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On Identification and Validation of Some Geothermal Models

LARISSA JU. FRADKIN

Physics and Engineering Laboratory, Department of Scientific and Industrial Research Lower Hutt, New Zealand

MICHAEL L. SOREY

U.S. Geological Survey, Menlo Park, California 94025

ALEX MCNABB

Applied Mathematics Division, Department of Scientific and Industrial Research Lower Hutt, New Zealand

Various distributed and lumped parameter models of the Wairakei geothermal reservoir, New Zealand, are discussed within a unifying mathematical framework. The need for proper system identification is emphasized. The best lumped parameter model obtained by system identification techniques is presented and interpreted as a slow drainage model. Validation of different models is conducted by studying their forecasting powers with identified parameter values and by testing these identified values for compatibility with additional data.

INTRODUCTION

With the investigation and exploitation of geothermal systems in various parts of the world during the past 20 years, numerous types of conceptual models have been developed to describe their significant features under natural and exploited conditions. In the case of the Wairakei geothermal field in New Zealand, for which pressure, temperature and discharge records have been available since 1953, a number of models have been tried. On the whole, we find that most of them, when expressed in mathematical form, have yielded results which are reasonably compatible with the observed response at Wairakei, be they zero-, one-, or two-dimensional. In such a situation the choice of model to be used for the prediction of future response or the optimization of field management procedures is not obvious. This is a typical problem encountered in modeling large real world systems, where many different processes are taking place at once and those that dominate the system's response are not easily discernable.

When data on a system's response are available, its statistical analysis can aid in deciding which mathematical equations best describe the system and can also provide estimates of the parameters in these equations. If different conceptual models lead to the same mathematical equations, the interpretation of parameters peculiar to each model may help in selecting the best model. Without such data on the system, the modeler can do little more than rely on experience gained in the analysis of other similar systems.

In this paper we point towards a mathematical framework which may encompass all models of the Wairakei response to production. We give a classification of existing conceptual models and discuss ways to validate them. We concentrate mostly on zero-dimensional models and use system identification techniques to find the best among them. We show that the latter may be interpreted as a slow drainage model in which the upper two-phase portion of the reservoir serves mainly as a source of liquid for the underlying liquid region from which most of the production is obtained. Finally, we compare this model to the existing distributed parameter models.

We emphasize that we are primarily concerned here with the appraisal of currently circulating Wairakei models. We do not claim to give appropriate credit to all of those who contributed to the development of these models. Also, our references usually only indicate that certain ideas had appeared elsewhere before, and their presentation by other authors should not be expected to be similar to ours.

WAIRAKEI RESERVOIR AND DATA DESCRIPTIONS

There is general agreement that hydrothermal phenomena at Wairakei are related to a plume of hot water rising through a cold-water-saturated environment from a magmatic heat source at a depth of the order of 10 km. The areal extent of the hot fluid column near ground surface is about 15 km^2 (a hot water feature of comparable area underlies the Tauhara region to the southeast and is connected hydraulically to the Wairakei field). Most of the production wells drilled at Wairakei range in depth from 500 m to 1500 m, and for the purposes of this paper the Wairakei reservoir is defined as the upper portion of the confined hot column between these limits. Also, the mean land surface (MLS) is introduced as a reference level 500 m above the mean sea level.

Prior to 1953 the vertical pressure gradient in the reservoir was slightly in excess of hydrostatic, producing an upflow of liquid to feed the natural surface discharge features [*McNabb and Dickinson*, 1975]. A two-phase zone in which the water was at its boiling point existed in the plume from about MLS 500 m below the surface up to MLS 100–200 m. Except near natural discharge vents the top layer appeared to be dominated by cool groundwater. By 1958 a steam zone had formed near MLS 400 m over portions of the reservoir, and by 1962 it had extended over most of the reservoir area at this level [*Grant*, 1979]. At the natural discharge vents liquid has been replaced by vapor. The depth to which two-phase conditions extend at present varies with location, but in general it has increased to MLS 600 m [*Donaldson and Grant*, 1980].

Production of hot water and steam from the Wairakei reservoir since 1953 has been obtained from a sequence of Pleistocene volcanic rocks consisting of pumice breccias, welded tuffs, and flows. Most of the permeable features tapped by wells are fault and fissure zones or contact zones between different volcanic units. An extensive mudstone layer is interbedded in these volcanic rocks at a depth of about 150 m, but analyses of pressure data before and after exploitation indicate that it does not have a significant influence on field behaviour [*McNabb and Dickinson,* 1975]. A significant feature of the response to fluid extraction at Wairakei is that differences in pressure between wells in various parts of the reservoir have remained significantly less than the average decline in pressure with time [*Bolton,* 1970]. Two factors could be responsible for this uniform nature of the pressure response. The first is a high degree of lateral permeability due to fractures intersecting wells at various levels. The second is the existence of single-phase (liquid) conditions in the deeper parts of the reservoir leading to much faster rates of diffusion of pressure changes across the reservoir.

The pressure history smoothed by *Bolton* [1970] has in the past been used for model verification. To obtain this curve Bolton first has averaged reservoir pressures for each month between 1959 and 1968 based on measurements from a group of 50 deep wells within what he called the Western Production Area. The pressures averaged in this way were either measured pressures at MLS 730 m or pressure values extrapolated by assuming that the hydrostatic conditions prevailing in shallow bores gave a meaningful extension. It is interesting that for any particular month the variations in pressures between wells at this level did not exceed 2 bars. We extended the representative pressure history up to year 1977. The resulting history, as well as the smoothed version presented by Bolton, can be seen in Figure 1.

Classification of Reservoir Models

It is convenient to classify conceptual geothermal models on the basis of assumed reservoir properties such as reservoir geometry, hydrologic character of the reservoir rock, and state of the reservoir fluid. The models of Wairakei developed to date differ slightly in their definition of the reservoir boundaries, and all treat the reservoir rock as a permeable medium.

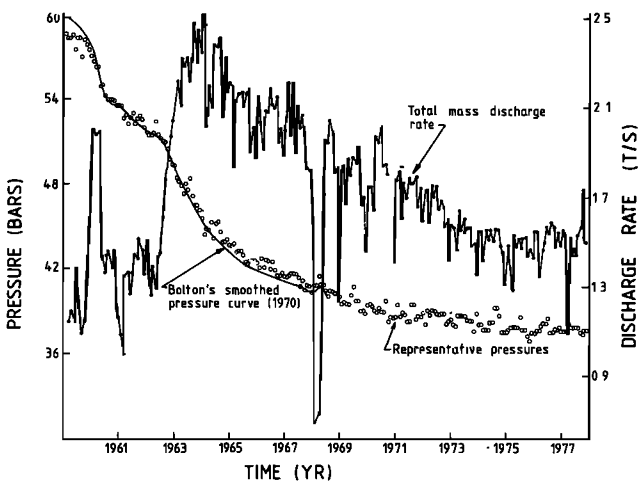

Fig. 1. Wairakei data on total discharge rate and representative pressures, with *Bolton*'s [1970] smoothed version of pressure history.

(Some models, such as the slow drainage model considered below, treat parts of the reservoir as a fractured permeable medium.) The main difference between these models then lies in the assumed (or deduced) state of the reservoir fluid. We concentrate here on that aspect.

The reservoir is correspondingly termed liquid, two-phase, or vapor if it is saturated with liquid, water and steam mixture, or vapor only. The reservoir is said to be of the mixed type if the state of liquid varies with location. It is not always easy to determine the extent to which two-phase conditions develop within a reservoir during exploitation. This is why the Wairakei reservoir has been modeled as two-phase by some and as liquid or mixed by others. The mathematical models of different reservoir types vary markedly in their complexity. We demonstrate this by considering two ideal cases of two-phase and liquid reservoirs.

If the reservoir is two-phase, then assuming the rock and fluid in local thermal equilibrium, the ground movement, the capillary pressures, the pressure work, and viscous dissipation negligible, the local mass and energy conservation equations are

$$\begin{aligned}\frac{\partial}{\partial t}(\phi\rho) &= \nabla\left[k\left(\frac{k_{rl}\rho_l}{\mu_l}+\frac{k_{rv}\rho_v}{\mu_v}\right)\nabla p\right] - \nabla\left[kg\left(\frac{k_{rl}\rho_l^2}{\mu_l}+\frac{k_{rv}\rho_v^2}{\mu_v}\right)\right] - q' \\ \frac{\partial}{\partial t}[(1-\phi)\rho_s h_s + \phi\rho h_f] &= \nabla\left[k\left(\frac{k_{rl}h_l\rho_l}{\mu_l}+\frac{k_{rv}h_v\rho_v}{\mu_v}\right)\nabla p\right] - \nabla\left[kg\left(\frac{k_{rl}h_l\rho_l^2}{\mu_l}+\frac{k_{rv}h_v\rho_v^2}{\mu_v}\right)\right] - \nabla(\xi\nabla T) - h_q q'\end{aligned} \tag{1}$$

with p and h_f dependent variables, where $\rho_f = S\rho_l + (1-S)\rho_v$ is the density of the fluid mixture in place, $h_f = (S\rho_l h_l + (1-S)\rho_v h_v)/\rho_f$ is the enthalpy of the fluid mixture in place, and the discharge rate q' is a distributed point sink. Other symbols used above are defined in the section on notations. These equations were derived essentially in this form by Elder and Kerr in 1954 (for details, see *Faust and Mercer* [1979]; also see *Garg and Pritchett* [1977] for a derivation of similar equations written in terms of density and internal energy as dependent variables).

We introduce here a convenient classification of mathematical models according to their complexity. Namely, when dealing with a full system of conservation equations (mass, energy, etc.) we shall call it a general model. On the other hand, when these equations are weakly coupled and the system may be approximated by one with a lesser number of equations, we shall call the resulting model reduced. When the system's dependent variables vary not only with time but with position too, we shall call the model distributed parameter. When dependent variables vary with time only, we shall call the corresponding zero-dimensional model lumped parameter. It is unfortunate that these commonly used terms refer to dependent variables as 'parameters.' In all other instances, only equation coefficients are called parameters here.

In the above terminology, system (1), completed with appropriate constitutive equations for $h_l(p)$, $h_v(p)$, $S(p, h_f)$, $T(p, h_f)$, $\rho_l(p, h_f)$, $\rho_v(p, h_f)$, $\rho(p, h_f)$, $\mu(T)$, $\phi(p)$, $k_{rl}(S)$, $k_{rv}(S)$ and initial and (possibly moving) boundary conditions, represents a general distributed parameter model of a geothermal reservoir (with negligible amounts of dissolved solids and gas as at Wairakei). The lumped parameter approximation is possible if the state of the reservoir may be described by a few numbers (which determine dependent variables). For example, assume that the pressure in the reservoir is hydrostatic due to high permeability and low viscosity and that it is also uniform across the reservoir (both conditions are approximately satisfied at Wairakei). In this case global (integrated over the reservoir volume) conservation equations constitute a general but lumped parameter model with representative pressure and enthalpy as dependent variables (functions of time only).

If a reservoir remains liquid during the exploitation period, it still may be necessary to use the general mathematical model for its description, distributed or lumped depending on the character of pressure variation across the field. However, in this case further simplification is possible if temperatures do not change significantly over the production period of, say, 20 years, as temperature measurements in deeper wells suggest is the case with Wairakei [*Bolton,* 1970]. Both local mass and energy conservation equations reduce then to one equation

$$\frac{\partial \phi \rho_l}{\partial t} = \nabla\left(\frac{k\rho_l}{\mu_l}\nabla p\right) - \nabla\frac{k\rho_l^2 \mathbf{g}}{\mu_l} - q' \tag{2}$$

Suitable constituent equations for $\rho_l(p)$, $\phi(p)$, and $\mu_l(T)$ and boundary and initial conditions complete this reduced distributed parameter model. Assuming further a horizontally uniform pressure drop and hydrostatic conditions vertically leads to the reduced lumped parameter model. Conceptual models for mixed reservoirs are a bit more involved, as they have to tie up different portions of the reservoir together; however, the corresponding mathematical models can still be classified in the manner described above.

REDUCED LUMPED PARAMETER MODELS OF WAIRAKEI

Wairakei reservoir is believed to be of mixed type (mainly liquid but two-phase at the top) by many modelers. Indeed, it must be liquid at deeper levels, because pressures there are significantly above saturation pressures and also because the areal pressure distribution is uniform (i.e. pressure disturbances propagate quickly) [*Elder,* 1966; *Bolton,* 1970]. The existence of the two-phase zone at the top is deduced from the presence of saturation conditions at the shallow levels and from the fact that total discharge enthalpy of the reservoir rose by 5% for 10 years [*McNabb,* 1975]. The fact that production is still coming mostly from the liquid zone is supported by the lack of significant variation in the total discharge enthalpy [*Donaldson and Grant,* 1980; *Pritchett et al.,* 1979]. Presence of the two-phase conditions in the upper portion of the reservoir suggests the choice of the general reservoir model for its mathematical description. However, further plausible assumptions on the nature of the reservoir lead to a simpler mathematical setup.

First of all we define the reservoir here as a vertical cylinder extending from the initial bottom level of the boiling surface, MLS 500 m (later referred to as $z = D$), down to the level MLS 1500 m ($z = 0$). The area of the reservoir cross section is 15 km^2.

We assume that vaporization effects, in particular changes in the mass of steam within the reservoir, are negligible. We also assume that the recharge is coming mostly into the liquid portion of the reservoir [*Marshall,* 1966]. For these conditions, integration of the mass conservation equation over the volume of the reservoir leads to the following global liquid mass conservation equation:

$$\dot{M}_l = r_l - q \tag{3}$$

where $M_l = A \int_0^D \phi\rho_l \, dz$ is the total liquid mass, $r_l = \int (k\rho_l/A'\mu_l)\,(\nabla p - \rho_l \mathbf{g})\, da'$ is the total liquid recharge, A' being the area of recharge, and $q = \int_v q' \, dv$ is the total mass discharge. Assuming further that porosity ϕ and liquid density ρ_l are constants and that saturation S varies with depth (and time) only, the total liquid mass becomes

$$M_l = A\phi\rho_l \int_0^D S\, dz \tag{4}$$

Finally, assuming that pressures below shallow levels are near hydrostatic (due to high permeability and low viscosity) and temperatures do not change quickly (as mentioned above), the total recharge rate can be written as [*McNabb,* 1981]:

$$r_l = \gamma(p_0 - p) \tag{5}$$

where γ is the recharge coefficient and p_0 is the initial reservoir pressure at MLS 730 m. Equation (5) gives the rate of recharge from the surrounding cold groundwater system induced by changes in pressure in the hot liquid portion of the reservoir. Below we will make use of the convenient approximation $\gamma = 0.4\pi Rk\rho_l/\mu_l$ obtained by *Grant* [1977].

Having done this preparatory work, we consider now three representative lumped parameter models of the response of the Wairakei reservoir to production. They differ in the choice of the factors that dominate this response.

Assuming that the reservoir remains liquid during the production period and that the water decompression is the dominant physical factor, it follows from (4) that the storage term in Equation (3) becomes

$$\dot{M}_l = AD\phi\dot{\rho}_l \tag{6}$$

By using the compressibility $\chi = (1/\rho_l)(\partial\rho_l/\partial p)_T$ and substituting (6) in (3), the following mathematical model for a liquid reservoir is obtained:

$$\dot{y} = -a^{(0)}y + b^{(0)}u \tag{7}$$

where $y = p_0 - p$, $u = q$, and

$$a^{(0)} = \gamma(AD\chi\phi\rho_l)^{-1} \qquad b^{(0)} = (AD\chi\phi\rho_l)^{-1} \tag{8}$$

Equation (8) represents the water decompression model of Wairakei [*Whiting and Ramey,* 1969].

Now let the pressure p at the reference level MLS 730 m drop by the amount δp at $z = 0$ during the time interval $(t, t + \delta t)$, and let pressures below the boiling surface be close to hydrostatic; then the boiling surface falls a distance $\delta\eta = (1/\rho_l g)\,\delta p = (1/\rho_l g)\,\dot{p}\delta t$. The derivative of the function $\eta(t)$ is therefore

$$\dot{\eta} = \frac{1}{\rho_l g}\dot{p} \tag{9}$$

By introducing a new variable

$$\tau = \eta^{-1}(z) \tag{10}$$

we obtain

$$\dot{M}_t = \frac{A\phi}{g}\dot{p}[1 - S(t, \eta+)] - \frac{A\phi}{g}\int_0^t \frac{\partial S}{\partial t}(t, \tau)\frac{dp}{d\tau}\,d\tau \qquad (11)$$

where $S(t, \tau) \equiv S(t, \eta(\tau))$ and τ has the meaning of the moment in time when the boiling surface reaches the level z.

Assuming that drainage is instantaneous, that is

$$\begin{aligned} S(t, z) &= 0 \qquad \eta \le z \le D \\ S(t, z) &= 1 \qquad 0 \le z \le \eta \end{aligned} \qquad (12)$$

the following model is obtained:

$$\dot{y} = -a^{(1)}y + b^{(1)}u \qquad (13)$$

where

$$a^{(1)} = \gamma\frac{g}{A\phi} \qquad b^{(1)} = \frac{g}{A\phi} \qquad (14)$$

Equations (13) and (14) represent the instantaneous drainage model of Wairakei [*McNabb,* 1975; *Grant,* 1977; *Robinson,* 1977].

Alternatively, if drainage from the two-phase zone is not instantaneous, $S = S(t - \tau)$ and the medium is considered as fractured permeable, some admittedly rough approximations lead to what we term the slow drainage model [*McNabb,* 1975]:

$$\dot{y} = a^{(2)}y + b^{(2)}u + c^{(2)}\dot{u} \qquad (15)$$

where

$$\begin{aligned} a^{(2)} &= \frac{a^{(1)}\tau_0^{-1}}{a^{(1)} + (1 - S_0)\tau_0^{-1}} \\ b^{(2)} &= \frac{b^{(1)}\tau_0^{-1}}{a^{(1)} + (1 - S_0)\tau_0^{-1}} \\ c^{(2)} &= \frac{b^{(1)}}{a^{(1)} + (1 - S_0)\tau_0^{-1}} \end{aligned} \qquad (16)$$

(See the notation section and the appendix.)

IDENTIFICATION AND VALIDATION OF REDUCED LUMPED PARAMETER MODELS BY DATA ANALYSIS

There are two sides to every model, mathematical and conceptual. Correspondingly, model validation should be carried out both by statistical means and by parameter interpretation.

Identification and Validation by Statistical Means

The first problem with any mathematical model is to see how well it matches the data. While doing this one usually simultaneously estimates the corresponding parameters. In the case of lumped parameter models, regression is often used for curve-fitting and parameter estimation [*Whiting and Ramey,* 1970; *Bolton,* 1970; *Grant,* 1977]. Sometimes the so-called direct approach is also employed [*McNabb,* 1975]. It involves fitting the representative pressure history by trial and error. We remind the reader here that, whatever the approach used, the goodness of fit may prove deceiving. It may happen that there exists a large domain of parameter values in some subspace of parameter space for which the data is well-matched. Consider, for example, equations like (13) with two parameters 'a' and 'b'. The standard sensitivity analysis shows that with 'b' fixed, small changes in 'a' lead to significant deterioration in the fit; the estimates of 'b' also appear to be rather sharp. However, very substantial changes in 'a' do not change the fit much if 'b' is changed appropriately.

Thus identified values may be biased, and this may be hard to detect. Such a situation can be expected when the nature of the data is not as assumed in the derivation of the chosen estimation algorithm. For example, analysis of the Wairakei data showed the presence of a linear feedback problem [*Fradkin,* this issue]. Indeed, the total discharge rate becomes linearly dependent on the (representative) reservoir pressure when the field operators leave the number of discharging wells unchanged for a while (as they did, for example, after 1968). This leads to an additional linear relationship between the discharge rate and the reservoir pressure of the form

$$q = \alpha p + \beta \qquad \alpha, \beta = \text{const} \qquad (17)$$

It is known that in the presence of linear feedback, arbitrarily large biases in parameter estimates can be introduced while the fit remains very good [*Johnston,* 1963]. Feedback and other identifiability problems account for the fact recognized by Wairakei modelers, that analysis of different parts of the Wairakei data using (13) often leads to significantly different results (different pairs (a, b) produce satisfying fit).

It is very useful, therefore, to compare the performance of various parameter estimation techniques derived under different assumptions on the nature of data. This allows us to form an opinion as to the nature of the analyzed data and hence helps to choose the best curve-fitting and estimation technique. A few simple algorithms, for example, those described by *Young* [1974], may be used to accomplish this task. The PL1 program SYSID, written for this purpose, also allows one to choose the best type of equation (model structure) within a specified class:

$$y_i = -a_1y_{i-1} - \cdots - a_ny_{i-n} + b_0u_i + \cdots + b_mu_{i-m} \qquad (18)$$

where y, u are functions of the measured variables, i is the time index, and lags n, $m \neq 0$ specify the order of each equation. We notice that the discretized form of (13) belongs to this class, with b_0 equal to zero and n, m equal to unity. To summarize, the data analysis problem may be treated as one of system identification, i.e., of finding the best type of equation describing the relationships between measured variables and the best parameter estimators without conceptualizing the system first. In this way the best of the existing mathematical models or even a new, better one may be identified.

The first part of this problem (finding the best equation) was already attacked by *Bolton* [1970]. A full analysis with additional data was done by *Fradkin* [this issue]. The best of all simple mathematical models proved to be the equation

$$\dot{y} = a^{(2)}y + b^{(2)}u + c^{(2)}\dot{u} \qquad (19)$$

It coincides with (15) (this is why the constitutive equation (A3) was chosen in the appendix and not (A2)). Regression was shown to be an appropriate technique for estimation of parameters $a^{(2)}$, $b^{(2)}$, $c^{(2)}$ provided that one integrated and then discretized (19) and carefully chose the periods for analysis afterwards (so as to avoid the ones during which only one feedback relationship, like (17), might have been operating) [*Fradkin,* this issue]. The corresponding fit is demonstrated in Figure 2.

In order to validate a mathematical model, it is necessary to test whether it has any forecasting power. For this purpose only part of the available data should be used for parameter estimation and another part for forecasting. The corresponding tests were carried out by *Fradkin* [this issue] and the model (19), properly identified, was shown to possess this forecasting power.

Validation by Parameter Interpretation

It is important to see whether the interpretation of parameters according to a conceptual model leads to any meaningful values for physical parameters. By using an equation like (7) or (13), one identifies the following parameter estimates:

$$a = (0.02 \pm 0.01)(\text{month}^{-1}) \qquad b = (0.78 \pm 0.4)(\text{m}^{-1}\text{s}^{-2}) \tag{20}$$

(The unorthodox units for a are due to the 1-month time step.) Interpretation of these values in terms of the decompression model (equations (7) and (8)) leads to the values

$$A = (700 \pm 380)\ \text{km}^2 \qquad k = (6 \pm 3) \times 10^{-15}\ \text{m}^2 \tag{21}$$

(assuming that $\phi = 0.2$). The area value is unreasonably large, as that suggested by the resistivity survey is 15 km^2.

Interpretation of the parameters in (20) in terms of the instantaneous drainage model (13) and (14) leads to the values

$$\phi = 0.09 \pm 0.05 \qquad k = (45 \pm 20) \times 10^{-15}\ \text{m}^2 \tag{22}$$

(assuming that $A = 15$ km^2). These values are not unreasonable, considering that the relatively low values of porosity reflect rapid drainage only in the fractures. However, the corresponding recharge coefficient $\gamma = 0.001$ m^{-1} s yields estimates of the mass of fluid recharging the reservoir which are in poor agreement with estimates based on the repeat gravity measurements discussed below.

By using (19), one identifies the following parameter values:

$$a^{(2)} = (0.01 \pm 0.005)(\text{mo}^{-1}) \qquad b^{(2)} = (0.39 \pm 0.2) \times 10^{-5}(\text{m}^{-1}\ \text{s}^{-2}) \qquad c^{(2)} = (1.3 \pm 1.2) \times 10^{2}(\text{m}^{-1}\ \text{s}^{-1}) \tag{23}$$

Although the estimate of $c^{(2)}$ itself is very unreliable, the enhancement of the model provides us with estimates (23) of parameters a and b superior to those in (20) both from statistical [*Fradkin*, this issue] and conceptual (see discussion on gravity data below) points of view.

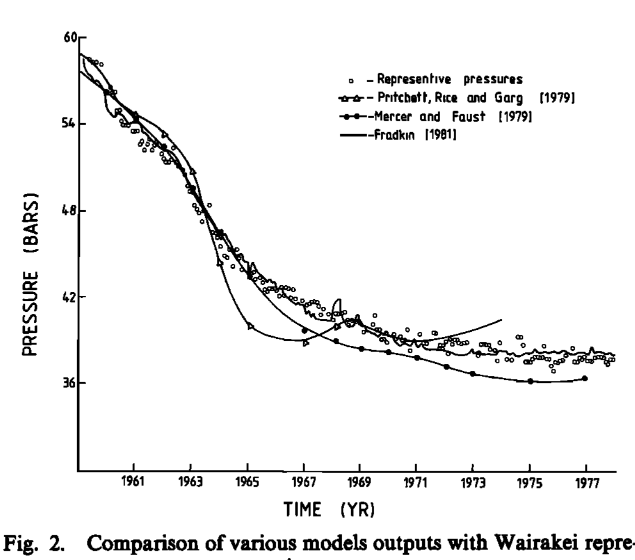

Fig. 2. Comparison of various models outputs with Wairakei representative pressures.

Interpretation of parameters (23) in terms of the slow drainage model (15) and (16) (see also (A4)) leads to the following field parameter values:

$$\phi = 0.2 \pm 0.1 \qquad k = (27 \pm 15) \times 10^{-15}\ \text{m}^2 \qquad \eta_0 = (20 \pm 20)\text{m} \tag{24}$$

(assuming that $A = 15$ km^2, $k_m = 10^{-15}$ m^2, and $S_0 = 0.3$). This porosity estimate is in good agreement with average values from laboratory tests on cores. The macroscopic permeability estimate is also consistent with values for the field permeability determined from other independent considerations [*McNabb et al.*, 1975]. The characteristic length estimate implies that the average block size of 50 m used by *McNabb* [1975] previously is a very rough but acceptable approximation (also see discussion on Tauhara below).

An additional constraint on the validity of the various Wairakei models is provided by the repeat gravity measurements of *Hunt* [1977]. Ratios of total mass recharge to total mass produced for various periods based on the gravity data are:

Time Period	Ratio
1958–1961	0.30 ± 0.15
1961–1967	0.35 ± 0.15
1967–1974	0.90 ± 0.15

On the field scale, differences between recharge and production represent changes in storage within the reservoir, due mainly to boiling and liquid drainage. Corresponding estimates of the ratio of recharge to discharge based on matching the slow drainage model to the Wairakei pressure history are:

Time Period	Ratio
1958–1961	0.28 ± 0.15
1961–1967	0.48 ± 0.25
1967–1974	0.80 ± 0.4

Agreement with the gravity data is considered good, although there are two complicating factors which should be noted. First, possible changes in the mass stored within the shallow groundwater system above the reservoir in response to rainfall variations were not accounted for in the gravity work. Second, the gravity surveys did not extend over the Tauhara area and hence could not detect changes in mass storage at Tauhara in response to production at Wairakei. Given the similarities in initial reservoir pressure and temperatures between the two areas, and the fact that pressure declines at Tauhara range from 70 to 90% of those at Wairakei, significant storage changes should have occurred at Tauhara. Thus the mass contribution from Tauhara to the production at Wairakei could be substantial and would be interpreted from the gravity data as an apparent recharge to Wairakei.

The effects of the Tauhara response on the Wairakei pressure history and hence on the results of models fitting that data are not easily discernable. With the lumped parameter models one could consider both areas as part of one larger reservoir ($A = 30$ km^2) and adjust the resultant parameter estimates accordingly. (This would halve the porosity and hence improve estimate (24) of η_0.) This of course involves neglecting the effects of the resistance to fluid transmission between the two areas. Alternatively, we could assume that the effects of Tauhara are analogous to that of the surrounding cold groundwater system in providing recharge and are therefore accounted for in the value of recharge coefficient γ, estimated by fitting the pressure data. Additional study of the fluid state within the Tauhara reservoir and its changes with time is needed to resolve the matter.

VALIDATION OF CURRENT GENERAL DISTRIBUTED PARAMETER MODELS

Distributed parameter model validation should follow the same steps taken for lumped parameter models; namely, after system identification an analysis of forecasting powers should be performed and followed by tests for compatibility of the identified parameter values with additional data on the reservoir. No proper system identification procedure has been developed for the analysis of distributed parameter models. The curve-fitting by the so-called direct approach is currently in use. It involves solving the initial boundary value problem, described in the section on classification of the reservoir models, for pressure and discharge enthalpy, assuming that all the constituent equations and parameters are known. If solutions do not fit the measured pressure and enthalpy history, adjustments of parameters are made by trial and error method until visually satisfying fits are obtained.

Representative Wairakei reservoir pressures simulated this way by *Mercer and Faust* [1979] and *Garg et al.* [1979] are shown in Figure 2. Mercer and Faust treat the reservoir as a horizontal layer of variable thickness containing both liquid and two-phase regions, and allow for recharge vertically from above and below. Garg et al. treat Wairakei in vertical cross section from land surface to a depth of 3 km, with mixed fluid conditions and recharge from below and from the sides of the model (simulating inflow from Tauhara). As seen in Figure 2, the slow drainage model fits the representative pressure history better than do either of the distributed parameter models, although simulated pressures from the model of Garg et al. fall below the observed pressures for the Western Production Area, in part because calculated pressures from grid blocks in the Eastern Production Area were averaged in. In neither of the distributed parameter models is the recovery following the partial shutdown in 1968 simulated. Obviously, these models do not possess forecasting power.

The average measured enthalpy of the Wairakei discharge rose from an initial value of 1100 kJ/kg (near that of liquid water at 253°C) to about 1160 kJ/kg by 1967 and fell slightly thereafter [*Donaldson and Grant*, 1980]. Thus most of the production has come from single-phase portions of the reservoir. An earlier distributed parameter modeling attempt by *Pritchett et al.* [1976] appeared to greatly overestimate the discharge enthalpy history, based on published saturation distributions, although no data on discharge enthalpy was given. The more recent model by *Garg et al.* [1979] presents a plot of simulated discharge enthalpy which is closer to the observed data but overestimates the rise in average enthalpy by a factor of about 2. Results from the *Mercer and Faust* [1979] model, although expressed in terms of mixture enthalpy h_f, indicate that the corresponding average discharge enthalpy history would be quite dissimilar to the observed one. This in part is a consequence of their vertical-averaging procedures.

For comparison with the recharge history based on the gravity data we have calculated the total recharge to the produced mass ratios from published results for *Garg et al.* [1979] model:

Time Period	Ratio
1958–1961	0.27
1961–1967	0.36
1967–1974	0.72

Agreement with the gravity data is good. It is impossible to cite the corresponding estimates for the *Mercer and Faust* [1979] model because their recharge includes both inflows from above and below, and their model does not allow us to estimate the recharge in the sense of this paper.

To summarize, both of the considered distributed parameter models need further improvement. We make the general comment, however, that when there exist lumped parameter models adequate for description of system input-output relationships, they should be considered superior to even perfect distributed parameter models, as they are parsimonious (have the smaller number of parameters).

CONCLUSIONS

This study shows that a properly identified lumped parameter model of such a system as the Wairakei geothermal reservoir may be quite adequate for description of the system's response. Indeed, the results of the model validation carried out in this paper are consistently in favor of the slow drainage model. This is so firstly from the statistical point of view, as the model matches the representative pressures well and also possesses some forecasting power. Secondly, the model is good from the physical point of view, as it supplies us with good estimates of the field parameters such as porosity, permeability, and the recharge-storage ratio. There is no denying that the slow drainage model offers a significantly simplified conception of the nature of the reservoir; however, it serves its purpose of describing the system as an input-output relationship and illustrates that the two-phase portions of the reservoir are better understood not as a permeable but as a fractured permeable medium.

In cases of geothermal fields with nonuniform horizontal pressure distribution the application of more complicated system identification methods to distributed parameter models may be unavoidable. However, the experience gained in studying Wairakei still may prove instructive in both mathematical and conceptual modeling of geothermal systems.

APPENDIX

Derivation of the Slow Drainage Model

When drainage from the two-phase zone is not instantaneous we assume the saturation continuous at the boiling surface. If, in addition, $S = S(t - \tau)$ then the integration of (11) by parts leads to the following model:

$$y = cu + d \int_0^t \frac{d^2S(t-\tau)}{d^2\tau} y(\tau)\, d\tau \qquad \text{(A1)}$$

where

$$c = \frac{b^{(1)}}{a^{(1)} + (\partial S/\partial\tau)(t,\tau)|_{\tau=t}} \qquad d = \frac{1}{a^{(1)} + (\partial S/\partial\tau)(t,\tau)|_{\tau=t}}$$

This model obviously requires knowledge of liquid saturation above the boiling surface as a function of time and depth. In order to derive this function, it is convenient to conceptualize the upper portion of the reservoir as fractured permeable, i.e., as consisting of permeable blocks surrounded by fractures (following *McNabb* [1975]). Then after the fall of the boiling surface the void space in fractures fills with steam released by vaporization. The picture is simplified further by assuming that the blocks are in the shape of parallelepipeds of height L and that drainage from a block starts when it is completely exposed, i.e., at the moment when the boiling surface $\eta(t)$ reaches the bottom of the block. By this time fractures have drained and the pressures in them become vapor-static. The time constant determining the propagation of the pressure disturbances in liquid (the state of water within the block immediately after the fall of the boiling surface) is small, so that within days vapor static conditions are established even inside of wide blocks. The vaporization caused by drainage of the liquid from the blocks leads to some cooling and a pressure drop that allows us to consider the pressure within the blocks to be roughly vapor static [*McNabb,* 1981]. Then, by assuming both temperature and pressure to be constant in the upper portion of the reservoir, the reduced (very rough) model of the state of fluid inside each block becomes as follows:

$$\phi\rho_l \frac{\partial S}{\partial t} = -\frac{k_m}{\mu_l}\frac{\partial}{\partial z}(k_{rl}(S)\rho g)$$

where k_m is a microscopic permeability of the medium inside of the blocks. Another (macroscopic) permeability k was implied in (5)–(16) as the fractures are also considered as the 'pores' of the medium below the boiling surface.

A constitutive equation for $k_{rl}(S)$ is needed to solve this equation. *McNabb* [1975] used the following one:

$$k_{rl} = \left(\frac{S - S_0}{1 - S_0}\right)^2 \tag{A2}$$

where S_0 is the residual saturation. For reasons explained in the section on identification and validation of lumped parameter models, we use instead the equation

$$k_{rl} = \frac{S - S_0}{1 - S_0} \tag{A3}$$

Finally, the 'initial' and boundary conditions for this model are

$$S(\tau, \eta(\tau)) = 1$$

$$S(\infty, z) = S_0$$

The solution of this problem is as follows:

$$S = (1 - S_0)\exp\left(-\frac{(t-\tau)}{\tau_0} + \frac{(z - \eta(\tau))}{\eta_0}\right) + S_0$$

where

$$\eta_0 = \frac{k_m \rho_l g}{\phi\mu_l(1 - S_0)}\tau_0 \tag{A4}$$

and η_0 and τ_0 are the characteristic length and time of the fractured permeable medium respectively.

If it is true that the length of the block L is smaller than characteristic length η_0, then the saturation may be considered as being constant with depth on a macroscopic scale:

$$S = (1 - S_0)\exp\left(-\frac{t-\tau}{\tau_0}\right) + S_0 \tag{A5}$$

The substitution of (A5) into (A1) leads to the slow drainage model (15).

Limitations of the Slow Drainage Model

One of the limitations of the slow drainage model is the assumption that vaporization effects are negligible. In the case of Wairakei, if the number of discharging bores had been increased after 1968, the decline in reservoir pressure would have been greater and a larger portion of the production zone would have become two-phase. In this case, more energy would have been mined from the reservoir itself (with a liquid production zone, energy is mined from the sides), reservoir temperatures would have fallen significantly with time, and hence more complicated conservation equations would be required to model he system. Up until the present, at least, these considerations do not appear to be crucial, as the simple models like (13) or (15) work well.

New complications are expected with the encroachment of cold groundwater from above, induced by pressure declines in the shallow portion of the reservoir. A few shallow wells have apparently stopped producing for this reason [*Hitchcock,* 1978]. At the present time there also appears to be a gradual falloff in the output from the deeper bores because of cooling of their liquid feeds at a time when reservoir pressures have stabilized [*Hitchcock,* 1978]. *Grant* [1978] relates this to two-phase effects at shallow levels. While such effects are not accounted for directly in the slow drainage model, their influence might be expected to show up in terms of changes in the parameter values obtained in the data analysis procedures.

NOTATION

- a, b, c model parameters, indexed according to model.
- A reservoir cross-sectional area.
- A' area of recharge.
- D reservoir depth.
- g gravity acceleration.
- h enthalpy.
- k macroscopic permeability.
- k_m microscopic permeability.
- L block size.
- M_l mass of the reservoir liquid.
- p representative reservoir pressure at MLS 735 m.
- p_0 initial reservoir pressure at MLS 735 m.
- q total mass discharge rate from the wells.
- q' discharge rate as a function of location.
- r recharge rate.
- R reservoir radius.
- S volumetric saturation.
- S_0 residual saturation.
- t time.
- T temperature.
- V reservoir volume.
- u_i model input at the moment i.
- y_i model output at the moment i.
- z variable depth.
- α, β feedback parameters.
- γ recharge coefficient.
- η position of the boiling surface.
- η_0 characteristic length of the fractured permeable medium.
- μ dynamic viscosity.
- ξ conduction-dispersion coefficient.

ρ density.
τ the moment of time the boiling surface reaches level z.
τ_0 characteristic time of the fractured permeable medium.
ϕ porosity.
χ compressibility.
f as a subscript, fluid.
i as a subscript, time index.
m, n as a subscript, maximum time lags for input and output, respectively
l as a subscript liquid water.
r as a subscript, relative.
s as a subscript, solid rock matrix.
v as a subscript, vapor.

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