based approach to eliminate the $O(n^3)$ term, while Dumbo-MVBA used erasure codes to handle large input sizes more efficiently, reducing the communication complexity to $O(ln + \lambda n^2)$. However, these optimizations often introduced additional rounds of communication and expensive cryptograhic computations.

Our focus on optimizing the classical MVBA protocol aims to address these challenges by reducing the number of broadcasting parties, thus lowering communication overhead while maintaining correctness and resilience.

Committee-Based Protocols. Committee-based protocols have emerged as a promising approach to reduce communication complexity in BA protocols. Algorand [20] introduces the concept of committee members, wherein a predefined set of parties collaborates to reach an agreement. Algorand operates within a permission-less (non-trusted) setting. In a similar vein, COINcidence [19] presents a protocol within a trusted setup, leveraging committee-based participation to achieve agreement with high probability.

Our approach diverges from both Algorand and COINcidence. Unlike Algorand, we operate within a trusted setup, and unlike both Algorand and COINcidence, our protocol involves selected parties undertaking a singular step. Furthermore, our protocol guarantees an agreement on a single party's requests with a probability of 1. Conversely, FasterDumbo [11] utilizes the committee concept to minimize the number of components in the agreement process, yielding outputs from n-f parties' requests. Beat [42] amalgamates the most effective elements of HoneyBadgerBFT's concrete implementation to showcase superior performance across varied settings.

Optimistic Fastlane Mechanisms. Certain works, such as those by Victor et al. [25] and Cachin et al. [17], have explored the incorporation of an optimistic 'fastlane' mechanism, designed for operation within partially synchronous networks. However, in instances of sluggish agreement processes, these mechanisms transition to asynchronous Byzantine agreement protocols. Recent endeavors, such as those by Joleton [39] and Boltdumbo [45], have adopted a similar strategy. Our work remains pertinent within this context as we rely on asynchronous protocols when the network exhibits Byzantine behavior, particularly within the pessimistic path.

Given our protocol's focus on outputting requests from a single party, we omit comparisons with DAG-rider [22] and Aleph [21], which utilize directed acyclic graphs for agreement alongside sequential ACS. Additionally, recent efforts have aimed to eliminate the private setup phase in asynchronous Byzantine agreement protocols, emphasizing asynchronous distributed key generation [40, 30, 7].

3 System Model

We use the standard notion [16, 13] to describe a distributed algorithm involving n parties and an adversary in an authenticated setting. This section explains the foundational assumptions, network model, adversary model, computational model, and cryptographic tools used in our protocol.

3.1 System and Network Assumptions

Parties and Their Setup: There are n parties, denoted as p_1, p_2, \ldots, p_n . We consider a trusted setup of threshold cryptosystems. The system provides a secret key and a public key to each party before the protocol starts. A party

uses its secret key to sign a message, and other parties use the corresponding public key to verify the signed message. The generation and distribution of these keys are out of the scope of this paper. We follow standard literature for key generation and distribution and refer interested readers to [9, 4, 6]. We use the node and party alternatively throughout the paper.

Network Model: Parties are connected via reliable, authenticated point-to-point channels. Here, reliable implies that if an honest party p_i sends a message to another honest party p_j , the adversary cannot modify or drop the message. However, the adversary can delay the message delivery to influence the protocol execution time. Since we consider an adaptive adversary that can corrupt a party at any time during the protocol execution, the adversary can corrupt a party p_i after it sends a message and then make the party p_i drop the message.

Adversary Model: We consider an adaptive adversary that can corrupt any party during the protocol execution. If the adversary takes control of a party, the party is corrupted, reveals its internal state to the adversary, behaves arbitrarily, and remains corrupted. An honest party follows the protocol, keeps its internal state secret from the adversary, and remains uncorrupted throughout the protocol execution. The adversary can corrupt f parties among the n parties, where $f < \frac{n}{3}$.

Computational Model: We adopt standard modern cryptographic assumptions and definitions from [16, 13]. The assumptions allow the parties and the adversary to be probabilistic polynomial-time interactive Turing machines. This means that upon receiving a message, a party carries out some computations, changes its state, generates outgoing messages if required, and waits for the next incoming message. To rule out infinite protocol executions and restrict the run time of the adversary, we require the message bits generated by honest parties to be probabilistic uniformly bounded by a polynomial in the security parameter λ . Therefore, we assume that the number of parties n is bounded by a polynomial in λ .

3.2 Validated Asynchronous Byzantine Agreement:

Multi-valued Byzantine Agreement (MVBA) allows parties to agree on an arbitrary string $\{0,1\}^l$, which must satisfy a predefined validity condition before the honest parties agree on it. This protocol ensures that the agreed-upon values are valid even when inputs come from malicious parties. This property is crucial for ensuring the protocol's correctness. Therefore, it is called the validated asynchronous byzantine agreement [13, 24, 47].

Multi-valued Byzantine Agreement The MVBA protocol provides a polynomial-time computable predicate Q. Each party inputs a value v that must satisfy the condition specified by Q, which is application-dependent. The MVBA protocol guarantees the following properties with negligible probability of failure:

- Liveness. If all honest parties are activated, then all honest parties will reach a decision.
- External Validity. If an honest party outputs a value v, then v satisfies the predicate Q(v) = true.
- Agreement. If two honest parties output values v and v', respectively, then v=v'.
- Integrity. If honest parties output a value v, then v must have been input by a party.
- Efficiency. The number of messages generated by honest parties is uniformly bounded with high probability.

3.3 Preliminaries

Asynchronous Binary Byzantine Agreement Biased Towards 1: The ABBA protocol allows parties to agree on a single bit $b \in \{0, 1\}$.

The ABBA protocol guarantees the following properties. Additionally, the biased external validity property applies to the biased ABBA protocol.

- Agreement. If an honest party outputs a bit b, then every honest party outputs the same bit b.
- **Termination.** If all honest parties receive input, then all honest parties will output a bit b.
- Validity. If any honest party outputs a bit b, then b was the input of at least one honest party.
- Biased External Validity. If at least $\langle f+1 \rangle$ honest parties propose 1, then any honest party that terminates will decide on 1.

(1, κ , ϵ)- Committee Selection: The committee selection protocol is executed among n parties (identified from 1 through n). This protocol ensures that an honest party outputs a κ -size committee set C with at least one honest member, given that at least f+1 honest parties participate. A protocol is a $(1, \kappa, \epsilon)$ -Committee Selection protocol if it satisfies the following properties with negligible probability of failure in the cryptographic security parameter λ :

- Termination. If \(\frac{f+1}{} \) honest parties participate in the committee selection and the adversary delivers the messages, then the honest parties will output the set \(C. \)
- Agreement. Any two honest parties will output the same set C.
- Validity. If any honest party outputs the set C, then:
 - (i) $|C| = \kappa$,
 - (ii) the probability of every party p_i being in C is the same, and
 - (iii) C contains at least one honest party with probability 1ϵ .
- Unpredictability. The probability of the adversary predicting the committee C before any honest party participates is at most $\frac{1}{{}^{n}C_{n}}$.

Guo et al. [11] constructed the $(1,\kappa,\epsilon)$ -Committee Selection protocol using a threshold coin-tossing mechanism (see Appendix A.3), which is derived from threshold signatures. The protocol ensures that at least one honest party is a committee member with overwhelming probability $1-\epsilon-\operatorname{neg}(\lambda)$, where $\operatorname{neg}(\lambda)$ is a negligible function in the cryptographic security parameter λ , and ϵ is $\exp(-\Omega\kappa)$.

Cryptographic Abstractions: We design a distributed algorithm in authenticated settings where we use robust, non-interactive threshold signatures to authenticate messages and a threshold coin-tossing protocol to select parties randomly [16].

- 1. Threshold Signature Scheme: We utilize a threshold signature scheme introduced in [43, 16]. The basic idea is that there are n parties, up to f of which may be corrupted. The parties hold shares of the secret key of a signature scheme and may generate shares of signatures on individual messages. t signature shares are both necessary and sufficient to construct a signature where $f < t \le (n f)$. The threshold signature scheme also provides a public key pk along with secret key shares sk_1, \ldots, sk_n , a global verification key vk, and local verification keys vk_1, \ldots, vk_n . Initially, a party p_i has information on the public key vk, a secret key sk_i , and the verification keys for all the parties' secret keys. We describe the security properties of the scheme and related algorithms in Appendix A.2.
- 2. Threshold Coin-Tossing Scheme: The threshold coin-tossing scheme, as introduced in [43, 16], involves parties holding shares of a pseudorandom function F that maps the name C (an arbitrary bit string) of a coin. We use a distributed pseudorandom function as a coin that produces k'' random bits simultaneously. The name C is necessary and sufficient to construct the value $F(C) \in \{0,1\}^{k''}$ of the particular coin. The parties may generate shares of a coin t coin shares are both necessary and sufficient where $f < t \le n f$, similar to threshold signatures. The generation and verification of coin-shares are also non-interactive. We describe the security properties of the scheme and related algorithms in Appendix A.3.

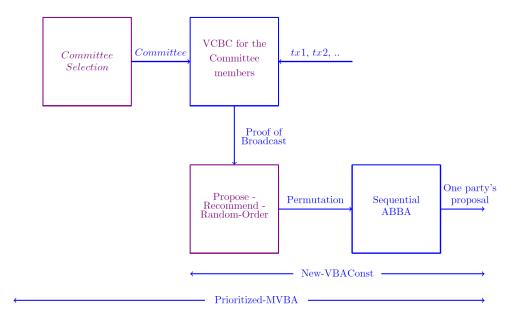
4 The Proposed Protocol

In this section, we present a protocol for multi-valued Byzantine agreement capable of tolerating up to $f < \frac{n}{3}$ Byzantine faults. The protocol features an expected communication bit complexity of $O(ln^2 + \lambda n^2)$ and an expected constant asynchronous round count. Our modular implementation comprises five distinct sub-protocols: committee selection, prioritized verifiable consistent broadcast protocol (pVCBC), Propose-Recommend, random order/permutation generation, and Sequential-ABBA. The framework of the proposed protocol is illustrated in Figure 1a. The violet-colored boxes indicate our contributions to the protocol. We break down the protocol into two main parts. The top part includes the committee selection and the VCBC protocol for the committee members, while the bottom part handles the distribution of the committee members' messages and the agreement protocol. Section 4.1 provides an overview and the construction of party selection and broadcast (top part of the framework), and Section 4.2 offers a detailed exposition of the protocol, including pseudocode and descriptions.

4.1 Optimizing Message and Communication Overhead

To enhance the efficiency of Byzantine agreement protocols, we propose a novel method to minimize the number of broadcast requests. Rather than requiring all n parties to broadcast their requests, we select a subset of parties to do so, ensuring that the protocol's progress and security properties are maintained.

Party Selection and Broadcast: In traditional asynchronous Byzantine agreement protocols, all n parties are typically allowed to broadcast their proposals. However, the main factor contributing to the $O(n^3)$ communication complexity in classic MVBA protocols is that each of the n parties must broadcast an O(n)-size array. These arrays include broadcast



(a) An overview of the proposed protocol framework, highlighting our contributions: committee selection, the VCBC protocol for committee members, recommendations, and Sequential ABBA.

information from all parties. Our approach aims to reduce the number of broadcasting parties, thereby eliminating the need for transmitting an O(n)-size array and decreasing the number of required Asynchronous Binary Byzantine Agreement (ABBA) instances. From this point forward, we use *proposal* instead of *request* and *propose* instead of *broadcasting requests*.

To achieve this reduction, we select at least f+1 parties to broadcast their proposals, ensuring that there is at least one honest party among them. This selection is crucial for guaranteeing the protocol's progress. We employ a committee selection protocol similar to FasterDumbo [11], which uses a cryptographic coin-tossing scheme to randomly select κ (a security parameter) parties. We set $\kappa=f+1$ to ensure the inclusion of at least one honest party, increasing the likelihood of multiple honest parties being selected.

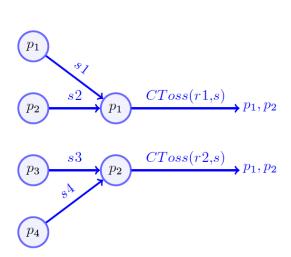
Dynamic selection of parties for each protocol instance prevents adversaries from corrupting fixed parties, ensures fair opportunity for all parties to propose, and mitigates the risk of starvation and Denial-of-Service (DoS) attacks. The committee selection process must be immune to adversary influence. The cryptographic coin-tossing scheme ensures random selection, requiring honest party participation after the completion of the previous instance. We used the committee member/selected party alternatively.

Committee Selection Protocol: The committee selection protocol (CS) is an integral part of our approach. It ensures that a reliable subset of parties is chosen to broadcast proposals, thus reducing communication complexity. The steps of the CS protocol are illustrated in Algorithm 1 and described as follows:

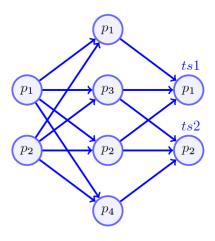
- Generating Coin-Shares: Upon invocation, a party generates a coin-share σ_i for the current instance and broadcasts it to all parties, waiting to receive f+1 coin-shares. (lines 3-5)
- Verifying Coin-Shares: When a party receives a coin-share for the first time from a party p_k , it verifies the coin-share and accumulates it in set Σ . This process continues until f+1 valid shares are collected. (lines 8-10)
- Selecting Parties: After receiving f+1 valid coin-shares, a party uses the CToss function along with the collected coin-shares to select f+1 parties. (lines 6-7)

We provide a illustration of the CS protocol in figure 2a and the cryptographic coin-tossing definitions (e.g. CShareVerify) in the Appendix A.3.

Broadcast protocol A committee member requires a verifiable proof of its proposal to demonstrate that it has broadcast the same proposal to at least f+1 honest parties. Typically, the VCBC is employed for this purpose, allowing every party to generate a verifiable proof. Since our protocol restricts proposal broadcasting to only the selected parties, we



(a) Committee-Selection protocol. Each party shares their $s_i = CShare(r_i)$ and a party requires f+1 shares. We see a party use $CToss(r_1, s)$ function to get the selected parties and each party gets the same set of parties. We show only for the two parties but it is true for every party.



(b) pVCBC protocol. Each selected party (here p_1 and p_2) propose its requests. Each receiving party verify the requests and add its sign-share on the message and returns to the sender. A party waits for 2f+1 sign-shares (here it is 3 because f = 1). The 2f+1 sign shares are necessary to create a threshold-signature (ts1).

Algorithm 1: Committee Selection Protocol for party p_i

```
1 Local variables initialization:
2 \Sigma \leftarrow \{\}
3 upon CS\langle id, instance \rangle invocation do
4 \sigma_i \leftarrow CShare\langle r_i \rangle
5 multi - cast\langle SHARE, id, \sigma_{id}, instance \rangle
6 wait until |\Sigma| = f + 1
7 return CToss\langle r_i, \Sigma \rangle
8 upon receiving \langle SHARE, id, \sigma_k, instance \rangle from a party p_k for the first time do
9 if CShareVerify\langle r_k, k, \sigma_k \rangle = true then
10 \Sigma \leftarrow \sigma_k \cup \Sigma
```

employ a slightly modified version of the verifiable consistent broadcast protocol, termed pVCBC. This adaptation ensures that if a party receives a verifiable proof from another party, it does not need to verify whether the proof originates from a selected party or not. The definition of the VCBC protocol and the construction of the pVCBC protocol can be found in the Appendix A.1 and Appendix C. We have also provided an illustration of the protocol in Figure 2b.

4.2 Prioritized MVBA

This section details the MVBA module in an asynchronous network. We propose a novel MVBA protocol called Prioritized-MVBA (pMVBA), which uses a committee as a prioritized parties and reduces the communication complexity of the classic MVBA protocol.

Overview of pMVBA The pMVBA protocol is inspired by the asynchronous binary Byzantine agreement methodology from Cachin et al. [13]. It significantly reduces the number of initial broadcasts, eliminating costly sub-components by leveraging the message distribution pattern of the adversary.

In classic MVBA, each party broadcasts an O(n)-size array to achieve O(1) expected constant runtime. Our pMVBA protocol as shown in Fig 3 and explained in Section 4.1, however, restricts this broadcasting to f+1 parties, significantly reducing communication complexity. The protocol replaces the O(n) commit broadcast with a single recommend message. We provide a details about classic MVBA protocol challenges in the Appendix B.2.

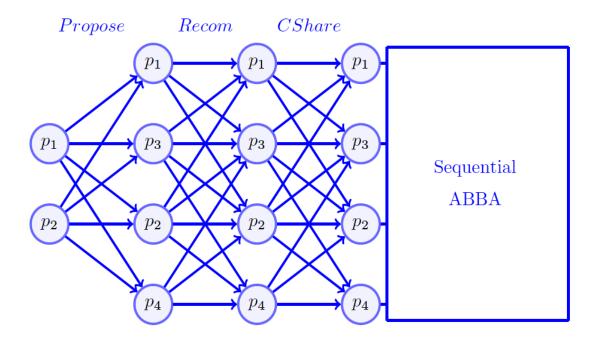


Figure 3: New-VBAConst protocol. After completing the pVCBC broadcast, each party proposes its value along with verifiable proof. Upon receiving verifiable proof, the party recommends the received proposal. Each party waits for n-f recommendations, then uses the coin-tossing scheme (CShare) to generate a permutation and initiates the ABBA protocol to reach an agreement on one of the proposals.

Since f+1 parties generate verifiable proofs and broadcast their proposals, a party must respond after receiving the first verifiable proof. If multiple parties complete the pVCBC protocol, different parties may receive proofs from different parties, hindering agreement. Therefore, we introduce the *recommend* step, where a party broadcasts received proofs to all parties, ensuring at least one proof reaches the majority of the parties (2f+1), which is necessary to employ ABBA protocol to reach an agreement. We use majority and threshold-number alternatively.

Replacing Commit with Recommend The key idea in pMVBA is to replace the commit step with the recommend step. The commit step in classic MVBA protocols involves collecting n-f verifiable proofs and creating an array of length n. In contrast, the recommend step involves broadcasting verifiable proofs directly, significantly reducing complexity.

Construction of the Recommend Step The recommend step consists of sending and collecting recommendations. This step ensures that at least one verifiable proof reaches the majority of parties. The detailed SCR protocol is shown in Algorithm 2:

- *Initialization:* A party initializes an empty set Σ .
- Broadcast Recommendation: A party creates a recommendation message and broadcasts it to all parties.
- Recommendation Collection: Upon receiving a recommendation, a party verifies the threshold-sign and instance. If valid, the party adds the proposal and σ to its set Σ .
- A party waits for n-f recommendation messages before proceeding.

Detailed pMVBA Protocol We now present the full details of the pMVBA protocol, inspired by the Cachin-MVBA protocol [13] but with fewer broadcasts. The protocol replaces the O(n) commit broadcast with one recommend message and runs an expected constant number of ABBA instances to reach an agreement on a party's proposal.

The formal description of the pMVBA protocol is shown in Algorithm 3. It consists of the following steps:

- Party/Committee Selection: When a new instance starts, a party invokes the CS protocol to select parties. (See Algorithm 1 lines 1-10 and Algorithm 3 line 10)
- **pVCBC Broadcast:** A selected party broadcasts its proposal using the pVCBC protocol and generates verifiable proofs of their broadcasts. (See Algorithm 3 line 13 and Algorithm 4 lines 1-16.)

Algorithm 2: Recommend Protocol for all parties

- **Propose and Recommend:** Upon completing the pVCBC protocol, a party broadcasts (as a propose) its verifiable proof. When a party receives a verifiable proof, it broadcasts the proof as a recommendation and a party waits for n-f recommendation messages. (See Algorithm 3 lines 14-22 and the Algorithm 2)
- **Permutation:** When a party receives n-f recommendations, it generates a random order of the selected parties using the cryptographic coin-tossing scheme. (See Algorithm 3 line 22 and Algorithm 4 lines 19-29)
- **Sequential-ABBA:** For the agreement phase, a party runs the ABBA protocol for the selected parties in the determined order, stopping once an agreement is reached. (See Algorithm 3 lines 32-58 and Algorithm 5 lines 1-28).

Algorithm 3: Prioritized-MVBA: Protocol for party p_i

```
1 Local variables initialization:
        instance \leftarrow 1
         PParties \leftarrow \bot
        \sigma \leftarrow \bot
        proposal \leftarrow \bot
        W \leftarrow \bot
         \Sigma \leftarrow \bot
    while true do
         ID \leftarrow \langle instance, id \rangle
         PParties \leftarrow CS\langle ID, instance \rangle
10
        if p_i \in PParties then
11
             proposal \leftarrow request
12
             \sigma \leftarrow pVCBC\langle \bar{I}D, requests \rangle
13
             multi-cast\langle "propose", ID, \sigma, proposal \rangle
14
         wait for a propose or a recommend message.
15
         upon receiving \langle propose, ID, \sigma, proposal \rangle for the first time do
16
              W \leftarrow SCR(ID, proposal, \sigma)
17
         upon receiving (recommendation, ID, proposal, \sigma) for the first time do
18
             if no recommendation is sent yet then
19
                   W \leftarrow SCR\langle ID, proposal, \sigma \rangle
20
        Permutation:
21
         \Sigma \leftarrow Permutation \langle ID, PParties \rangle
22
        Sequential-ABBA:
23
        proposal \leftarrow Sequential - ABBA\langle \Sigma, W \rangle
24
25
         decide\langle proposal \rangle
         instance \leftarrow instance + 1
26
```

5 Evaluation and Analysis of the Proposed Protocol

This section evaluates the proposed protocol based on its correctness and efficiency. Correctness ensures that the protocol maintains predefined security properties, making it resilient against adversarial attacks. Efficiency evaluates

the protocol's performance in terms of resource utilization and execution time. In this Section, we conduct extensive evaluation and analysis to answer:

- What are the formal proofs supporting the correctness of the Prioritized-MVBA protocol? (§ 5.1)
- How does the Prioritized-MVBA protocol perform in terms of message complexity, communication complexity, and running time? (§ 5.2)
- What are the observed behaviors and outcomes of the protocol under different network conditions and adversarial strategies? (§ 5.3)
- How does the performance of the Prioritized-MVBA protocol compare to other MVBA protocols in terms of key metrics? (§ 5.4)

5.1 Proof of the Prioritized-MVBA Protocol

The correctness of the proposed protocol is critical to ensure that it adheres to the standard MVBA outcomes. Our goal is to prove that reducing the number of broadcasts does not compromise the protocol's reliability. The output of a party's request depends on the output of an ABBA instance within our protocol's agreement loop, typically resulting in a value of I. For the ABBA protocol to output I, at least one honest party must input I. We substantiate our protocol's integrity by demonstrating that at least 2f+1 parties receive a verifiable proof of a party's proposal, thus ensuring the necessary input for the ABBA instance.

Lemma 5.1. At least one party's proposal reaches 2f + 1 parties.

Proof. Since the selected parties can be byzantine and the adversary can schedule the message delivery to delay the agreement, we have considered the following scenarios:

- 1. Among $\langle f+1 \rangle$ selected parties, f parties are non-responsive.
- 2. Selected $\langle f+1 \rangle$ parties are responsive, but other f non-selected parties are non-responsive.
- 3. Every party is responsive, including the selected $\langle f+1 \rangle$ parties.
- 4. Selected $t < \langle f+1 \rangle$ parties are responsive, and total m parties responsive, where $\langle 2f+1 \rangle \leq m \leq n$.

We prove that in every scenario, at least one party's proposal reaches $\langle 2f+1 \rangle$ parties.

- 1. Since among $\langle f+1 \rangle$ selected parties, f parties are non-responsive, only one party completes the pVCBC protocol and broadcasts the verifiable proof. If any party receives a verifiable proof, then the verifiable proof is from the responsive selected party. Since every party receives the verifiable proof for the same party's proposal, the proposal reaches $\langle 2f+1 \rangle$ parties.
- 2. The $\langle f+1 \rangle$ selected parties are responsive and complete the pVCBC protocol. Each selected party broadcasts the verifiable proof, and $\langle 2f+1 \rangle$ parties receive the verifiable proof (another f number of parties are non-responsive). Any party receives a verifiable proof, recommends the received verifiable proof, and waits for $\langle 2f+1 \rangle$ recommendations. If a party receives $\langle 2f+1 \rangle$ recommendations, then these recommendations include all $\langle f+1 \rangle$ parties' verifiable proofs because among the $\langle 2f+1 \rangle$ received recommendations, $\langle f+1 \rangle$ number of recommendations are from the selected parties. So, every proposal reaches $\langle 2f+1 \rangle$ parties.
- 3. The proof is by contradiction. Let no proposal reaches more than 2f parties. Since we assume every party is responsive, every party receives a proposal in propose step. We also know that no party participate in Sequential-ABBA protocol until the party receives $\langle 2f+1\rangle$ recommendations and get the order to Sequential-ABBA. There must be a $\langle 3f+1\rangle * \langle 2f+1\rangle$ recommendation messages. If no proposal can be recommended to more than 2f parties, the total number of recommendations is $\langle 3f+1\rangle * \langle 2f\rangle < \langle 2f+1\rangle * \langle 2f+1\rangle$ (Though a proposal can be recommended by more than one party we assume that every party recommends to the same 2f parties otherwise it would fulfil the requirement of $\langle 2f+1\rangle$ proposals). But, the honest parties must send enough recommendation messages to ensure the protocol's progress and the adversary delivers the messages eventually. Therefore, at least one party's proposal reaches $\langle 2f+1\rangle$ parties, a contradiction.
- 4. The proof is by contradiction. Let no proposal reaches to more than 2f parties. If $1 \le t \le \langle f+1 \rangle$ parties distribute their verifiable proof to $\langle 2f+1 \rangle \le m < \langle 3f+1 \rangle$ parties and no proposal reaches more than

2f parties, then there must be no more than m*2f recommendations. However, m parties must receive $m*\langle 2f+1\rangle$ recommendations greater than m*2f, a contradiction.

This lemma ensures that if 2f + 1 parties receive a verifiable proof, at least one honest party will input I to the ABBA instance, ensuring the protocol reaches an agreement on I.

Lemma 5.2. Without any permutation, the adversary can cause at most f + 1 iterations of the agreement loop in the Sequential-ABBA protocol.

Proof. The proof works by counting the total number A of vote = 0 messages (line 44 of Algorithm 4) that are generated by honest parties (over all 0 vote in the agreement loop). Since every honest party has received a verifiable proof from one party in the propose broadcast step (line 18 of Algorithm 3), the party generates vote = 0 messages for at most f number of selected parties. Thus $A \leq (f)(n-f)$ (all (n-f) votes are zero). For the asynchronous binary byzantine agreement protocol to decide 0 for a particular party's proposal and to cause one more iteration of the loop, at least $\langle f+1 \rangle$ honest parties must propose 0 for the binary agreement (line 46 of Algorithm 4). Since honest parties only propose 0 if they have received $\langle 2f+1 \rangle$ vote=0 messages, there must be at least $\langle f+1 \rangle$ honest parties who have generated a vote=0 message in this iteration. Let R denote the number of iterations of the loop where the binary agreement protocol decides 0. From the preceding argument, we have $A \geq R(f+1)$. Combining these two bounds on A, we obtain $R(f+1) \leq f*(n-f)$.

$$R \leq \frac{(f)(n-f)}{f+1}$$

$$R \leq \frac{(f)(2f+1)}{f+1}$$

$$R \leq \frac{(f)(f+1+f)}{f+1}$$

$$R \leq f + \frac{f^2}{f+1}$$

$$(1)$$

However, there are only $\langle f+1 \rangle$ number of selected parties' proposals, and from Lemma 5.1, at least one verifiable proof reaches $\langle f+1 \rangle$ honest parties. Therefore, $R < \langle f+1 \rangle$ and the adversary can cause at most R+1 or $\langle f+1 \rangle$ number of iterations.

Lemma 5.3. Let $\overline{A} \subseteq \{1, 2, ..., f+1\}$ be the set of selected parties for which at least f+1 honest parties receive the verifiable proof, and let Π be a random permutation of the f+1 selected parties. Then, except with negligible probability:

- For every party $p \in \overline{A}$, the ABBA protocol on ID|p will decide 1.
- $|\overline{A}| \geq 1$.
- There exists a constant $\beta > 1$ such that for all $t \geq 1$, $Pr[\Pi[1] \notin \overline{A} \wedge \Pi[2] \notin \overline{A} \wedge ... \wedge \Pi[t] \notin \overline{A}] \leq \beta^{-t}$.

Proof. The binary agreement protocol biased towards 1 decides 0 if the honest parties input 0. From the Sequential-ABBA protocol, each party must receive $\langle n-f\rangle$ vote=0 messages in order to input 0 to an ABBA instance and to decide 0. However, from Lemma 5.1 at least $\langle f+1\rangle$ honest parties receive at least one verifiable proof from a selected party p, then it is not the case for the party $p\in \overline{A}$. This proves the first claim.

To prove the second claim, let A denote the total number of 0-votes cast by honest parties. Since an honest party votes 0 for at most f number of parties' proposals, we have $A \leq f(n-f)$. On the other hand, a party can cast vote = 0 for $f+1-|\overline{A}|$ parties' proposals, and at least $\langle f+1 \rangle$ honest parties cast vote = 0 to decide 0 then $A \geq (f+1-|\overline{A}|)(f+1)$. These bounds on A are the same as in Lemma 5.2 with $R = f+1-|\overline{A}|$. Using the same argument, it follows that $|\overline{A}| \geq 1$, since R is at most f^1 .

The third claim follows now because $|\overline{A}|$ is at least a constant fraction of f+1 and thus, there is a constant $\beta>1$ such that $Pr[\Pi(i))\notin \overline{A}]\leq \frac{1}{\beta}$ for all $1\leq i\leq t$. Since the probability of the t first elements of Π jointly satisfying the condition is no larger than for t independently and uniformly chosen values, we obtain $Pr[\Pi[1]\notin \overline{A}\wedge\Pi[2]\notin \overline{A}\wedge\dots\wedge\Pi[t]\notin \overline{A}]\leq \beta^{-t}$.

Theorem 5.4. Given a protocol for biased binary Byzantine agreement and a protocol for verifiable consistent broadcast, the Prioritized-MVBA protocol provides multi-valued validated Byzantine agreement for n > 3f and invokes a constant expected number of binary Byzantine agreement protocols.

Proof. Agreement: If an honest party outputs a value v, then every honest party outputs the value v. From Lemma 5.1, at least one proposal reaches 2f+1 parties, and the ABBA protocol reaches an agreement on 1 if f+1 honest parties input 1. The agreement property of the ABBA protocol ensures all honest parties output 1 for that ABBA instance and receive the same value v, which satisfies $threshold-validate\langle v,\sigma\rangle=true$.

Liveness: If honest parties participate and deliver messages, all honest parties decide. From Lemma 5.1, at least one proposal reaches 2f + 1 parties, ensuring the ABBA protocol decides 1. Lemma 5.3 confirms the protocol reaches an agreement after a constant expected number of iterations.

External-validity: If an honest party terminates and decides on a value v, then $externally - valid \langle v, \sigma \rangle = true$. The validity of the ABBA protocol ensures at least one honest party inputs 1 to the ABBA instance, meaning the honest party received a valid $threshold - signature \sigma$ for v.

Integrity: If honest parties decide on a value v, then v was proposed by a party. The ABBA protocol returns 1 if at least one honest party inputs 1, which requires a valid $threshold - signature \sigma$. Honest parties reply with sign - shares only if a value v is proposed, ensuring v was proposed by a party.

5.2 Efficiency Analysis

The efficiency of a Byzantine Agreement (BA) protocol depends on message complexity, communication complexity, and running time. We analyze the proposed protocol's efficiency by examining its sub-components: the pVCBC sub-protocol, committee-selection and permutation generation, propose-recommend steps, and the Sequential-ABBA sub-protocol.

Running Time: Each sub-protocol and step, except for Sequential-ABBA, has a constant running time. The running time of the proposed protocol is dominated by the Sequential-ABBA sub-protocol, which has an expected constant number of asynchronous rounds.

Message Complexity: In all sub-protocols and steps, except for pVCBC and propose steps, each party communicates with all other parties. Thus, the message complexity is $O(n^2)$. The expected message complexity of the Sequential-ABBA protocol is also $O(n^2)$.

Communication Complexity: The communication complexity of each sub-protocol and step is $O(n^2(l+\lambda))$, where l is the bit length of input values and λ is the bit length of the security parameter. The expected communication complexity of the Sequential-ABBA protocol is also $O(n^2(l+\lambda))$.

In conclusion, the Prioritized-MVBA protocol achieves optimal efficiency and correctness by reducing the number of broadcasts and ensuring all security properties of a multi-valued Byzantine agreement protocol. The detailed analysis confirms its resilience, integrity, liveness, and external validity, making it suitable for robust decentralized infrastructures.

5.3 Case Study

The key contribution of this paper is the reduction in the number of requesting parties and the subsequent utilization of fewer broadcasts to eliminate the expensive sub-components of the classic MVBA protocol. The main challenge is to provide sufficient information to the parties to maintain the protocol's progress while removing the costly elements. It is crucial to ensure that at least one selected party's proposal reaches at least 2f + 1 parties. This is achieved through

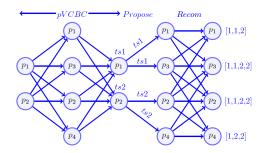
¹To prove the Lemma 5.2 and Lemma 5.3, we have used the same technique of Lemma 7 and Lemma 9 from [13]. We also refer interested readers to Lemma 10 of the same paper for the pseudorandom generation.

the pVCBC step and the propose-recommend step. To illustrate the protocol's effectiveness, we present a case study in two parts. Section 5.3.1 demonstrates how the protocol achieves the desired properties with the minimum number of nodes a system can have. To further clarify that the protocol can maintain these properties with a larger number of nodes, Section 5.3.2 provides a case study using charts for a system with three times the number of faulty nodes.

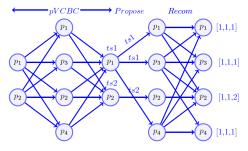
5.3.1 Message Flow in Different Stages

In an asynchronous network, the presence of a faulty node and the adversary's ability to manipulate message delivery can delay the agreement process. This section shows how a faulty party can affect message delivery patterns and how the adversary can delay specific messages to prevent a node's proposal from reaching the majority. Despite these challenges, the goal is to ensure that nodes can still reach an agreement on a party's proposal.

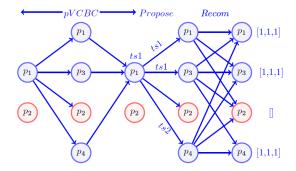
Figure 4a assumes no faulty nodes, with the adversary delivering messages uniformly, allowing the protocol to reach an agreement on the first try. Figure 4b considers a scenario where one node gets a threshold-signature (ts) early, or the adversary prioritizes one node's ts delivery over others, preventing one selected node's proposal from reaching the majority. Figure 5a assumes the selected node p_2 is either faulty or completely isolated by the adversary, and Figure 5b assumes a non-selected node is faulty or isolated, positively impacting the ts delivery of the selected nodes.



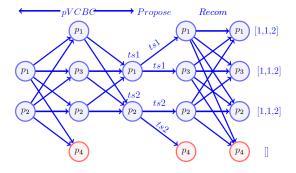
(a) Every selected node is non-Byzantine and the adversary delivers the threshold-signature (ts1, ts2) uniformly (both p_1 and p_2 's threshold-signature are received by the same number of parties). Both p_1 's and p_2 's proposals are received by every party.



(b) Every selected node is non-Byzantine and the adversary delivers the messages non-uniformly (Node p_1 's threshold-signature is received by three parties but node p_2 's threshold-signature is received by one party). p_1 's proposal is received by every party, but node p_2 's proposal is received by only one party.



(a) One selected node is non-Byzantine (p_1) , and the adversary delivers the messages from this node to every other node. Consequently, only node p_1 completes the pVCBC protocol and proposes the threshold-signature. Therefore, every party has the proposal except p_2 (either Byzantine or non-responding). The red node represents either the node is Byzantine or non-responding or the network is not delivering the messages from that node.



(b) Every selected node is non-Byzantine, and the adversary delivers the messages uniformly (both p_1 and p_2 's threshold-signature are received by the same number of parties). Both p_1 's and p_2 's proposals are received by every party. The non-selected node p_4 is not responding, which does not affect the overall criteria. The red node represents either the node is Byzantine or non-responding or the network is not delivering the messages from that node.

In conclusion, the above figures collectively demonstrate that regardless of the message delivery pattern or the presence of faulty nodes, the protocol consistently achieves agreement, thereby proving its robustness and effectiveness.

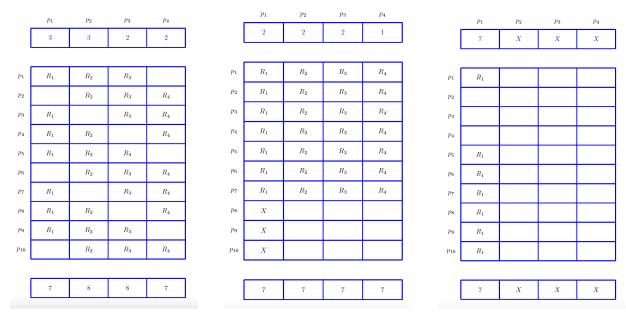


Figure 6: All selected parties complete their pVCBC and broadcast their proposals. All parties are non-Byzantine; therefore, all of them recommend their received proposal. Every party's proposal reaches at least 7 parties.

Figure 7: All selected parties complete their pVCBC and broadcast their proposals. Three non-selected faulty parties cannot recommend; consequently, a party receives recommendations for all selected parties. Therefore, all selected parties proposals reach at least 7 parties.

Figure 8: All selected parties are Byzantine or non-responding except P_1 . The non-Byzantine party completes the pVCBC and proposes its requests. Since there is only one proposal, every party receives the same proposal, which is received by every non-Byzantine party. Therefore, the proposal reaches 7 parties.

5.3.2 Message Distribution for More Than One Faulty Node

For this study, we assume a total of n=3f+1=10 parties, with f=3 being faulty. This configuration allows us to explore various message distribution patterns. We require at least one party's proposal to reach 2f+1=7 parties, with a total of f+1=4 selected parties. We examine the impact of different numbers of faulty parties and message distribution patterns. In each figure, the top box indicates selected parties and the number of parties that can recommend the proposal to others (e.g., Party p_1 can receive 3 recommendations for the first selected party (R_1) , 3 for the second selected party (R_2) , and 2 for the third selected party (R_3) , totaling 7). The bottom box counts the number of recommendations received for each selected party (e.g., p_1 receives a total of 7 R_1 recommendations).

Figure 6 assumes no faulty nodes, with the adversary delivering messages uniformly, ensuring that every selected node's proposal reaches the threshold number of parties. Figure 7 assumes three non-selected non-responding nodes, which also allows every selected node's proposal to reach the threshold. Figure 8 assumes one honest selected node or the adversary delivering messages from that node, ensuring only the honest selected node's proposal reaches the threshold. Figure 9 and Figure 10 show scenarios with two and three honest selected nodes, respectively, where their proposals reach the threshold. Figure 11 explores non-uniform message distribution, which may prevent some selected nodes' proposals from reaching the threshold, but only when all selected nodes are active.

In conclusion, the above scenarios illustrate that the protocol can maintain its effectiveness and achieve agreement even with varying numbers of faulty nodes and different message distribution patterns.

5.4 Analysis of Communication Complexity and Resilience

This section evaluates our protocol's performance by comparing its communication complexity with classic MVBA protocols and assessing its resilience, termination, and safety properties against committee-based protocols.

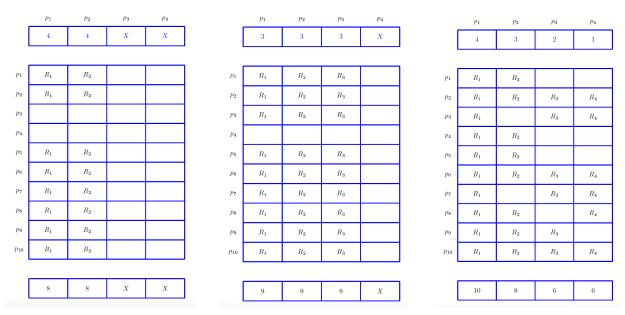


Figure 9: Two selected parties are non-Byzantine and complete the pVCBC protocol, proposing their proposals, which are received by every non-Byzantine party. Each selected party's proposal reaches more than 7 parties.

Figure 10: Three selected parties are non-Byzantine and complete the pVCBC protocol, proposing their proposals, which are received by every non-Byzantine party. Each selected party's proposal reaches more than 7 parties.

Figure 11: This study tests whether non-uniform message distribution affects recommendation reception. Even with non-uniform distribution, at least half of the selected parties' proposals reach 7 parties.

5.4.1 Comparison of communication complexity with classic protocols.

Our work focuses on the first polynomial time asynchronous byzantine agreement protocol and removes the expensive sub-component from that protocol. We compare our work with the classic MVBA protocol and the traditional MVBA protocols. Table 1 highlights these comparisons. Our techniques differ from the recent improvements in the MVBA protocol. Therefore, we compare our protocol with the classic protocol and the following improvement. Our protocol differs from the Cachin-MVBA [13] protocol in terms of communication complexity. VABA [23] is a view-based protocol where each view is an instance (a complete execution of the protocol) of the protocol. VABA does not guarantee that the parties will reach an agreement at each instance. In our protocol, parties can reach an agreement on a valid proposal in every instance. We also present a comparison between our proposed protocol and the Dumbo-MVBA [29] protocols. The Dumbo-MVBA protocol is intended for large inputs. It uses the erasure code technique to remove the expensive term. The erasure code first disperses the message and then recovers the message at the end of the agreement, which is expensive in terms of latency and computation and is not worth it for small-size inputs. Our protocol achieves the desired communication complexity (removal of the $O(n^3)$ term) without the utilization of information dispersal techniques (erasure code) and in one instantiation of the protocol. We can use the information dispersal technique like Dumbo-MVBA in our protocol to reduce the communication complexity to $O(ln + \lambda n^2)$, but our protocol does not depend on the erasure code to remove the $O(n^3)$ term.

Table	1: Comparison for po				•
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Protocols	Comm. (bit)	Word	Time	Msg.	ErasureCode.	Instance
Cachin [13]	$O(\ln^2 + \lambda n^2 + n^3)$	$O(n^3)$	O(1)	$O(n^2)$	No	1
VABA [23]	$O(\ln^2 + \lambda n^2)$	$O(n^2)$	O(1)	$O(n^2)$	No	≥ 1
Dumbo-MVBA [29]	$O(ln + \lambda n^2)$	$O(n^2)$	O(1)	$O(n^2)$	Yes	1
Dumbo-MVBA* [29]	$O(ln + \lambda n^2)$	$O(n^2)$	O(1)	$O(n^2)$	Yes	1
Our work	$O(\ln^2 + \lambda n^2)$	$O(n^2)$	O(1)	$O(n^2)$	No	1

5.4.2 Comparison of Resilience, Termination, and Safety with Committee-Based Protocols

We compare our work with notable committee-based protocols, specifically focusing on resilience, termination, and safety properties. Table 2 highlights these comparisons. COINcidence [19] assumes a trusted setup and violates optimal resilience. It also does not guarantee termination and safety with high probability (whp). Algorand [20] assumes an untrusted setup, with resilience dependent on network conditions, and does not guarantee termination whp. The Dumbo [11] protocol uses a committee-based approach, but its committee-election protocol does not guarantee the selection of an honest party, thus failing to ensure agreement or termination. Our protocol achieves optimal resilience and guarantees both termination and safety, as our committee-election process ensures the selection of at least one honest party. This guarantees that the protocol can make progress and reach agreement despite adversarial conditions.

Protocols	n>	Termination	Safety
COINcidence [19]	4.5f	whp	whp
Algorand [20]	*	whp	w.p. 1
Dumbo1 [11]	3f	whp	w.p. 1
Dumbo2 [11]	3f	whp	w.p. 1
Our work	3f	w.p. 1	w.p. 1

Table 2: Comparison for performance metrics of the committee based protocols

In conclusion, we have compared our protocol with both the classical protocol and the committee-based protocol. Though our protocol differs from the atomic broadcast protocol in a number of proposals, we provide a comparison of our protocol with the atomic broadcast protocol in Appendix B.1.

6 Discussions and Conclusion

In this paper, we introduced pMVBA, a novel MVBA protocol designed to address the high communication complexity inherent in the classic MVBA protocol. By leveraging a committee-based approach combined with the ABBA protocol, pMVBA significantly reduces communication overhead while maintaining optimal resilience and correctness. Our protocol achieves several key improvements over existing MVBA protocols. By dynamically selecting a subset of parties (f + 1) to broadcast proposals, we ensure that at least one honest party is always included, thereby enhancing the protocol's resilience. The integration of the pVCBC protocol allows for efficient proposal broadcasting and verifiable proof generation. The recommend step replaces the commit step used in the classic MVBA protocol, reducing the need for extensive communication bit exchanges and achieving agreement with fewer communication rounds. Theoretical analysis and case studies demonstrated the effectiveness of pMVBA, showing that it achieves an expected constant asynchronous round count and optimal message complexity of $O(n^2)$, where n represents the number of parties. The communication complexity is reduced to $O((l + \lambda)n^2)$, making pMVBA suitable for large-scale decentralized applications.

Limitations and Future Work. While pMVBA presents significant advancements, it is not without limitations. One limitation is the assumption of a trusted setup for cryptographic keys, which may not be practical in all decentralized environments. Additionally, the protocol's performance in extremely large networks with highly dynamic membership has not been thoroughly tested and may present scalability challenges. Another limitation is the reliance on the ABBA protocol for agreement, which, while efficient, could still be optimized further to handle more adversarial conditions. Moreover, the security parameters and their impact on the overall performance need more comprehensive analysis under various network conditions. Future work will focus on addressing these limitations. We plan to explore alternative setups that do not require a trusted third party for key distribution, thus enhancing the protocol's applicability in trustless environments. We will also investigate adaptive mechanisms to improve scalability in large and dynamic networks. Further optimizations to the pMVBA protocol will be considered to enhance its resilience and efficiency. Additionally, extensive empirical evaluations under diverse network conditions will be conducted to better understand the protocol's performance and robustness.

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A Definitions

A.1 Verifiable Consistent Broadcast

A protocol completes a verifiable consistent broadcast if it satisfies the following properties:

- Validity. If an honest party sends m, then all honest parties eventually delivers m.
- Consistency. If an honest party delivers m and another honest party delivers m', then m=m'.
- Integrity. Every honest party delivers at most one request. Moreover, if the sender p_s is honest, then the request was previously sent by p_s .

A.2 Threshold Signature Scheme

The (f + 1, n) non-interactive threshold signature scheme is a set of algorithms used by n parties, with up to f potentially faulty. The threshold signature scheme satisfies the following security requirements, except with negligible probabilities:

- Non-forgeability. To output a valid signature, a party requires t signature shares. Therefore, it is computationally infeasible for an adversary to produce a valid signature, as an adversary can corrupt up to f parties (f < t) and thus cannot generate enough signature shares to create a valid signature proof for a message.
- **Robustness.** It is computationally *infeasible* for an adversary to produce t valid *signature shares* such that the output of the share combining algorithm is not a valid signature.

The scheme provides the following algorithms:

- Key generation algorithm: KeySetup($\{0,1\}^{\lambda}, n, f+1$) $\rightarrow \{UPK, PK, SK\}$. Given a security parameter λ , this algorithm generates a universal public key UPK, a vector of public keys $PK := (pk_1, pk_2, \dots, pk_n)$, and a vector of secret keys $SK := (sk_1, sk_2, \dots, sk_n)$.
- Share signing algorithm: SigShare_i $(sk_i, m) \to \sigma_i$. Given a message m and a secret key share sk_i , this deterministic algorithm outputs a signature share σ_i .
- Share verification algorithm: VerifyShare_i $(m,(i,\sigma_i)) \to 0/1$. This algorithm takes three parameters as input: a message m, a signature share σ_i , and the index i. It outputs 1 or 0 based on the validity of the signature share σ_i (whether σ_i was generated by p_i or not). The correctness property of the signing and verification algorithms requires that for a message m and party index i, $\Pr[VerifyShare_i(m,(i,SigShare_i(sk_i,m)))=1]=1$.
- Share combining algorithm: CombineShare_i $(m, \{(i, \sigma_i)\}_{i \in S}) \to \sigma/\bot$. This algorithm takes two inputs: a message m and a list of pairs $\{(i, \sigma_i)\}_{i \in S}$, where $S \subseteq [n]$ and |S| = f + 1. It outputs either a signature σ for the message m or \bot if the list contains any invalid signature share (i, σ_i) .
- Signature verification algorithm: Verify_i $(m,\sigma) \to 0/1$. This algorithm takes two parameters: a message m and a signature σ , and outputs a bit $b \in \{0,1\}$ based on the validity of the signature σ . The correctness property of the combining and verification algorithms requires that for a message $m, S \subseteq [n]$, and |S| = f + 1, $\Pr[\operatorname{Verify}_i(m, \operatorname{Combine}_i(m, \{(i,\sigma_i)\}_{i \in S})) = 1 \mid \forall i \in S, \operatorname{VerifyShare}_i(m, (i,\sigma_i)) = 1] = 1$.

A.3 Threshold Coin-Tossing

We assume a trusted third party has an unpredictable pseudo-random generator (PRG) $G: R \to \{1, \dots, n\}^s$, known only to the dealer. The generator takes a string $r \in R$ as input and returns a set $\{S_1, S_2, \dots, S_s\}$ of size s, where $1 \le S_i \le n$. Here, $\{r_1, r_2, \dots, r_n\} \in R$ are shares of a pseudorandom function F that maps the coin name C. The threshold coin-tossing scheme satisfies the following security requirements, except with negligible probabilities:

- **Pseudorandomness.** The probability that an adversary can predict the output of F(C) is $\frac{1}{2}$. The adversary interacts with the honest parties to collect *coin-shares* and waits for t *coin-shares*, but to reveal the coin C and the bit b, the adversary requires at least $\langle t-f\rangle$ *coin-shares* from the honest parties. If the adversary predicts a bit b, then the probability is $\frac{1}{2}$ that F(C) = b ($F(C) \in \{0,1\}$). Although the description is for single-bit outputs, it can be trivially modified to generate k-bit strings by using a k-bit hash function to compute the final value.
- Robustness. It is computationally *infeasible* for an adversary to produce a coin C and t valid *coin-shares* of C such that the share-combine function does not output F(C).

The dealer provides a private function $CShare_i$ to every party p_i , and two public functions: CShareVerify and CToss. The private function $CShare_i$ generates a share σ_i for the party p_i . The public function CShareVerify can verify the share. The CToss function returns a unique and pseudorandom set given f+1 validated coin shares. The following properties are satisfied except with negligible probability:

- For each party $i \in \{1, ..., n\}$ and for every string r_i , $CShareVerify(r_i, i, \sigma_i) = \text{true}$ if and only if $\sigma_i = CShare_i(r_i)$.
- If p_i is honest, then it is impossible for the adversary to compute $CShare_i(r)$.
- For every string r_i , $CToss(r, \Sigma)$ returns a set if and only if $|\Sigma| \geq f+1$ and each $\sigma \in \Sigma$ and $CShareVerify(r, i, \sigma) = \text{true}$.

B Miscellaneous

B.1 Comparison with Atomic Broadcast Protocol

As discussed earlier, when the inputs of each party are nearly identical, outputting the requests of n-f parties is not a viable solution. This approach results in higher computational effort without increasing the number of accepted transactions. Table 3 provides a comparison of the communication complexity of our protocol with atomic broadcast protocols. Notably, no atomic broadcast protocol can eliminate the multiplication of $O(n^3)$ terms. Additionally, atomic broadcast protocols require extra rounds of message exchanges. Here, we focus solely on the communication complexity.

	* ·
Protocols	Communication Complexity
HB-BFT/BEAT0 [5]	$O(ln^2 + \lambda n^3 log n)$
BEAT1/BEAT2 [42]	$O(ln^3 + \lambda n^3)$
Dumbo1 [11]	$O(ln^2 + \lambda n^3 log n)$
Dumbo2 [11]	$O(ln^2 + \lambda n^3 log n)$
Speeding Dumbo [10]	$O(ln^2 + \lambda n^3 log n)$
Our Work	$O(ln^2 + \lambda n^2)$

Table 3: Comparison of the communication complexity with the atomic broadcast protocols

B.2 The Challenge of Classical MVBA Designs

To maintain a message complexity of $O(n^2)$, the classic MVBA protocol incorporates the Verifiable Consistent Broadcast (VCBC) protocol. Additionally, it introduces the concept of *external validity*, wherein an input is deemed valid if it satisfies certain criteria. The protocol operates as follows:

Each party utilizes the VCBC protocol to broadcast their request and generate a corresponding verifiable proof. Upon completion of this step, the party broadcasts both the verifiable proof and the request, providing evidence that the request has been broadcast to every other party.

Upon receiving verifiable proof from n-f parties, signaling the completion of the VCBC protocol by the threshold number of parties, a party can initiate the ABBA protocol. However, there exists the possibility that other parties have not received sufficient verifiable proof, or that the adversary manipulates the distribution of proofs in a manner that prevents the majority of ABBA instances from receiving adequate proof.

To address this, each party communicates with others by transmitting an n-bit array, indicating receipt of verifiable proof from n-f parties. Upon receipt of n-f verifiable proofs, a party generates a permutation of the parties and invokes ABBA instances based on the order of permutation, ensuring that the number of ABBA instances remains constant on average.

C Deferred protocols

Construction of the pVCBC

- Upon invocation of the pVCBC protocol, a party creates a message using the *ID* and the requests. The party then authenticates the message using its private key and multicasts the message (lines 03-06).
- Upon receiving a signed message (msg_{signed}) from a party p_j , a party checks whether the sender is a selected party and whether the message is authentic (signed by the sender). If the sender is a selected party and the message is authentic, the party adds its signature share (sign share) to the message, resulting in ρ . The party then replies with ρ to the sender (lines 13-16).
- Upon receiving a signature share ρ_k from a party p_k , a selected party checks the validity of ρ_k (ensuring it was added using p_k 's key). If ρ_k is valid, the party adds the signature share ρ_k to its set Σ (lines 09-11).
- A selected party waits for $\langle n-f \rangle$ valid signature shares. Upon receiving $\langle n-f \rangle$ signature shares, the party combines them to generate a threshold signature σ (proof that the party has sent the same request to at least $\langle f+1 \rangle$ honest parties) and returns σ to the caller (lines 07-08).

Construction of the Permutation Here is the pseudocode for the permutation protocol (see Algorithm 4). Below is a step-by-step description of the protocol:

- Upon invocation of the Permutation protocol, a party generates a coin-share σ_i for the instance and broadcasts σ_i to every party, then waits for 2f + 1 coin-shares (lines 22-24).
- When a party receives a coin-share from another party p_k for the first time, it verifies the coin-share (ensuring it is from p_k) and accumulates the coin-share in the set Σ . The party continues to respond to coin-shares until it has received 2f + 1 valid shares (lines 27-29).
- Upon receiving 2f + 1 valid coin-shares, a party uses its CToss function and the collected coin-shares to generate a permutation of the n parties (lines 24-25).

Sequential-ABBA The purpose of the Sequential-ABBA protocol is to run an agreement loop to agree on a selected party's proposal. We use the asynchronous binary Byzantine agreement protocol biased towards 1 for each selected party until the parties reach an agreement on one of the party's proposals.

Construction of the Sequential-ABBA Since the Sequential-ABBA protocol guarantees that the parties will eventually reach an agreement on one of the selected party's proposals, the protocol runs a loop for the selected parties until it agrees on a valid proposal. The pseudocode of the Sequential-ABBA protocol is given in Algorithm 5, and a step-by-step description is provided below:

- The Sequential-ABBA protocol takes two arguments: (i) Σ , the permutation of the selected parties, and (ii) W, the list of verifiable proofs the party has received (line 1).
- Upon invocation of the protocol, a party declares two variables: (i) l, an index number to access the selected
 parties one by one from the array Σ, and (ii) b, initially set to zero to indicate that the parties have not reached
 any agreement (lines 2-3).
- While loop: Parties are chosen one after another according to the permutation Σ of $\{1, \ldots, f+1\}$. Let l denote the index of the party selected in the current loop (the selected party p_l is called the candidate). Each party p_l follows these steps for the candidate p_l (lines 4-22):
 - Broadcasts a vote message to all parties containing $u_l=1$ if party p_i has received p_l 's proposal and verifiable proof (including the proposal in the vote), and $u_l=0$ otherwise.
 - Waits for $\langle n-f \rangle$ vote messages but does not count votes indicating $u_l=1$ unless a valid proposal from p_l has been received—either directly or included in the vote message.
 - Runs a binary validated Byzantine agreement biased towards 1 (see Algorithm 6) to determine whether p_l has properly broadcast a valid proposal. Vote 1 if p_i has received a valid proposal from p_l and adds the protocol message that completes the verifiable broadcast of p_l 's proposal to validate this vote. Otherwise, if p_i has received n-f vote messages containing $u_l=0$, then vote 0; no additional information is needed. If the agreement loop decides 1, exit the loop.
- Upon reaching an agreement, a party returns the threshold signature of the agreed proposal.
- Upon receiving $\langle ID, vote, party, u_l, m \rangle$, a party checks whether $u_l = 1$. If it receives $u_l = 1$, then it assigns $m_l = m$.

Algorithm 4: Protocol for party p_i

```
1 upon pVCBC\langle ID, requests, PParties \rangle invocation do
         \Sigma \leftarrow \{\}
         msg \leftarrow createMsg\langle ID, requests \rangle
 3
         multi-cast \langle msq \rangle
         wait until |\Sigma| = n - f
         return \sigma \leftarrow CombineShare_{id}\langle requests, \Sigma \rangle
    upon receiving \langle m, \sigma_k \rangle from the party p_k for the first time do
         if VerifyShare_k\langle m, (k, \sigma_k) \rangle then
               \Sigma \leftarrow \sigma_k \cup \Sigma
10
11
    upon receiving \langle msg \rangle from the party p_i for the first time do
13
         if p_i \in PParties then
               reguests \leftarrow msq.reguests
14
               \sigma_{id} \leftarrow SigShare_{id}\langle sk_{id}, requests \rangle
15
16
               reply\langle\sigma_{id}\rangle
17
18
19
    Local variables initialization:
20
         \Sigma \leftarrow \{\}
21 upon Permutation \langle id, instance \rangle invocation do
         \sigma_i \leftarrow CShare\langle r_{id} \rangle
22
         multi - cast \langle SHARE, id, \sigma_{id}, instance \rangle
23
         wait until |\Sigma| = 2f + 1
24
         return CToss\langle r_{id}, \Sigma \rangle
25
26
27 upon receiving \langle SHARE, id, \sigma_k, instance \rangle from a party p_k for the first time do
         if CShareVerify\langle r_k, k, \sigma_k \rangle = true then
28
               \Sigma \leftarrow \sigma_k \cup \Sigma
29
30
31
```

D Agreement protocol

D.1 Asynchronous Binary Byzantine Agreement (ABBA)

The ABBA protocol allows parties to agree on a single bit $b \in \{0,1\}$ [27, 36, 2]. We have adopted the ABBA protocol from [3], as given in Algorithm 6. The expected running time of the protocol is O(1), and it completes within O(k) rounds with probability $1-2^{-k}$. Since the protocol uses a common coin, the total communication complexity becomes $O(kn^2)$. For more information on how to realize a common coin from a threshold signature scheme, we refer interested readers to the paper [5].

Algorithm 5: Protocol for party p_i

```
1 upon Sequential-ABBA\langle \Sigma, W \rangle invocation do
          l \leftarrow \bar{0}
 2
          b \leftarrow 0
 3
          while b=0 do
                \Sigma_{vote} \leftarrow \{\}
 5
                l \leftarrow l + 1
                party \leftarrow \Sigma \langle l \rangle
                u_l \leftarrow 0
 8
                proposal_l \leftarrow \perp; where 1 \leq l \leq f+1
 9
                if l \in W then
10
                       u_l \leftarrow 1
11
                       m_l \leftarrow W[l]
12
                       multi-cast \langle ID, vote, party, u_l = 1, m_l \rangle
13
                else
14
                       multi-cast \langle ID, vote, party, u_l = 0, \perp \rangle
15
                wait until |\Sigma_{vote}| = n - f
16
17
                if u_l = 1 then
18
                      v \leftarrow \langle 1, m_l \rangle
19
                else
20
                       v \leftarrow \langle 0, \perp \rangle
21
                 \langle b, \sigma \rangle \leftarrow ABBA \langle v \rangle biased towards 1
22
23
          return \langle b, \sigma \rangle
    upon receiving \langle ID, vote, party, u_l, m \rangle for the first time from party p_k do
24
25
          if u_l = 1 then
26
                u_l \leftarrow 1
                m_l \leftarrow m
27
          \Sigma_{vote} \leftarrow \Sigma_{vote} + 1
28
```

Algorithm 6: ABBA: protocol for party p_i

```
1 upon receiving b_{input}, set est_0 := b_{input} and proceed as follows in consecutive epochs, with increasing rounds r:
    do
       multi-cast BVAL_r(est_r)
2
      bin_values_r := \{\}
3
      upon receiving BVAL_r(b) messages from f+1 parties do
4
          if BVAL_r(b) has not been sent then
              multi-cast BVAL_r(b)
6
      upon receiving BVAL_r(b) messages from 2f+1 parties do
7
          bin\_values_r := bin\_values_r \cup \{b\}
8
       while bin\_values_r \neq \phi do
9
          multi-cast AUX_r(v), where v \in bin\ values_r
10
          wait until at least (n-f) AUX_r messages have been received, such that the set of values carried by these
11
            messages, vals are a subset of bin\_values_r.
           S \leftarrow Coin_r.GetCoin() see [5, 16]
12
          if vals = \{b\} then
13
14
              est_{r+1} := b
15
              if (b = S\%2) then
16
                  output b
17
          else
18
              est_{r+1} := S\%2
19
       continue looping until both a value b is output in some round r, and the value Coin_{r'} = b for some round
20
```