Baryon number fluctuation and critical end point

Shi Yin,[1] Rui Wen,[1] and Wei-jie Fu[1,*]
[1]*School of Physics, Dalian University of Technology, Dalian, 116024, P.R. China*

We investigate the relationship between the peak value of baryon number fluctuation kurtosis and the critical baryon chemical potential. At the same time, the freeze-out curves under different position of the critical end point. We control the position of the critical end point by a new cutoff scale which can remove the fermion vacuum fluctuation from the flow equations. This work is done under the low energy Polyakov-quark-meson model with the functional renormalization group approach.

PACS numbers: 11.30.Rd, 11.10.Wx, 05.10.Cc, 12.38.Mh

INTRODUCTION

The location of the critical end point (CEP) of the QCD phase diagram is a popular research direction in the field of high energy physics. However, the physical property at high baryon chemical potential that is the high density area is hard to study in both theoretical and experimental. In the experimental field, the Relativistic Heavy Ion Collider (RHIC) that provides us with a lot of experimental data at the high density part, see [1–3].

This paper has been divided into three parts. Firstly, we give a simple discuss of the baryon number fluctuation and the relationship between the fermion vacuum fluctuation and the position of the CEP. Secondly, the theoretical framework of the FRG as well as the Polyakov-quark-meson model are introduced. At the same time, the method we use to cutoff the fermion vacuum fluctuation is also discussed. Finally, we give our numarical results and the comparison of the lattice results, then end with a summary.

BARYON NUMBER FLUCTUATIONS AND THE FERMION VACUUM FLUCTUATION

The kurtosis of the baryon number fluctuation is the core of our work, for the significant role it plays in the experiment area of the QCD phase structure. The calculation of the baryon number fluctuation can be done using the following formula,

$$\chi_n^B = \frac{\partial^n}{\partial(\mu_B/T)^n}\frac{p}{T^4}\,, \tag{1}$$

in which the $\mu_B = 3\mu$ is the beryon chemical potential that is triple of the quark chemical potential. Then we can obtain the first to fourth order of the beryon number fluctuation known as the generalized susceptibilities,

$$\chi_2^B = \frac{1}{VT^3}\langle(\delta N_B)^2\rangle\,, \tag{2}$$

$$\chi_4^B = \frac{1}{VT^3}\Big(\langle(\delta N_B)^4\rangle - 3\langle(\delta N_B)^2\rangle^2\Big)\,, \tag{3}$$

The meaning of the angle brackets stands for the average value, and the δN_B stands for the difference between the N_B and $\langle N_B\rangle$ which reads $\delta N_B := N_B - \langle N_B\rangle$.

For the purpose of comparing our calculation with the experimental and lattice results, we divide the fourth and second order of the baryon number fluctuations to get the kurtosis which is the observable in the experiments, e.g.,$\kappa\sigma^2 = \chi_4^B/\chi_2^B$. More details discussion about baryon number fluctuation see e.g., [4]. Here we investigate the relationship of the maximum value of the kurtosis and the fermion vacuum fluctuation.

We find that the position of the critical end point would change with different fermion vacuum contribution we involve in our calculation. In this work we add a new cutoff scale into the functional renormalisation group flow equation to restrict the fermion vacuum fluctuation and study the behavior of the position of the critical end point under the different cutoff. From the previous work of the low energy effective theory under the FRG, the flow equation of the effective potential contain the full contribution of the fermion vacuum part throughout the integral interval which is cover from the infrared point to the ultraviolet point. The effect of the fermion vacuum contribution is studied in the mean field approximation, see [5]. We can tell that the fermion vacuum fluctuation can suppress the baryon number fluctuation at finite temperature and density. If the new cutoff scale get the value of the UV scale, the flow return to the previous which includes all the fermion vacuum term. If the cutoff scale get the value of the IR scale, the flow get the result of the mean-field approximation. The neglect of the fermion vacuum part is known as the no-sea approximation.

POLYAKOV-QUARK-MESON MODEL AND THE CUTOFF

This work is done under the two quark flavor Polyakov-quark-meson model with functional renormalisation group approach. We give the effective action

$$\Gamma_k = \int_x \Big\{ Z_{q,k}\bar{q}\Big[\gamma_\mu\partial_\mu - \gamma_0(\hat{\mu}+igA_0)\Big]q + \frac{1}{2}Z_{\phi,k}(\partial_\mu\phi)^2 + h_k\bar{q}\big(T^0\sigma + i\gamma_5\vec{T}\cdot\vec{\pi}\big)q + V_k(\rho) - c\sigma\Big\}, \quad (4)$$

with the 4 dimension integral $\int_x = \int_0^{1/T} dx_0 \int d^3x$. The k is the FRG infrared (IR) cutoff scale which is running from the ultraviolet (UV) scale to 0. The meson field is defined as $\phi = (\sigma, \vec{\pi})$, and $\rho = \phi^2/2$. $\vec{T}$ is the generators of the $SU(N_f)$ group here we have $N_f = 2$. The generators satisfy $\mathrm{Tr}(T^iT^j) = \frac{1}{2}\delta^{ij}$, $T^0 = \frac{1}{\sqrt{2N_f}}\mathbb{1}_{N_f\times N_f}$. The $-c\sigma$ gives the chiral symmetry breaking in our theory. In this work we get the results under the local potential approximation (LPA), under which $\partial_t Z_{\phi,q} = 0$, $\partial_t h_k = 0$. The t is RG time with $t = \ln(k/\Lambda)$. We choose the UV cutoff scale Λ to 700MeV. The effective potential $V_k(\rho)$ involves the infermation of the meson chiral symmetry breaking. Here we solve the flow equation of the effective potential by the Taylor expansion around the expansion point κ. The expansed meson potential is $V_k(\rho) = \sum_{n=0}^{N_v} \frac{\lambda_{n,k}}{n!}(\rho-\kappa_k)^n$. In this work, we choose $N_v = 5$ for the good convergence of the fix expansion point $\partial_t\kappa_k = 0$. In order to get the pressure, the thermodynamical potential should be calculated. The definition of the thermodynamical potential is $\Omega[T,\mu] = V_{k=0}(\rho) + V_{glue}(L,\bar{L}) - c\sigma$. The glue potential is a function of the traced Polyakov loop L and the complex conjugate $\bar{L}$. They are in concerning with the gluonic background field A_0 by $L = \frac{1}{N_c}\langle\mathrm{Tr}\mathcal{P}\rangle$, $\bar{L} = \frac{1}{N_c}\langle\mathrm{Tr}\mathcal{P}^\dagger\rangle$ with $\mathcal{P} = \mathcal{P}\mathrm{exp}\big(ig\int_0^\beta d\tau A_0(\tau)\big)$.

To obtain the CEP position of our system under different cutoff scale Λ_2, we use the Pion decay constant as the criterion. The Pion decay constant is the order parameter of the phase transition, given by $f_\pi = \langle\sigma\rangle$. The peak value of $\partial f_\pi/\partial T$ is used to determine the crossover line at low baryon chemical potential. When a relatively large difference of the f_π appear appear between two adjacent temperatures, we believe the crossover is already convert to the first order phase transition. The temperature and baryon chemical potential between these two kinds of phase transition fix the position of the CEP.

In the LPA situation, we focus on the flow of effective potential. The flow equation can be written as

$$\partial_t V_k(\rho) = \frac{k^4}{4\pi^2}\big[(N_f^2-1)l_0^{B,4}(\bar{m}_{\pi,k}^2, \eta_{\phi,k}; T) + l_0^{(B,4)}(\bar{m}_{\sigma,k}^2, \eta_{\phi,k}; T) - 4N_cN_f l_0^{(F,4)}(\bar{m}_{q,k}^2, \eta_{q,k}; T, \mu)\big], \quad (5)$$

The anomalous dimensions of the fermion and boson is all 0 here. The $l_0^{(F,4)}$ in the flow equation stands for the fermion threshold function. The analytical form of the threshold function is

$$l_0^{(F,d)}(\bar{m}_{q,k}^2, \eta_{q,k}; T, \mu) = \frac{1}{z_q(d-1)\sqrt{1+\bar{m}_{q,k}^2}}\Big(1-\frac{\eta_{q,k}}{d}\Big) \times \big(1 - n_F(\bar{m}_{q,k}^2, z_q; T, \mu) - n_F(\bar{m}_{q,k}^2, z_q; T, \mu)\big) \quad (6)$$

which contains all contribution of the fermion vacumm fluctuation. We take place this with another threshold function with no fermion vacuum part

$$j_0^{(F,d)}(\bar{m}_{q,k}^2, \eta_{q,k}; T, \mu) = \frac{1}{z_q(d-1)\sqrt{1+\bar{m}_{q,k}^2}}\Big(1-\frac{\eta_{q,k}}{d}\Big) \times \big(-n_F(\bar{m}_{q,k}^2, z_q; T, \mu) - n_F(\bar{m}_{q,k}^2, z_q; T, \mu)\big) \quad (7)$$

Then we devided the integral of the flow equation into two parts by the new cutoff scale Λ_2 between the IR and UV scales. The integral between the IR scale and Λ_2 is the flow equation with the fermion vacuum contribution the other side of the integral that between the Λ_2 and UV scale is the flow equation without the fermion vacuum contribution. Now we can control the quantity of the fermion vacuum part by changing the new cutoff Λ_2.

RESULTS AND SUMMARY

We will give some numarical results and summarize the outcome in the last section. As mentioned in the previous section, we add a new cutoff scale to remove part of the fermion vacuum contribution from solving our FRG effective potential flow equation. Then we calculate the two and four order generalized susceptibilities and their ratio which is the kurtosis. The position of the CEP is also obtained by the f_π under several baryon chemical potential.

The Tab. I gives the relationship between the new cutoff scale and the position of the CEP. We can easily tell from the table, with the decrease of the Λ_2 the baryon chemical potential of the CEP b gradually gets low and the temperature of the CEP increases. In other words, the CEP is moving towards the left upper corner of the QCD phase diagram. When the cutoff comes to zero namely the no-sea approximation, the CEP even reached $\mu_B = 335$ MeV that is a vary small value. It is clear, there is a significant correlation between the CEP and the fermion vacuum fluctuation.

ACKNOWLEDGMENTS

The work was supported by the National Natural Science Foundation of China under Contracts Nos.

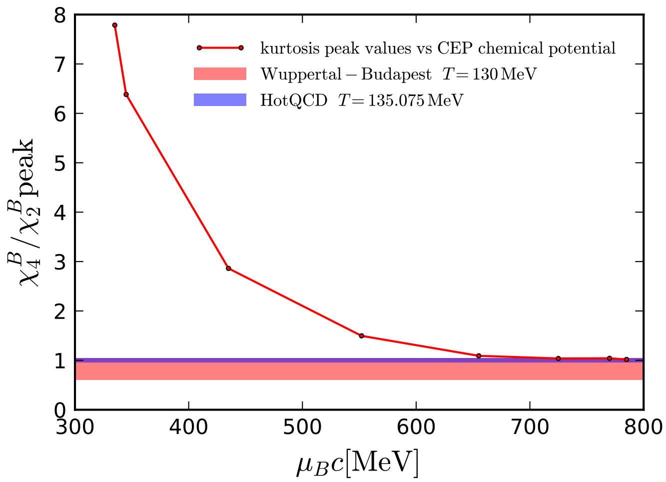

FIG. 1. The relation between the maximum baryon number fluctuation kurtosis value under $\mu_B = 0$ and the baryon chemical potential of the critical end point. The lattice data comes from the [6, 7] which are the results of HotQCD Collaboration and [8] by Wuppertal-Budapest Collaboration

Λ_2 (MeV)	0	100	200	300	400	500	600	700
T_c (MeV)	124	123	117	110	98	97	95	89
$\mu_B c$ (MeV)	335	345	435	552	655	725	770	785

TABLE I. The position of the critical point and the corresponding value of the cutoff Λ_2.

11775041.

* wjfu@dlut.edu.cn

[1] L. Adamczyk *et al.* (STAR), Phys. Rev. Lett. **112**, 032302 (2014), arXiv:1309.5681 [nucl-ex].
[2] X. Luo (STAR), *Proceedings, 9th International Workshop on Critical Point and Onset of Deconfinement (CPOD 2014): Bielefeld, Germany, November 17-21, 2014*, PoS **CPOD2014**, 019 (2015), arXiv:1503.02558 [nucl-ex].
[3] X. Luo and N. Xu, Nucl. Sci. Tech. **28**, 112 (2017), arXiv:1701.02105 [nucl-ex].
[4] W.-j. Fu and J. M. Pawlowski, Phys. Rev. **D92**, 116006 (2015), arXiv:1508.06504 [hep-ph].
[5] V. Skokov, B. Friman, E. Nakano, K. Redlich, and B. J. Schaefer, Phys. Rev. **D82**, 034029 (2010), arXiv:1005.3166 [hep-ph].
[6] A. Bazavov *et al.*, Phys. Rev. **D95**, 054504 (2017), arXiv:1701.04325 [hep-lat].
[7] A. Bazavov *et al.* (HotQCD), Phys. Rev. **D96**, 074510 (2017), arXiv:1708.04897 [hep-lat].
[8] S. Borsanyi, Z. Fodor, S. D. Katz, S. Krieg, C. Ratti, and K. K. Szabo, Phys. Rev. Lett. **111**, 062005 (2013), arXiv:1305.5161 [hep-lat].