Net-proton number fluctuations and the Quantum Chromodynamics critical point

Observations from collisions of heavy-ion at relativistic energies have established the formation of a new phase of matter, Quark Gluon Plasma (QGP), a deconfined state of quarks and gluons [1] in a specific region of the temperature versus baryonic chemical potential phase diagram of strong interactions. A program to study the features of the phase diagram, such as a possible critical point, by varying the collision energy ($\sqrt{s_{NN}}$), is performed at the Relativistic Heavy-Ion Collider (RHIC) facility. Non-monotonic variation with $\sqrt{s_{NN}}$ of moments of the net-baryon number distribution, related to the correlation length and the susceptibilities of the system, is suggested as a signature for a critical point [2,3]. We report the first evidence of a non-monotonic variation in kurtosis × variance of the net-proton number (proxy for net-baryon number) distribution as a function of $\sqrt{s_{NN}}$ with 3.0σ significance, for head-on (central) gold-on-gold (Au+Au) collisions measured using the STAR detector [4] at RHIC. Non-central Au+Au collisions and models of heavy-ion collisions without a critical point show a monotonic variation as a function of $\sqrt{s_{NN}}$.

One of the fundamental goals in physics is to understand the properties of matter when subjected to variations in temperature and pressure. Currently, the study of the phases of strongly interacting nuclear matter is the focus of many research activities worldwide, both theoretically and experimentally [5,6]. The theory that governs the strong interactions is Quantum Chromodynamics (QCD), and the corresponding phase diagram is called the QCD phase diagram. From

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different examples of condensed-matter systems, experimental progress in mapping out phase diagrams is achieved by changing the material doping, adding more holes than electrons. Similarly it is suggested for the QCD phase diagram, that adding more quarks than antiquarks (the energy required is defined by the baryonic chemical potential, μ_B), through changing the heavy-ion collision energy, enables a search for new emergent properties and a critical point in the phase diagram. Figure 1 shows a conjectured QCD phase diagram. It has at least two distinct phases: a QGP at higher temperatures, and a state of confined quarks and gluons at lower temperatures called the hadronic phase [9,10]. It is inferred from lattice QCD calculations [11] that the transition is consistent with being a cross over at small μ_B, and that the transition temperature is about 155 MeV [12–14]. An important predicted feature of the QCD phase structure is a critical point [2,15], followed at higher μ_B by a first order phase transition. The goal of this work is to search for the possible signatures of the critical point by scanning the temperature (T) versus μ_B in the QCD phase diagram by varying the collision energy $\sqrt{s_{NN}}$ of the heavy-ion collisions [3].
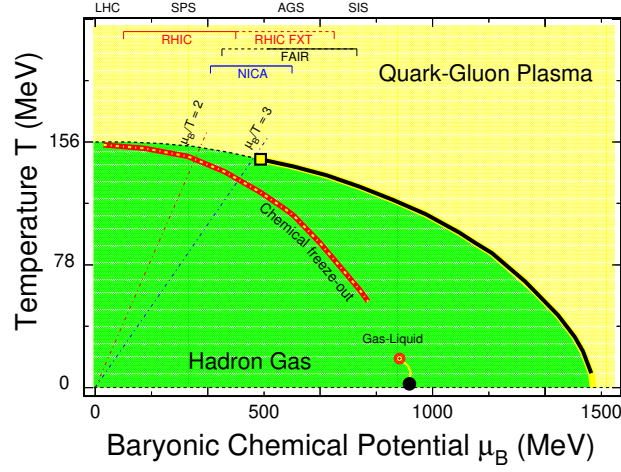
Figure 1: Conjectured QCD phase diagram. The phase boundary (solid line) between the hadronic gas phase and the high-temperature quark-gluon phase is a first-order phase transition line, which begins at large μ_B and small T and curves towards smaller μ_B and larger T. This line ends at the QCD critical point whose conjectured position, indicated by a square, is uncertain both theoretically and experimentally. At smaller μ_B there is a cross over indicated by a dashed line. The regions of $\mu_B/T \leq 2$ and 3 are shown as red and blue dot-dashed lines, respectively. A comparison between RHIC data and lattice QCD calculations disfavors the possible QCD critical point being located at $\mu_B/T \leq 2$ [7,8]. The red-yellow dotted line corresponds to the chemical freeze-out (where inelastic collisions among the constituents of the system cease) inferred from particle yields in heavy-ion collisions using a thermal model. The liquid-gas transition region features a second order critical point (red-circle) and a first-order transition line (yellow line) that connect the critical point to the ground state of nuclear matter ($T \sim 0$ and $\mu_B \sim 925$ MeV) [9]. The regions of the phase diagram accessed by past (AGS and SPS), ongoing (LHC, RHIC, SPS and RHIC operating in fixed target mode), and future (FAIR and NICA) experimental facilities are also indicated.

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Upon approaching a critical point, the correlation length diverges and thus renders, to a large extent, microscopic details irrelevant. Hence observables like the moments of the conserved net-baryon number distribution, which are sensitive to the correlation length, are of interest when searching for a critical point. A non-monotonic variation of these moments as a function of $\sqrt{s_{NN}}$ has been proposed as an experimental signature of a critical point [2,3]. However, considering the complexity of the system formed in heavy-ion collisions, signatures of a critical point are detectable only if they can survive the evolution of the system, including the effects of finite size and time [16]. Hence, it was proposed to study higher moments of distributions of conserved quantities (N) due to their stronger dependence on the correlation length [15]. The promising higher moments are the skewness, $S = \left\langle (\delta N)^3 \right\rangle / \sigma^3$, and kurtosis, $\kappa = [\left\langle (\delta N)^4 \right\rangle / \sigma^4] - 3$, where $\delta N = N - M$, M is the mean and σ is the standard deviation. The magnitude and the sign of the moments, which quantify the shape of the multiplicity distributions, are important for understanding the critical point [3,17]. An additional crucial experimental challenge is to reconstruct, on an event-by-event basis, all of the baryons produced within the acceptance of a detector [18–20]. However, theoretical calculations have shown that the proton-number fluctuations can also reflect the baryon-number fluctuations at the critical point [21].

The measurements reported here are from Au+Au collisions recorded by the STAR detector [4] at RHIC from the years 2010 to 2014. The data is presented for $\sqrt{s_{NN}}$ = 7.7, 11.5, 14.5, 19.6, 27, 39, 62.4 and 200 GeV as part of phase-I of the Beam Energy Scan (BES) program at RHIC [22]. These $\sqrt{s_{NN}}$ values correspond to μ_B values ranging from 420 MeV to 20 MeV at chemical freeze-out [22]. All valid Au+Au collisions occurring within 60 cm (80 cm for $\sqrt{s_{NN}}$ = 7.7

GeV) of the nominal interaction point, having vertex position in the transverse plane within 2 cm (1 cm for $\sqrt{s_{\mathrm{NN}}}$ = 14.5 GeV) of the beam axis, and having signals in trigger detectors above a noise threshold (called minimum bias) are included in the analysis [22]. For the results presented here, the number of minimum bias Au+Au collisions ranges between 3 million for $\sqrt{s_{\mathrm{NN}}}$ = 7.7 GeV and 238 million at $\sqrt{s_{\mathrm{NN}}}$ = 200 GeV. These statistics are found to be sufficient in order to compute the moments of the net-proton distributions up to the fourth order [23]. The collisions are further characterised by their impact parameter, which is indirectly determined from the measured multiplicity of charged particles other than protons and anti-protons in the pseudo-rapidity range $|\eta| < 1$, where $\eta = -\ln[\tan(\theta/2)]$, with θ being the angle between the momentum of the particle and the positive direction of the beam axis. We exclude protons and anti-protons specifically to avoid self-correlation effects [24]. The effect of resonance decays and the pseudo-rapidity range for centrality determination have been understood and optimised using model calculations [6,26]. The results presented here correspond to two event classes: central collisions (impact parameters $\sim$ 0-3 fm, obtained from the top 5% of the above-mentioned multiplicity distribution) and peripheral collisions (impact parameters $\sim$ 12-13 fm, obtained from the 70-80% region of the multiplicity distribution).

The protons (p) and anti-protons ($\bar{p}$) are identified, along with their momentum, by reconstructing their tracks in the Time Projection Chamber (TPC) placed within a solenoidal magnetic field of 0.5 Tesla, and by measuring their ionization energy loss (dE/dx) in the sensitive gas-filled volume of the chamber. The selected kinematic region for protons covers all azimuthal angles for the rapidity range $|y| < 0.5$, where rapidity is the arctanh of the component of speed parallel to the

beam direction in units of the speed of light, with full azimuthal angle. The precise measurement of dE/dx with a resolution of 7% in Au+Au collisions allows for a clear identification of protons up to 800 MeV/c in transverse momentum (p_T, the component of momentum perpendicular to the beam direction). The identification for larger p_T (up to 2 GeV/c, with purity above 98%) was made by a Time Of Flight detector (TOF) (see Methods) having a timing resolution of better than 100 ps. A minimum p_T threshold of 400 MeV/c and a maximum distance of closest approach to the collision vertex of 1 cm for each $p(\bar{p})$ candidate track is used to suppress contamination from secondaries (for example protons from interactions of energetic particles produced in the collisions with detector materials) [22,27]. This p_T acceptance accounts for approximately 80% of the total p + $\bar{p}$ multiplicity at mid-rapidity. This is a significant improvement from the results previously reported [27] which only had the $p + \bar{p}$ measured using the TPC ($0.4 < p_T$ (GeV/c) < 0.8), whereas the current analysis includes the addition of the TOF detector, where the p_T acceptance is increased up to 2 GeV/c. The observation of non-monotonic variation of the kurtosis $\times$ variance is much more significant with the increased acceptance.

Figure 2 shows the event-by-event net-proton ($N_p - N_{\bar{p}} = \Delta N_p$) distributions obtained by measuring the number of protons (N_p) and anti-protons ($N_{\bar{p}}$) at mid-rapidity ($|y| < 0.5$) in the transverse momentum range $0.4 < p_T$ (GeV/c)< 2.0 for Au+Au collisions at various $\sqrt{s_{NN}}$. To study the shape of the event-by-event net-proton distribution in detail, cumulants (C_n) of various orders are calculated, where $C_1 = M$, $C_2 = \sigma^2$, $C_3 = S\sigma^3$ and $C_4 = \kappa\sigma^4$.

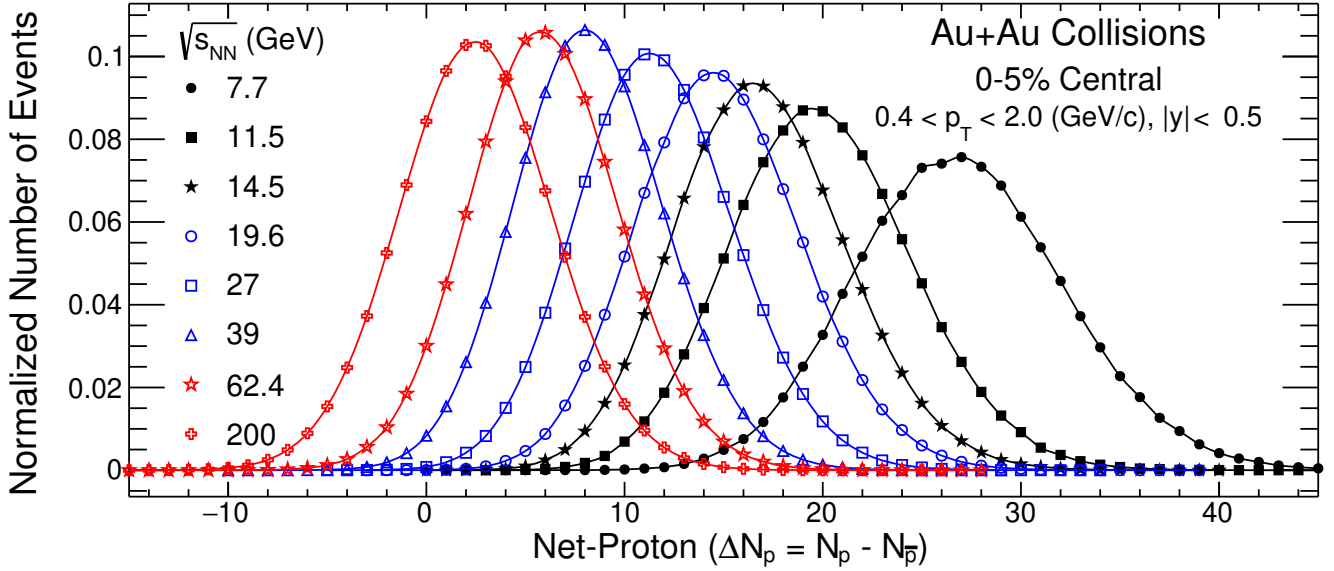Figure 3 shows the variation of net-proton cumulants (C_n) as a function of $\sqrt{s_{NN}}$ for cen-

Figure 2: Event-by-event net-proton number distributions for head-on (0-5% central) Au+Au collisions for nine $\sqrt{s_{NN}}$ values measured by the STAR detector at RHIC. The distributions are normalized to the total number of events at each $\sqrt{s_{NN}}$. The statistical uncertainties are smaller than the symbol sizes and the lines are to guide the eye. The distributions in this figure are not corrected for proton and anti-proton detection efficiency.

tral and peripheral Au+Au collisions. The cumulants are corrected for the multiplicity variations arising due to finite impact parameter range for the measurements [6]. These corrections suppress the volume fluctuations considerably [6,28]. The cumulants are also corrected for finite track reconstruction efficiencies of the TPC and TOF detectors. This is done by assuming a binomial probability distribution to reconstruct particles out of those produced [27,29]. A cross-check using a different method based on unfolding (see Methods) of the distributions for central Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV has been found to give values consistent with the cumulants shown in Fig. 3. Typical values for the efficiencies in the TPC (TOF) for the momentum range studied in
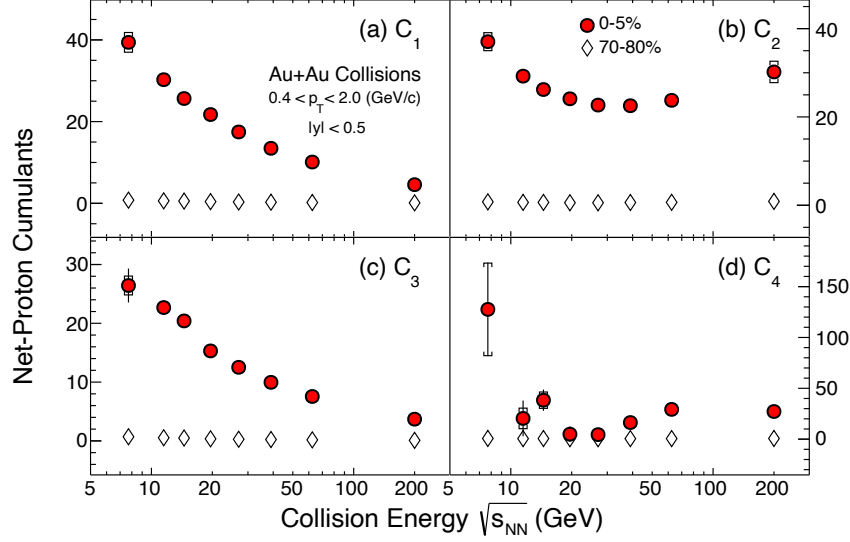
Figure 3: Cumulants (C_n) of the net-proton distributions for central (0-5%) and peripheral (70-80%) Au+Au collisions as a function of collision energy. The p_T range for the measurements is from 0.4 to 2 GeV/c and the rapidity (y) range is $\pm$ 0.5. The vertical bars represent the statistical uncertainties and the caps correspond to the systematic uncertainties.

0-5% central Au+Au collisions at $\sqrt{s_{NN}}$ = 7.7 GeV are 83%(72%) and 81%(70%) for the protons and anti-protons, respectively. The corresponding efficiencies for $\sqrt{s_{NN}}$ = 200 GeV collisions are 62%(69%) and 60%(68%) for the protons and anti-protons, respectively. The statistical uncertainties are obtained using both a bootstrap approach [23,29] and the Delta theorem [23,29,30] method. The systematic uncertainties are estimated by varying the experimental requirements to reconstruct p ($\bar{p}$) in the TPC and TOF. These requirements include the distance of the proton and anti-proton tracks from the primary vertex position, track quality reflected by the number of TPC space points used in the track reconstruction, the particle identification criteria, and the uncertainties in estimating the reconstruction efficiencies.
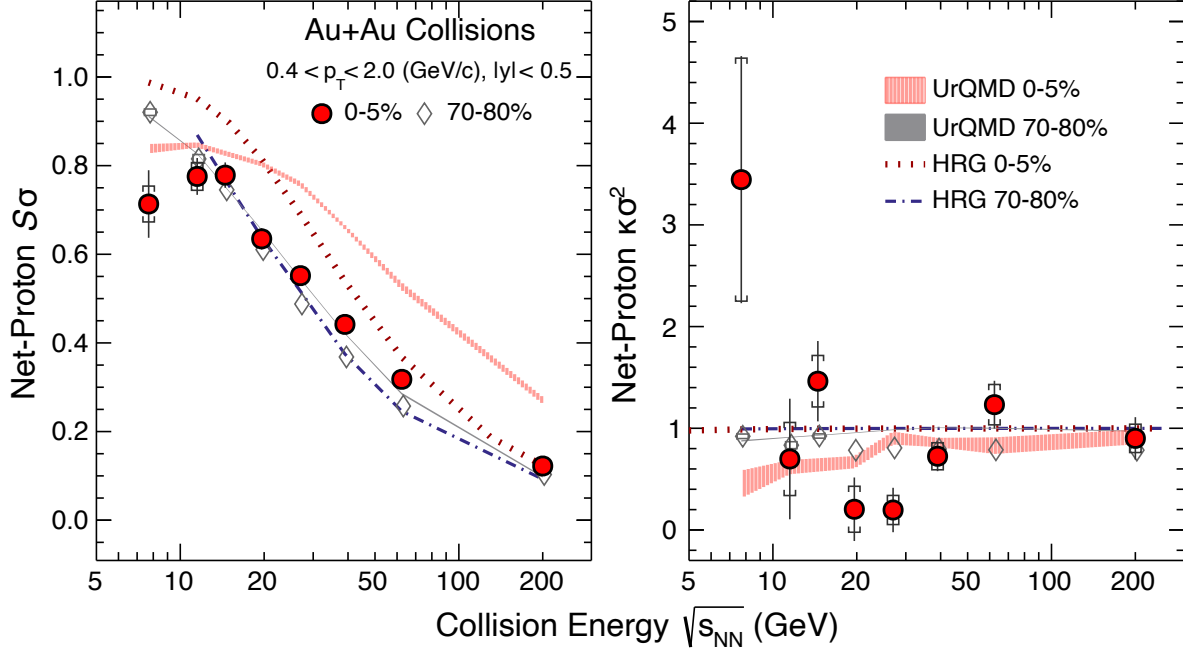
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Figure 4: $S\sigma$ (left panel) and $\kappa\sigma^2$ (right panel) as a function of collision energy for net-proton distributions measured in Au+Au collisions. The results are shown for central (0-5%) and peripheral (70-80%) collisions within $0.4 < p_{\mathrm{T}}$ (GeV/c) < 2.0 and $|y| < 0.5$. The error bars and caps show statistical and systematic uncertainties, respectively. The vertical-dashed (0-5%) and dash-dotted (70-80%) lines correspond to results from a hadron resonance gas (HRG) model [26]. The orange (0-5%) and black (70-80%) shaded bands are the results from a transport model calculation (UrQMD [33]). These model calculations utilize the experimental acceptance, and incorporate conservation laws for strong interactions, but do not include a phase transition or a critical point.

The large values of C_3 and C_4 for central Au+Au collisions show that the distributions have non-Gaussian shapes, a first possible indication of enhanced fluctuations arising from a possible critical point [15,17]. The corresponding values for peripheral collisions are small and close to zero. For central collisions, the C_1 and C_3 monotonically decrease with $\sqrt{s_{\mathrm{NN}}}$, while the C_2 and C_4 show

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a non-monotonic variation, with a possible minimum between $\sqrt{s_{NN}}$ of 11.5 and 39 GeV.

We employ ratios of cumulants in order to cancel volume variations to first order. Further, these ratios of cumulants are related to the ratio of baryon-number susceptibilities (χ_n^B, where n is the order) at a given T and μ_B, computed in QCD and QCD-based models as: $C_3/C_2 = S\sigma = (\chi_3^B/T)/(\chi_2^B/T^2)$ and $C_4/C_2 = \kappa\sigma^2 = (\chi_4^B)/(\chi_2^B/T^2)$ [31]. Close to the critical point, QCD-based calculations predict the net-baryon number distributions to be non-Gaussian and susceptibilities to diverge, causing moments, especially higher-order quantities like $\kappa\sigma^2$, to have non-monotonic variation as a function of $\sqrt{s_{NN}}$ [31,32].

Figure 4 shows the variation of $S\sigma$ (or C_3/C_2) and $\kappa\sigma^2$ (or C_4/C_2) as a function of $\sqrt{s_{NN}}$ for central and peripheral Au+Au collisions. In central collisions a non-monotonic variation with beam energy is observed for $\kappa\sigma^2$. The $\kappa\sigma^2$ values go below unity (statistical baseline) and then rise to values above unity with decrease in beam energy. The peripheral collisions on the other hand show a monotonic variation with $\sqrt{s_{NN}}$ and $\kappa\sigma^2$ values are always below unity. It is worth noting that in peripheral collisions, the system formed may not be hot and dense enough to undergo a phase transition or come close to the QCD critical point. The central Au+Au collision data for $\kappa\sigma^2$ ($S\sigma$), in the collision energy range of 7.7 – 62.4 GeV, are well described by a polynomial function of order three (four) in $\sqrt{s_{NN}}$, with χ^2/NDF ~ 1. The derivative of the polynomial function changes sign (see Methods) with $\sqrt{s_{NN}}$ thereby indicating a non-monotonic variation of the measurement with the collision energy. The uncertainties of the derivatives are obtained by varying the data points randomly at each energy within the statistical and systematic uncertainties separately. The

overall significance of the change in the sign of the slope for C_4/C_2 versus $\sqrt{s_{NN}}$, based on the third order polynomial function fitting procedure, from $\sqrt{s_{NN}} = 7.7$ to 62.4 GeV is 3.0σ. This significance is obtained by generating 3 million sets of points, where for each set, the measured C_4/C_2 value at a given $\sqrt{s_{NN}}$ is randomly varied within the total Gaussian uncertainties (systematic and statistical uncertainties added in quadrature). Then for each new C_4/C_2 versus $\sqrt{s_{NN}}$ set of points, a third order polynomial function is fitted and the derivative values calculated at different $\sqrt{s_{NN}}$ (as discussed above). The probability that at least one derivative at a given $\sqrt{s_{NN}}$ has a different sign from the remaining ones is found to be 0.99846009, which corresponds to a 3.0σ. The significances are calculated using statistical and systematic uncertainties of the derivatives, added in quadrature.

The expectations from an ideal statistical model of hadrons assuming thermodynamical equilibrium, called the Hadron Resonance Gas (HRG) model [26], calculated within the experimental acceptance, are also shown in Fig. 4. The HRG results are similar to those from a system of totally uncorrelated and statistically random particle production. The HRG results are close to unity for $\kappa\sigma^2$ without any dependence on $\sqrt{s_{NN}}$ and they describe the data for 70-80% Au+Au collisions better than 0-5% central collisions. For $S\sigma$, HRG calculations deviate significantly from measurements for Au+Au collisions at 0-5% centrality below $\sqrt{s_{NN}} = 62.4$ GeV, while they are closer to data for 70-80% centrality. Corresponding $\kappa\sigma^2$ ($S\sigma$) results for 0-5% and 70-80% Au+Au collisions from a transport-based UrQMD model [33] calculation, which incorporates conservation laws and most of the relevant physics apart from a phase transition or a critical point, and which is calculated within the experimental acceptance, show a monotonic decrease (increase) with decreasing

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collision energy. The UrQMD model values for 70-80% centrality are closer to the correspond-ing measurements than the 0-5% values. Neither of the model calculations explains the measured dependence of the $\kappa\sigma^2$ and $S\sigma$ of the net-proton distribution on $\sqrt{s_{NN}}$ for central Au+Au collisions.

In conclusion, we have presented measurements of net-proton cumulant ratios with the STAR detector at RHIC over a wide range in μ_B (20 to 420 MeV) which are relevant to a QCD critical point search in the QCD phase diagram. We have observed a non-monotonic behaviour, as a function of $\sqrt{s_{NN}}$, in net-proton $\kappa\sigma^2$ in central Au+Au collisions with a significance of 3.0σ. In contrast, monotonic behaviour with $\sqrt{s_{NN}}$ is observed for the statistical hadron gas model, and for a nuclear transport model without a critical point, as observed experimentally in peripheral collisions. The deviation of the measured $\kappa\sigma^2$ from several baseline calculations with no critical point, and its non-monotonic dependence on $\sqrt{s_{NN}}$, are qualitatively consistent with expectations from a QCD-based model which includes a critical point [3,15]. Our measurements can also be compared to the baryon-number susceptibilities computed from QCD to understand various other features of the QCD phase structure as well as to obtain the freeze-out conditions in heavy-ion collisions. Higher event statistics, which will allow for a more differential measurement of these experimental observables in y-p_T along with comparison to theoretical QCD calculations which includes the dynamics associated with heavy-ion collisions, will help in establishing the critical point.

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Methods

(a) Proton and anti-proton identification in STAR detector:

Figure 5 (left panel) shows a typical distribution of the square of the mass associated with each track in an event obtained from the TOF [1,2] as a function of the product of the momentum and the charge of the track determined by the TPC [3]. The proton candidates are well separated from other hadrons like kaons and pions. The right panel of Fig. 5 shows p_T versus y for protons in the STAR detector. The white dashed rectangular box is the region selected for the results presented here. It may be noted that STAR, being a collider experiment, has a p_T versus y acceptance near mid-rapidity that is uniform across all beam energies studied. Uniform acceptance allows for the results to be directly compared across all the $\sqrt{s_{NN}}$.
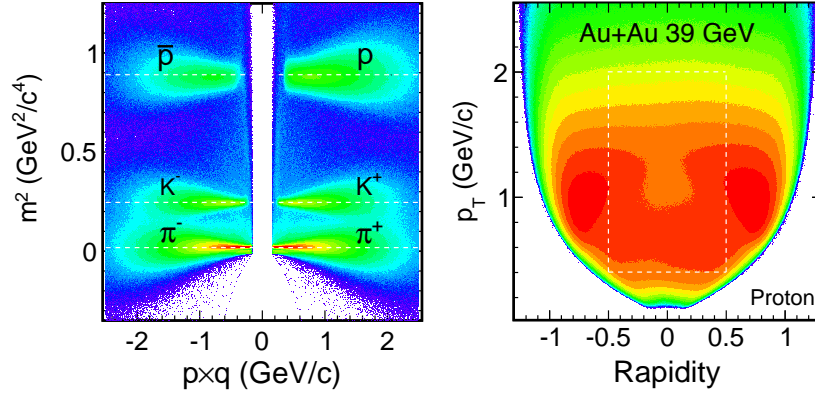
Figure 5: Left panel: Square of the mass of the charged particles, requiring timing information from the TOF, as a function of the product of the momentum (p) and the ratio of the particles charge to the elementary charge e (q), both measured using the TPC in Au+Au collisions at $\sqrt{s_{NN}}$ = 39 GeV. The white dashed lines correspond to the expected square of the mass of each particle species. Right panel: The transverse momentum (p_T) versus the rapidity (y) for protons measured in the STAR detector for Au+Au collisions.

(b) Efficiency corrections using unfolding of net-proton multiplicity distributions:

The unfolding method was applied to a data set that provides the most dense charged particle environment in the detectors (0-5% central Au+Au collisions at $\sqrt{s_{NN}}$ = 200 GeV), where one expects the maximum non-binomial detector effects. Detector-response matrices were determined based on detector simulations with respect to generated and measured protons and anti-protons [4]. All possible non-binomial effects, including multiplicity dependent efficiency, were corrected by utilizing the response matrices. The detector response in such cases was found to be best described by a beta-binomial distribution. Even in this situation, the differences in the binomial [5] and

unfolding methods of efficiency correction were at a level of less than one σ of the uncertainties.
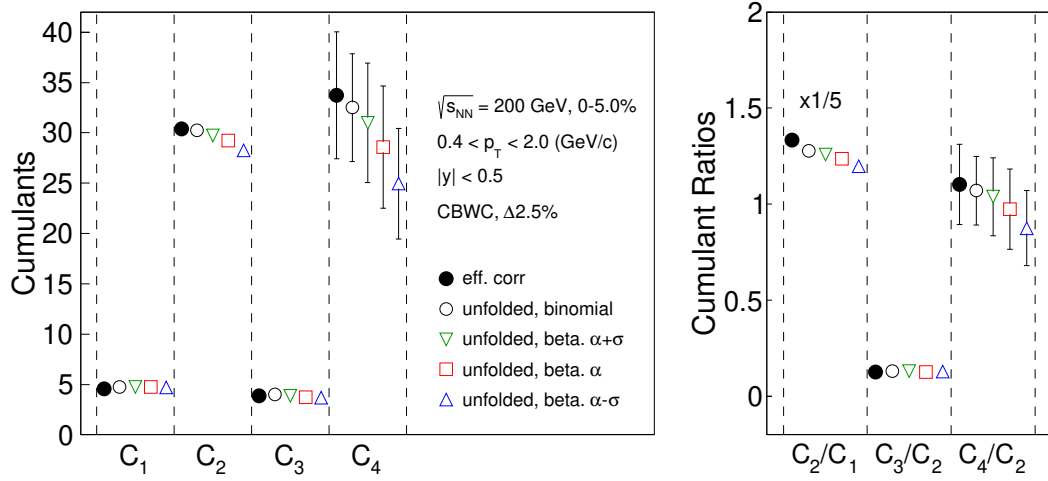

Figure 6: Cumulants and their ratios up to the fourth order, corrected for proton and anti-proton reconstruction efficiencies in $\sqrt{s_{NN}} = 200$ GeV Au+Au collisions at 0-5% centrality. Results from the conventional efficiency correction are shown as black filled circles, results from the unfolding with the binomial detector response are shown as black open circles, and results from beta-binomial detector response with $\alpha + \sigma$, α and $\alpha - \sigma$ are shown as green triangles, red squares and blue triangles, respectively. The parameter α quantifies the deviation from binomial effects, obtained from simulation. C_2/C_1 is scaled by a constant factor.

Cumulants and their ratios up to the fourth order, corrected for the detector efficiencies using the unfolding method, are shown in Fig. 6 for 0-5% central Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV. The results are obtained by using centrality bin width correction (CBWC) [6] at 2.5% bin width. For each column, the first point is efficiency corrected using the binomial model method (as employed in the present analysis), the next point is the result corrected for the binomial detector response using the unfolding technique, and the last three points are from unfolding using the beta-binomial

response with three values of the non-binomial parameter. The results are ordered from left to right in terms of increasing deviations of the response function compared to the binomial distribution. Checks using unfolding of the distributions for central Au+Au collisions have been found to yield values consistent with the cumulants obtained using the default binomial method of efficiency correction, within the current statistics of the measurements.

(c) Values of polynomial function fit to $\kappa\sigma^2$ and $S\sigma$ and their derivatives versus $\sqrt{s_{\mathrm{NN}}}$:

The values of the parameters of the polynomial functions for $\kappa\sigma^2$ and $S\sigma$ at $\sqrt{s_{\mathrm{NN}}} = 7.7$, 11.5, 14.5, 19.6, 27, 39 and 62.4 GeV are given in Table 1. The uncertainties on the parameters are from the fitting procedure taking into account both the statistical and systematic uncertainties on the data. The χ^2/NDF = 1.0 for the third order polynomial fit to $\kappa\sigma^2$ versus $\sqrt{s_{\mathrm{NN}}}$ and the χ^2/NDF = 1.4 for the fourth order polynomial fit to $S\sigma$ versus $\sqrt{s_{\mathrm{NN}}}$. The $\sqrt{s_{\mathrm{NN}}} = 200$ GeV data point is not included to quantify the non-monotonic variations as the polynomial function fits either did not converge or yielded larger χ^2 values. It may also be noted that the possible critical point is predicted to exist at baryon chemical potential values larger than those corresponding to $\sqrt{s_{\mathrm{NN}}} = 200$ GeV.

Table 1: Values of the parameters of third (fourth) order polynomial that describes the collision energy dependence of $\kappa\sigma^2$ ($S\sigma$) at various $\sqrt{s_{NN}}$ along with their uncertainties. The polynomials are of the form $\sum_n p_n(\sqrt{s_{NN}})^n$, n = 0-3 for third order polynomial and 0-4 for fourth order polynomial and p_n are the parameters.

Parameters	$\kappa\sigma^2$	$S\sigma$
p_0	5.33 ± 1.88	0.48 ± 0.35
p_1	−0.47 ± 0.19	0.07 ± 0.06
p_2	0.013±0.006	−0.0047 ± 0.0031
p_3	−0.00011±0.00005	0.00011 ± 0.00007
p_4	–	−0.0000008 ± 0.0000005

The values of the derivatives of the polynomial functions for $\kappa\sigma^2$ and $S\sigma$ at $\sqrt{s_{\text{NN}}} = 7.7$, 11.5, 14.5, 19.6, 27, 39 and 62.4 GeV are given in Table 2. The uncertainties on the derivatives are obtained by varying the data points randomly at each energy within the statistical and systematic uncertainties separately. This process assumes that the systematic uncertainties on the data points are fully uncorrelated. In addition, we also provide an estimate of systematic uncertainty on the derivative at each $\sqrt{s_{\text{NN}}}$ which assumes the systematic uncertainties on the data points to be fully correlated. Also shown in the Table 2. are the significance values of the derivative from being non-zero at each $\sqrt{s_{\text{NN}}}$, calculated using the statistical and the uncorrelated systematic uncertainties added in quadrature.

Table 2: Values of the derivative of the third (fourth) order polynomial that describes the collision energy dependence of $\kappa\sigma^2$ ($S\sigma$) at various $\sqrt{s_{\mathrm{NN}}}$. The first uncertainty on the derivative is statistical, the second is the fully correlated systematic uncertainty and the third is the uncorrelated systematic uncertainty. Also shown are the significances of the derivative values from zero.

$\sqrt{s_{\mathrm{NN}}}$ (GeV)	Derivative of polynomial ($\kappa\sigma^2$)	Sig.	Derivative of polynomial ($S\sigma$)	Sig.
7.7	$-0.286 \pm 0.0916 \pm 0.0472 \pm 0.066$	2.5	$0.0122 \pm 0.0177 \pm 0.0061 \pm 0.0105$	0.6
11.5	$-0.208 \pm 0.066 \pm 0.0365 \pm 0.047$	2.6	$-0.0034 \pm 0.0087 \pm 0.0033 \pm 0.0052$	0.3
14.5	$-0.154 \pm 0.049 \pm 0.0289 \pm 0.035$	2.6	$-0.0115 \pm 0.0044 \pm 0.0017 \pm 0.0029$	2.2
19.6	$-0.075 \pm 0.026 \pm 0.0177 \pm 0.018$	2.4	$-0.01794 \pm 0.0032 \pm 0.00003 \pm 0.0026$	4.4
27	$0.009 \pm 0.013 \pm 0.0052 \pm 0.011$	0.5	$-0.0156 \pm 0.00273 \pm 0.00051 \pm 0.00153$	5.0
39	$0.069 \pm 0.0216 \pm 0.0054 \pm 0.016$	2.6	$-0.00025 \pm 0.0048 \pm 0.00079 \pm 0.0038$	0.04
62.4	$-0.086 \pm 0.054 \pm 0.008 \pm 0.043$	1.2	$-0.0391 \pm 0.0203 \pm 0.0044 \pm 0.0147$	1.6

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