Nontrivial dispersion relation and QCD thermodynamics in the low energy effective model

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We study the...

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I. INTRODUCTION

The past years have ....

II. LOW-ENERGY EFFECTIVE MODELS UNDER THE FRG

As is shown in the previous work(see[??]), the low-energy effective theory can be studied with the flow equation of the scale-dependent effective action $\Gamma_k[\Phi]$, Φ is the superfield, it can be written as $\Phi = (A_\mu, c, \bar{c}, q, \bar{q}, \phi, ...)$. The flow equation within the framework of FRG can be written as

$$\partial_t \Gamma_k[\Phi] = \frac{1}{2} Tr G_{\Phi\Phi}[\Phi] \partial_t R_k^\Phi, \qquad t = ln(k/\Lambda) \tag{1}$$

where

$$G_{\Phi_i \Phi_j}[\Phi] = \left(\frac{1}{\frac{\delta^2 \Gamma_k[\Phi]}{\delta \Phi^2} + R_k^\Phi} \right)_{ij} \tag{2}$$

is the propagator which is full field-dependent. The effective action that depends on the scale of the quark-meson model can be written like this

$$\begin{aligned} \Gamma_k = \int_x \{ & Z_{q,k} \bar{q} (\gamma_\mu \partial_\mu - \gamma_0(\mu + igA_0)) q + \frac{1}{2} Z_{\phi,k} (\partial_\mu \phi)^2 \\ & + h_k \bar{q} (T^0 \sigma + i\gamma_5 \vec{T} \cdot \vec{\pi}) q + V_k(\rho) - c\sigma \} + \cdots \end{aligned} \tag{3}$$

here we omited the higher-order terms. The integral sign can be written as $\int_x = \int_x^{1/T} dx_0 \int d^3x$.The meson field is $\phi = (\sigma, \vec{\pi})$. $V_k(\rho)$ is field-dependent effective potential which is $O(4)$ invariant, with $\rho = \phi^2/2$.The k is the infrared cutoff scale in FRG, see [??]; Λ is some reference scale ; μ is the chemical potential of quark. $(T^0$,$\boldsymbol{T})$ is the generators of flavor space with $\mathrm{tr}(T^i T^j) = \frac{1}{2}\delta^{ij}$ and $T^0 = \frac{1}{\sqrt{2N_f}} \mathbb{1}_{N_f \times N_f}$. The linear term $-c\sigma$ breaks the chiral symmetry. At the same time, the linear breaking parameter c is proportional to the mass of $\vec{\pi}$; h_k is the Yukawa coupling. The flow equation in Eq. (1) is shown

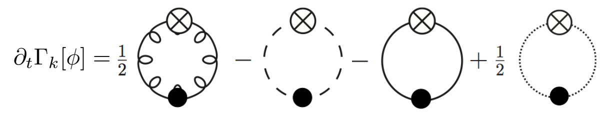

FIG. 1. The first two diagrams stand for the glue contribution; the other two stand for the quark and mesonic contribution

in Fig. 1. More details see [**?**].
Then, we can write the effective action like

$$\begin{aligned} &\Gamma_k[\Phi] = \Gamma_{glue,k}[\Phi] + \Gamma_{matt,k}[\Phi], \\ &\Gamma_{matt,k} = \Gamma_{q,k} + \Gamma_{\phi,k} \end{aligned} \tag{4}$$

III. POLYAKOV-QUARK-MESON MODEL UNDER THE FRG APPROXIMATION

A. Flow equation

$$\partial_t \Gamma_k[\Phi] = \frac{1}{2} Tr G_{\phi\phi}[\Phi] \partial_t R_k^\phi - Tr G_{q\bar{q}}[\Phi] \partial_t R_k^q \tag{5}$$

$$G_{\phi\phi/q\bar{q}}[\Phi] = \left(\frac{1}{\frac{\delta^2 \Gamma_k[\Phi]}{\delta \Phi^2} + R_k^\Phi} \right)_{\phi\phi/q\bar{q}} \tag{6}$$

$$\bar{\phi} = Z_{\phi,k}^{\frac{1}{2}} \phi, \qquad \bar{h}_k = \frac{h_k}{Z_{q,k} Z_{\phi,k}^{\frac{1}{2}}} \tag{7}$$

B. The flow equations of the effective potential

The 3d regulators have been used in our calculation. The details see Appendix A. Through the derivation of the effective action, 5 and 6 we can get the flow equation of the effective potential under the constant mesonic

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fields:

$$
\begin{aligned}
\partial_t V_k(\rho) =& \frac{k^4}{4\pi^2}[(N_f^2-1) l_0^{(B,4)}(\bar{m}_{\pi,k}^2, \eta_{\phi,k}^{\perp}; T) \\
&+ l_0^{(B,4)}(\bar{m}_{\sigma,k}^2, \eta_{\phi,k}^{\perp}; T) \\
&- 4N_c N_f l_0^{(F,4)}(\bar{m}_{q,k}^2, \eta_{q,k}; T, \mu)]
\end{aligned}
\tag{8}
$$

The $l_0^{(B/F,n)}$ is the threshold functions. The form of the threshold functions are given in the Appendix A. Below are the quark mass and meson masses which are renormalized and dimensionless

$$
\begin{aligned}
\bar{m}_{\pi,k}^2 &= \frac{V_k'(\rho)}{k^2 Z_{\phi,k}^{\perp}} \\
\bar{m}_{\sigma,k}^2 &= \frac{V_k'(\rho) + 2\rho V_k''(\rho)}{k^2 Z_{\phi,k}^{\perp}} \\
\bar{m}_{q,k}^2 &= \frac{h_k^2 \rho}{2k^2 Z_{q,k}^2}
\end{aligned}
\tag{9}
$$

In the finite temperature the meson wave function renormalization $Z_{\phi,k}$ is split into $Z^{\parallel}$ and $Z^{\perp}$. In the past calculation we use a approximation that we assume $Z^{\parallel} = Z^{\perp}$. In this work, we abolished this approximation and observe if the change have any influence on the results. By considering the difference between $Z_{\phi,k}^{\perp}$ and $Z_{\phi,k}^{\parallel}$ we can calculate the anomalous dimensions respectively.The defination of the anomalous dimensions is given by

$$
\eta_{\phi,k}^{\perp} = -\frac{\partial_t Z_{\phi,k}^{\perp}}{Z_{\phi,k}^{\perp}}, \qquad \eta_{\phi,k}^{\parallel} = -\frac{\partial_t Z_{\phi,k}^{\parallel}}{Z_{\phi,k}^{\parallel}}
\tag{10}
$$

and the definition of the quark anomalous dimension is

$$
\eta_{q,k} = -\frac{\partial_t Z_{q,k}}{Z_{q,k}}
\tag{11}
$$

The frequency and spatial momentum are independent when we calculating the anomalous dimensions. And the frequency and spatial momenta are low when we deducing the anomalous dimensions. Some details of the meson anomalous dimensions are discussed in Appendix B and quark anomalous dimension in Appendix C.

Here we consider two expansion ways of the effective potential. On one hand, the Taylor expansion of the effective potential is about a field value κ which is unrenormalised and fixed. The effective potential can be written as

$$
\bar{V}_k(\bar{\rho}) = \sum_{n=0}^{N} \frac{\bar{\lambda}_{n,k}}{n!}(\bar{\rho} - \bar{\kappa}_k)^n
\tag{12}
$$

with $\bar{\lambda}_{n,k} = \lambda_{n,k}/Z_{\phi,k}^n$ and $\bar{\kappa}_k = Z_{\phi,k}\kappa$. The other hand, we make the expansion point κ depends on the cutoff scales k. Then the $\bar{\kappa}_k$ can be written as

$$
\bar{\kappa}_k = Z_{\phi,k}\kappa_k
\tag{13}
$$

C. Baryon number fluctuation

The definition of the baryon number fluctuation is given by

$$
\chi_n^B = \frac{\partial^n}{\partial(\mu_B/T)^n}\frac{p}{T^4}
\tag{14}
$$

the baryon chemical potential $\mu_B = 3\mu$. The quadratic and quartic fluctuations can be expressed using the average baryon number fluctuation $\langle N_B \rangle$

$$
\begin{aligned}
\chi_2^B &= \frac{1}{VT^3}\langle \delta N_B{}^2 \rangle \\
\chi_4^B &= \frac{1}{VT^3}\left(\langle \delta N_B{}^4 \rangle - 3\langle \delta N_B{}^2 \rangle^2\right)
\end{aligned}
\tag{15}
$$

Appendix A: the regulator functions and the threshold functions

Three-d flat regulators have been used in this work. The regulator functions of the meson field and quark field can be written as

$$
\begin{aligned}
R_k^{\phi}(q_0, \vec{q}) &= Z_{\phi,k}^{\perp}\vec{q}^2 r_B(\vec{q}^2/k^2), \\
R_k^{q}(q_0, \vec{q}) &= Z_{q,k} i\vec{\gamma}\cdot\vec{q} r_F(\vec{q}^2/k^2)
\end{aligned}
\tag{A1}
$$

in which

$$
\begin{aligned}
r_B(x) &= \left(\frac{1}{x} - 1\right)\Theta(1-x), \\
r_F(x) &= \left(\frac{1}{\sqrt{x}} - 1\right)\Theta(1-x)
\end{aligned}
\tag{A2}
$$

Then we give a definition to the meson and quark propagator

$$
\begin{aligned}
G_{\phi}(q, \bar{m}_{\phi,k}^2) &= \frac{1}{z_{\phi}\tilde{q}_0^2 + 1 + \bar{m}_{\phi,k}^2}, \\
G_q(q, \bar{m}_{q,k}^2) &= \frac{1}{z_q^2(\tilde{q}_0 + i\tilde{\mu})^2 + 1 + \bar{m}_{q,k}^2}
\end{aligned}
\tag{A3}
$$

in the equation above we have $\tilde{q}_0 = q_0/k$, $\tilde{\mu} = \mu/k$ and for the fermions we have $q_0 = (2n_q+1)\pi T (n_q \in \mathbb{Z})$ for the bosons we have $q_0 = 2n_q \pi T$. Here $z = Z^{\parallel}/Z^{\perp}$ is the ratio of the two components of the wave function renormalizations. In this work we choose $z_q = 1$ which means $Z_q^{\parallel} = Z_q^{\perp}$ and $z_{\phi} \neq 1$ which means $Z_{\phi}^{\parallel} \neq Z_{\phi}^{\perp}$. To obtain the threshold functions, we define

$$
\begin{aligned}
\mathcal{F}_{(1)}(\bar{m}_{q,k}^2, z_q; T, \mu) &= \frac{T}{k}\sum_{n_q} G_q(q, \bar{m}_{q,k}^2), \\
\mathcal{B}_{(1)}(\bar{m}_{\phi,k}^2, z_{\phi}; T) &= -\frac{T}{k}\sum_{n_q} G_{\phi}(q, \bar{m}_{\phi,k}^2)
\end{aligned}
\tag{A4}
$$

By summing the propagator up over the Matsubara frequencies, we can get the form of the definitions above

$$\mathcal{F}_{(1)}(\bar{m}^2_{q,k}, z_q; T, \mu) = \frac{1}{2z_q\sqrt{1+\bar{m}^2_{q,k}}} \times (1 - n_F(\bar{m}^2_{q,k}, z_q; T, \mu) - n_F(\bar{m}^2_{q,k}, z_q; T, -\mu)) \tag{A5}$$

and

$$\mathcal{B}_{(1)}(\bar{m}^2_{\phi,k}, z_\phi; T) = \frac{1}{z_\phi^{1/2}\sqrt{1+\bar{m}^2_{\phi,k}}} (\frac{1}{2} - n_B(\bar{m}^2_{\phi,k}, z_\phi; T) \tag{A6}$$

in the equations above the form of the distribution functions are

$$n_B(\bar{m}^2_{\phi,k}, z_\phi; T) = \frac{1}{\mathbf{exp}\{\frac{1}{T}\frac{k}{z_\phi^{1/2}}(1+\bar{m}^2_{\phi,k})^{1/2}\} - 1} \tag{A7}$$

and

$$n_F(\bar{m}^2_{q,k}, z_\phi; T) = \frac{1}{\mathbf{exp}\{\frac{1}{T}\frac{k}{z_q}(1+\bar{m}^2_{q,k})^{1/2} - \mu\} + 1} \tag{A8}$$

In the effective potential's flow equation, there are bosonic and fermionic threshold functions. The anomalous dimension of the mesonic field has change into the transverse component of it. The form of the threshold functions are

$$l_0^{(B,d)}(\bar{m}^2_{\phi,k}, \eta^\perp_{\phi,k}; T) = \frac{2}{d-1}\left(1 - \frac{\eta^\perp_{\phi,k}}{d+1}\right)\mathcal{B}_{(1)}(\bar{m}^2_{\phi,k}, z_\phi; T) \tag{A9}$$

and

$$l_0^{(F,d)}(\bar{m}^2_{q,k}, \eta_{q,k}; T, \mu) = \frac{2}{d-1}\left(1 - \frac{\eta_{q,k}}{d}\right)\mathcal{F}_{(1)}(\bar{m}^2_{q,k}, z_q = 1; T, \mu) \tag{A10}$$

Then, we define

$$\mathcal{F}_{(n)}(\bar{m}^2_{q,k}, z_q; T, \mu) = \frac{T}{k}\sum_{(n_q)}(G_q(q, \bar{m}^2_{q,k}))^n \tag{A11}$$

and through the equation below we can deduce the expression of any value of n

$$\mathcal{F}_{(n+1)}(\bar{m}^2_{q,k}, z_q; T, \mu) = -\frac{1}{n}\frac{\partial}{\partial \bar{m}^2_{q,k}}\mathcal{F}_{(n)}(\bar{m}^2_{q,k}, z_q; T, \mu) \tag{A12}$$

The definition of the threshold function $\mathcal{BB}_{(1,1)}$ is

$$\mathcal{BB}_{(1,1)}(\bar{m}^2_{\phi_a,k}, \bar{m}^2_{\phi_b,k}, z_\phi; T) = -\frac{T}{k}\sum_{n_q} G_\phi(q, \bar{m}^2_{\phi_a,k}) G_\phi(q, \bar{m}^2_{\phi_b,k}) \tag{A13}$$

then we can obtain the $\mathcal{BB}_{(2,2)}$ in a same way

$$\mathcal{BB}_{(2,2)}(\bar{m}^2_{\phi_a,k}, \bar{m}^2_{\phi_b,k}, z_\phi; T) = \frac{\partial^2}{\partial \bar{m}^2_{\phi_a,k}\partial \bar{m}^2_{\phi_b,k}}\mathcal{BB}_{(1,1)}(\bar{m}^2_{\phi_a,k}\bar{m}^2_{\phi_b,k}, z_\phi; T) \tag{A14}$$

The expression of the $\mathcal{BB}_{(1,1)}$ is

$$\begin{aligned}\mathcal{BB}_{(1,1)}(\bar{m}^2_{\phi_a,k}, \bar{m}^2_{\phi_b,k}, z_\phi; T) = -\frac{1}{z_\phi^{1/2}}\Bigg\{&\left(\frac{1}{2} + n_B(\bar{m}^2_{\phi_a,k}, z_\phi; T)\right)\frac{1}{(1+\bar{m}^2_{\phi_a,k})^{1/2}} \\ &\times\frac{1}{\bar{m}^2_{\phi_a,k} - \bar{m}^2_{\phi_b,k}} + \left(\frac{1}{2} + n_B(\bar{m}^2_{\phi_b,k}, z_\phi; T)\right) \\ &\times\frac{1}{(1+\bar{m}^2_{\phi_b,k})^{1/2}}\frac{1}{\bar{m}^2_{\phi_b,k} - \bar{m}^2_{\phi_a,k}}\Bigg\}\end{aligned} \tag{A15}$$

then we can get the expression of the threshold functions of any n. At the same time, in our calculation there are also some other kind of threshold functions.

$$\mathcal{BB}\tilde{q}^2_{0\,(1,1)}(\bar{m}^2_{\phi_a,k}, \bar{m}^2_{\phi_b,k}, z_\phi; T) = -\frac{T}{k}\sum_{n_q} G_\phi(q, \bar{m}^2_{\phi_a,k}) G_\phi(q, \bar{m}^2_{\phi_b,k})\tilde{q}_0^2 \tag{A16}$$

$$\mathcal{F}\tilde{q}^2_{0\,(1)}(\bar{m}^2_{q,k}, z_q; T, \mu) = \frac{T}{k}\sum_{n_q} G_q(q, \bar{m}^2_{q,k})\tilde{q}_0^2 \tag{A17}$$

and

$$\mathcal{F}\tilde{q}^4_{0\,(1)}(\bar{m}^2_{q,k}, z_q; T, \mu) = \frac{T}{k}\sum_{n_q} G_q(q, \bar{m}^2_{q,k})\tilde{q}_0^4 \tag{A18}$$

The form of the threshold functions $\mathcal{F}\tilde{q}_0^2$, $\mathcal{F}\tilde{q}_0^4$ and $\mathcal{BB}\tilde{q}_0^2$ is like

$$\begin{aligned}&\mathcal{BB}\tilde{q}^2_{0\,(1,1)}(\bar{m}^2_{\phi_a,k},\bar{m}^2_{\phi_b,k},z_\phi;T)=\\&-Z_\phi^{1/2}\left\{\left(\frac{1}{2}+n_B(\bar{m}^2_{\phi_a,k},z_\phi;T)\right)\frac{(1+\bar{m}^2_{\phi_a,k})^{-1/2}}{\bar{m}^2_{\phi_b,k}-\bar{m}^2_{\phi_a,k}}\right.\\&\left.+\left(\frac{1}{2}+n_B(\bar{m}^2_{\phi_b,k},z_\phi;T)\right)\frac{(1+\bar{m}^2_{\phi_a,k})^{-1/2}}{\bar{m}^2_{\phi_a,k}-\bar{m}^2_{\phi_b,k}}\right\}\end{aligned}\tag{A19}$$

$$\begin{aligned}&\mathcal{F}\tilde{q}^2_{0\,(1)}(\bar{m}^2_{q,k},z_q;T,\mu)=\\&-\frac{\sqrt{1+\bar{m}^2_{q,k}}}{2z_q}\times(1-n_F(\bar{m}^2_{q,k},z_q;T,\mu)\\&-n_F(\bar{m}^2_{q,k},z_q;T,-\mu))\end{aligned}\tag{A20}$$

$$\begin{aligned}&\mathcal{F}\tilde{q}^4_{0\,(1)}(\bar{m}^2_{q,k},z_q;T,\mu)=\\&-\frac{(1+\bar{m}^2_{q,k})^{3/2}}{2z_q}\times(1-n_F(\bar{m}^2_{q,k},z_q;T,\mu)\\&-n_F(\bar{m}^2_{q,k},z_q;T,-\mu))\end{aligned}\tag{A21}$$

By using the method above we can get the threshold functions that contain the summation of fermion and boson propagators $\mathcal{FB}$ in the quark anomalous dimension.

$$\begin{aligned}&\mathcal{FB}_{(1,1)}(\bar{m}^2_{q,k},\bar{m}^2_{\phi,k},z_q,z_\phi;T,\mu,p_0)\\&=\frac{T}{k}\sum_{n_q}G_\phi(p-q,\bar{m}^2_{\phi,k})G_q(q,\bar{m}^2_{q,k})\\&=\frac{1}{2}\frac{k^2}{z_\phi z_q^2}\left\{-n_B(\bar{m}^2_{\phi,k},z_\phi;T)\frac{z_\phi^{1/2}}{(1+\bar{m}^2_{\phi,k})^{1/2}}\frac{1}{\left(ip_0-\mu+\frac{k}{z_\phi^{1/2}}(1+\bar{m}^2_{\phi,k})^{1/2}\right)-(1+\bar{m}^2_{q,k})(\frac{k}{z_q})^2}\right.\\&-(n_B(\bar{m}^2_{\phi,k},z_\phi;T)+1)\frac{z_\phi^{1/2}}{(1+\bar{m}^2_{\phi,k})^{1/2}}\frac{1}{\left(ip_0-\mu+\frac{k}{z_\phi^{1/2}(1+\bar{m}^2_{\phi,k})}\right)^2-(1+\bar{m}^2_{q,k})(\frac{k}{z_q})^2}\\&+n_F(\bar{m}^2_{q,k},z_q;T,-\mu)\frac{z_q}{(1+\bar{m}^2_{q,k})^{1/2}}\frac{1}{\left(ip_0-\mu-\frac{k}{z_q}(1+\bar{m}^2_{q,k})^{1/2}\right)^2-(1+\bar{m}^2_{\phi,k})\frac{k^2}{z_\phi}}\\&\left.+(n_F(\bar{m}^2_{q,k},z_q;T,\mu)-1)\frac{z_q}{(1+\bar{m}^2_{q,k})^{1/2}}\frac{1}{\left(ip_0-\mu+\frac{k}{z_q}(1+\bar{m}^2_{q,k})^{1/2}\right)^2-(1+\bar{m}^2_{\phi,k})\frac{k^2}{z_\phi}}\right\}\end{aligned}\tag{A22}$$

Appendix B: mesonic anomalous dimensions

In order to obtain the flow equation of the effective potential, the mesonic anomalous dimensions are needed. Because we have divided the wave function renormalizations into the transverse and longitudinal components of it, so the anomalous dimensions should be divided either.

Appendix C: Flow equations of the fermion

The flow of the quark wave function renormalization are given by

$$\begin{aligned}\eta_{q,k}(p_0,\vec{p})=&\frac{1}{Z_{q,k}(p_0,\vec{p})}\frac{1}{4N_cN_f}\\&Re\left[\frac{\partial^2}{\partial|p|^2}Tr\left(i\vec{\gamma}\cdot\vec{p}\left(-\frac{\delta^2\partial_t\Gamma_k}{\delta\bar{q}(-p)\delta q(p)}\right)\right)\Big|_{\rho=\kappa}\right]\end{aligned}\tag{C1}$$

and the flow of the Yukawa coupling can be written as

$$\partial_t h_k(p_0,\vec{p}) = \frac{\sqrt{2N_f}}{\sigma}\frac{1}{4N_cN_f} \times Re\left[Tr\left(-\frac{\delta^2\partial_t\Gamma_k}{\delta\bar{q}(-p)\delta q(p)}\right)\Bigg|_{\rho=\kappa}\right] \tag{C2}$$

Now we can obtain the form of the quark anomalous dimension

$$\eta_{q,k} = \frac{1}{24\pi^2 N_f}(4-\eta_{\phi,k})\bar{h}_k^2 \times \{(N_f^2-1)\mathcal{FB}_{(1,2)}(\bar{m}_{q,k}^2,\bar{m}_{\pi,k}^2;T,\mu,p_{0,ex}) + \mathcal{FB}_{(1,2)}(\bar{m}_{q,k}^2,\bar{m}_{\sigma,k}^2;T,\mu,p_{0,ex})\} \tag{C3}$$

Appendix D: The results of the kurtosis