COMMENTS

The authors study how the splitting of the longitudinal ($Z_{\parallel}$) and transverse ($Z_{\perp}$) wave function renormalizations would influence various thermal quantities. They focus on the Z's of the mesons within a 2-flavor PQM model using the FRG approach. While the splitting is clearly observable within the model, its effects on thermal quantities are found to be minimal. They also explore the difference between a fixed point and a running expansion scheme in solving the flow equation. There they find a difference in the calculated kurtosis χ_4^B/χ_2^B, which appears to be sensitive to the splitting effect under study.

Overall the paper is well written. And the topic on the correct treatment of the wave function renormalization within a quasi-particle thermal model is in my opinion very interesting. For example, the simplest NJL model usually assumes $Z = 1$ while in most DSE (and lattice) studies a momentum dependent wave function renormalization and mass function are found. The actual influence of the effect on the thermodynamics vary between observables, and for many the deviation is small. This should, nevertheless, not be used as an argument to neglecting the effect for thermal study in general. In many cases, the thermal pressure of a broad resonance give similar result as a point mass particle. This does not mean widths are negligible in thermal study, as revealed by studying more specific thermal quantity. I think similar argument applies to the current paper, and hence I disagree with one of the statements in the conclusion:

"We conclude that the splitting of the mesonic wave function renormalization can be safely neglected in the studies of equilibrium thermodynamical bulk properties ... "
and made some suggestions to clarify the issue.

In addition, I would like to see the following issues addressed before I could recommend this manuscript for publication:

A. physics questions

- 1.) The major problem I have is the physical origin of the splitting of $Z_{\parallel}$ and $Z_{\perp}$. By Lorentz invariance, they should be the same at $T = 0$, and at finite T, as the apparent Lorentz invariance is lost with the introduction of temperature, the 0- and i- components can be different. E.g. in the study of wave function renormalization of the gluons, the splitting grows as temperature is turned on. See, e.g. Physics Reports

524 (2013).

As explained in the text, the splitting at $T = 0$ is due to the use of the 3D cutoff. Can the authors comment on how they can justify the subsequent calculations they make are physically correct and not due to the flaw in the problem set up, i.e., the use of a 3D cutoff. Will an improved regulator help to obtain a more physical result?

Alternatively, it would also be good if the authors can show the deviations in thermal quantities if the splitting is artificially increased (say 10 x)

- 2.) The smallness of the effects reported could also be due to the smallness of the ϕ contributions to the quantities studied. For a meaningful comparison, I think it would be useful to single out the contribution from the ϕ part, e.g. the quasi-particle pressure due to ϕ, with and without splitting. If the thermal quantities are dominated by quarks or the gluons, compare the different treatment on the ϕ contribution may be more appropriate. It would also be useful to show the explicit Z dependence on P_ϕ.

B. questions concerning the text

- 1.) LPA′ needs to be defined explicitly.
- 2.) Can the authors justify why $Z_\sigma = Z_\pi$, while their mass functions are very different?
- 3.) Fig.1: Please clarify whether all the Z's at UV point are the same, and what are the values?
- 4.) Fig.1: Please comment on whether there are interesting differences at finite μ.
- 5.) I assume the Polyakov loop value is obtained from a mean field calculation similar to the study in Ref. Phys.Rev.C83:054904,2011. Please clarify.
- 6.) p.6 right column: spacial (or spatial?)
- 7.) Fig.8: At low temperature, the Polyakov loop is more appropriately approximated as 0 rather than 1. Can the authors comment on what happens to the scheme dependence there?