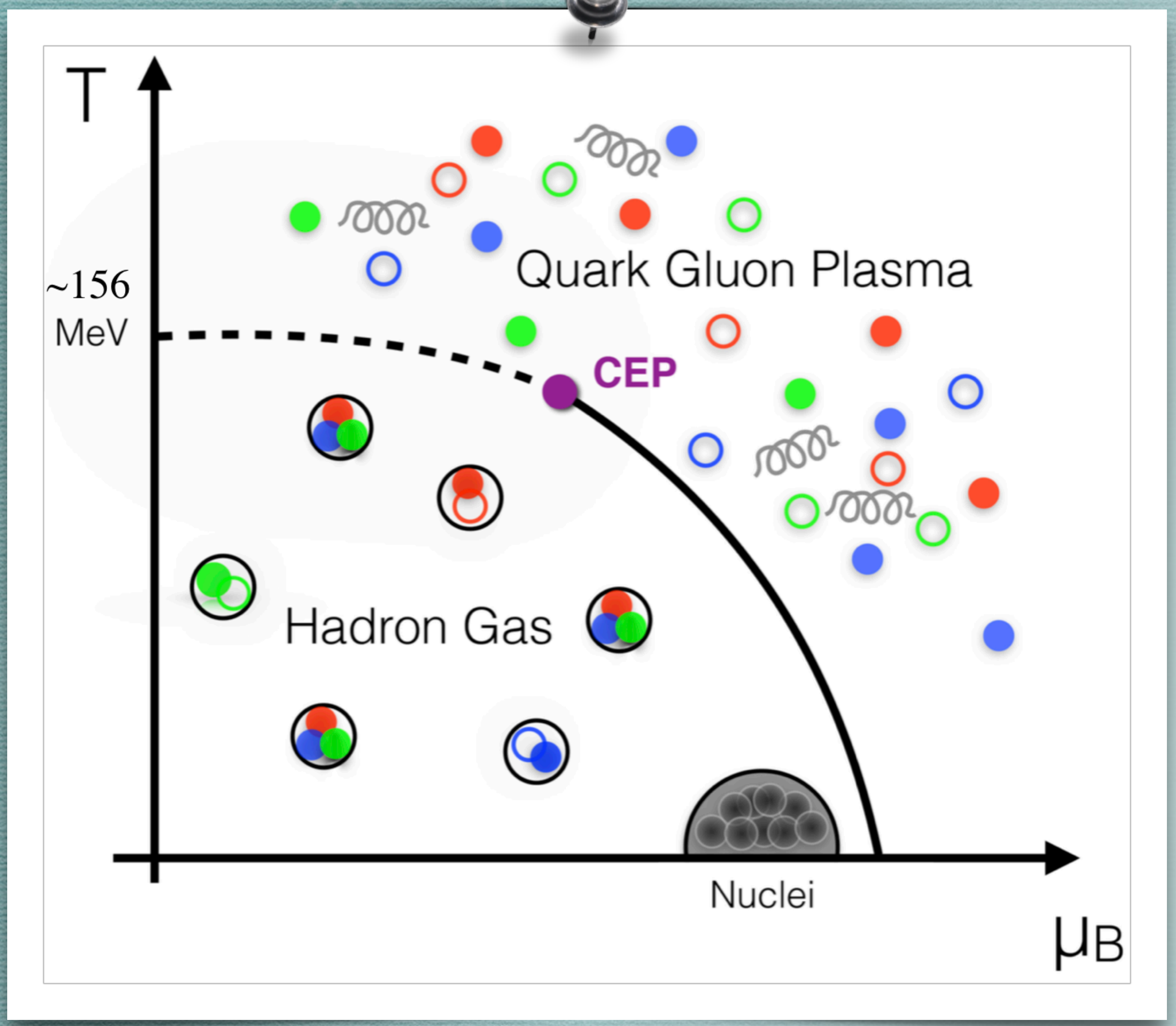
Nuclear Science Computing Center at CCNU

New Developments in Lattice QCD

Heng-Tong Ding (丁亨通)

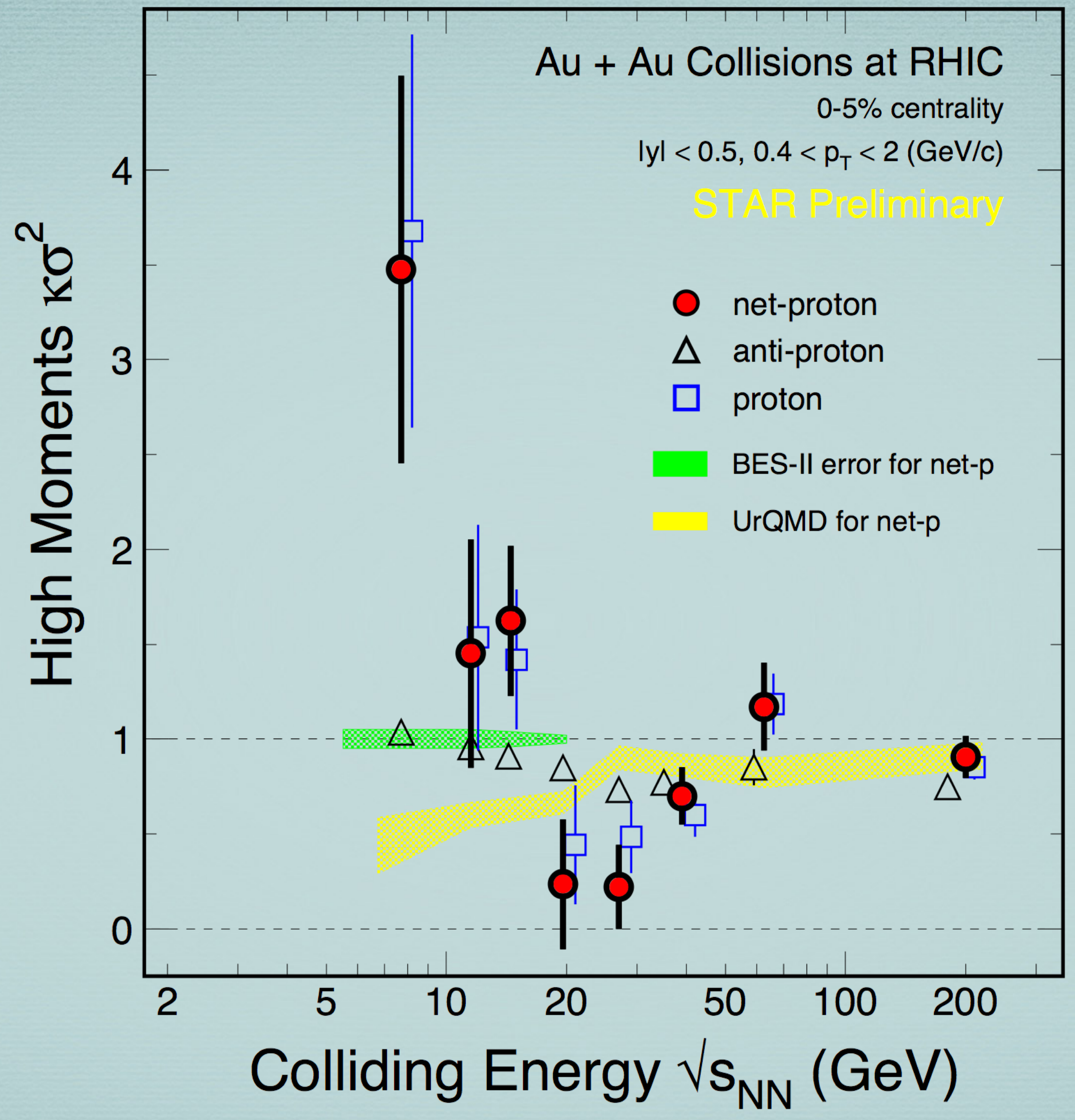
Central China Normal University (华中师范大学)

Search for Criticality



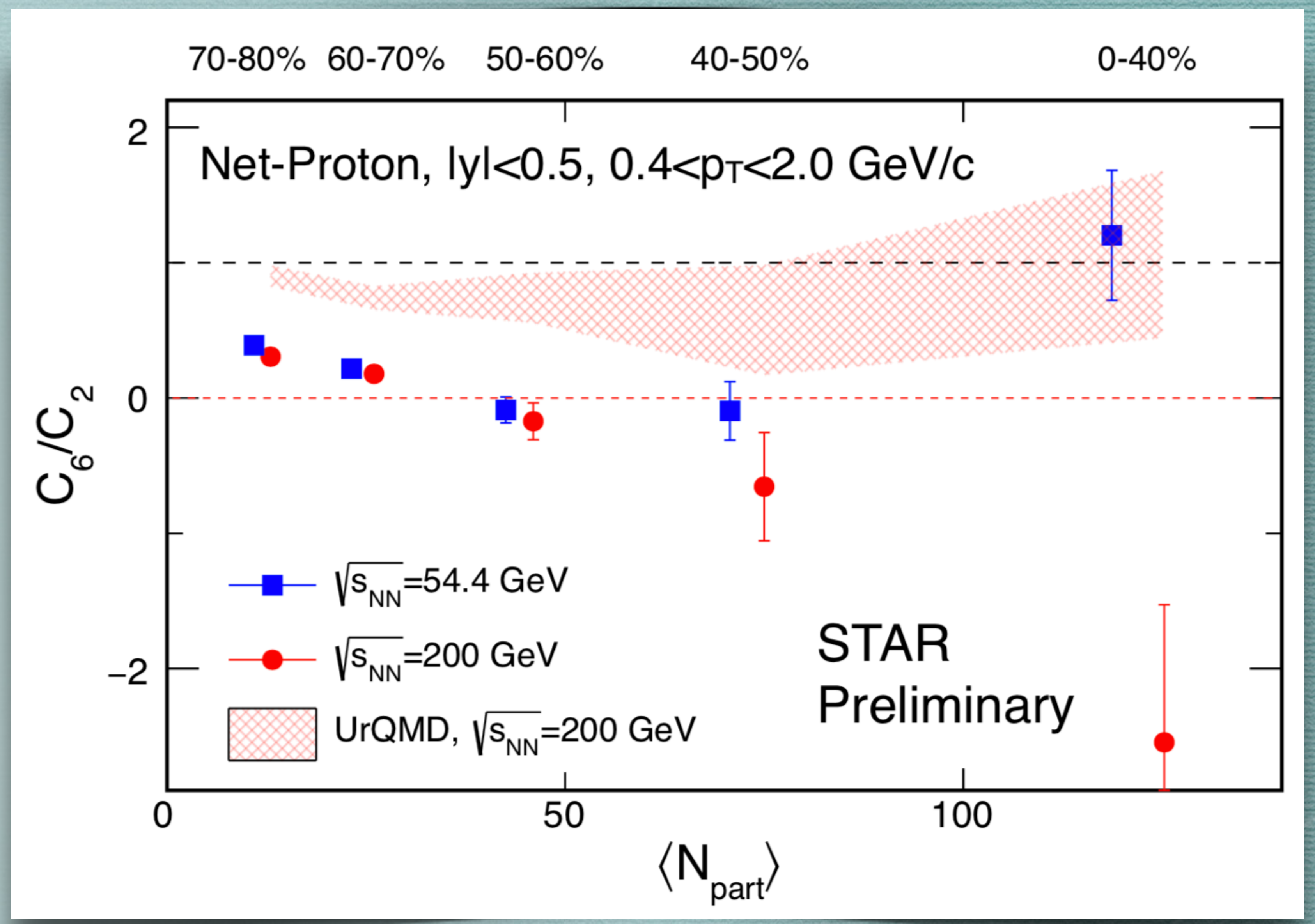
HTD, F. Karsch, S. Mukherjee, arXiv:1504.05274

4th to 2nd order cumulant ratio



STAR data: X.F. Luo, 1503.02558, X.F. Luo and N. Xu, 1701.02105

6th to 2nd order cumulant ratio

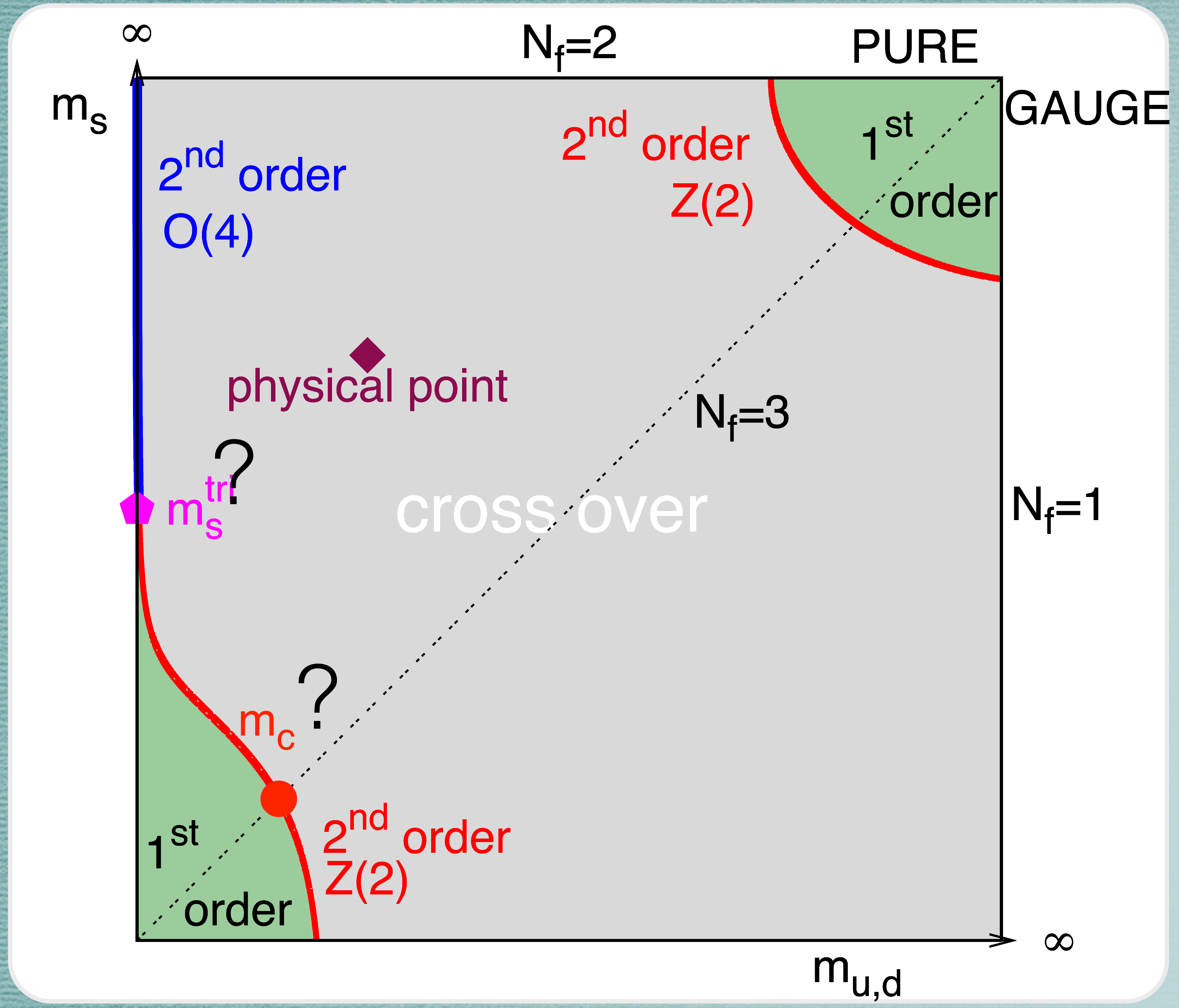


Toshihiro Nonaka, ATHIC 2018, EMMI 2019
Ashish Pandav, 11:20 Tue
Toshihiro Nonaka, Poster

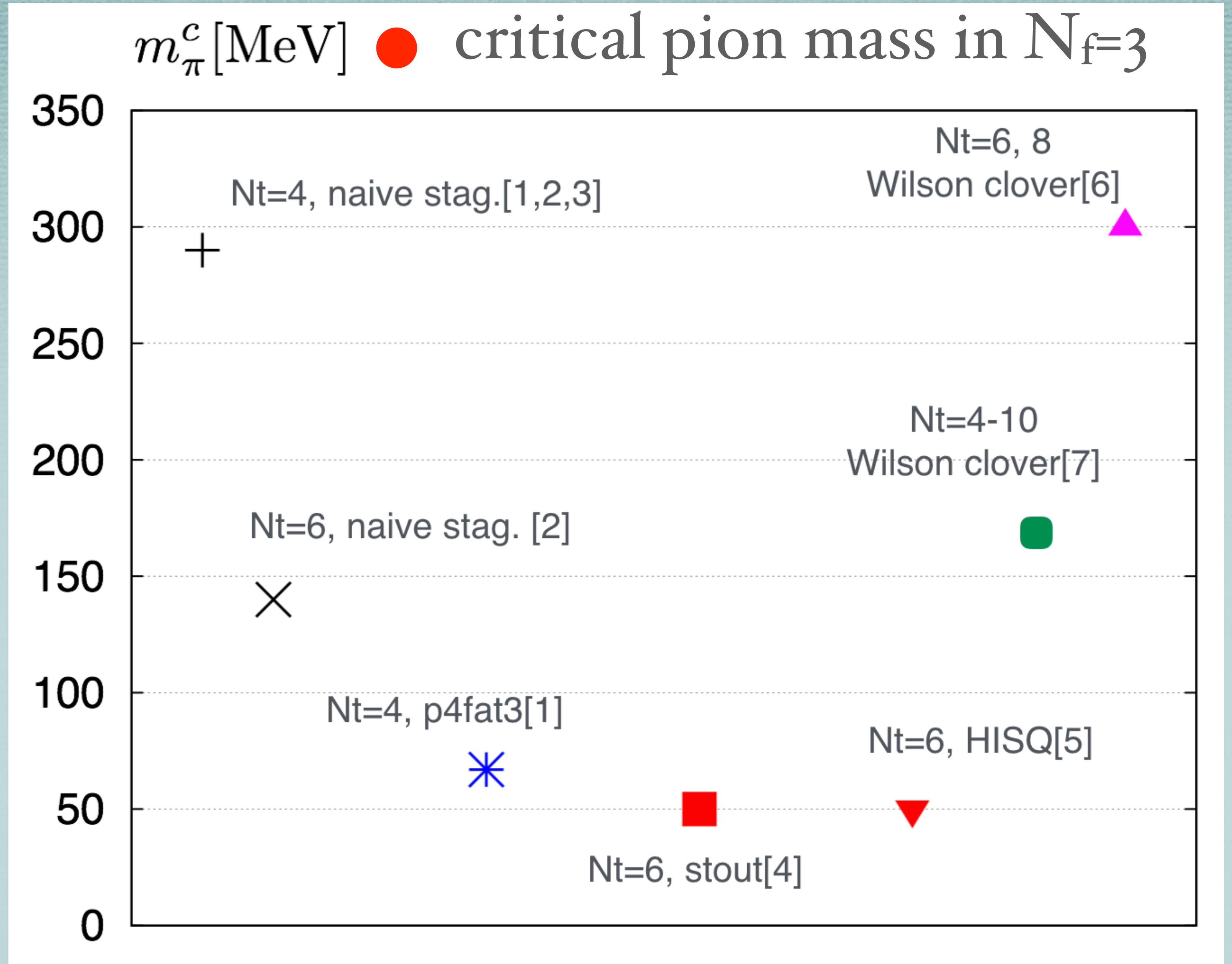
- Sign Problem at μ_B=/=0
- Taylor Expansion
- Imaginary μ_B

QCD criticality at μ_B=0: relevance to CEP

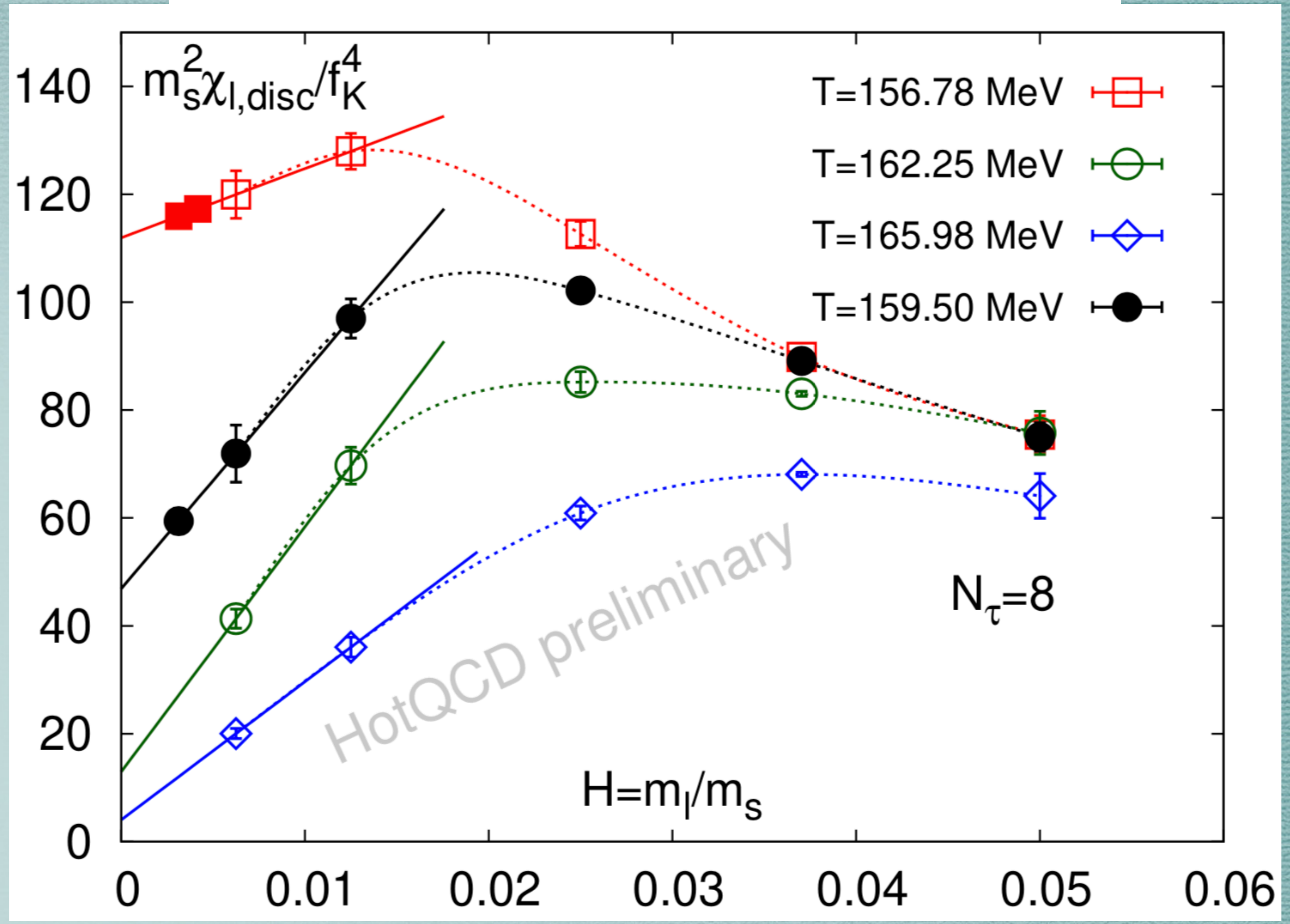
Columbia plot:
QCD phase diagram in quark mass plane



critical pion mass in N_f=3



disconnected chiral susceptibility



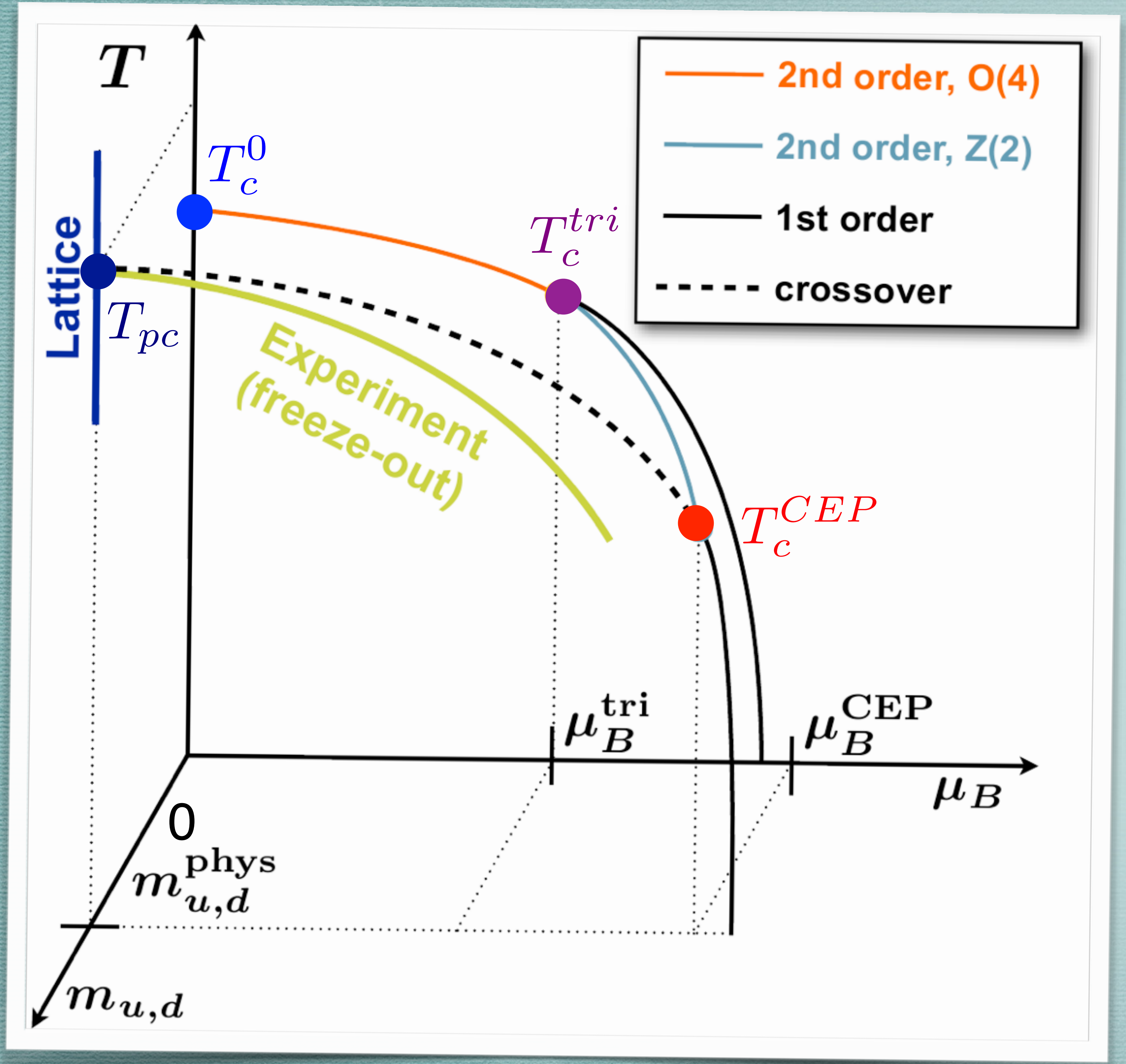
[1]F. Karsch et al., Nucl.Phys.Proc.Suppl. 129 (2004) 614 [2] P. de Forcrand et al, PoS LATTICE2007 (2007) 178 [3]D. Smith & C. Schmidt, PoS LATTICE2011 (2011) 216
[4]Endrodi et al., PoS LAT2007 (2007) 228 [5] Bazavov et al., PRD95 (2017) 074505 [6]Nakamura et al., PRD92 (2015)114511 [7] Jin et al., PRD96 (2017) 034523,1909. 05441

Not relevant: 1st order chiral phase transition region
as it becomes small and is away from the physical point

Relevant: 2nd order chiral phase transition belonging to O(4) universality class
as axial U(1) symmetry is not effectively restored at the critical temperature ($\chi_{l,disc} \neq 0$)

See more about $U_A(1)$ at T=/=0: H. Fukaya EPJ Web Conf. 175 (2018) 01012, Lattice 2017

QCD phase diagram in 3D: quark mass, μB, T



T_{pc} : 156.5(1.5)MeV, chiral crossover T at μB=0

Patrick Steinbrecher, QM2018,
Bazavov et al., [HotQCD] Phys. Lett. B795 (2019) 15

T_c^{CEP}: transition T at the critical end point

T_c^0 : chiral phase transition T at m_q=0 and μB=0

T_c^{tri} : transition T at the tri-critical point

- Random Matrix Model & NJL suggests:

$$T_c^{tri} - T_c^{CEP}(m_q) \propto m_q^{2/5}$$

Y. Hatta & T. Ikeda, PRD67 (2003) 014028
M. A. Halasz et al, PRD 58 (1998) 096007
M. Buballa, S. Carignano, PLB791(2019)361

- $T_c^0(\mu_B)$ decreases as μB up to NLO from LQCD

O. Kaczmarek et al., PRD83 (2011) 014504
P. Hegde & HTD, PoS LATTICE2015 (2016) 141

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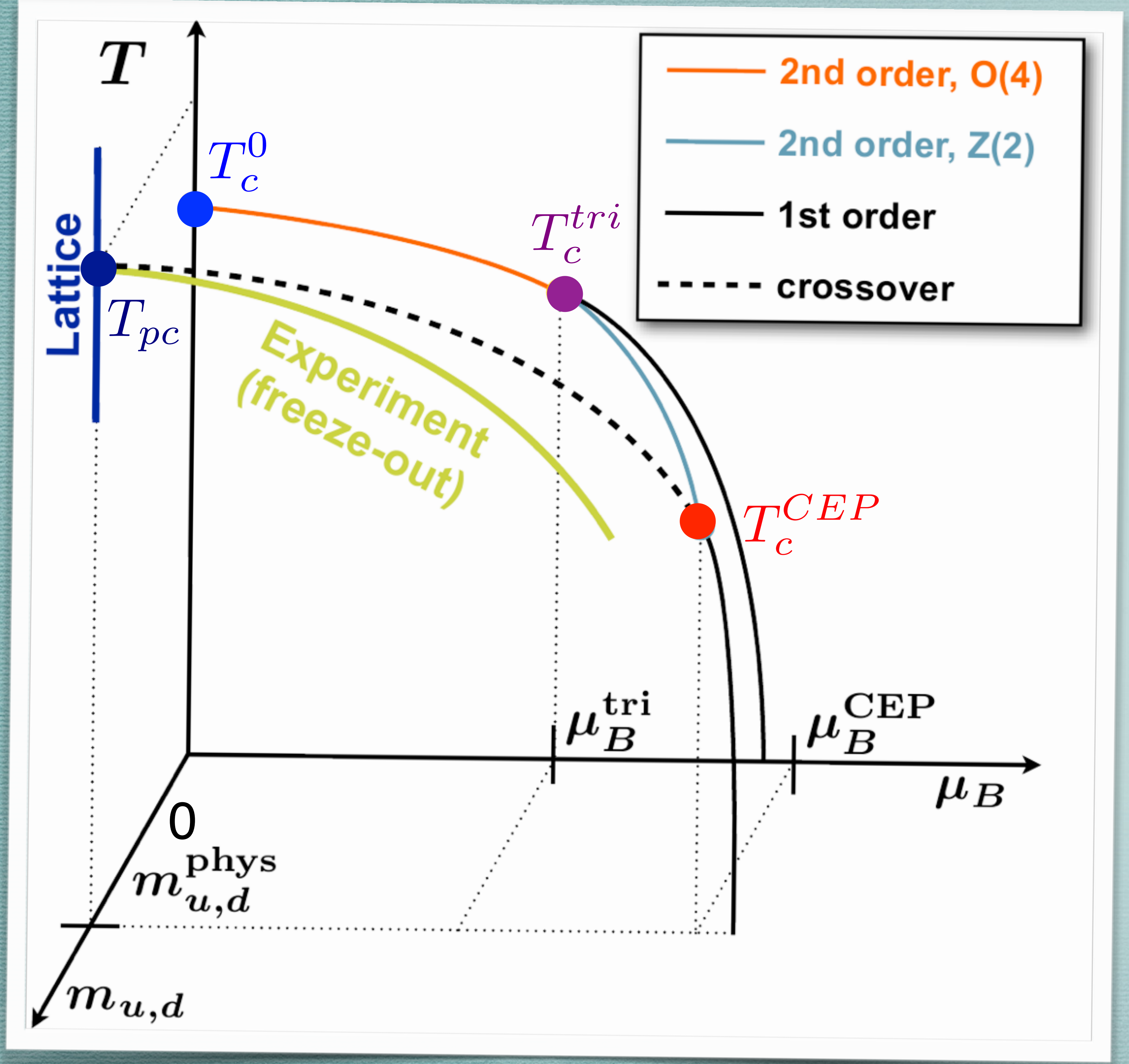

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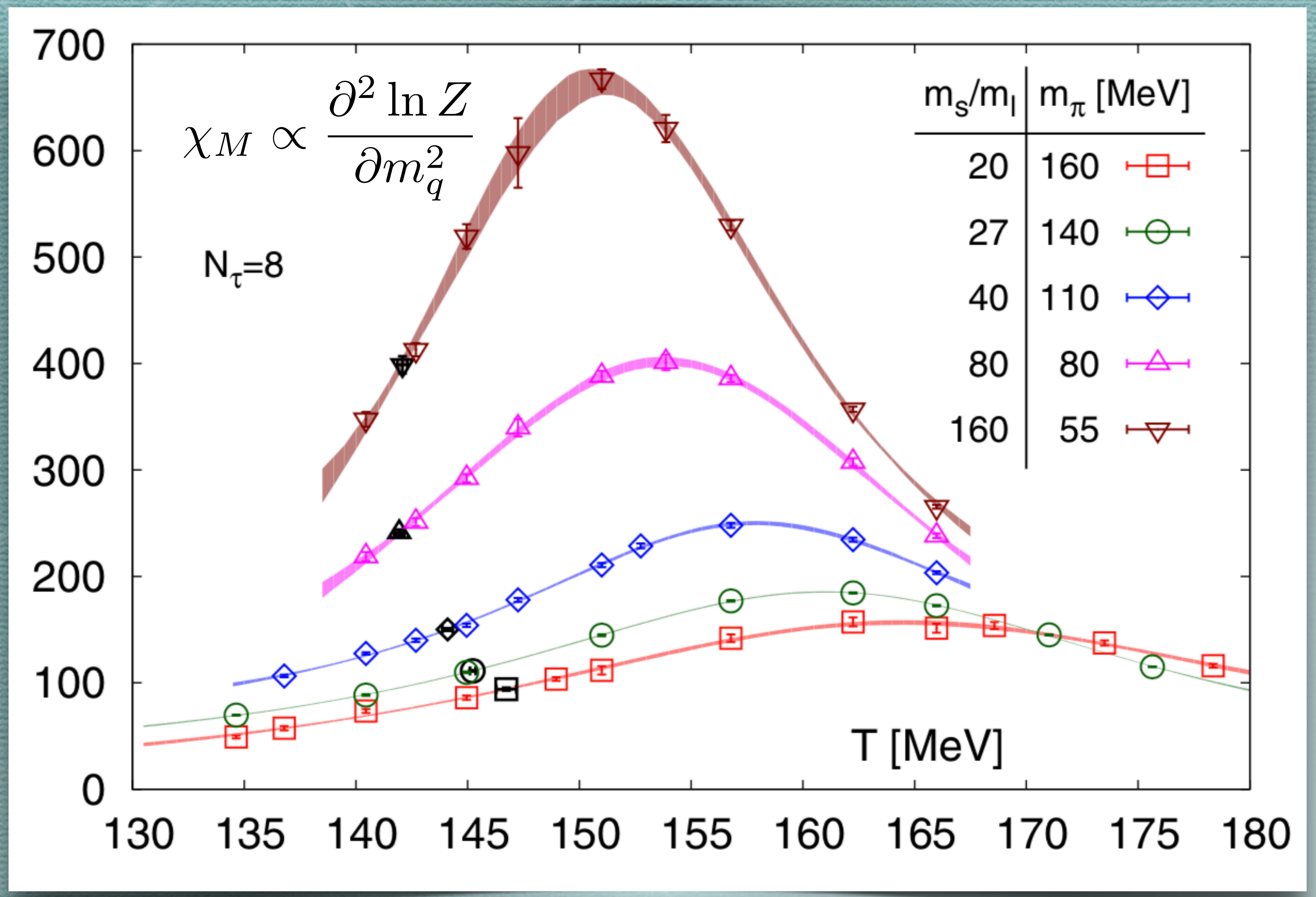
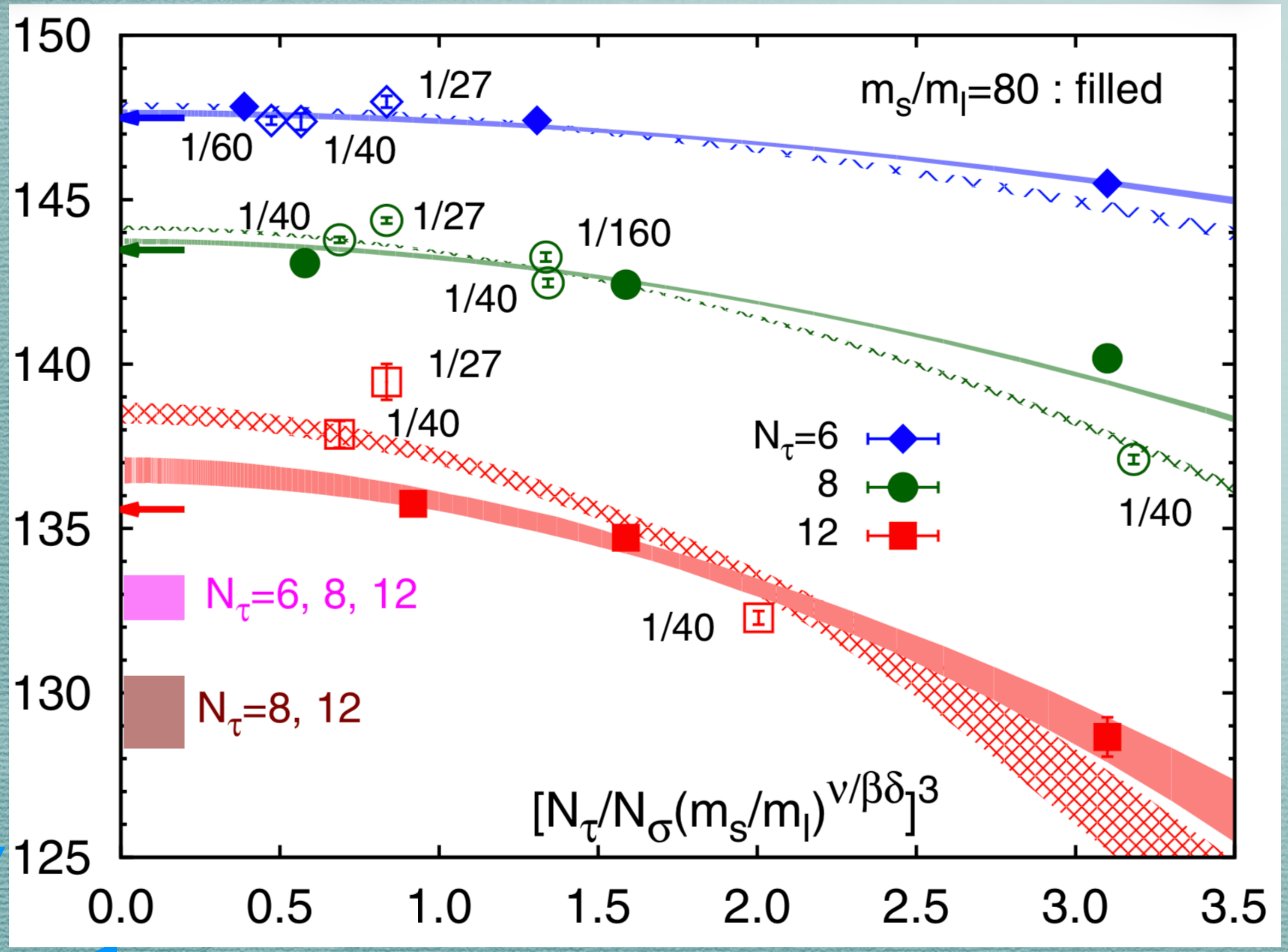
Indication

$$T_c^0 > T_c^{tri} > T_c^{CEP}$$

order parameter susceptibility

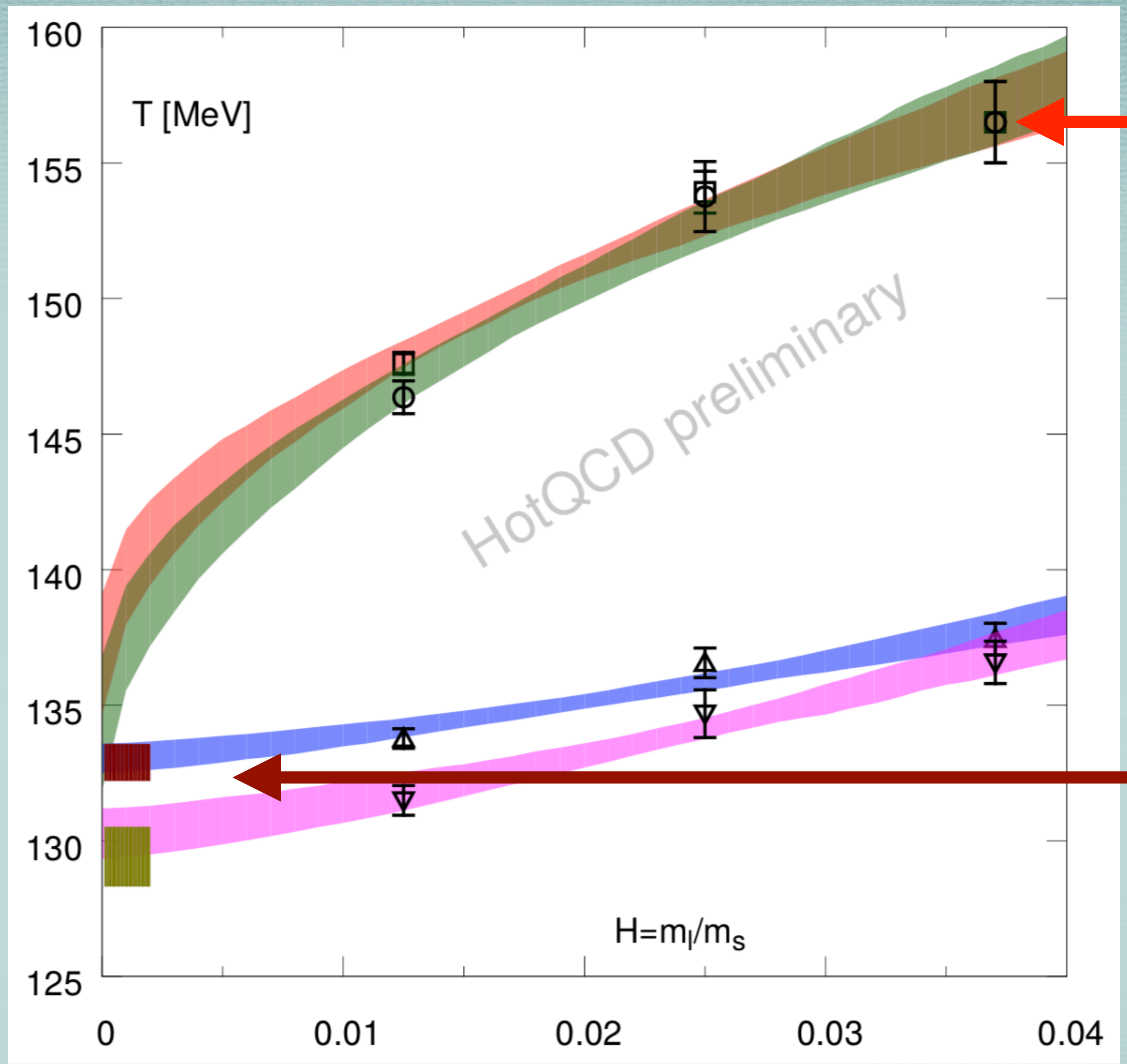




Continuum extrapolation

Infinite volume & chiral extrapolation

O(4) scaling analyses: Nf=2+1, Nt=6,8 & 12 lattices, $M_\pi \gtrsim 55$ MeV
thermodynamic, continuum & chiral extrapolated



Chiral crossover

T_{pc}^{phys} = 156.5(1.5) MeV

$$T_{pc}(H) = T_c^0 \left(1 + \frac{z_p}{z_0} H^{\frac{1}{\beta\delta}}\right)$$

Chiral phase transition T

$$T_c^0 = 132^{+3}_{-6}\ \text{MeV}$$

HTD, P. Hegde, O. Kaczmarek et al.[HotQCD], Phys.Rev. Lett. 123 (2019) 062002

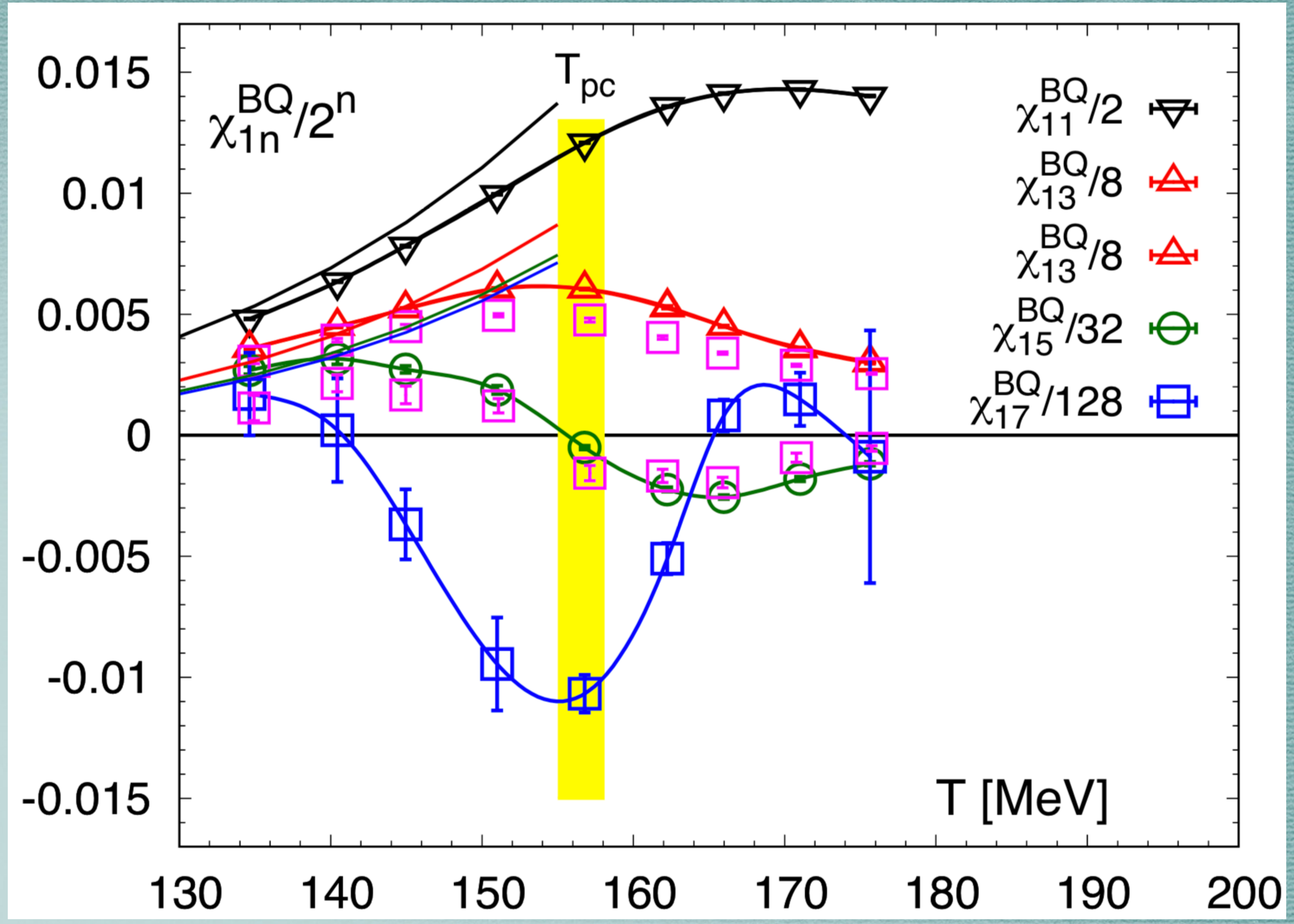
T_c^0 is ~ 25 MeV smaller than the chiral crossover T!

See QCD-inspired model calculations:
e.g. J. Berges, D. U. Jungnickel and C. Wetterich, Phys. Rev.D59, 034010 (1999)
J. Braun, B. Klein, H.-J. Pirner and A. H. Rezaeian,Phys. Rev. D73, 074010 (2006)

High order fluctuations & critical behavior

Jishnu Goswami [HotQCD], Poster

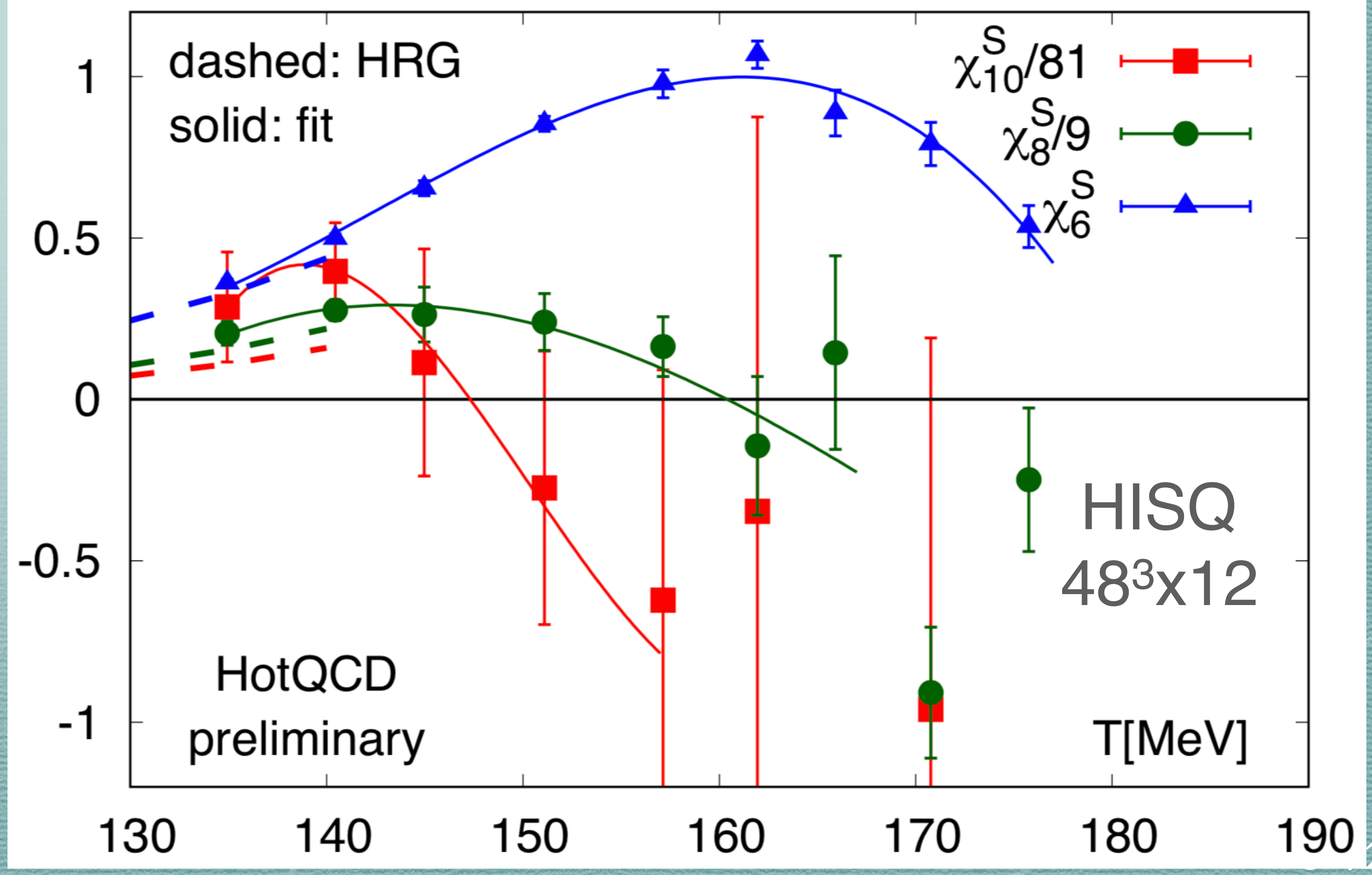
$$\chi_{lmn}^{BQS} \equiv \chi_{lmn}^{BQ}(T) = \frac{\partial^{l+m+n} P(T,\hat{\mu})/T^4}{\partial \hat{\mu}_B^l \partial \hat{\mu}_Q^m \partial \hat{\mu}_S^n}\bigg|_{\hat{\mu}=0}$$



Many 8th order fluctuations turn to be negative at T ⩾ 135-140 MeV

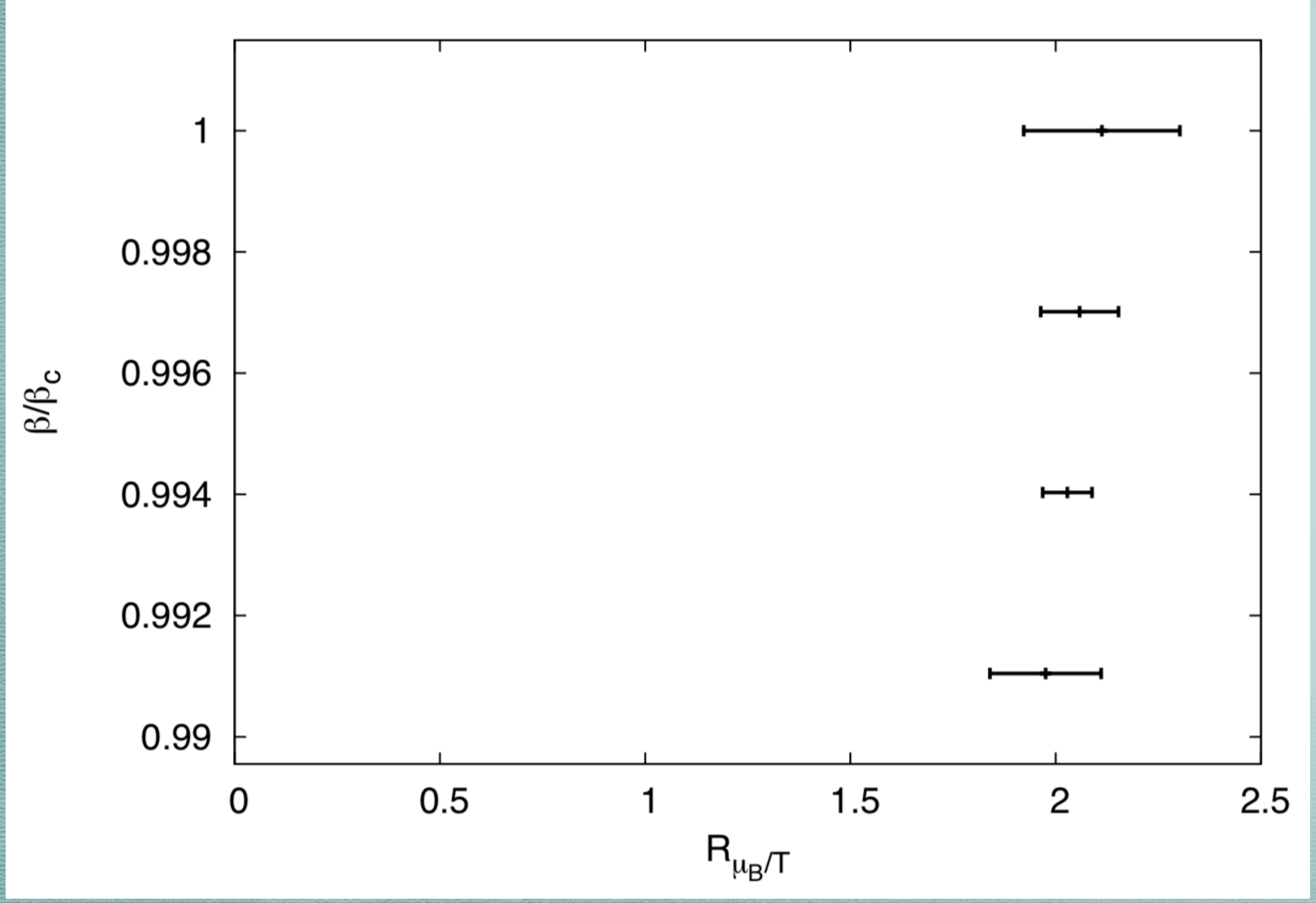
Suggests zeros in the complex plane: no phase transition in the above T window

Supports for $T_c^{CEP} < 135 - 140\text{MeV} \sim T_c^0$



Singularity in the complex plane: Radius of convergence in μ_B

Idea testing on Nt=4 coarse lattices
using 2-stout improved staggered fermions



M. Giordano & A. Pasztor, PRD99(2019)114510

Try to have a direct determination of the leading singularity of the pressure, i.e. the closest Lee-Yang zero, via reweighting to a complex chemical potential

Attila Pasztor
12:20 Wed

Singularity in the complex plane: Radius of convergence in μ_B

based on O(4) universality and LQCD results on chiral phase transition

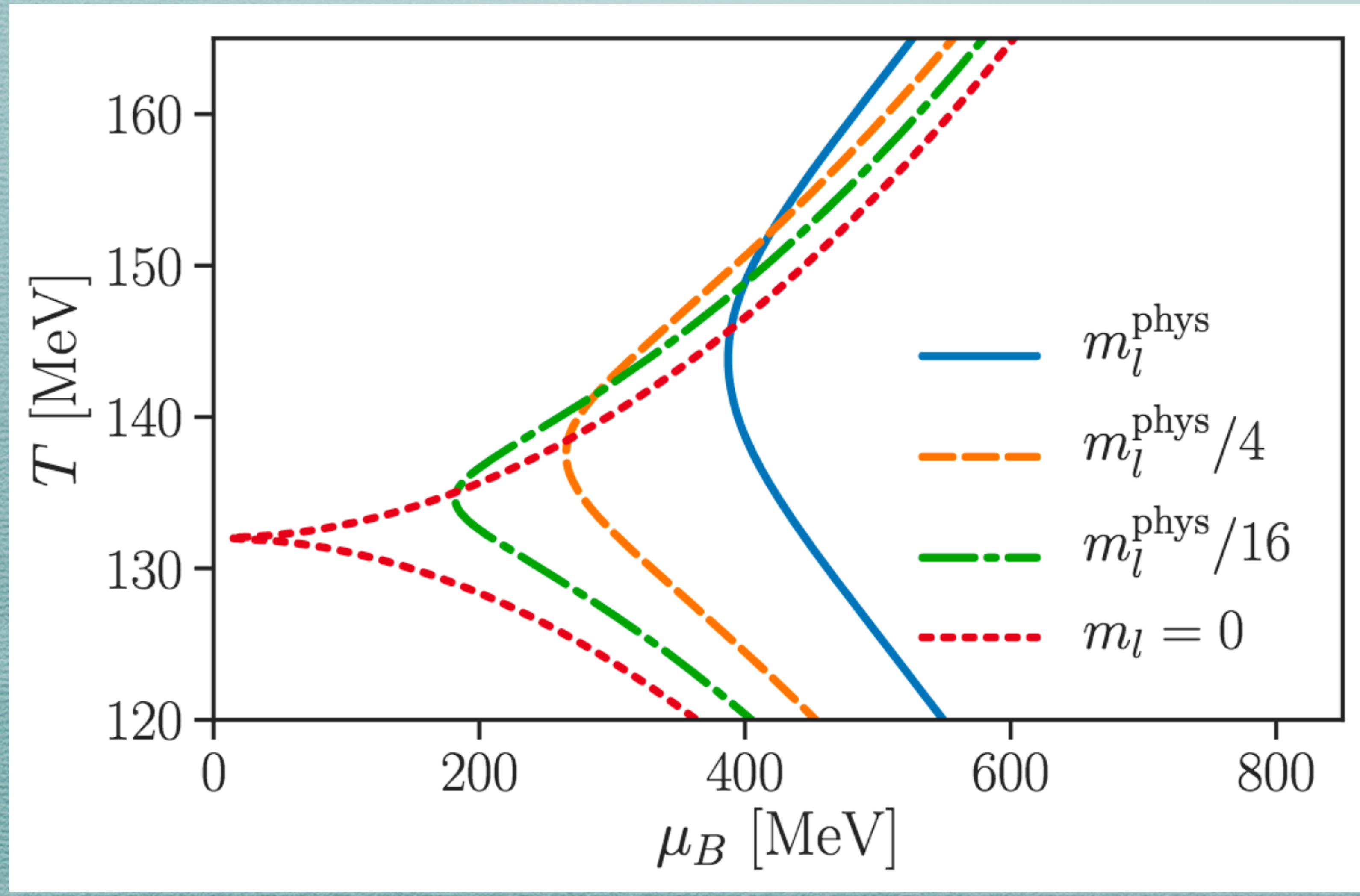
Universal parameters: O(4) critical exponents, scaling functions

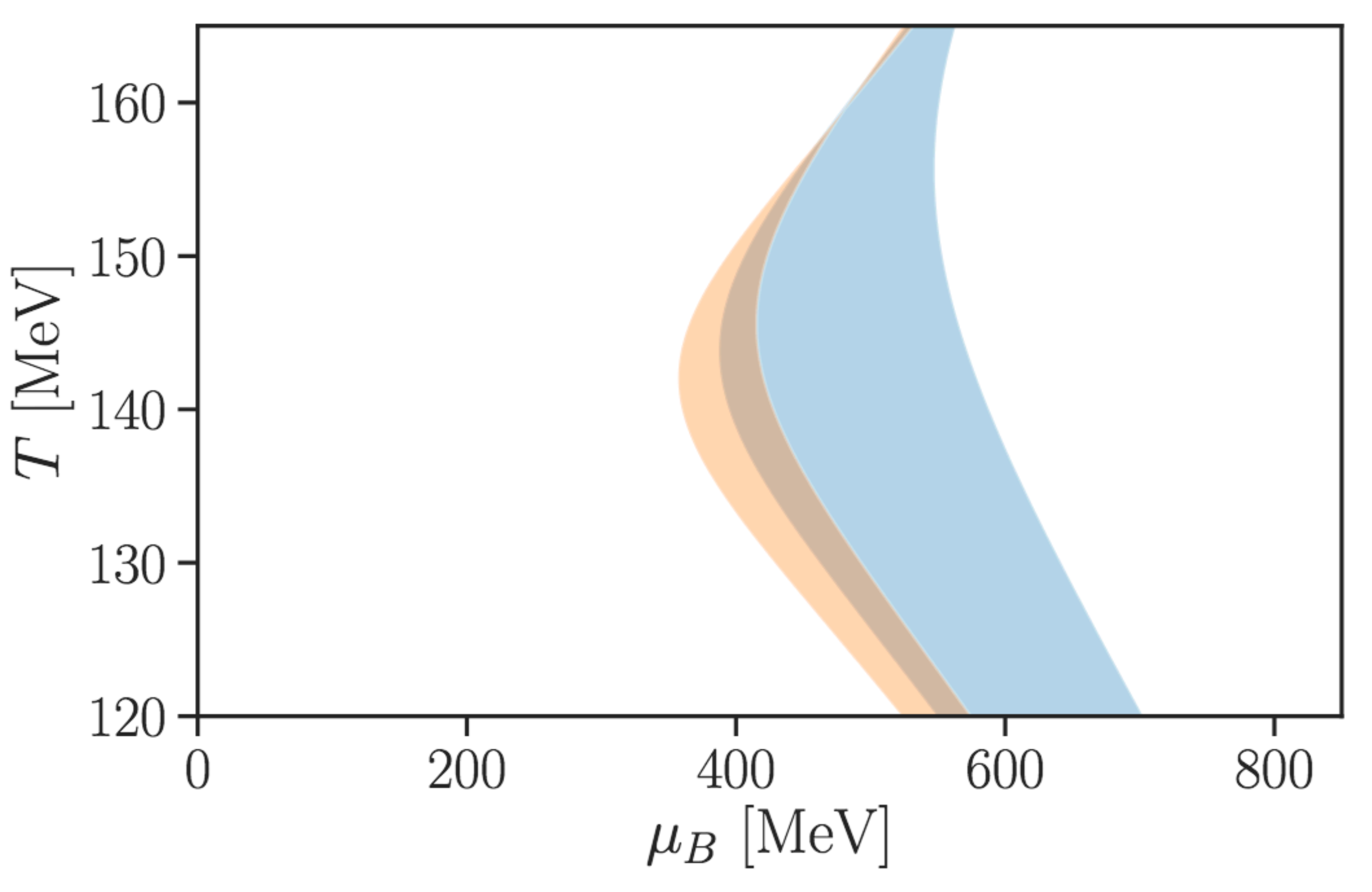
Scaling functions: singularity in the complex plane—Lee-Yang edge singularity

The singularity limits the convergence of the Taylor series in μ_B

Non-universal parameters:

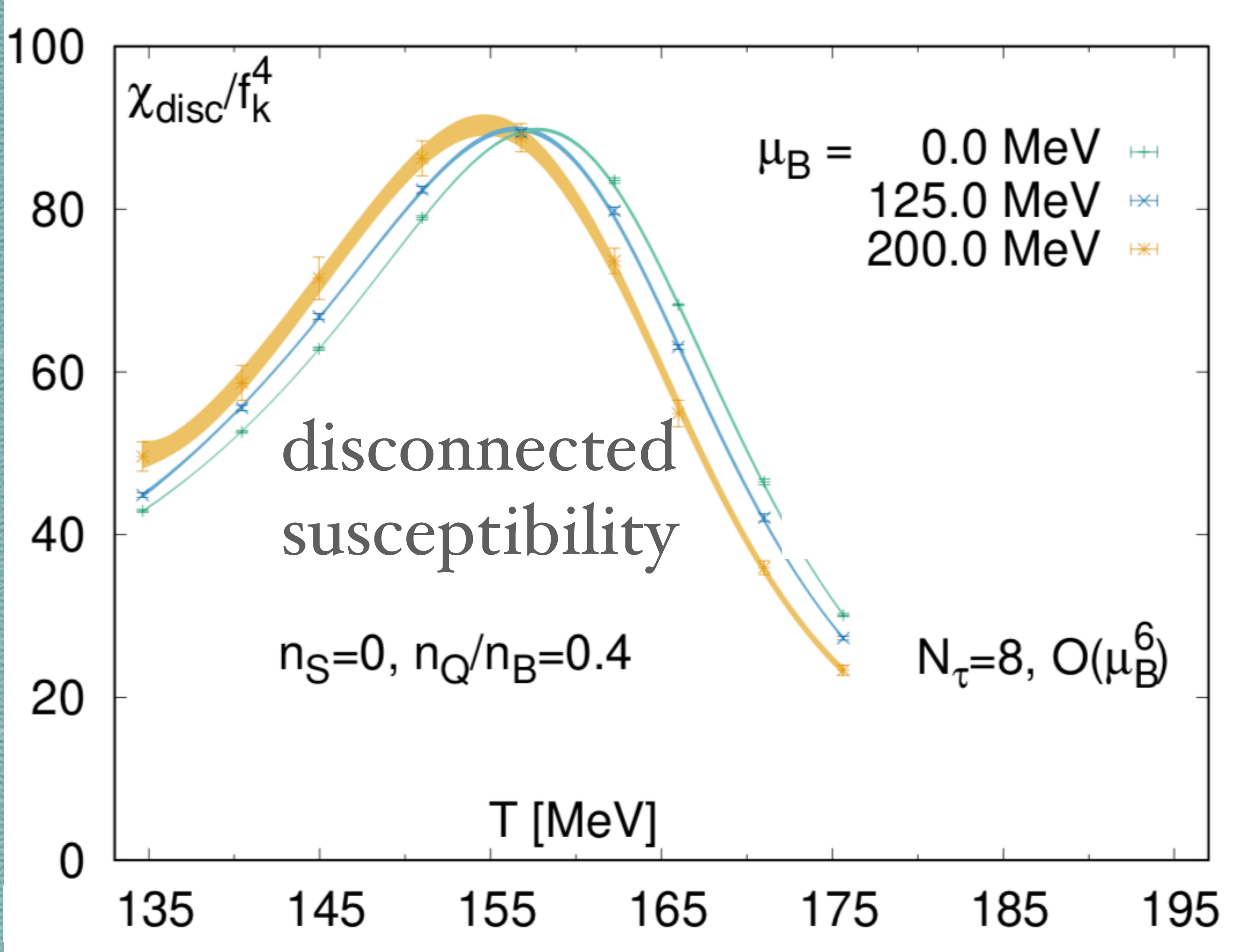
LQCD-determined chiral phase transition T_c^0, curvature of transition line

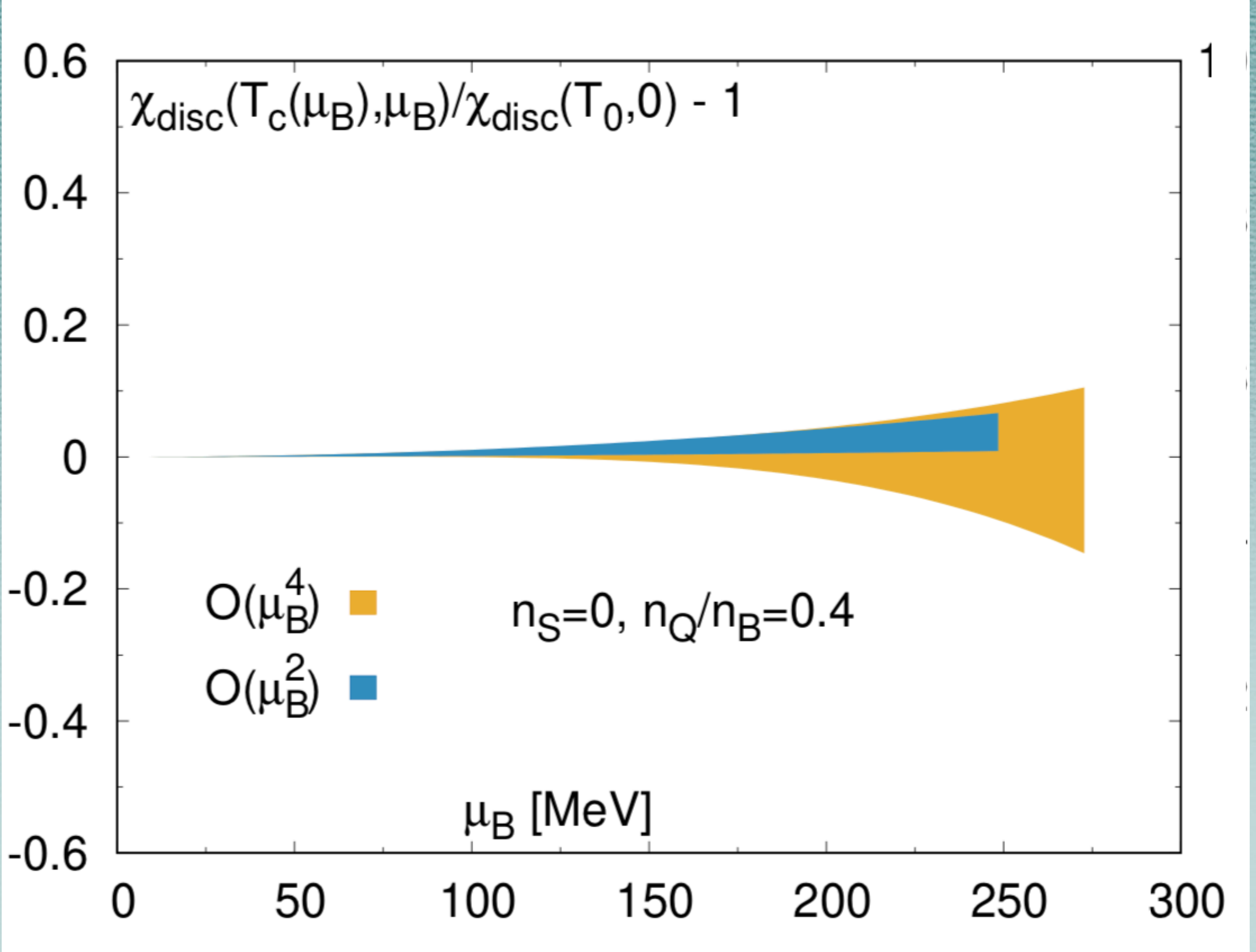




Vladimir Skokov
16:40 Wed

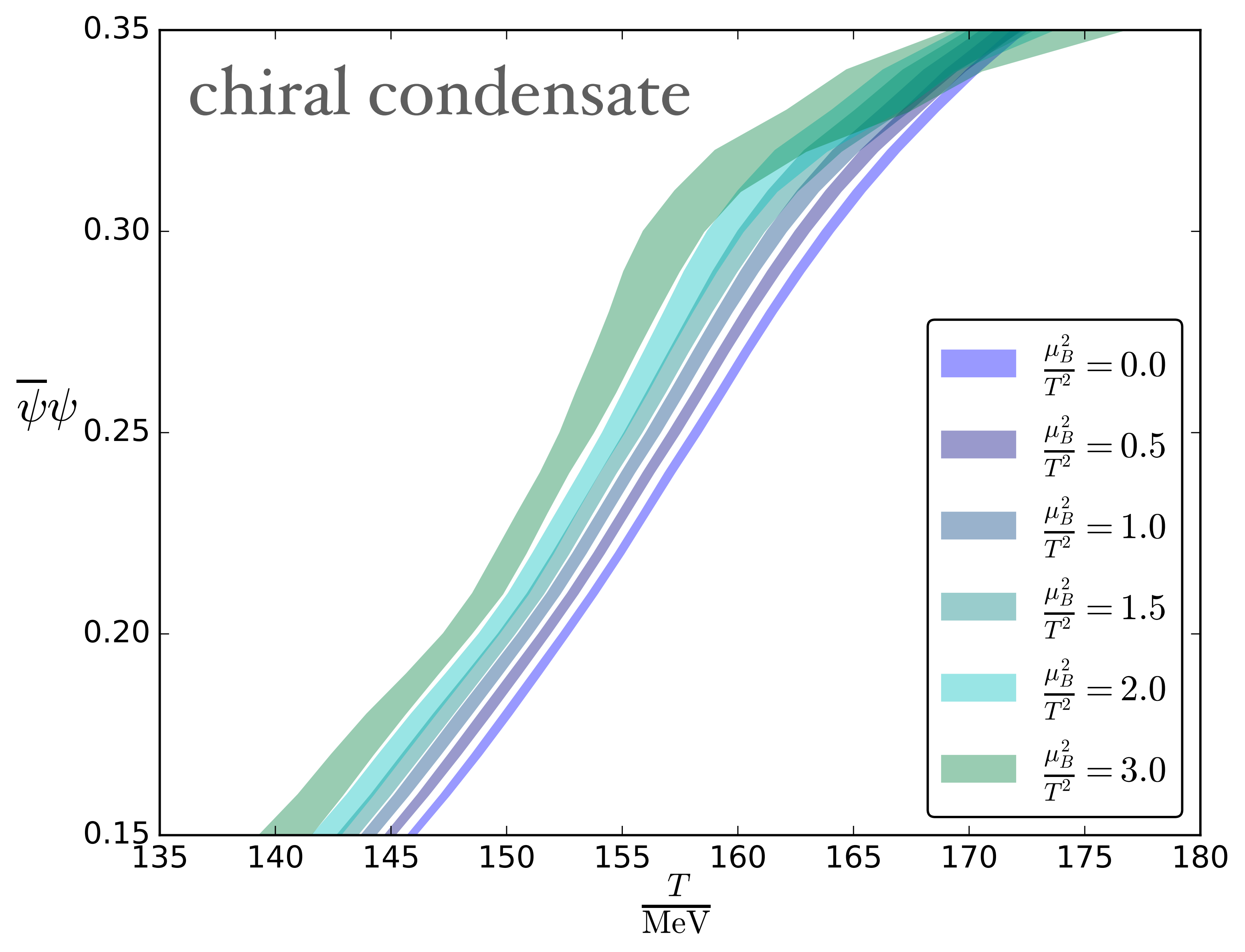
S. Mukherjee & V. Skokov, arXiv:1909.04639

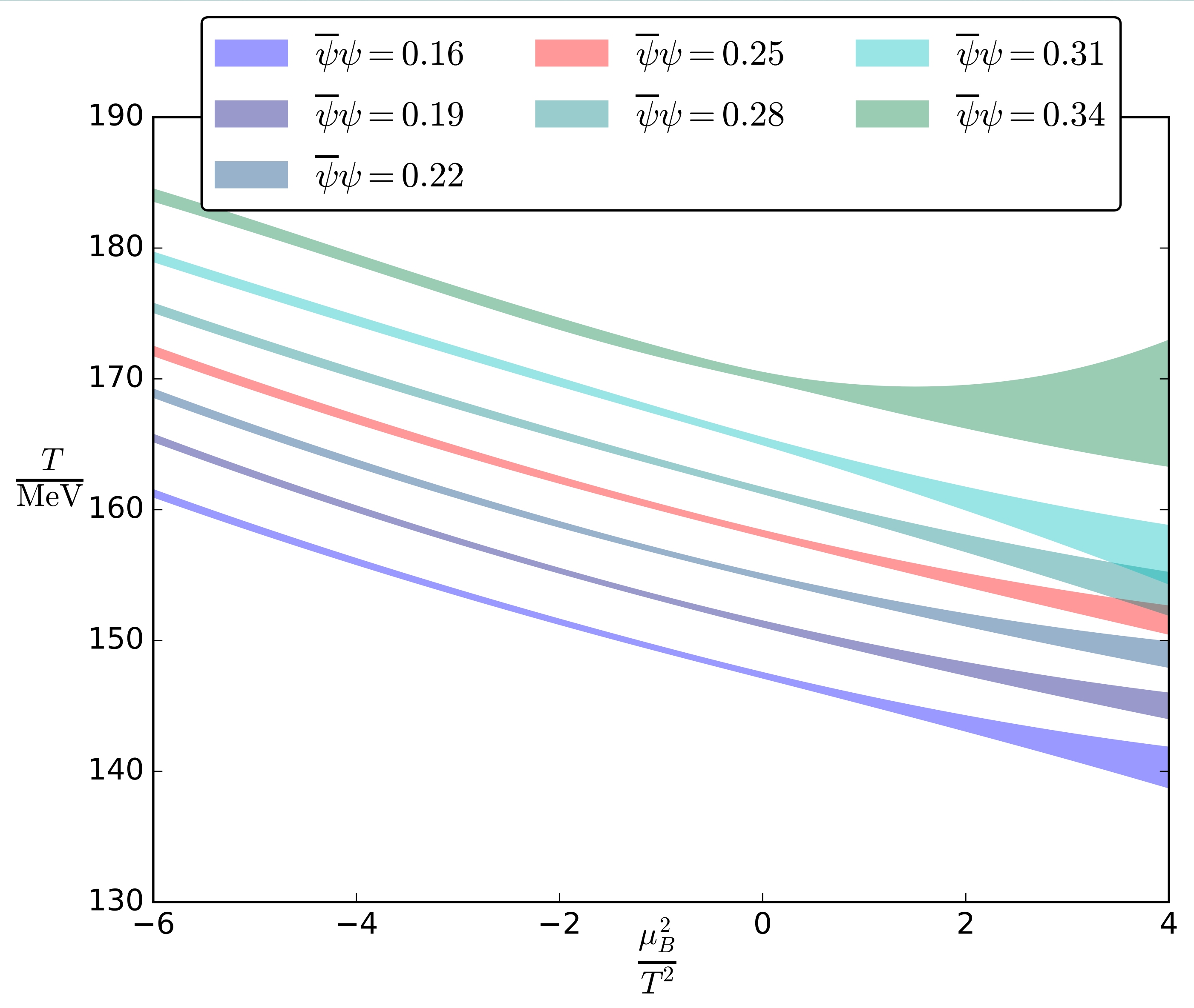




Taylor Expansion, P. Steinbrecher QM 2018,
Bazavov, HTD, P. Hegde et al. [HotQCD], Phys. Lett. B795 (2019) 15

Strength of QCD transition does not increase at all up to μ_B~300 MeV





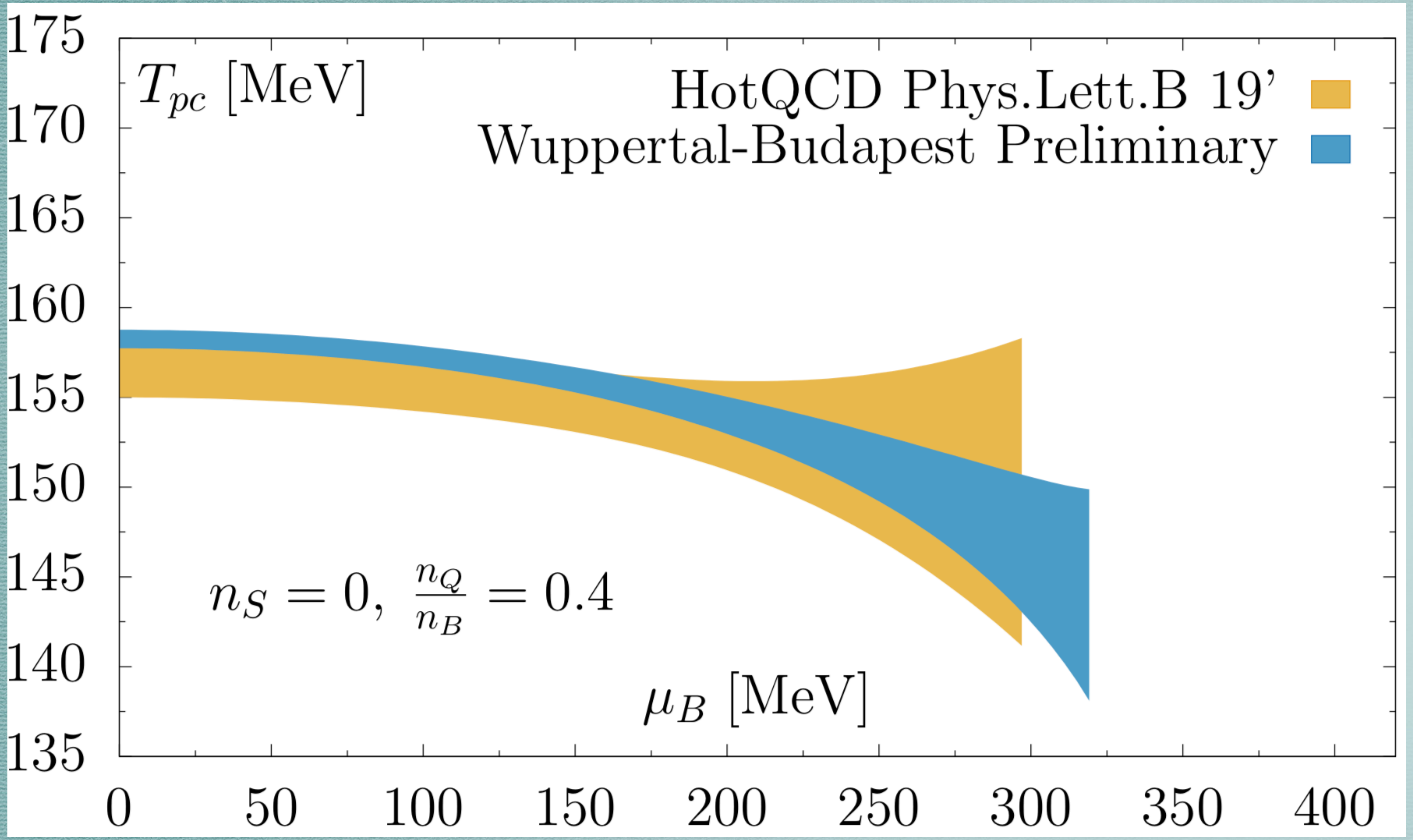
Lattice data suggests

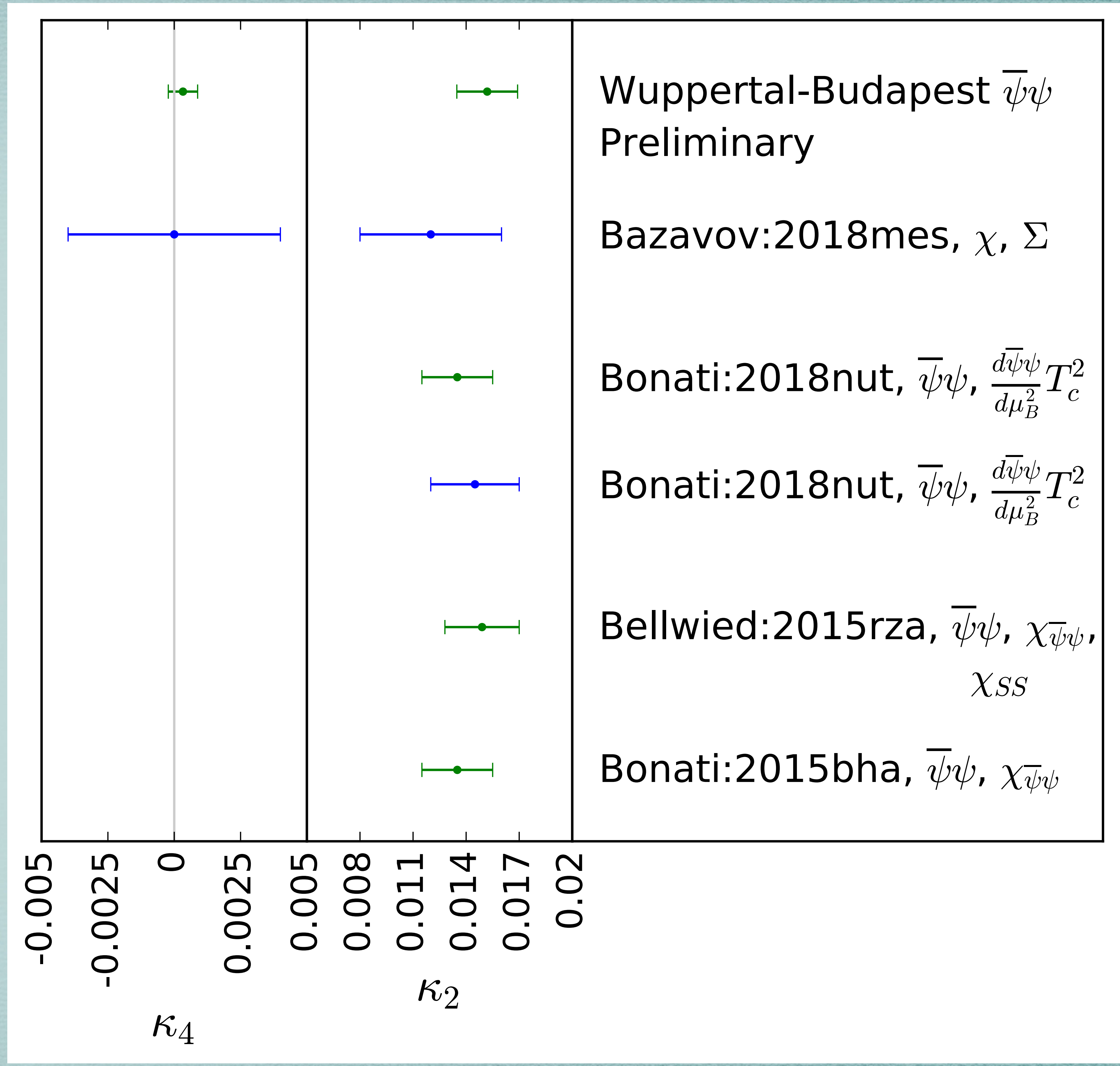
$$T_c^{CEP} < 135-140\mathrm{MeV}$$

$$\mu_B^{CEP} > 300\mathrm{MeV}$$

Imaginary μ_B, Ruben Kara [Wuppertal-Budapest], Poster

Chiral crossover line: $T_{pc}(\mu_B) = T_{pc}(0)\left(1 - \kappa_2\left(\frac{\mu_B}{T}\right)^2 - \kappa_4\left(\frac{\mu_B}{T}\right)^4\right)$





courtesy from Jana Guenther

Taylor expansion: HotQCD: Bazavov, HTD, Hegde et al., Phys. Lett. B795 (2019) 15

Imaginary muB: Wuppertal-Budapest preliminary: Jana Guenther, 11:40 Tue

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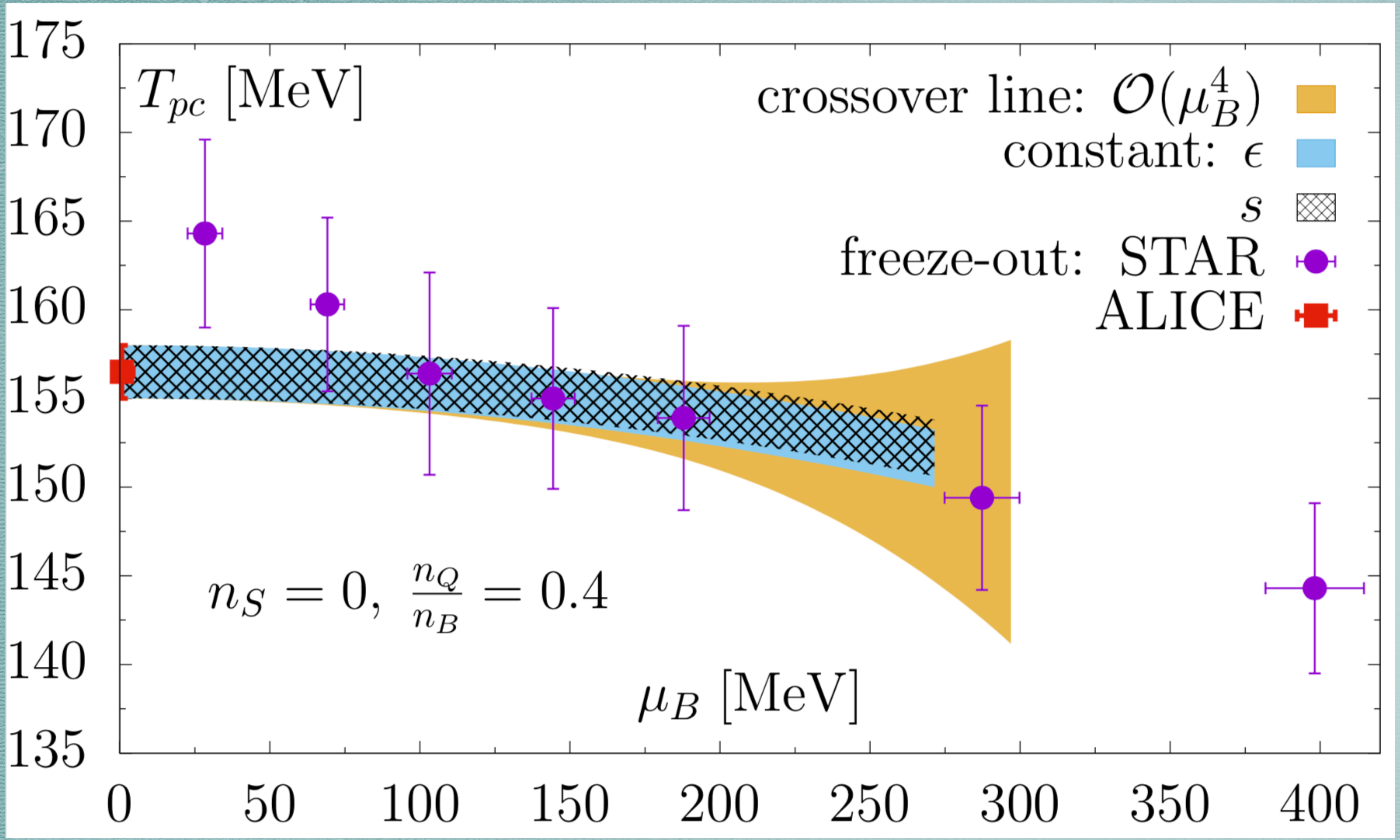

Bazavov, HTD, P. Hegde et al. [HotQCD], Phys. Lett. B795 (2019) 15

ALICE data point:

$T_f = 156.5(1.5)$ MeV

Andronic et al, Nature 561 (7723) (2018) 321

STAR data points:

Adamczyk et al., Phys. Rev. C 96 (4) (2017) 044904

Similar chiral crossover line: see from Jana Guenther, 11:40 Tue

Explore the QCD phase diagram through fluctuations of conserved charges x=B,Q,S

$$\frac{M_x(\sqrt{s})}{\sigma_x^2(\sqrt{s})} = \frac{\langle N_x \rangle}{\langle(\delta N_x)^2\rangle} = \frac{\chi_1^x(T,\mu_B)}{\chi_2^x(T,\mu_B)} = R_{12}^x(T,\mu_B)$$

HIC

mean: M_x

variance: σ_x^2

skewness: S_x

kurtosis: κ_x

hyper-skewness: S_x^h

hyper-kurtosis: κ_x^h

$$\frac{S_x(\sqrt{s})\,\sigma_x^3(\sqrt{s})}{M_x(\sqrt{s})} = \frac{\langle(\delta N_x)^3\rangle}{\langle N_x\rangle} = \frac{\chi_3^x(T,\mu_B)}{\chi_1^x(T,\mu_B)} = R_{31}^x(T,\mu_B)$$

$$\kappa_x(\sqrt{s})\,\sigma_x^2(\sqrt{s}) = \frac{\langle(\delta N_x)^4\rangle}{\langle(\delta N_x)^2\rangle} = \frac{\chi_4^x(T,\mu_B)}{\chi_2^x(T,\mu_B)} = R_{42}^x(T,\mu_B)$$

LQCD

generalized susceptibilities

$$\chi_n^x(T,\mu_B) = \frac{1}{VT^3}\frac{\partial^n \ln Z(T,\vec{\mu})}{\partial(\mu_x/T)^n}$$

$$\frac{S_x^h(\sqrt{s})\,\sigma_x^5(\sqrt{s})}{M_x(\sqrt{s})} = \frac{\langle(\delta N_x)^5\rangle}{\langle N_x\rangle} = \frac{\chi_5^x(T,\mu_B)}{\chi_1^x(T,\mu_B)} = R_{51}^x(T,\mu_B)$$

Proxies:
proton, charge particles,
kaons

$$\kappa_x^h(\sqrt{s})\,\sigma_x^4(\sqrt{s}) = \frac{\langle(\delta N_x)^6\rangle}{\langle(\delta N_x)^2\rangle} = \frac{\chi_6^x(T,\mu_B)}{\chi_2^x(T,\mu_B)} = R_{62}^x(T,\mu_B)$$

Many caveats

Non-equilibrium effects

S. Mukherjee, R. Venugopalan, Yi Yin PRL(2016). ...

Proton v.s. Baryon

M. Kitazawa and M. Asakawa, PRC(2012)...

Detector effects: cuts in acceptance & kinematics...

V. Koch, S. Jeon, PRL (2000)
A.Bzdak,V.Koch, PRC (2012)
V. Skokov et al., PRC (2013)...

Final-state interactions in the hadronic phase

J.Steinheimer et al., PRL (2013)...

... ...

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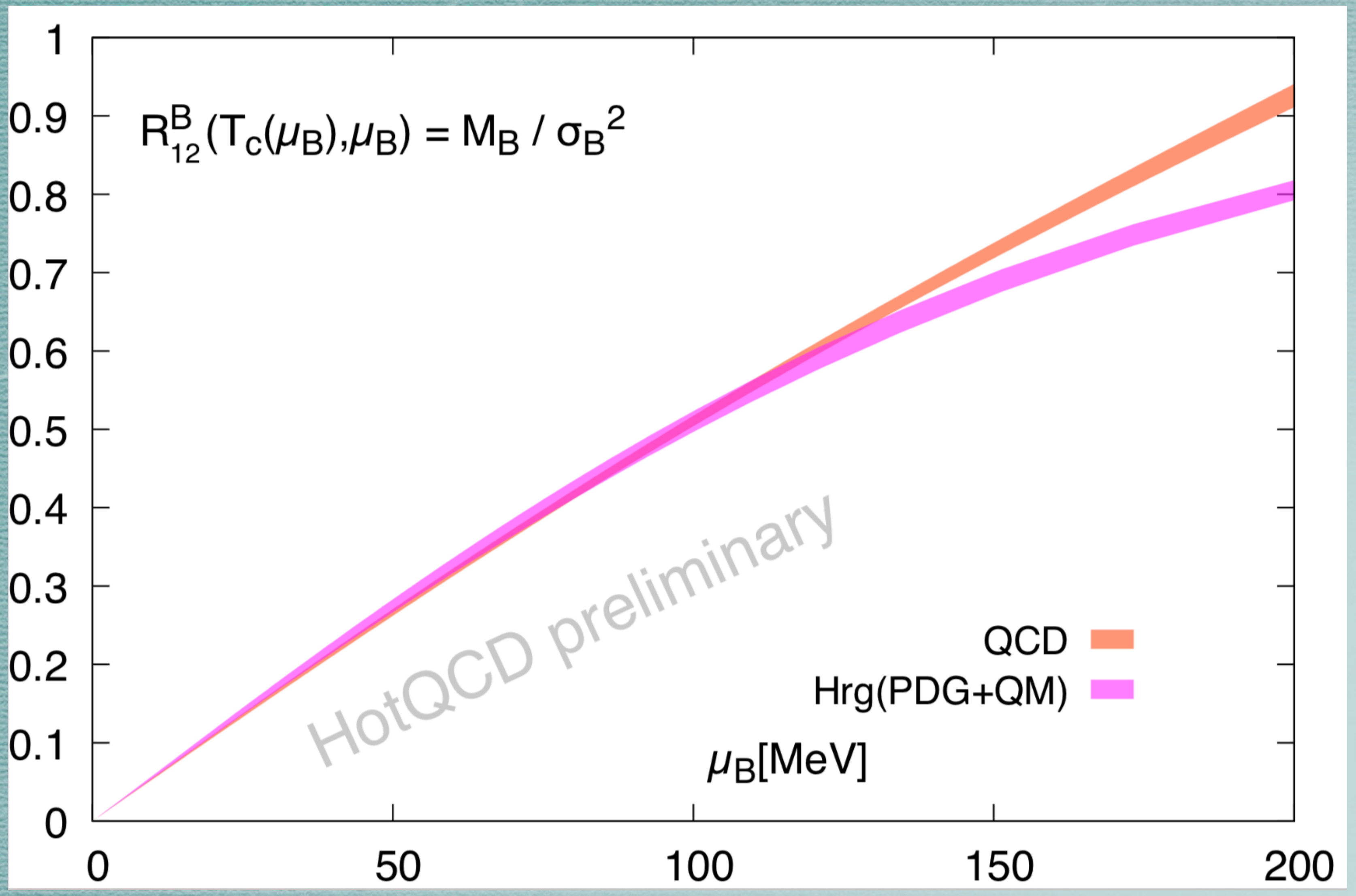
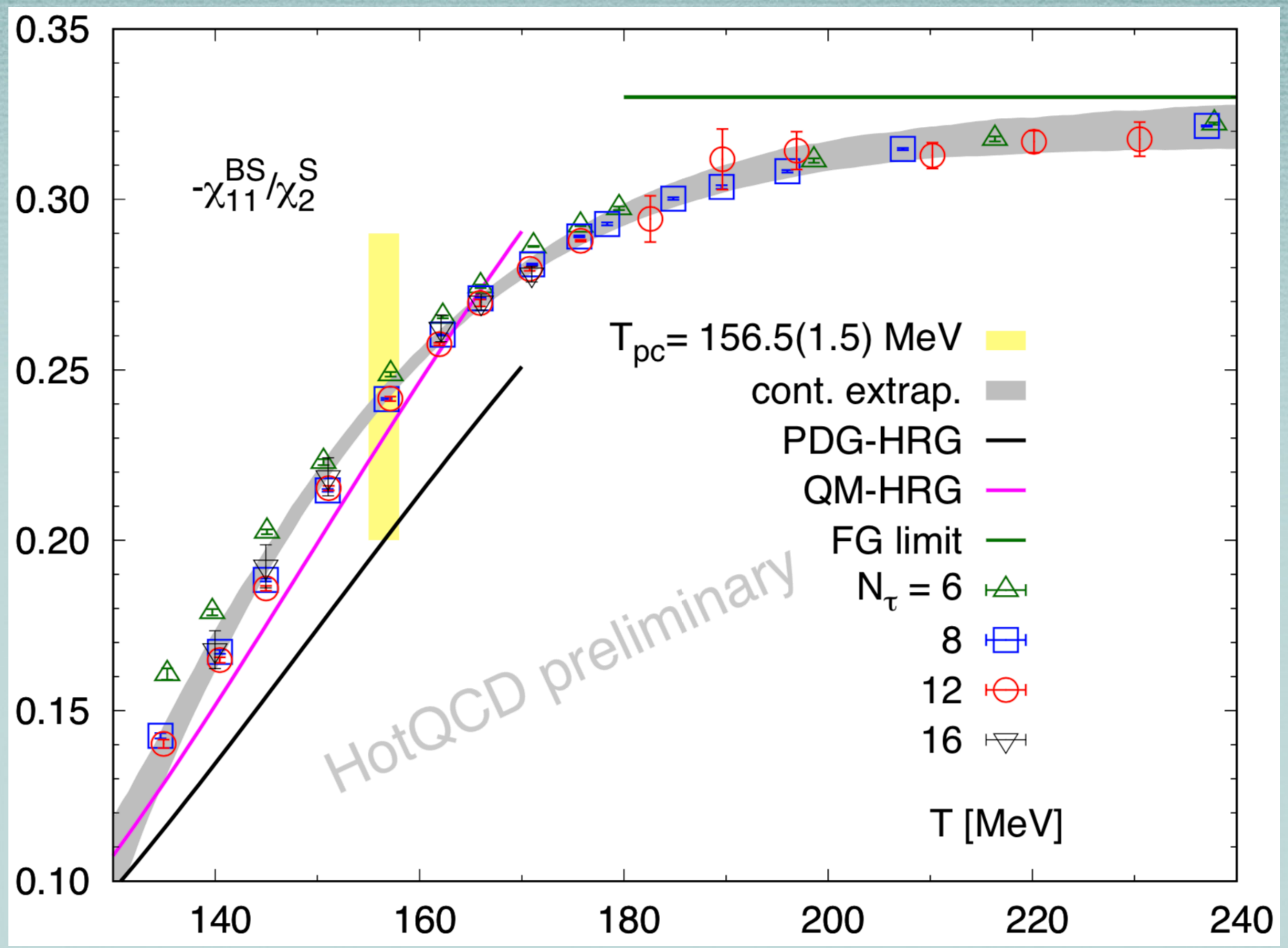
... ...

Baseline from thermal equilibrated QCD

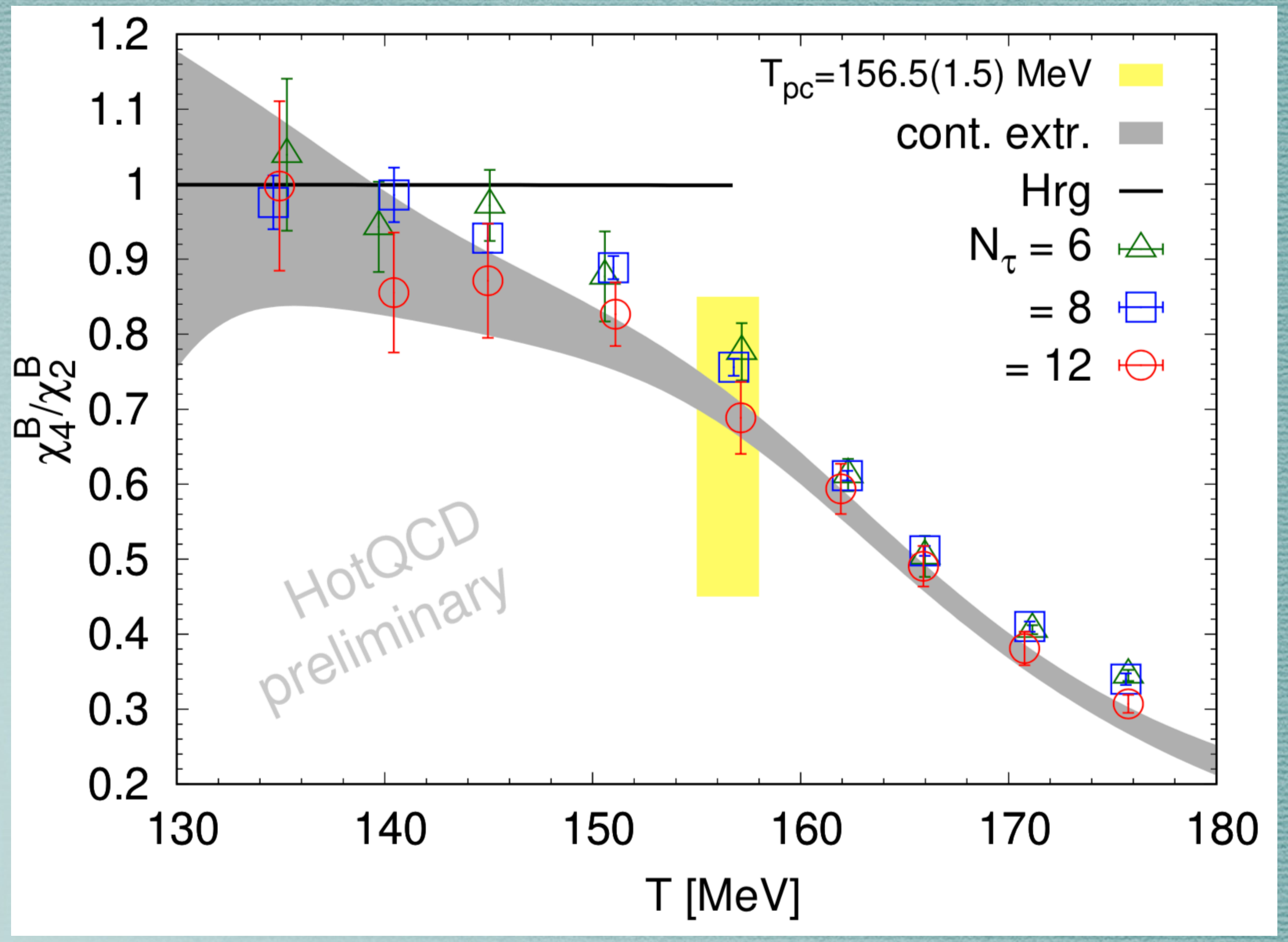
Comparison to Hadron Resonance Gas Model (HRG)

See various 2nd & 4th order cumulant ratios in Jishnu Goswami's Poster





for continuum extrapolated results of 2nd order off-diagonal correlations, see also Bellwied et al., 1910.14592



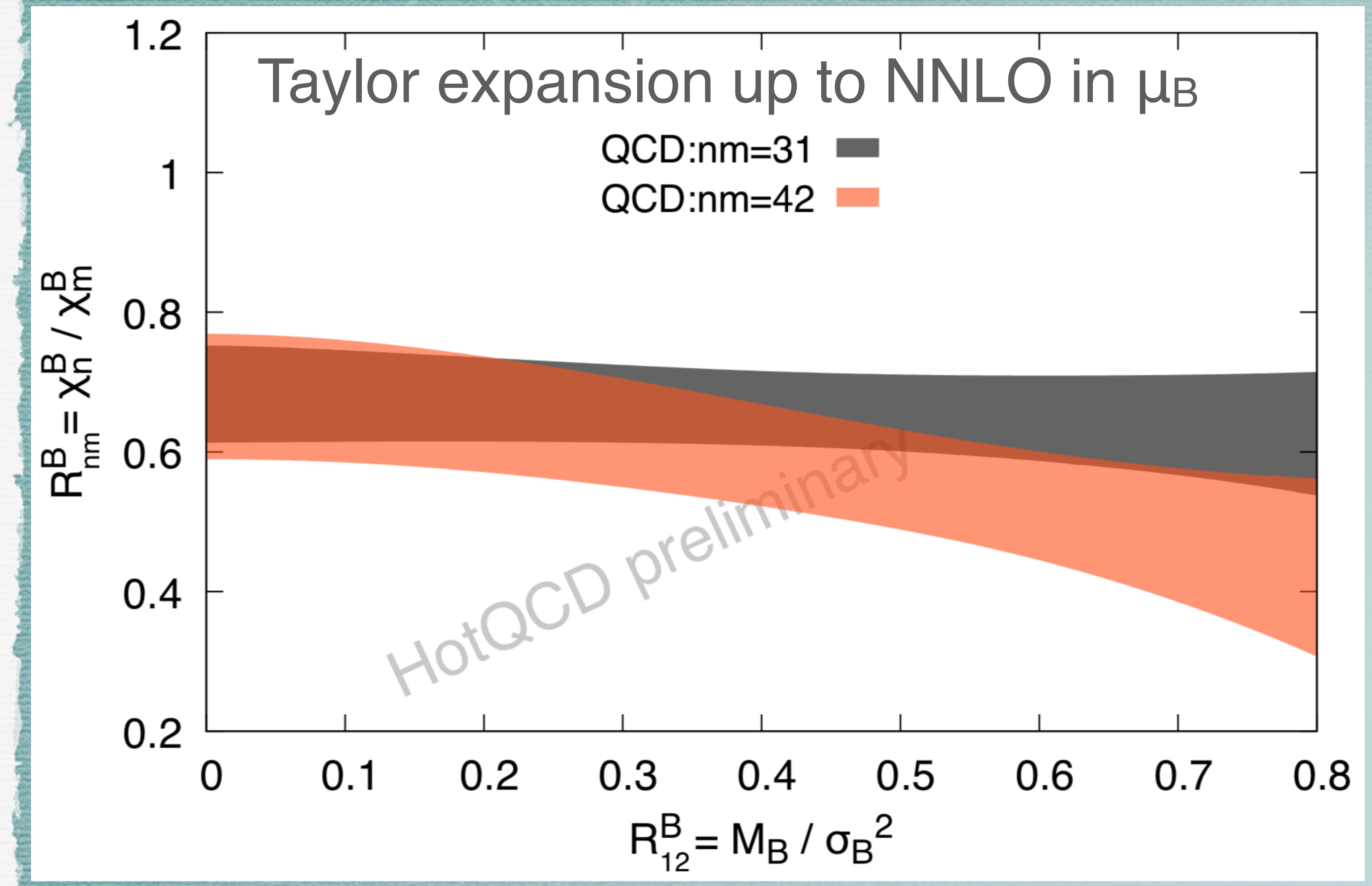
- At T~T_{pc} ideal HRG describes cumulants up to 2nd order at $\mu_B \lesssim$ 120 MeV
- QM-HRG needed to describe 2nd cumulant ratios where e.g. Not-PDG-listed strange-baryons' contributions manifest
- An ideal HRG cannot describe cumulants for >2nd orders
- Cumulant ratios at μ_B=0 : contact to ALICE data interesting

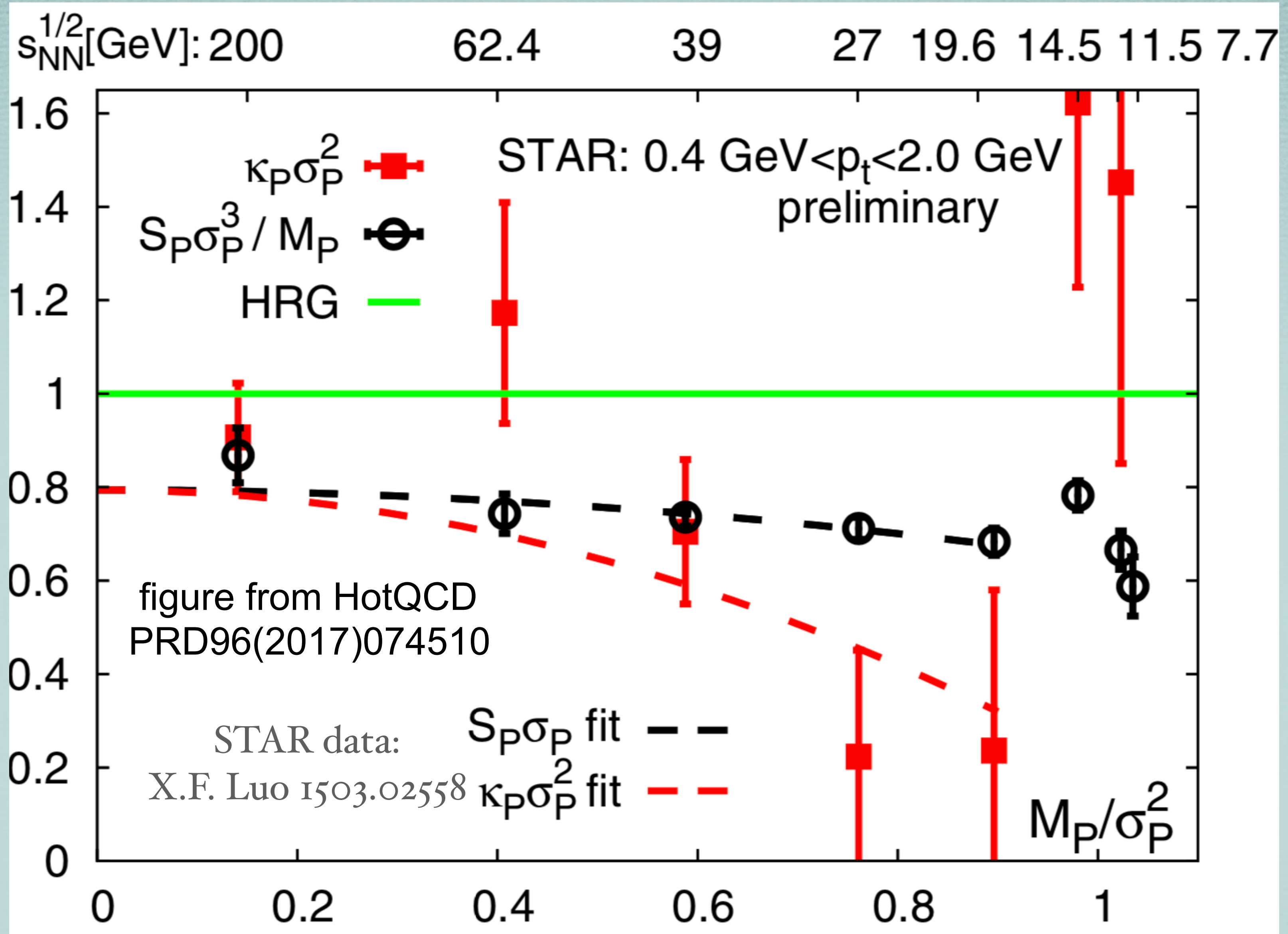
See modified versions of HRG e.g.: Alba & Liva PRC19', Aarts et al., PRD19', Dash et al., PRC19', Lo et al., PLB18', Huovinen & Petreczky PLB18', Vovchenko et al., PRL 17', PRC 17'...

Anar Rustamov Plenary Wed, Mesut Arslandok 11:00 Tue

QCD v.s. Experimental data: skewness (R_{31}) & kurtosis (R_{42}) ratios

Dennis Bollweg 17:50 Wed





Bands in QCD:
chiral crossover T region

$$T_{pc}(\mu_B) = T_{pc}(0)\left(1 - \kappa_2\left(\frac{\mu_B}{T}\right)^2 - \kappa_4\left(\frac{\mu_B}{T}\right)^4\right)$$

QCD v.s. Experimental data: skewness (R_{31}) & kurtosis (R_{42}) ratios

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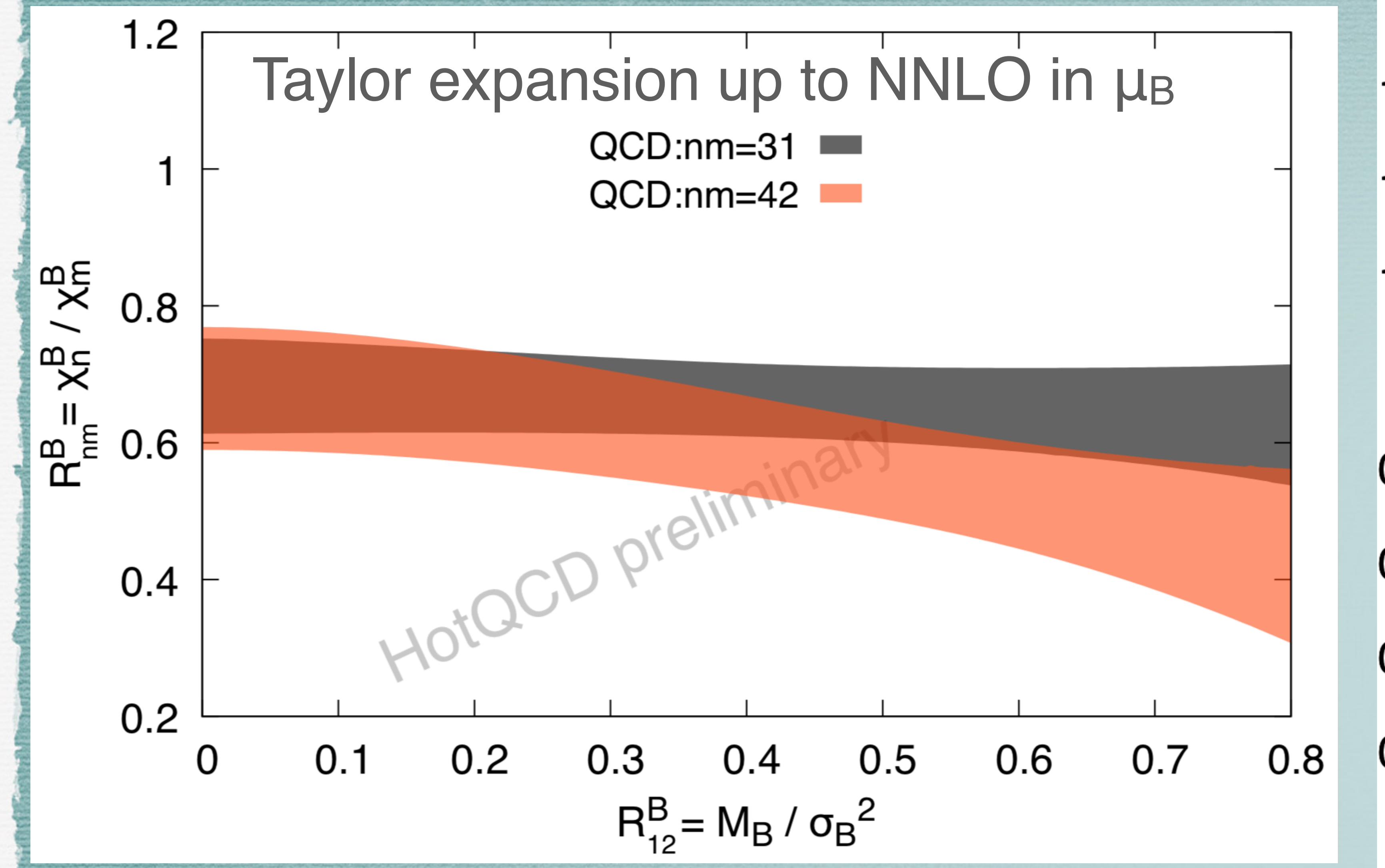

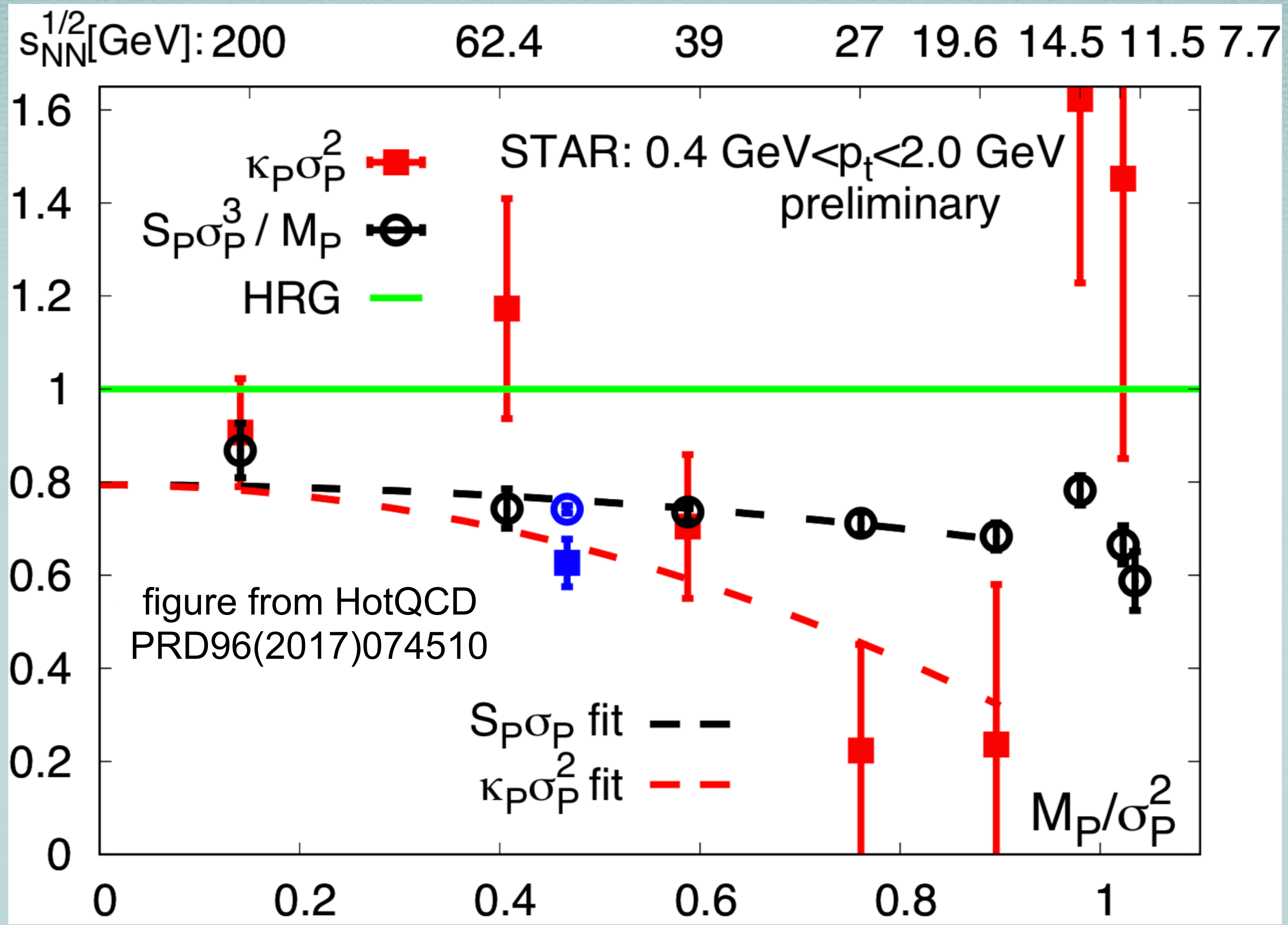

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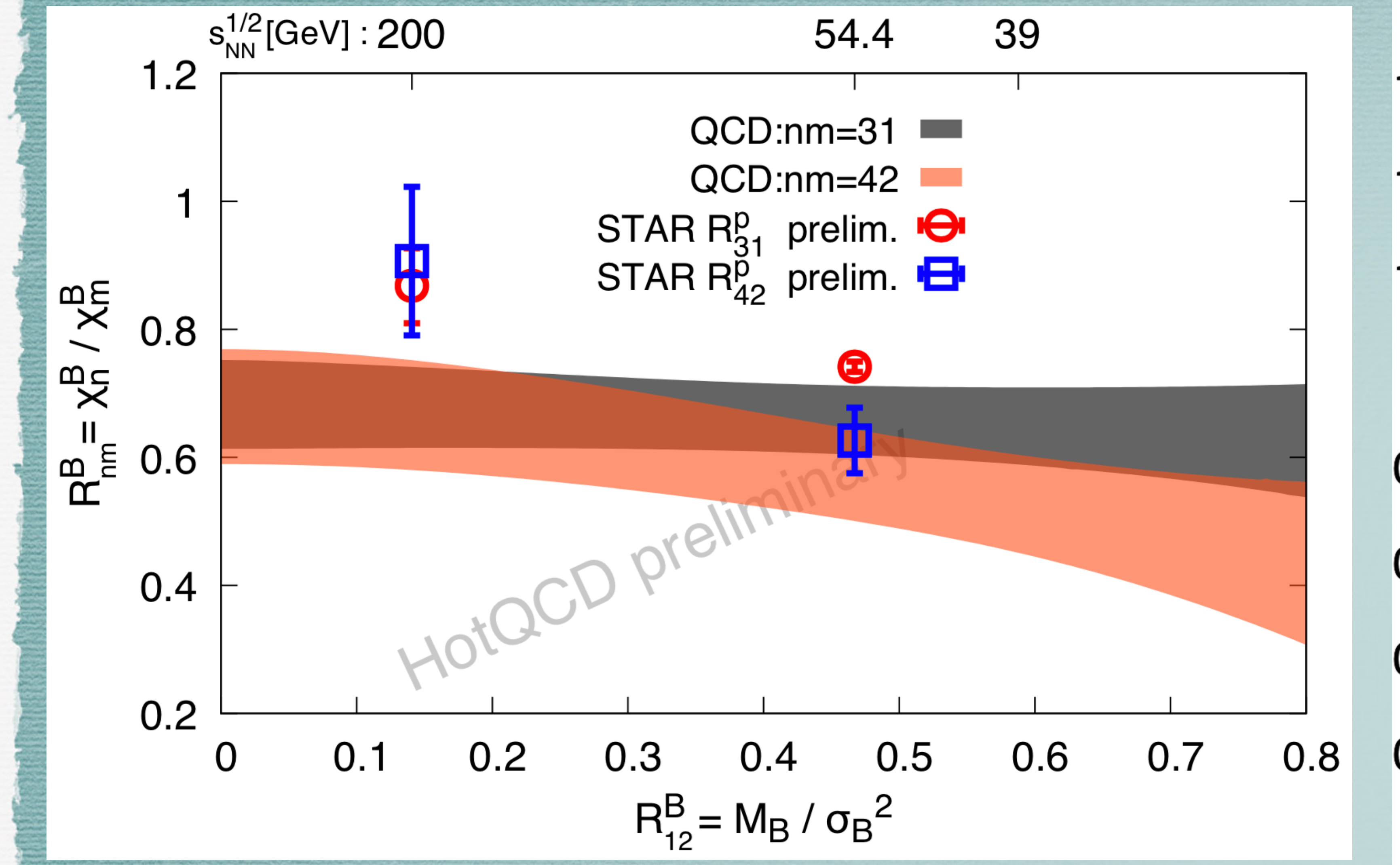
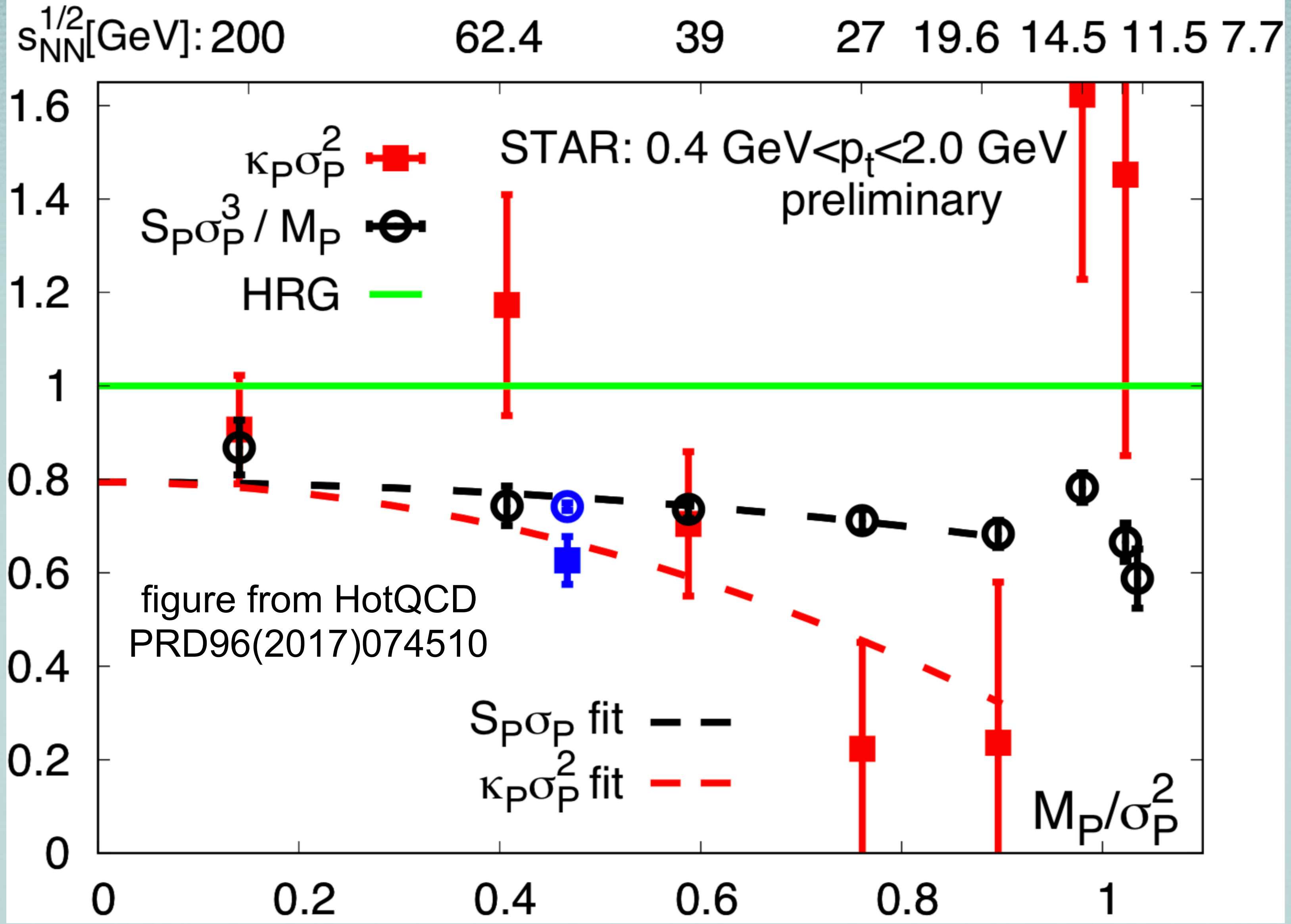
$\sqrt{s_{NN}}$=54.4 GeV data points fall on the fits to the old data

QCD v.s. Experimental data: skewness (R_{31}) & kurtosis (R_{42}) ratios

Dennis Bollweg 17:50 Wed



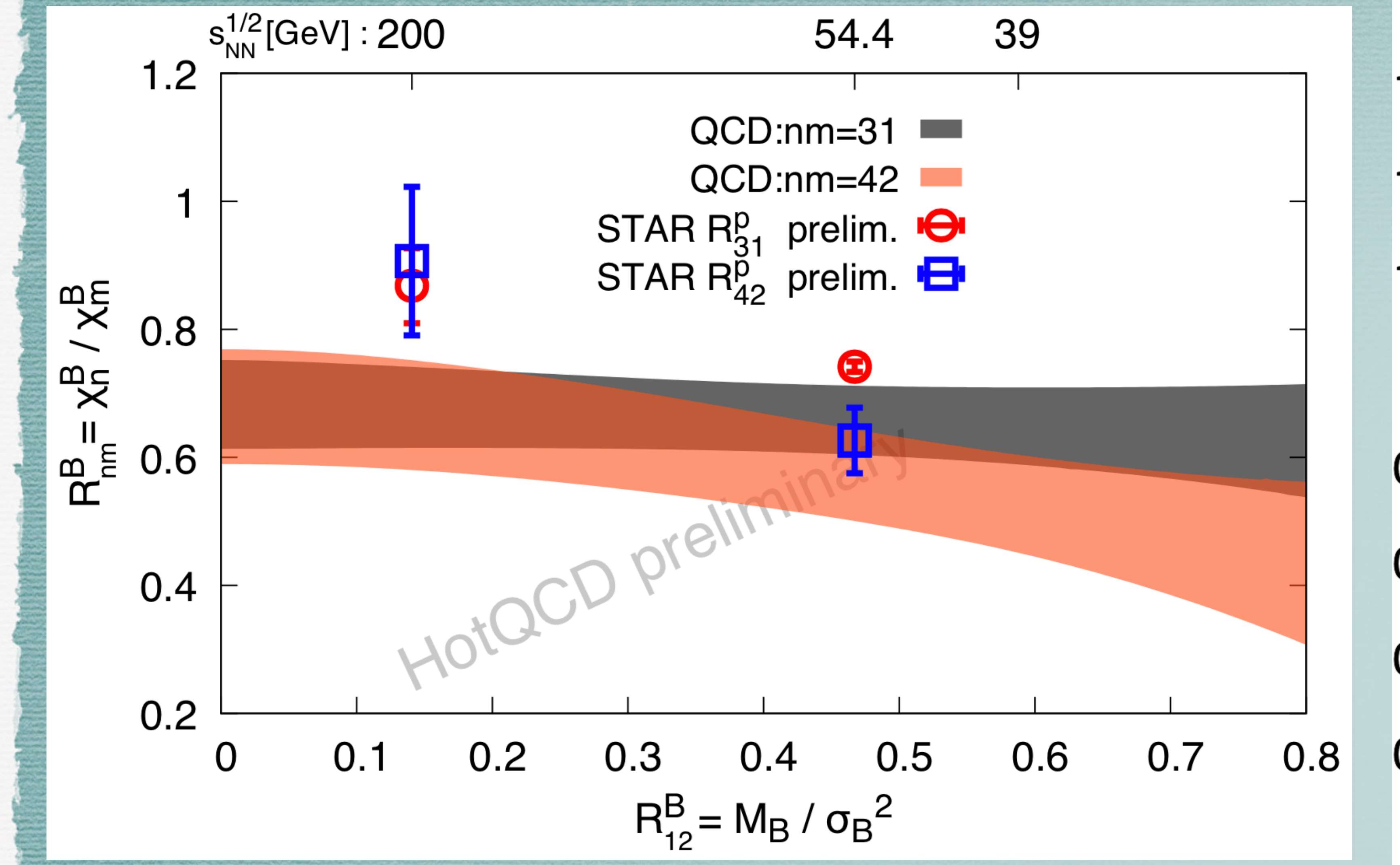


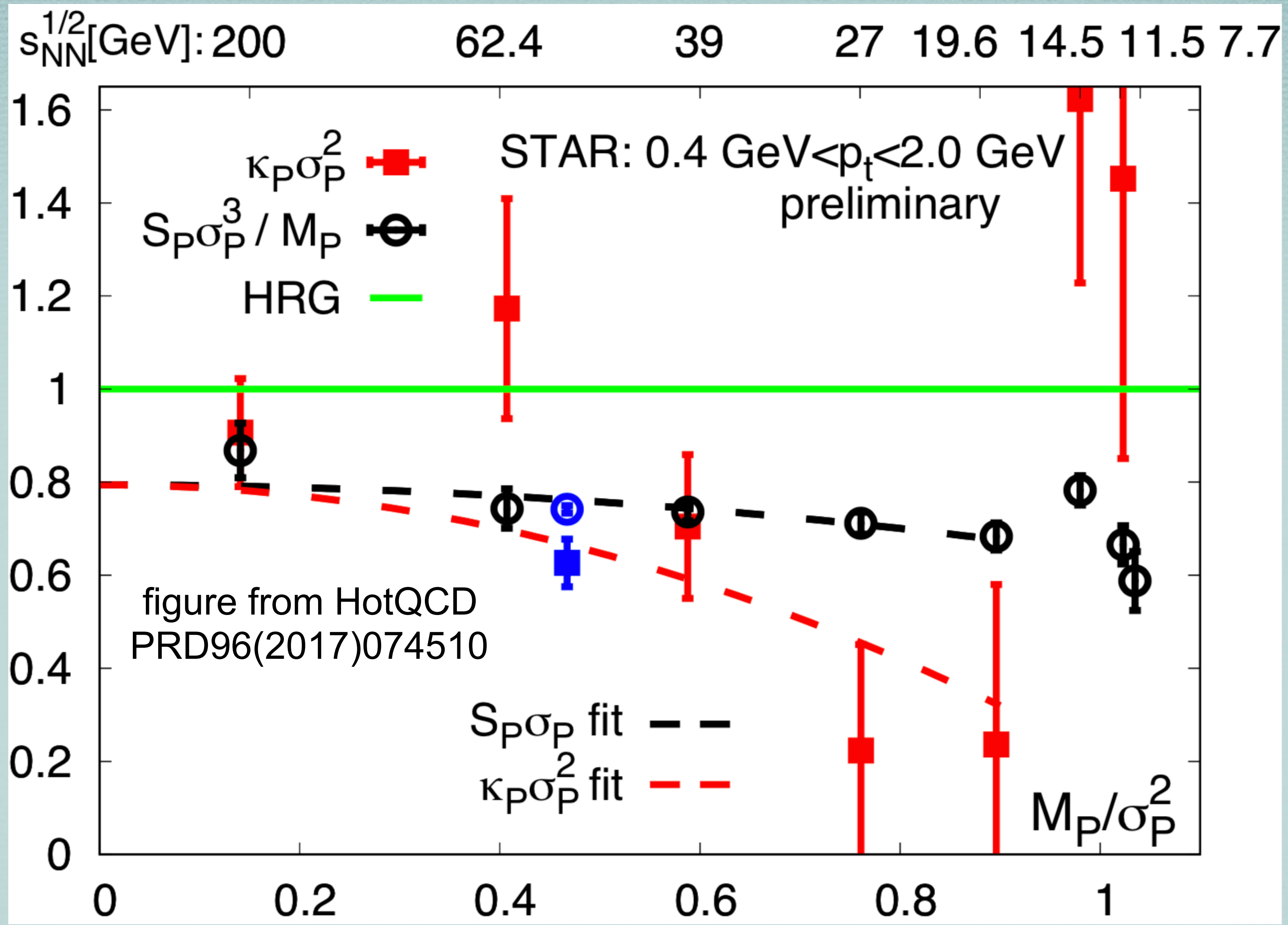
Consistency of $\sqrt{s_{NN}}$=54.4 GeV data with QCD at T=155-158 MeV

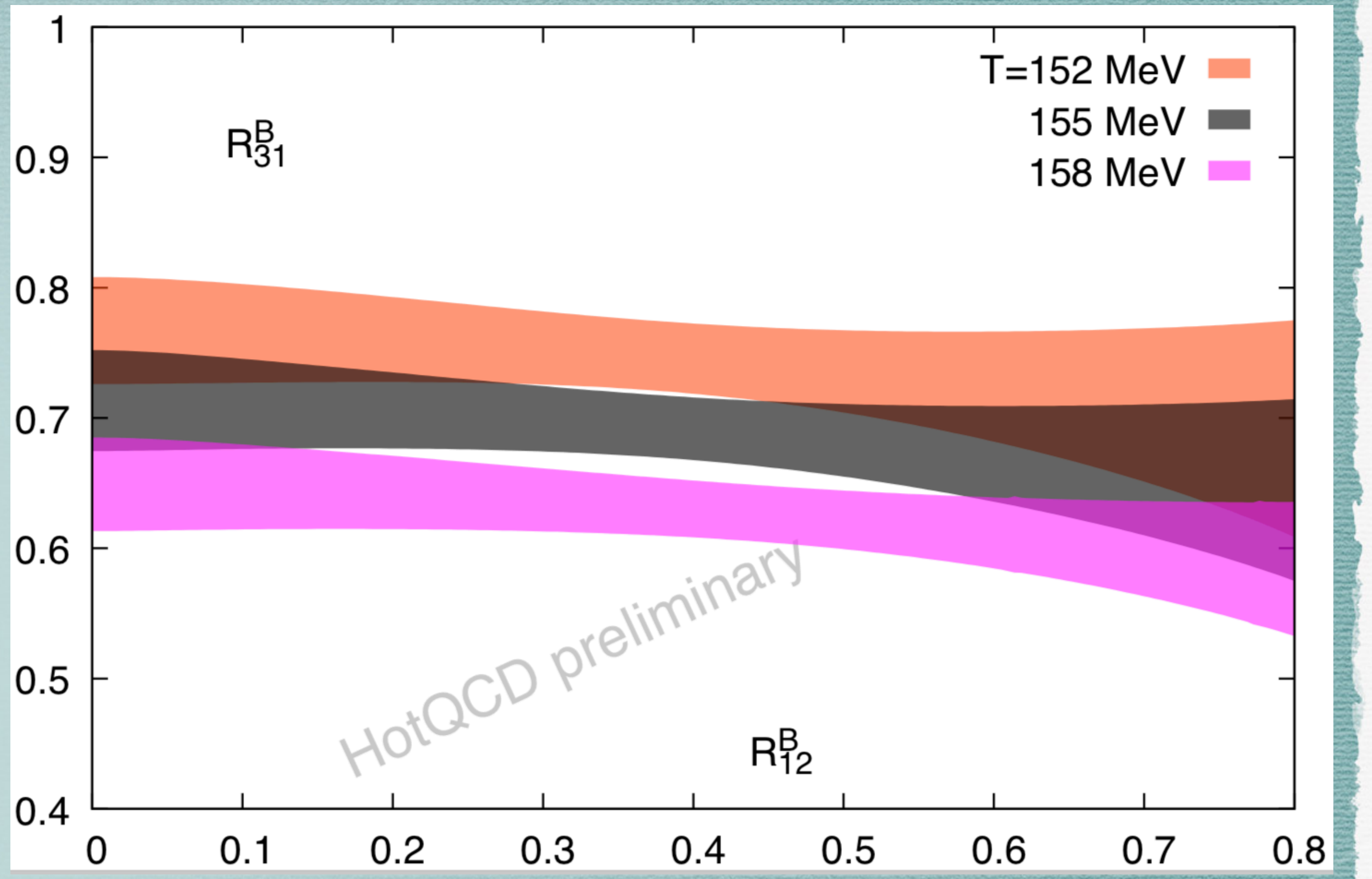
$\sqrt{s_{NN}}$=54.4 GeV data points fall on the fits to the old data

QCD v.s. Experimental data: skewness (R_{31}) & kurtosis (R_{42}) ratios

Dennis Bollweg 17:50 Wed







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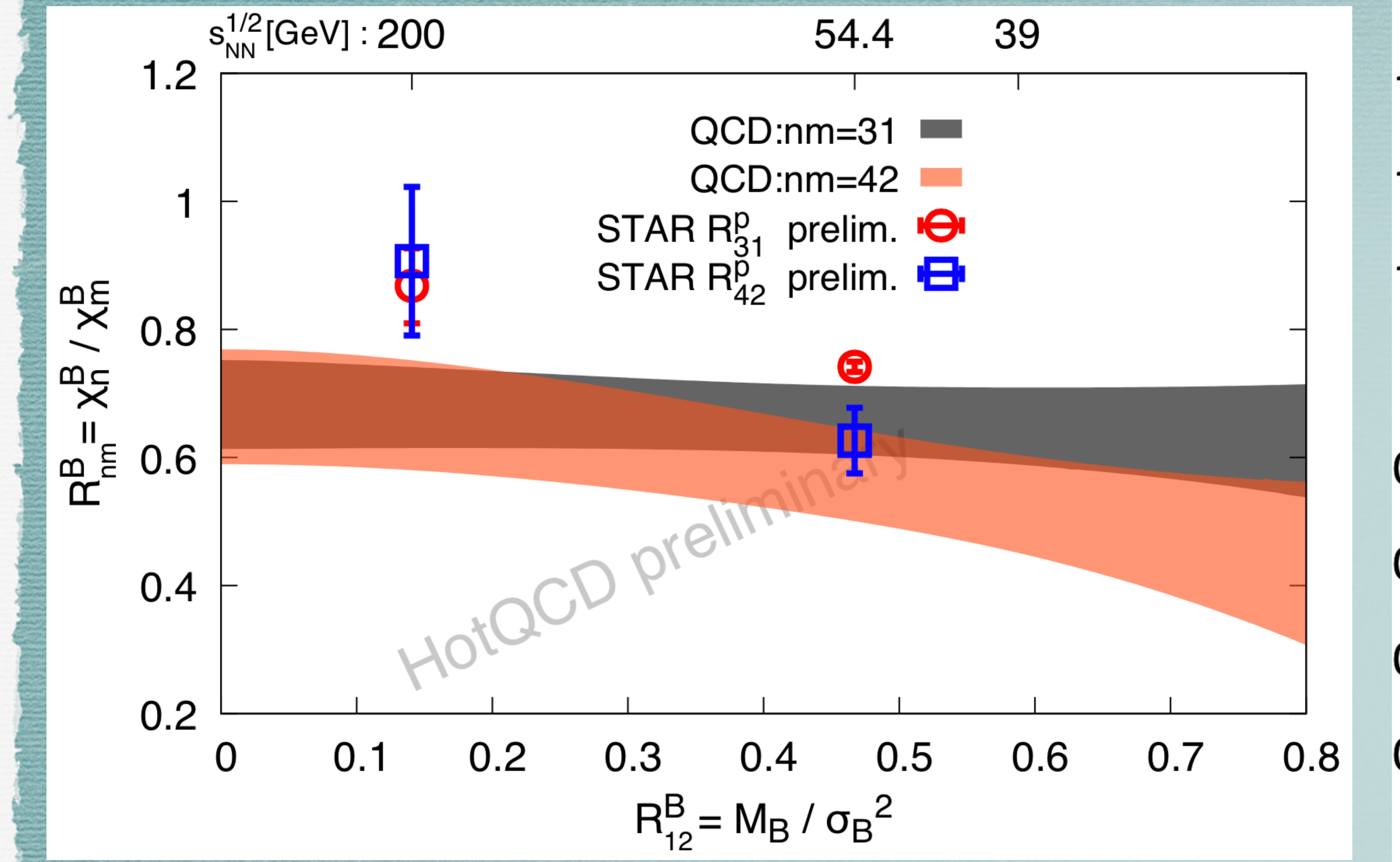
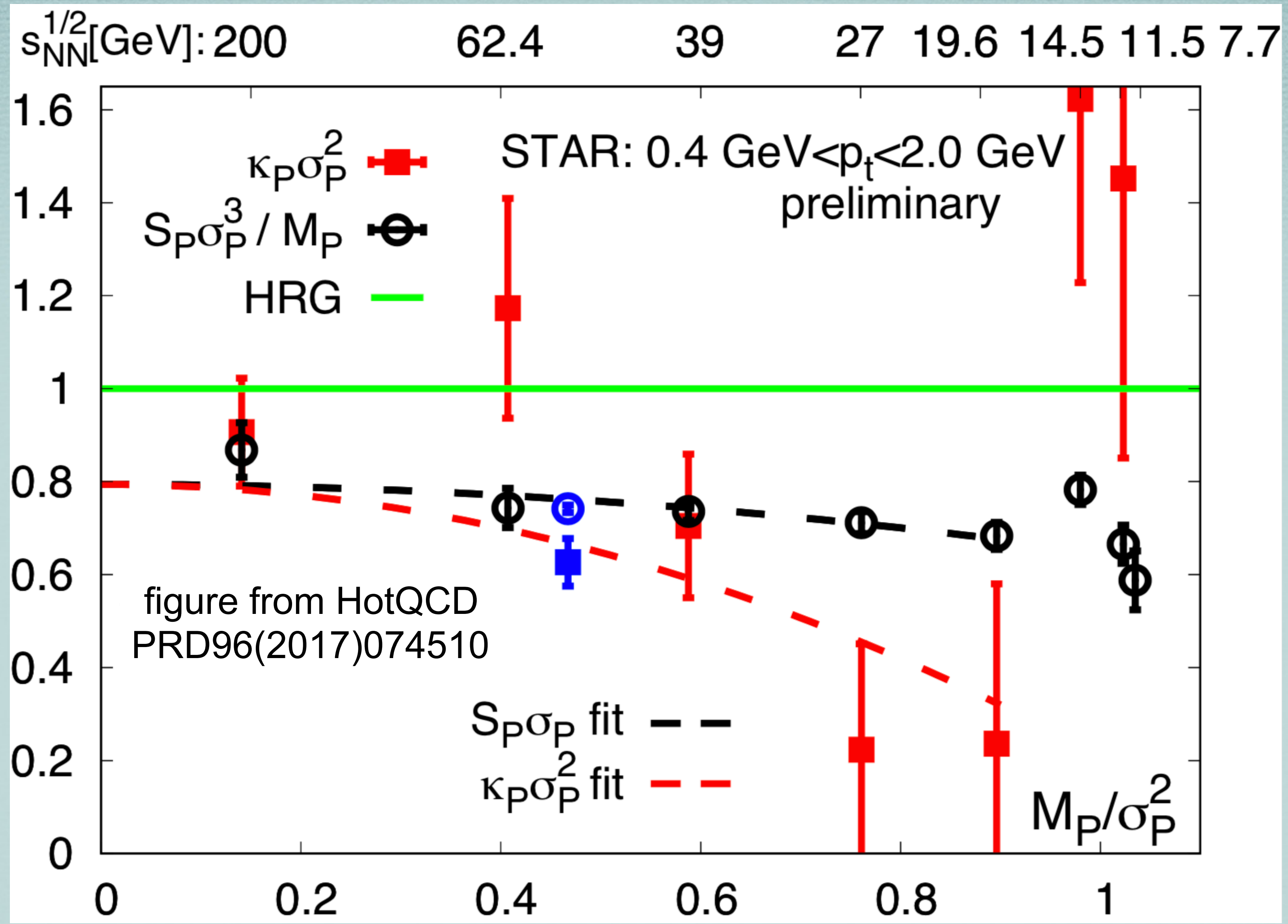
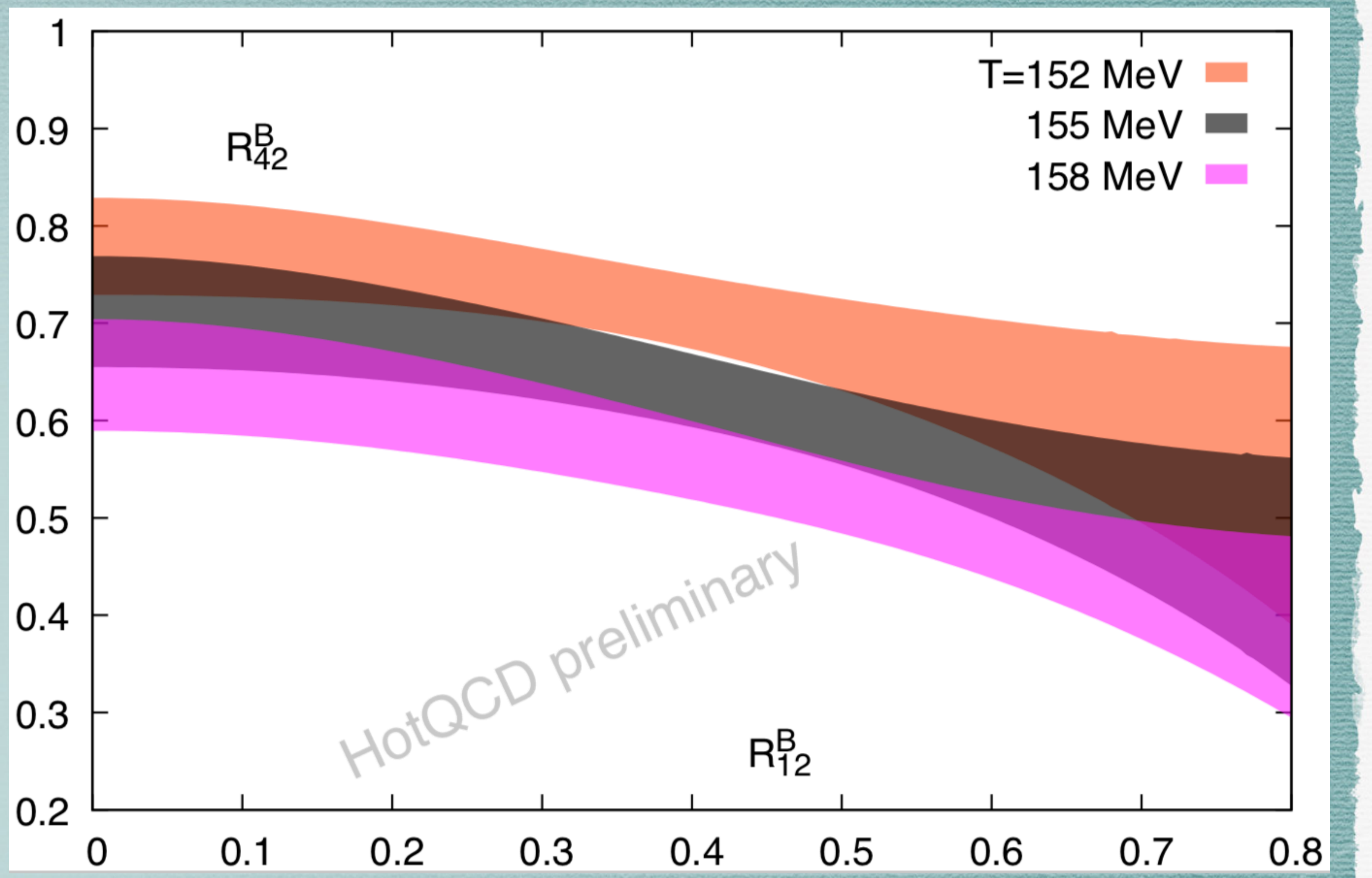
Dennis Bollweg 17:50 Wed







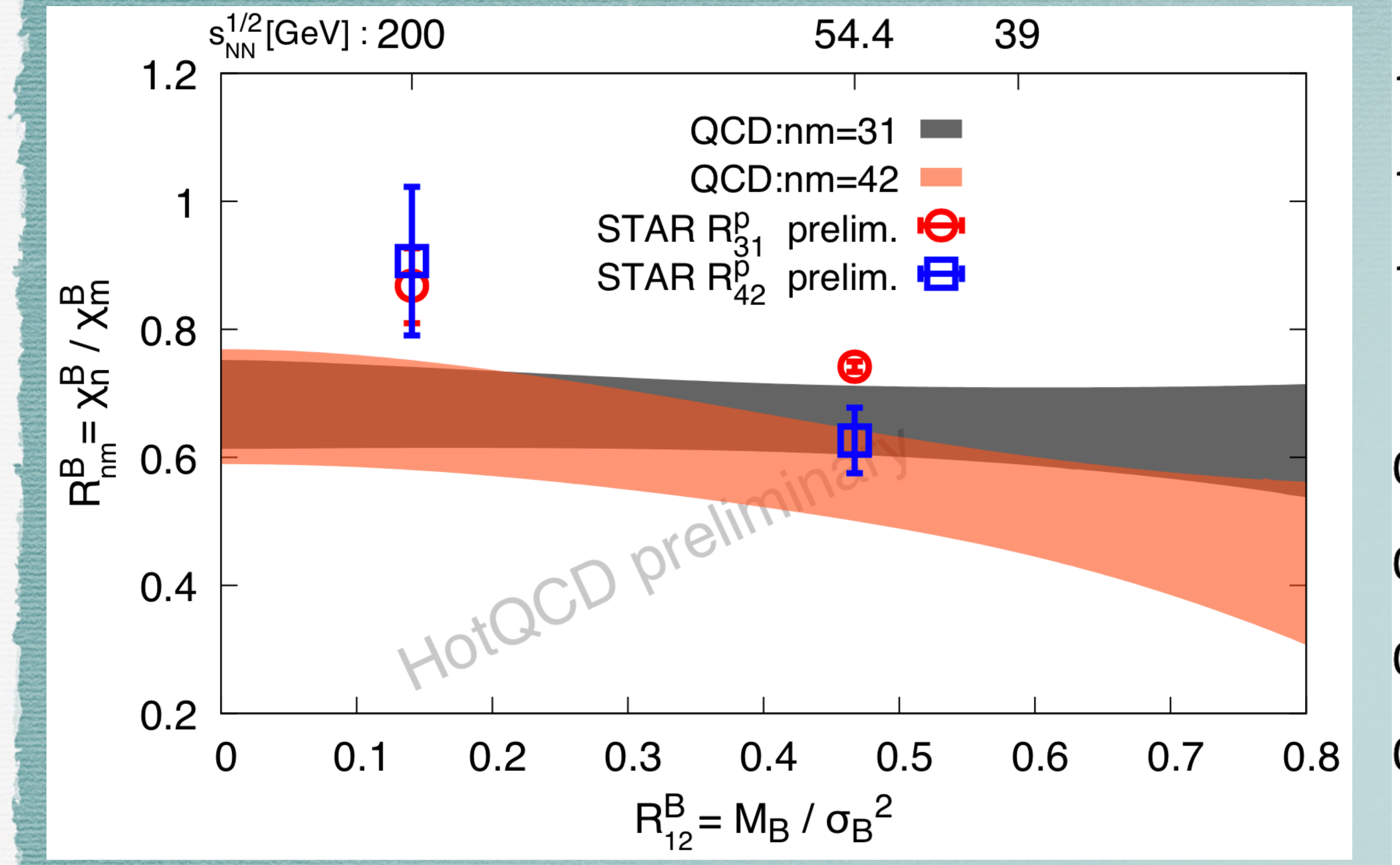
Consistency of $\sqrt{s_{NN}}$=54.4 GeV data with QCD at T=155-158 MeV

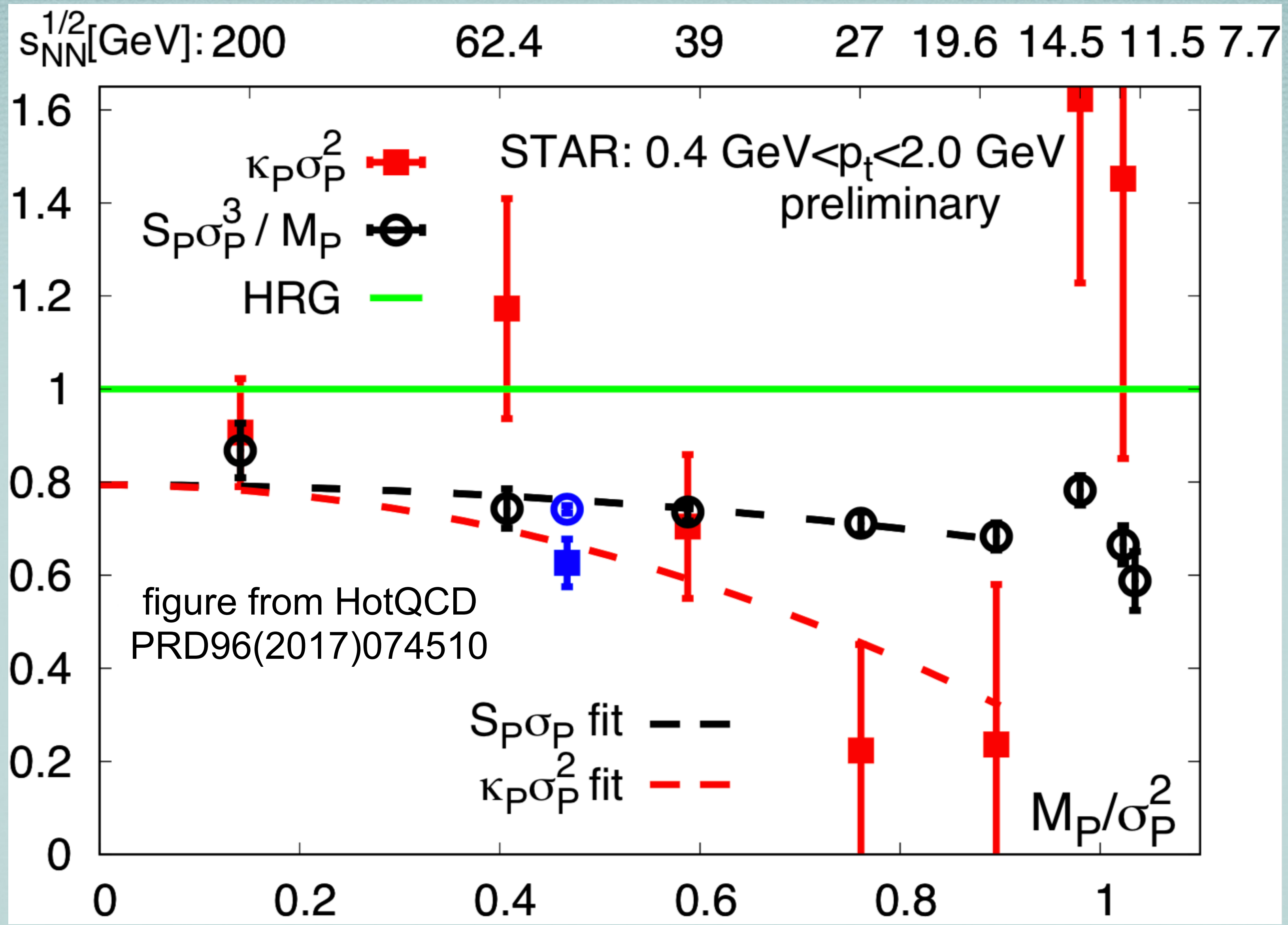
$\sqrt{s_{NN}}$=54.4 GeV data points fall on the fits to the old data

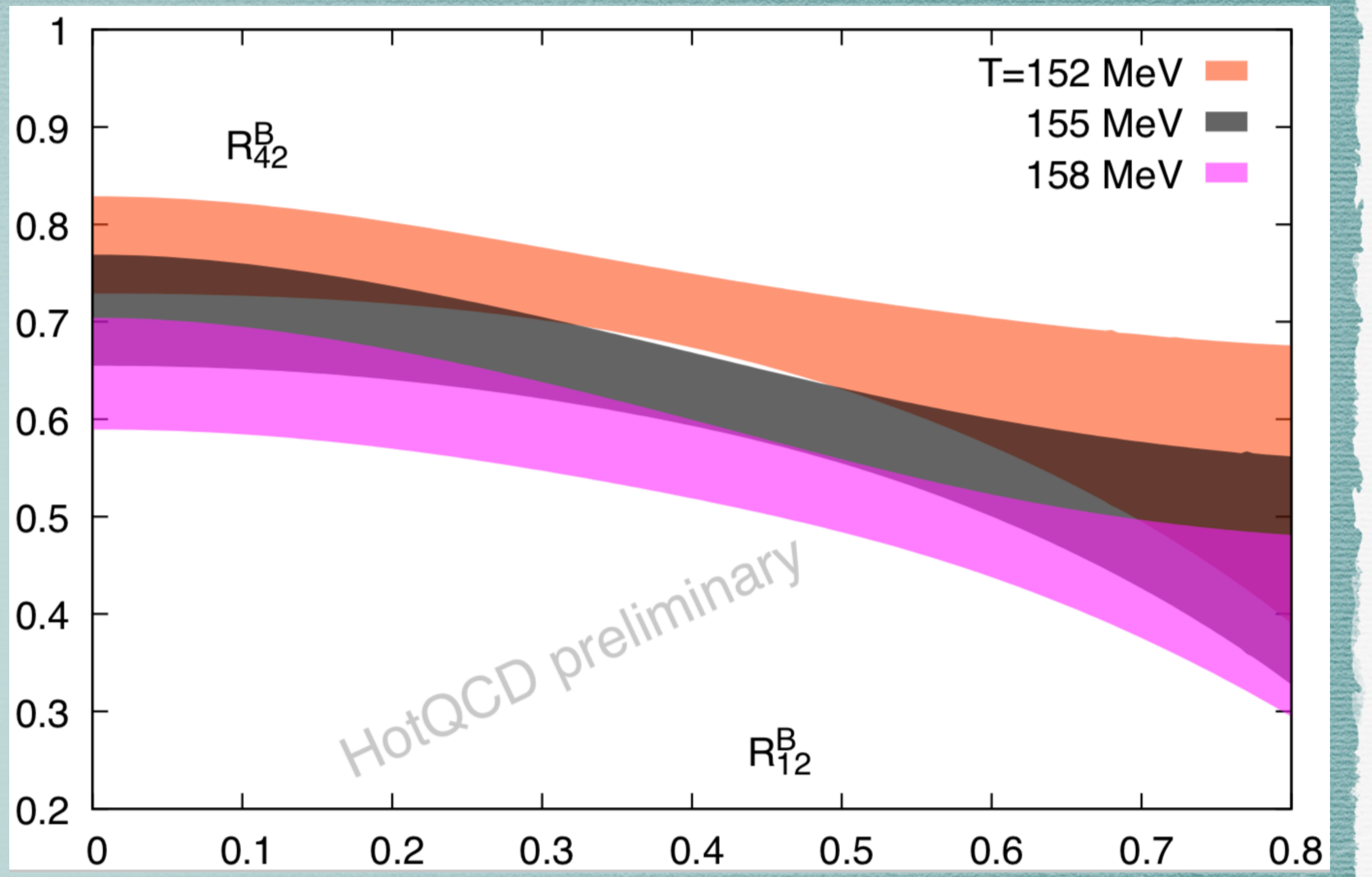
At fixed R_{12}, R_{31} & R_{42} increase with decreasing T

QCD v.s. Experimental data: skewness (R_{31}) & kurtosis (R_{42}) ratios

Dennis Bollweg 17:50 Wed







Consistency of $\sqrt{s_{NN}}$=54.4 GeV data with QCD at T=155-158 MeV

$\sqrt{s_{NN}}$=54.4 GeV data points fall on the fits to the old data

At fixed R_{12}, R_{31} & R_{42} increase with decreasing T

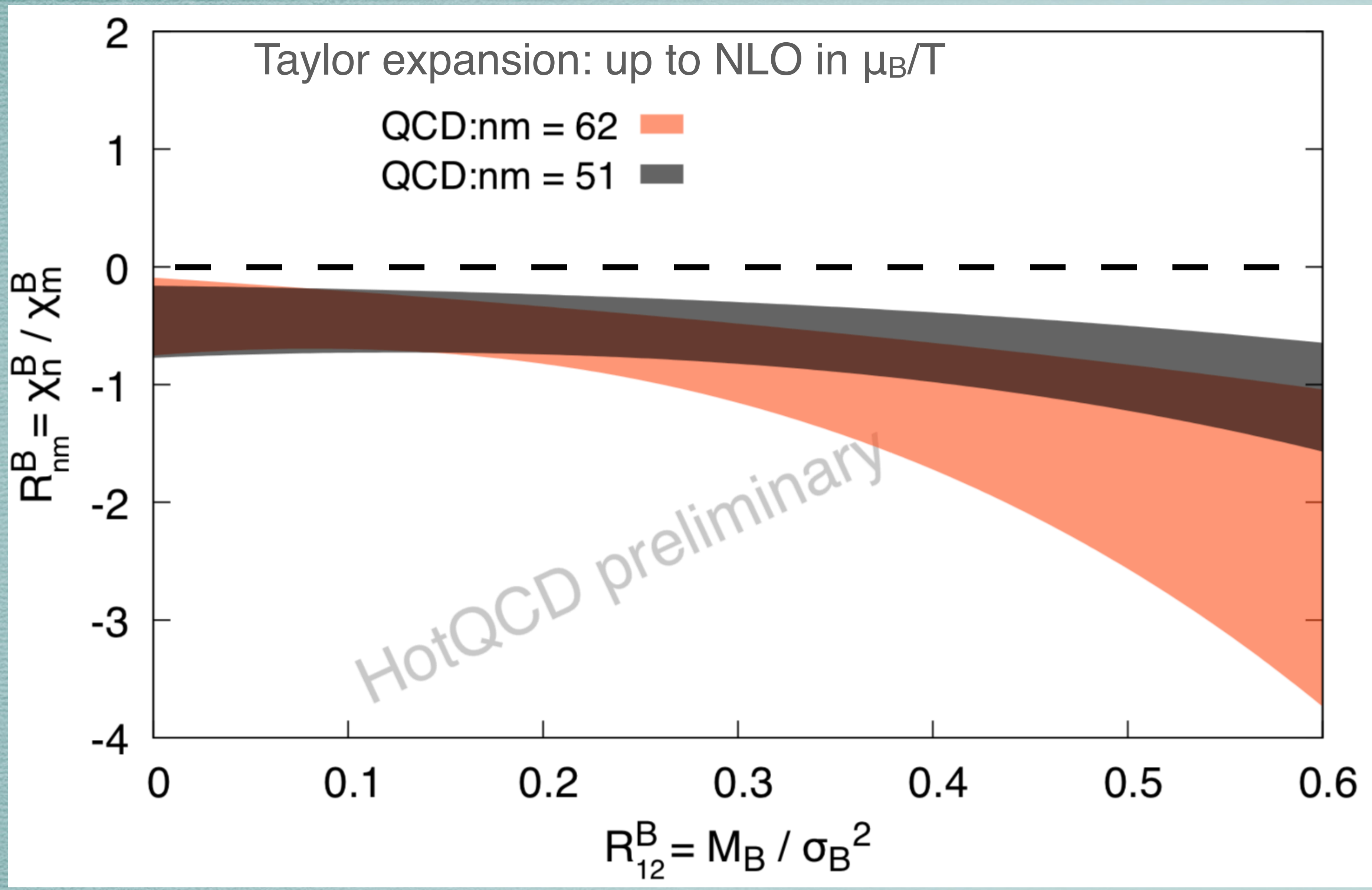
Inconsistency of T_f=165(4) MeV from yields and their ratios at $\sqrt{s_{NN}}$=200 GeV

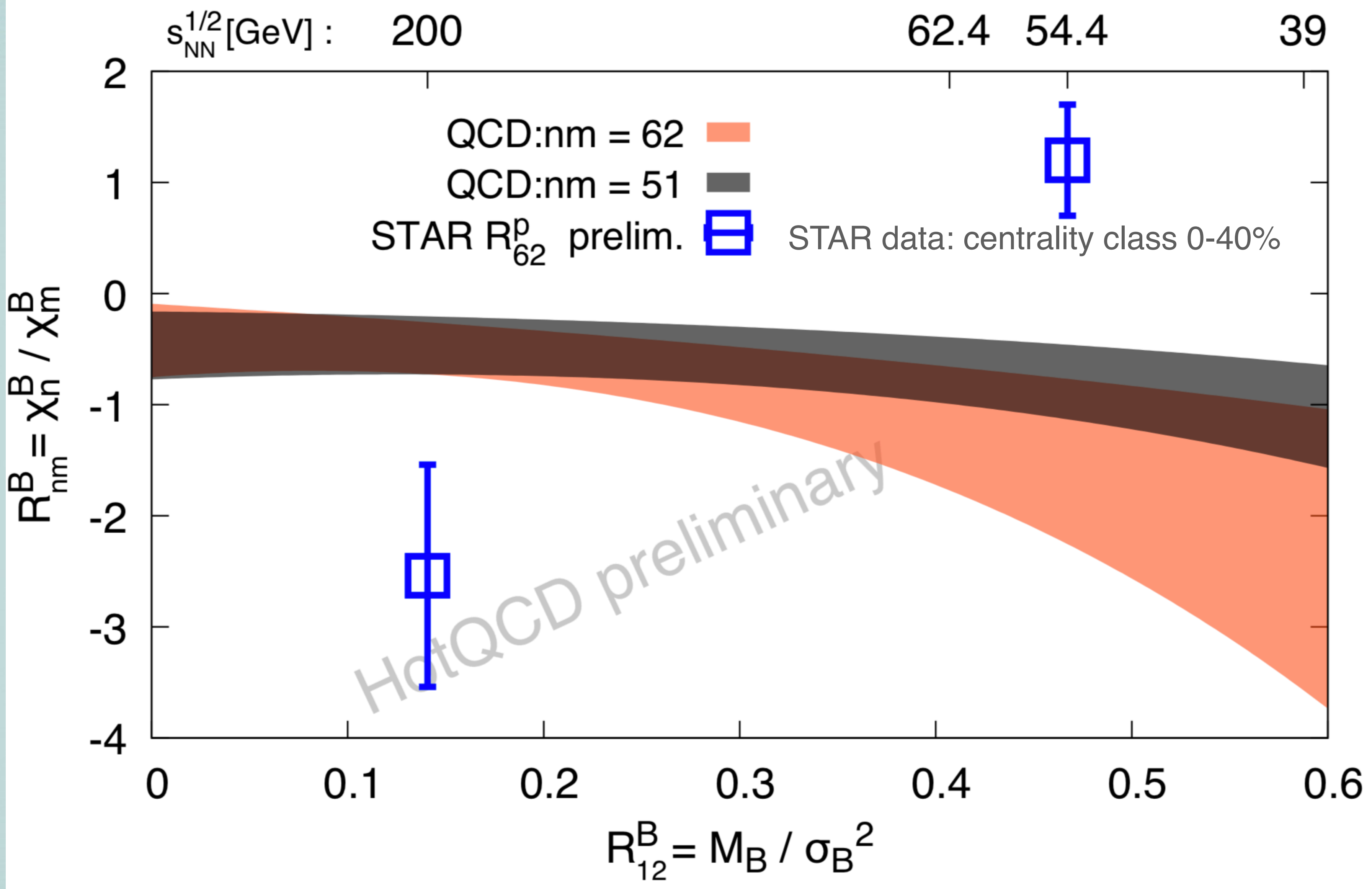
Statistics ? Non-equilibrium?…?…?

QCD v.s. Experimental data: hyper-skewness (R_{51}) & hyper-kurtosis (R_{62}) ratios

Dennis Bollweg 17:50 Wed

$$R^B_{62}(\mu_B/T) = \frac{\chi^B_6(\mu_B/T)}{\chi^B_2(\mu_B/T)} = \frac{\chi^B_6(0)}{\chi^B_2(0)} + \frac{1}{2}\left(\frac{\mu_B}{T}\right)^2 \left(\frac{\chi^B_8(0)}{\chi^B_2(0)} - \frac{\chi^B_6(0)}{\chi^B_2(0)}\frac{\chi^B_4(0)}{\chi^B_2(0)}\right) + \mathcal{O}\left(\frac{\mu_B}{T}\right)^4$$





R_{51} and R_{62}: Statistics-hungry quantities

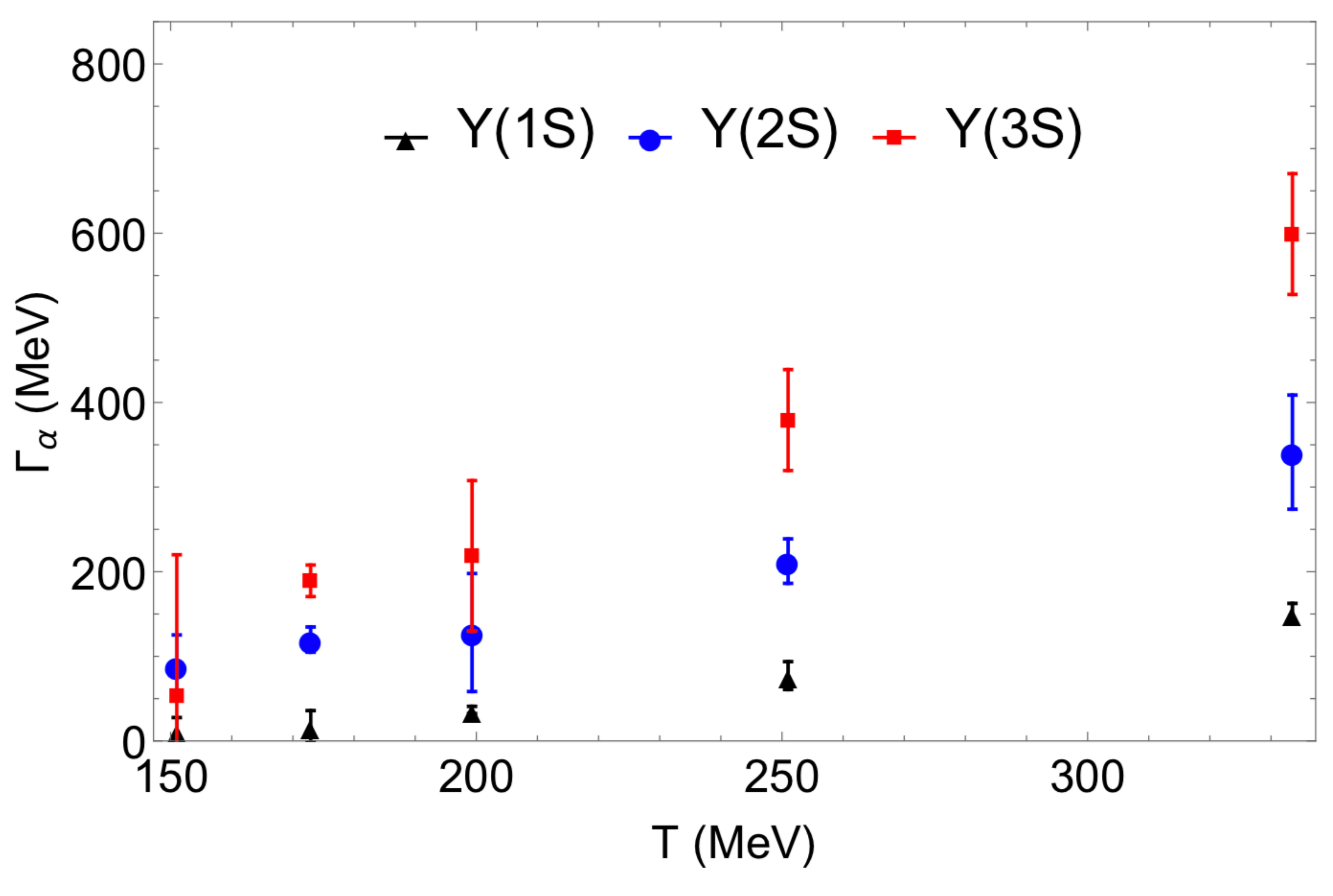
LQCD: Both hyper-skewness and hyper-kurtosis are negative down to $\sqrt{s_{NN}}$=39 GeV

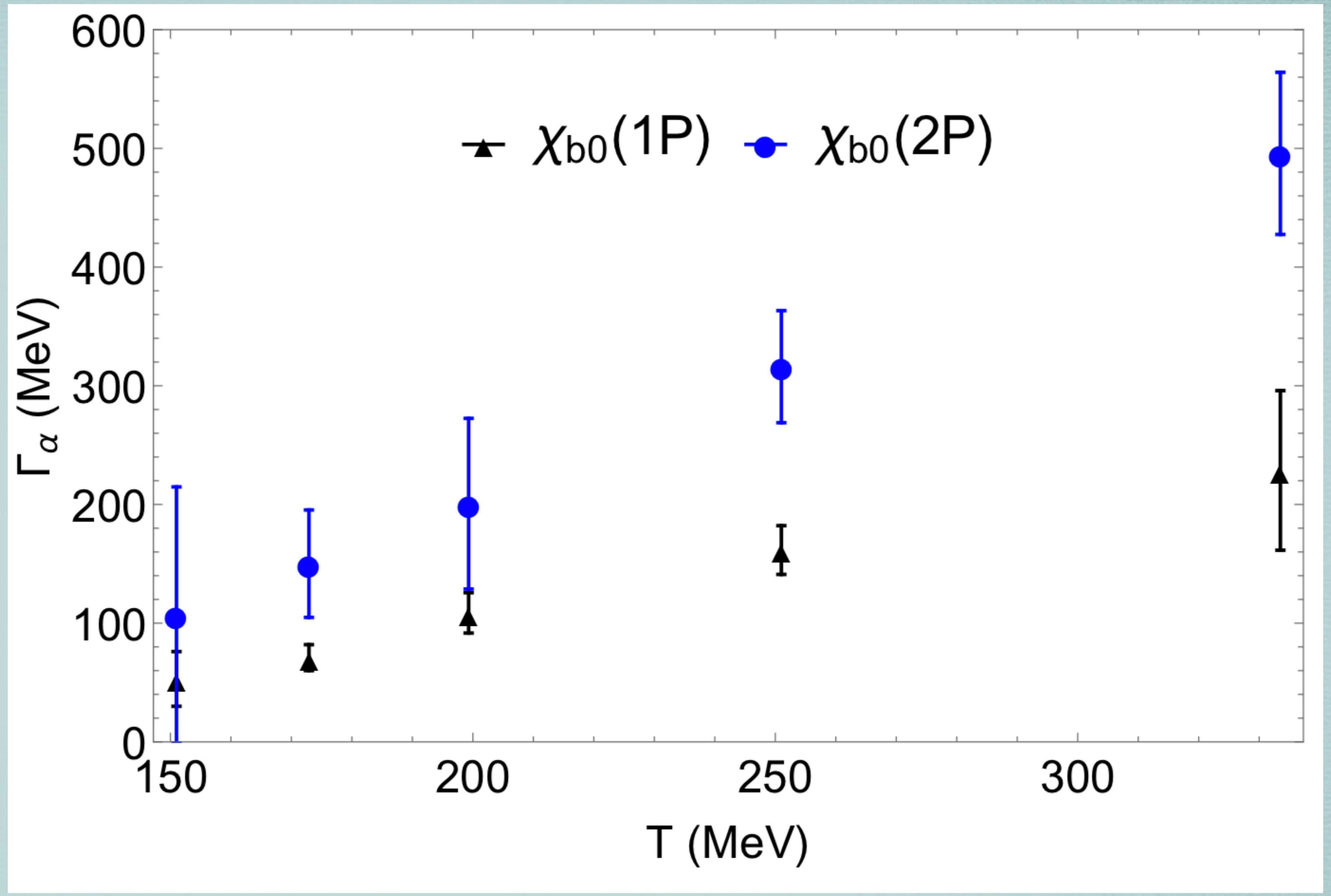
Nice consistency seen in skewness & kurtosis at $\sqrt{s_{NN}}$=54.4 GeV with QCD while deviations seen at both $\sqrt{s_{NN}}$=54.4 & 200 GeV

Excited bottomonia up to 3S and 2P

NRQCD, 48^3x12 lattices,
1st lattice QCD study of up to 3S and 2P bottomonia

More news about Quarkonia from LQCD
see Alexander Rothkopf Plenary Fri





Rasmus Larsen et al., 1910.07374, 1908.08437

Rasmus Larsen
17:00 Wed

Thermal width extracted from temporal correlators:

$$\Gamma_{\Upsilon(1S)}(T) < \Gamma_{\chi_{b0}(1P)}(T) < \Gamma_{\Upsilon(2S)}(T) < \Gamma_{\chi_{b0}(2P)}(T) < \Gamma_{\Upsilon(3S)}(T)$$

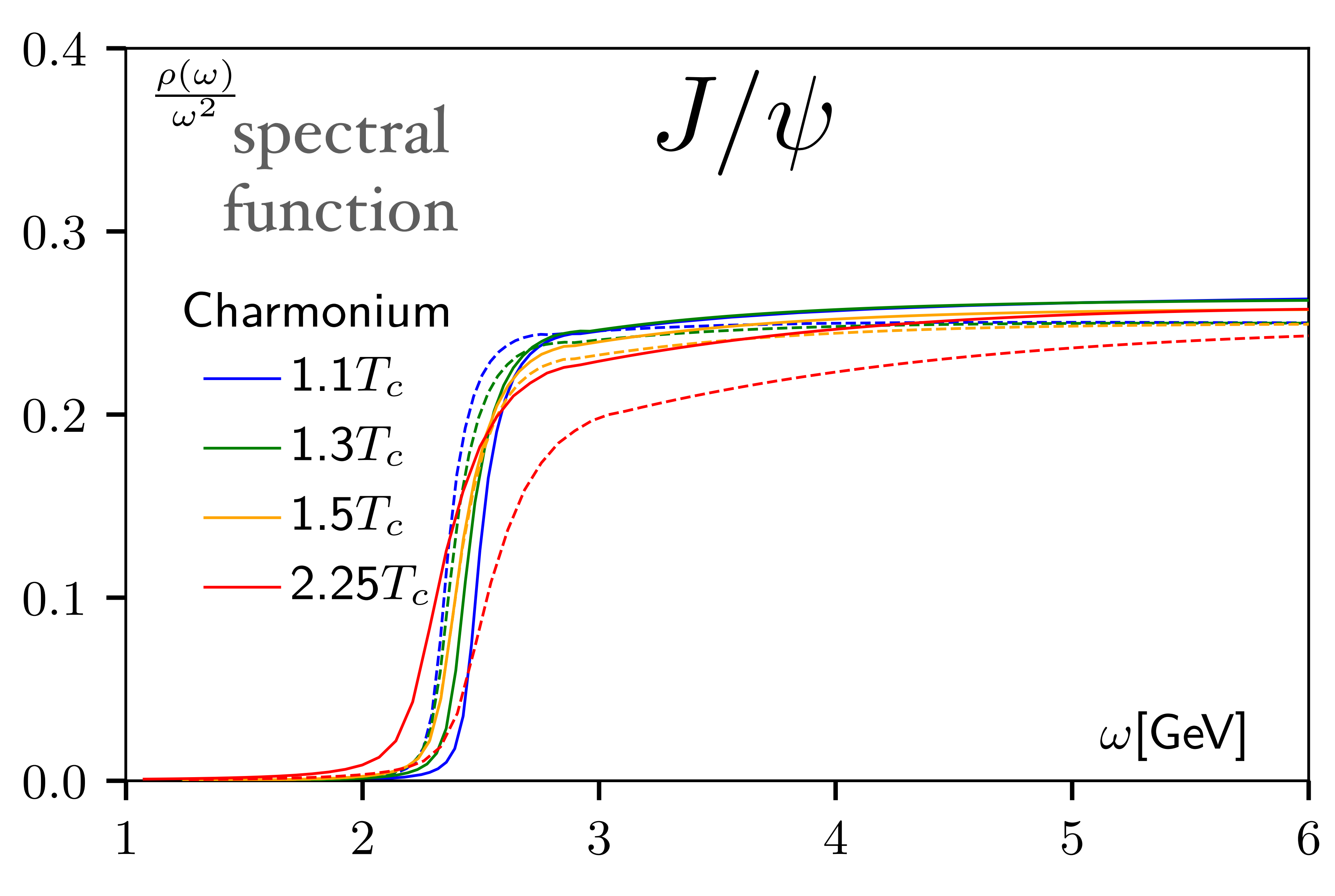
Compatible to Sequential dissociation picture

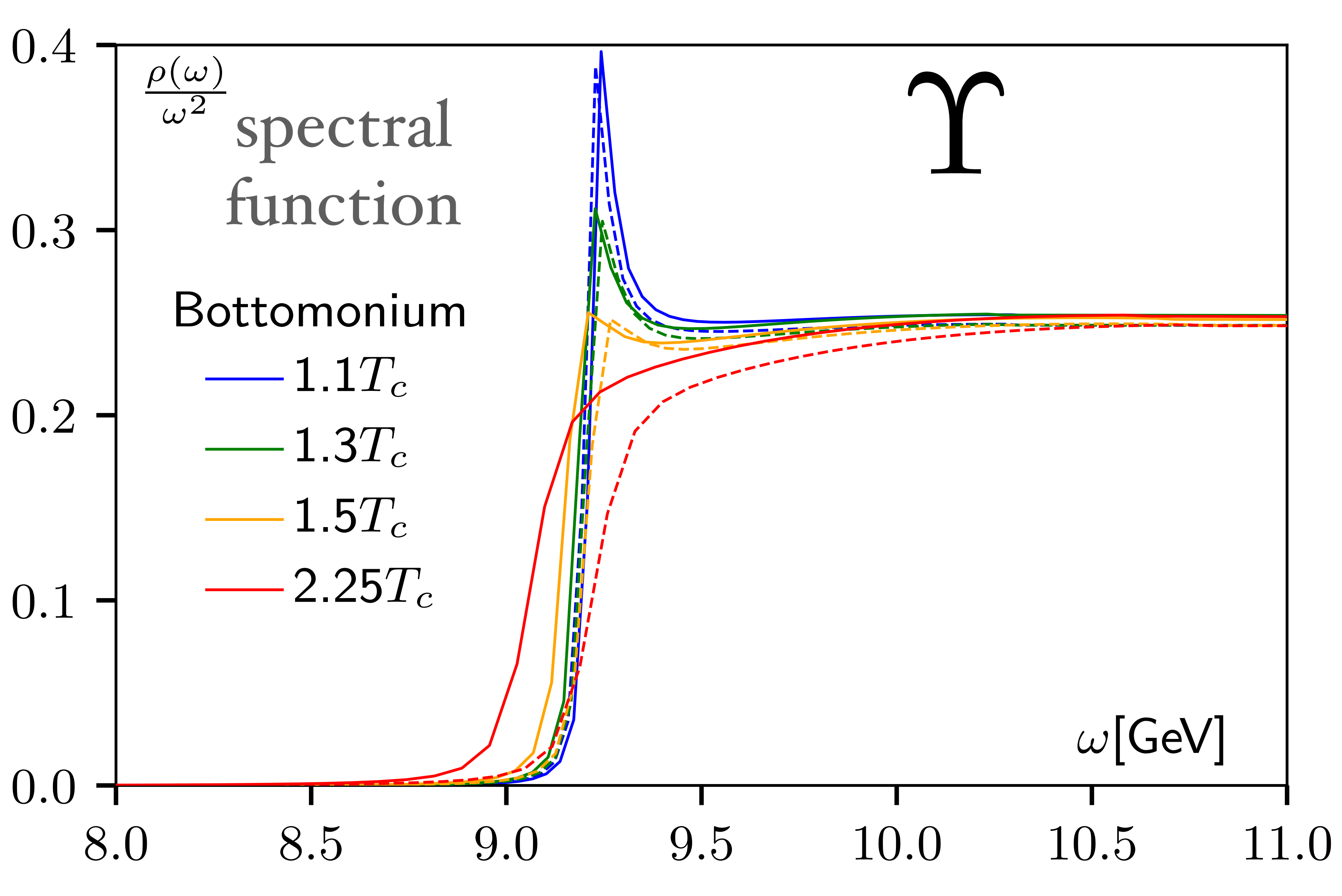
Fate of J/Ψ and ϒ

More news about Quarkonia from LQCD
see Alexander Rothkopf Plenary Fri

Quenched QCD, 1st continuum extrapolated results

e.g. T=1.1 T_c continuum extrapolation based on 192^3x64, 144^3x48, 120^3x40, 96^3x32 lattices





Broken lines:
spectral function (SPF)
from pNRQCD

Solid lines
fit to difference of neighboring temporal correlators
using pNRQCD +cont SPF
as a ansatz

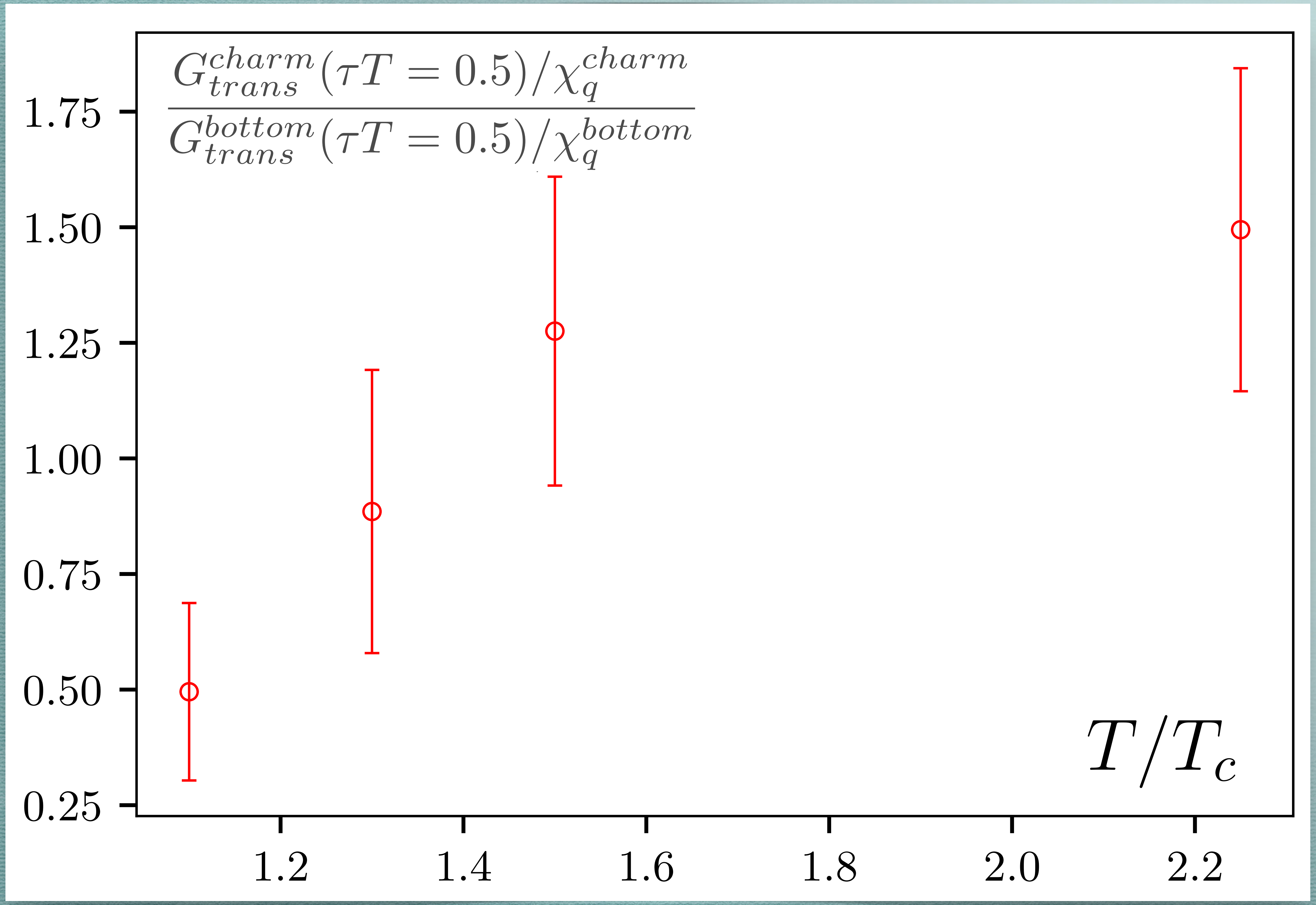
pNRQCD fits to
Pseudo-scalar correlators
Burnier et al., JHEP11(2017)206

- At T>=1.1T_c : No resonance peaks of J/Ψ is needed
- At T>=1.5T_c: No resonance peaks of ϒ is needed

Olaf Kaczmarek
14:40 Tue

Flavor hierarchy of drag coefficients η

Ratios of vector correlators contributed from transport peaks between charm and bottom quarks



Quenched QCD, continuum extrapolated results

G_{trans}: correlators after resonance and continuum contribution fitted using pNRQCD+cont are subtracted

Using Lorentzian ansatz for the transport peak:

$$\frac{G^{charm}_{trans}/\chi_q^{charm}/T}{G^{bottom}_{trans}/\chi_q^{bottom}/T} \approx \frac{M_{bottom}}{M_{charm}} \frac{\tan^{-1}(T/\eta^{charm})}{\tan^{-1}(T/\eta^{bottom})}$$

$M_{bottom}/M_{charm} \approx 3$

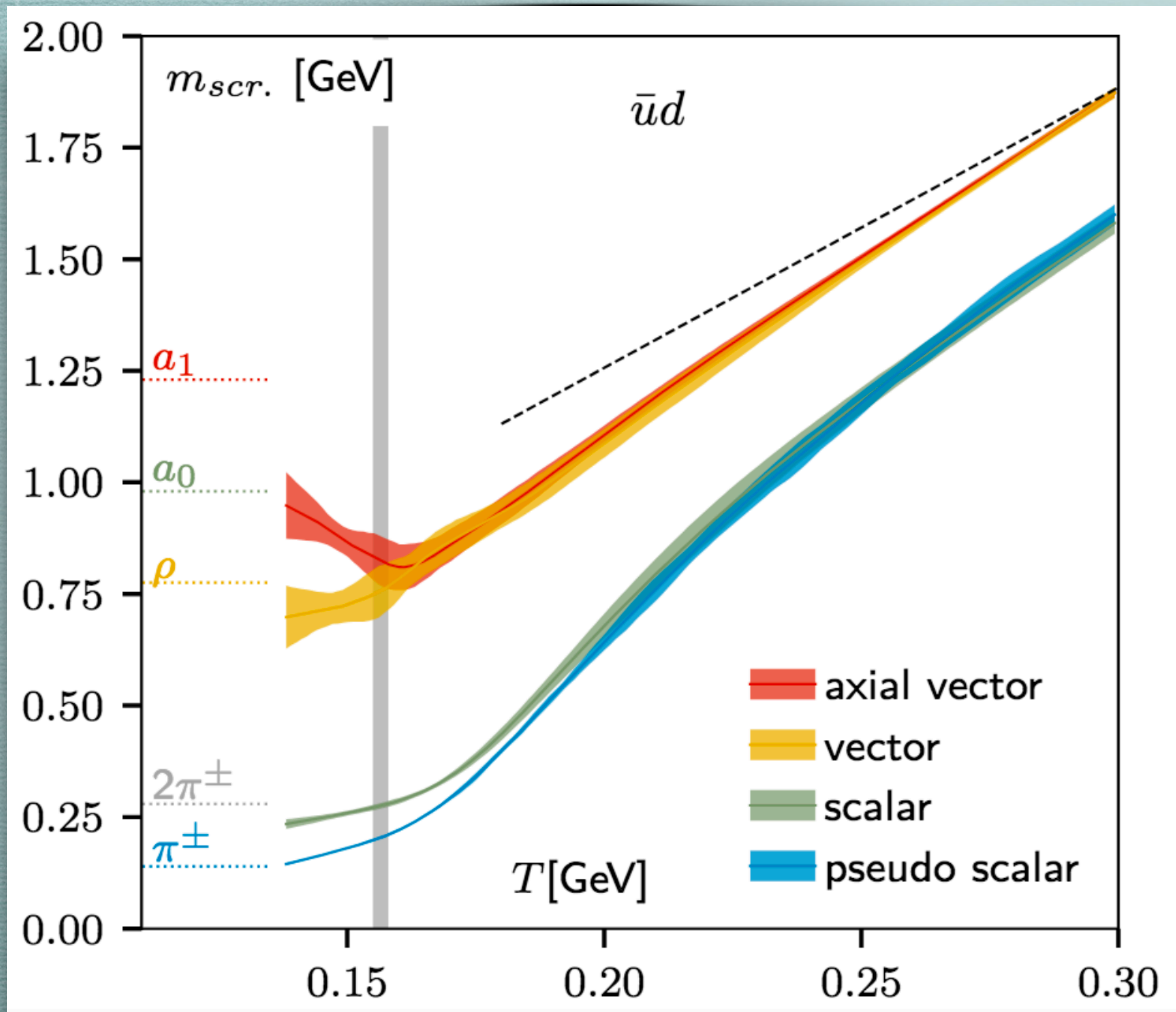
$$\frac{\tan^{-1}(T/\eta^{charm})}{\tan^{-1}(T/\eta^{bottom})} < 1 \quad \Rightarrow \quad \eta^{charm} > \eta^{bottom}$$

Olaf Kaczmarek
14:40 Tue

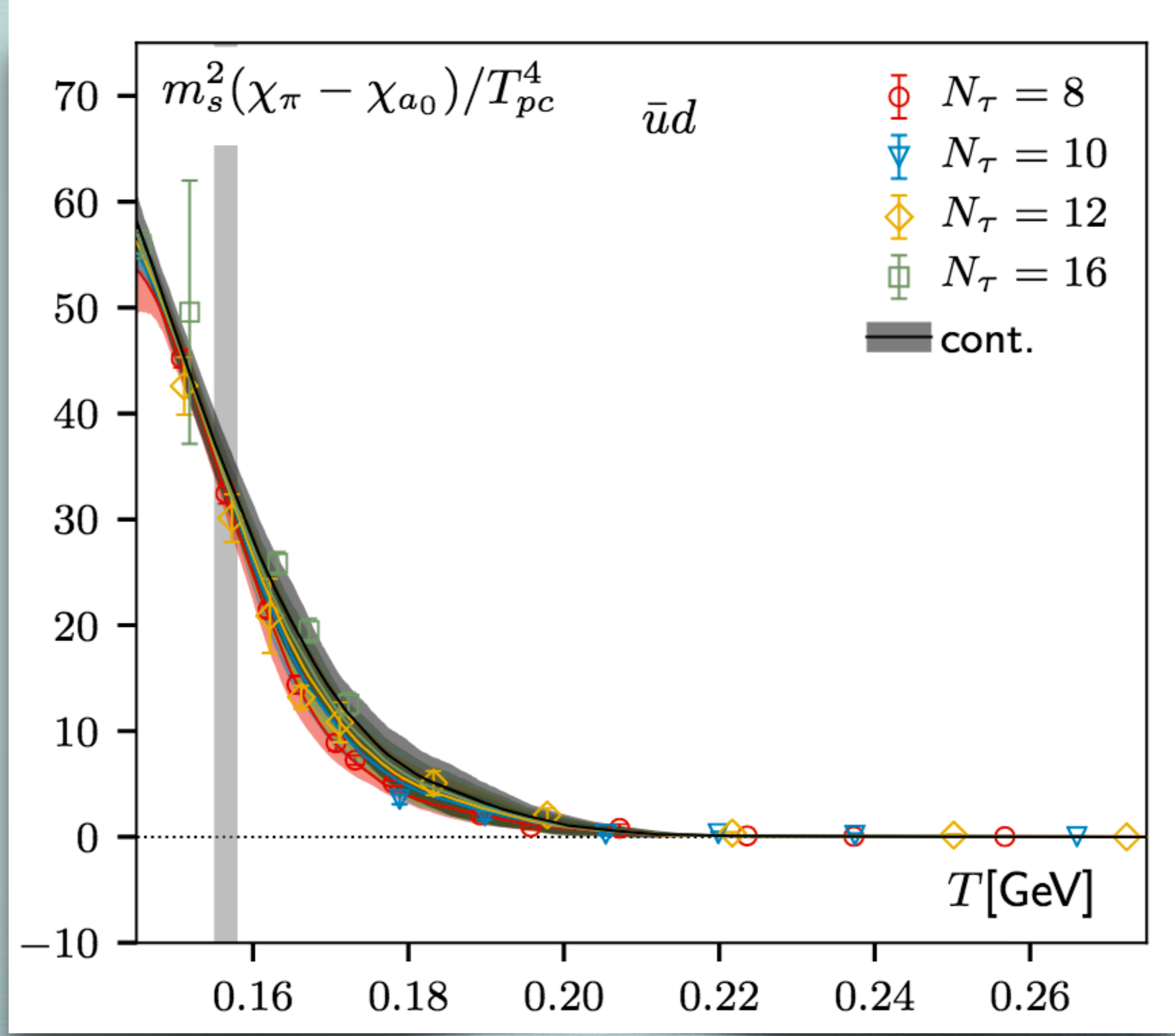
SU$_L$(2)xSU$_R$(2) and U$_A$(1) Symmetries

Continuum extrapolated results in 2+1 flavor QCD with m_π=140 MeV

Meson screening masses



U$_A$(1) susceptibility



SU$_L$(2)xSU$_R$(2) chiral symmetry: Degeneracy of Vector and Axial Vector at T≃T$_{pc}$

Axial U(1) symmetry: Degeneracy of Pseudo scalar and Scalar at T~200 MeV

[HotQCD] arXiv:1908.09552 [hep-lat]

Summary

- Negative 6th and 8th order cumulants as well as $T_c^0 \approx 132$ MeV suggests a possible critical end point can located only at

$$T_c^{CEP} < 135 - 140\text{MeV} \qquad \mu_B^{CEP} > 300\text{MeV}$$

- hyper-skewness and hyper-kurtosis ratios are obtained in NLO in μ_B

$\sqrt{s_{NN}}$=200 GeV: $R_{51}^B = -0.5(3), \qquad R_{62}^B = -0.7(3)$

$\sqrt{s_{NN}}$=54.4 GeV: $R_{51}^B = -0.7(4), \qquad R_{62}^B = -2(1)$

- Great progress achieved in understanding the fate of quarkonia states as well as heavy quark transports