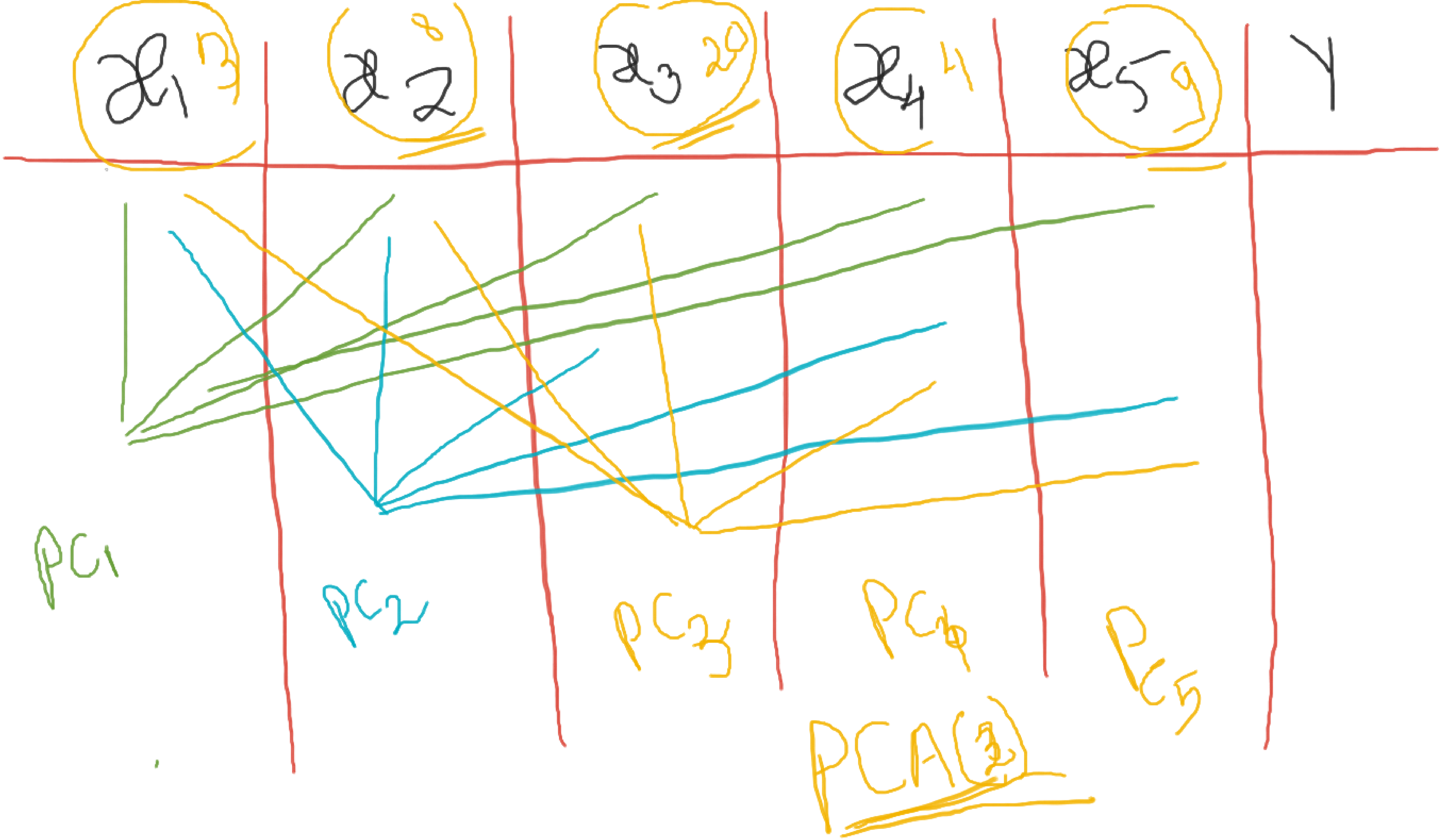
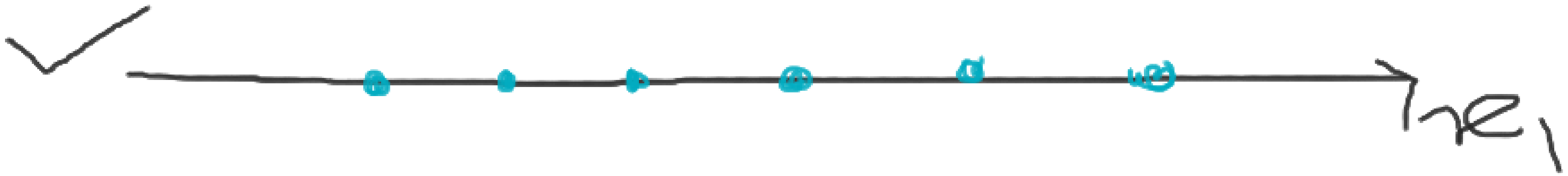
x_1	x_2	x_3	x_4	x_5	Y
1		10	0		
2		8	1		
4		9	2		
5		12	3		
6		14	4		



x_1
x_2
8
x_3
20
x_4
4
x_5
9
Y
PC1
PC2
PC3
PC4
PC5
PCA(3)

PC_1	PC_2	PC_3	PC_4	PC_5	Y

Var = x_1



Var of x_1 > x_2

Var = Spread

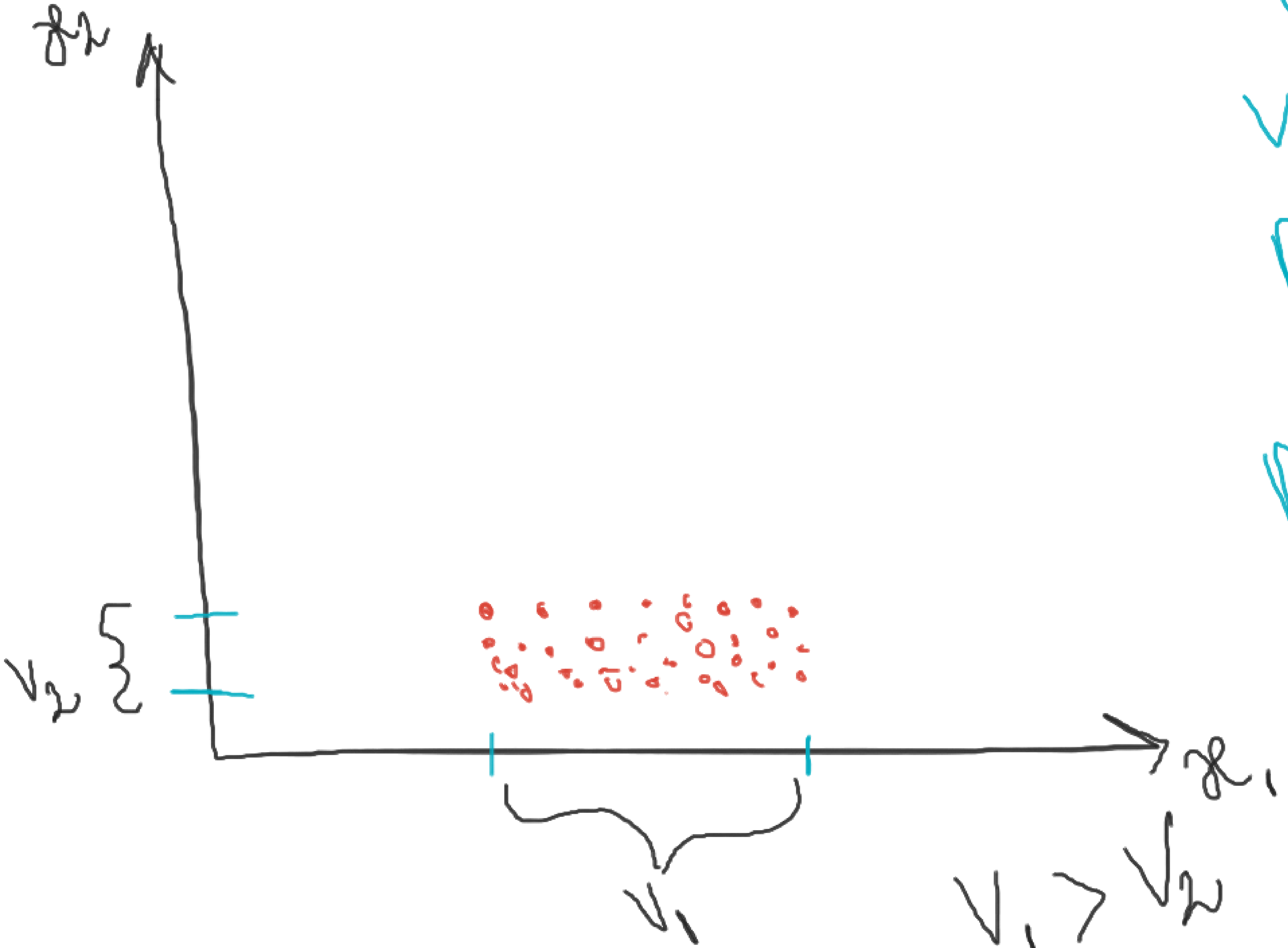
COV = spread + direction

Mean =

Eigenvalues

&

Eigenvectors



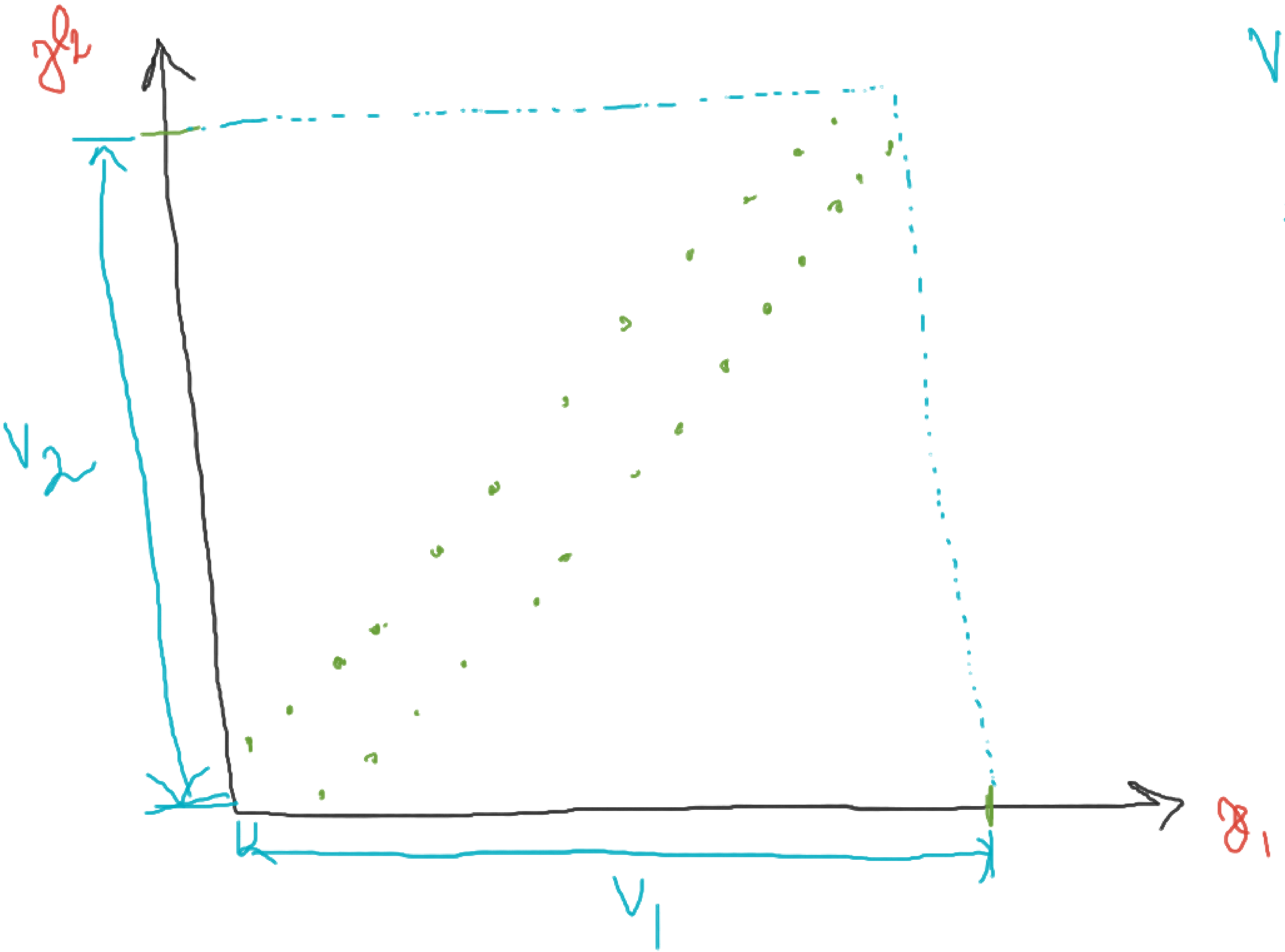
$V_1 > V_2$

$V_1 > V_2$

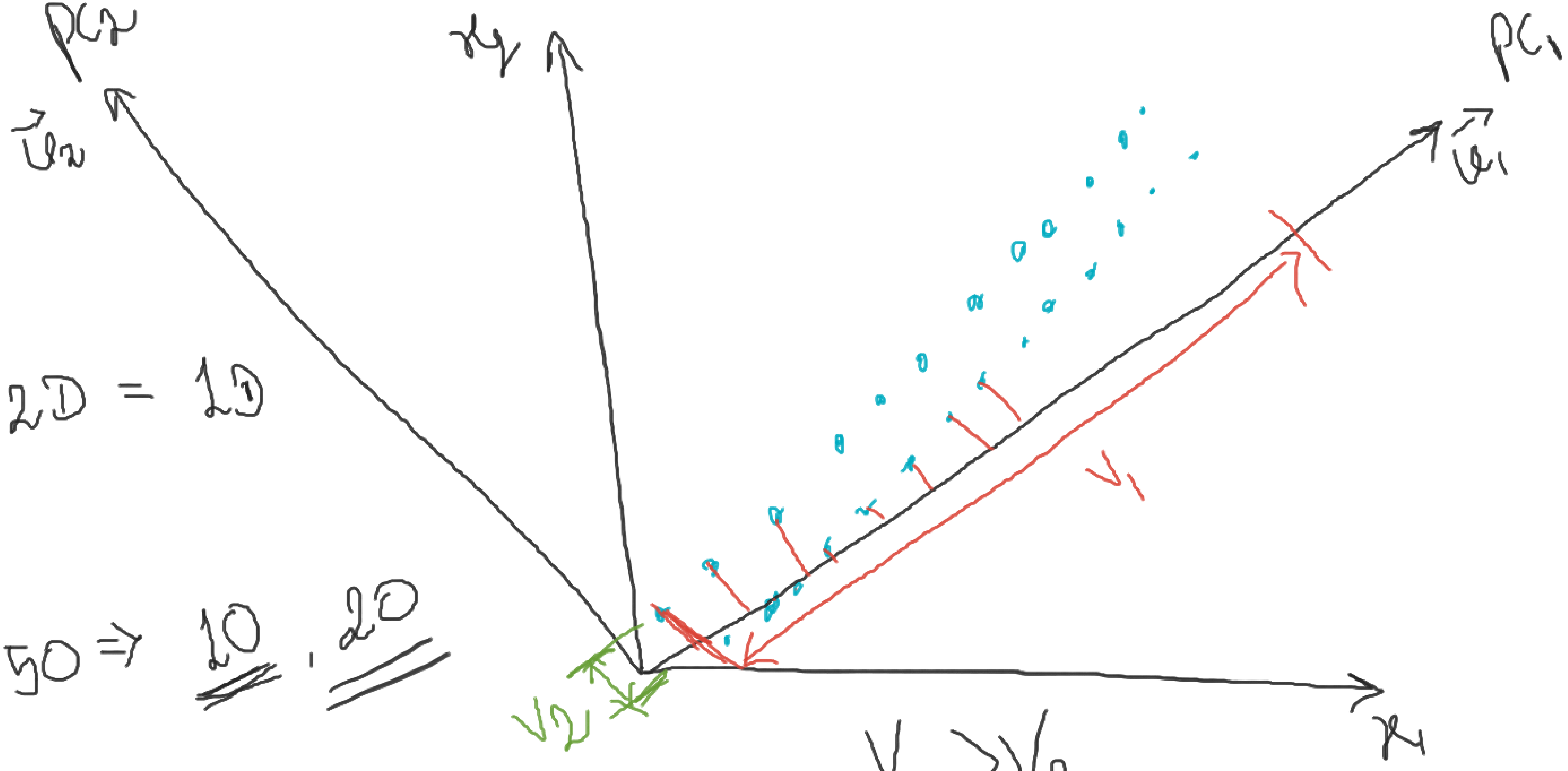
$V_1 =$

PCA(2)

PCA(1) $= x_1$



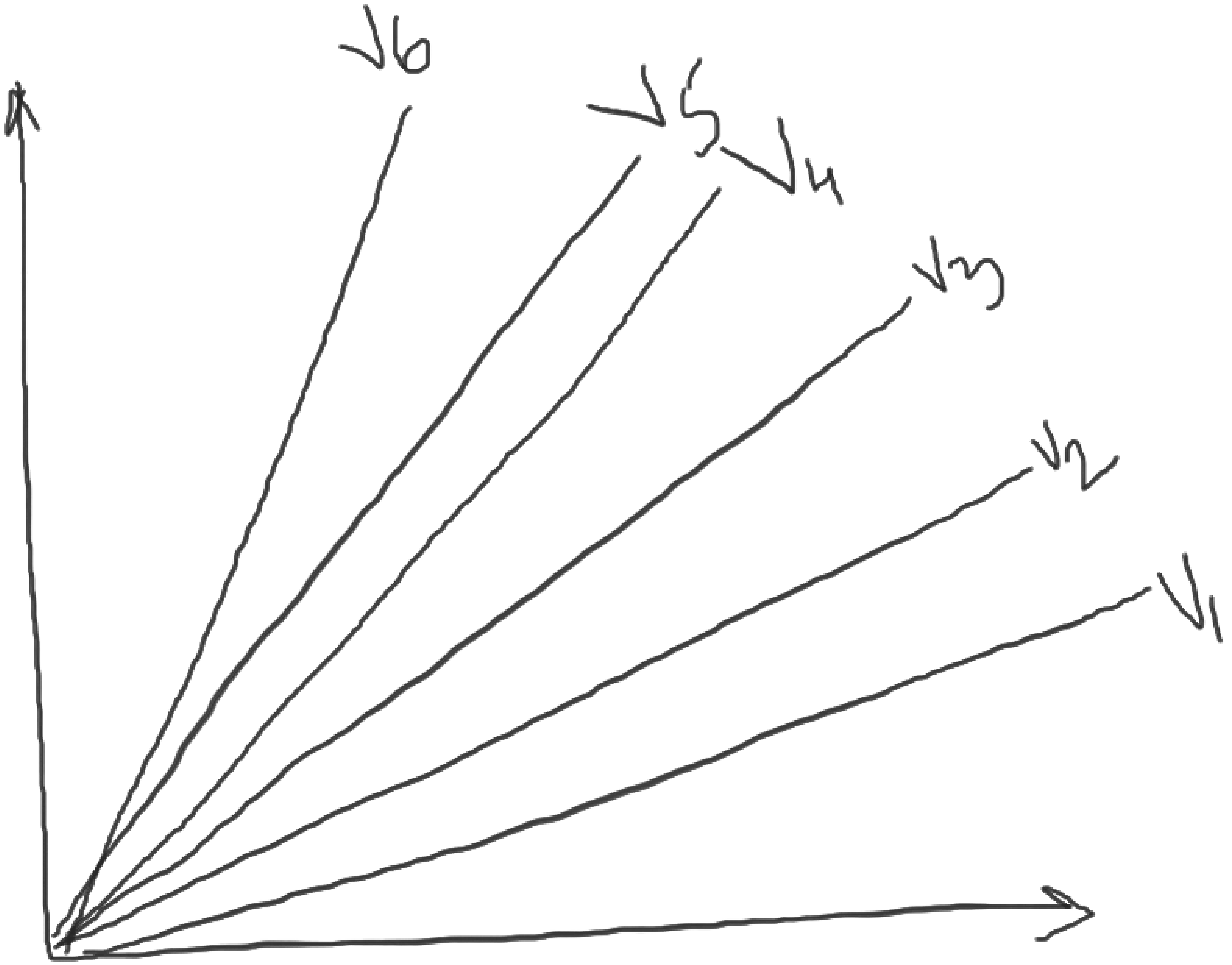
$V_1 \simeq V_2$



2D = 1D

50 ⇒ 10, 20

$V_1 > V_2$


v6
v5
v4
v3
v2
v1

$V_1 = 20$

$V_2 = 25$

$V_3 = 10$

$V_4 = 30$

$V_5 = 18$

$V_6 = 39$

$V_6 = 39$

$V_4 = 30$

$V_2 = 25$

$V_1 = 20$

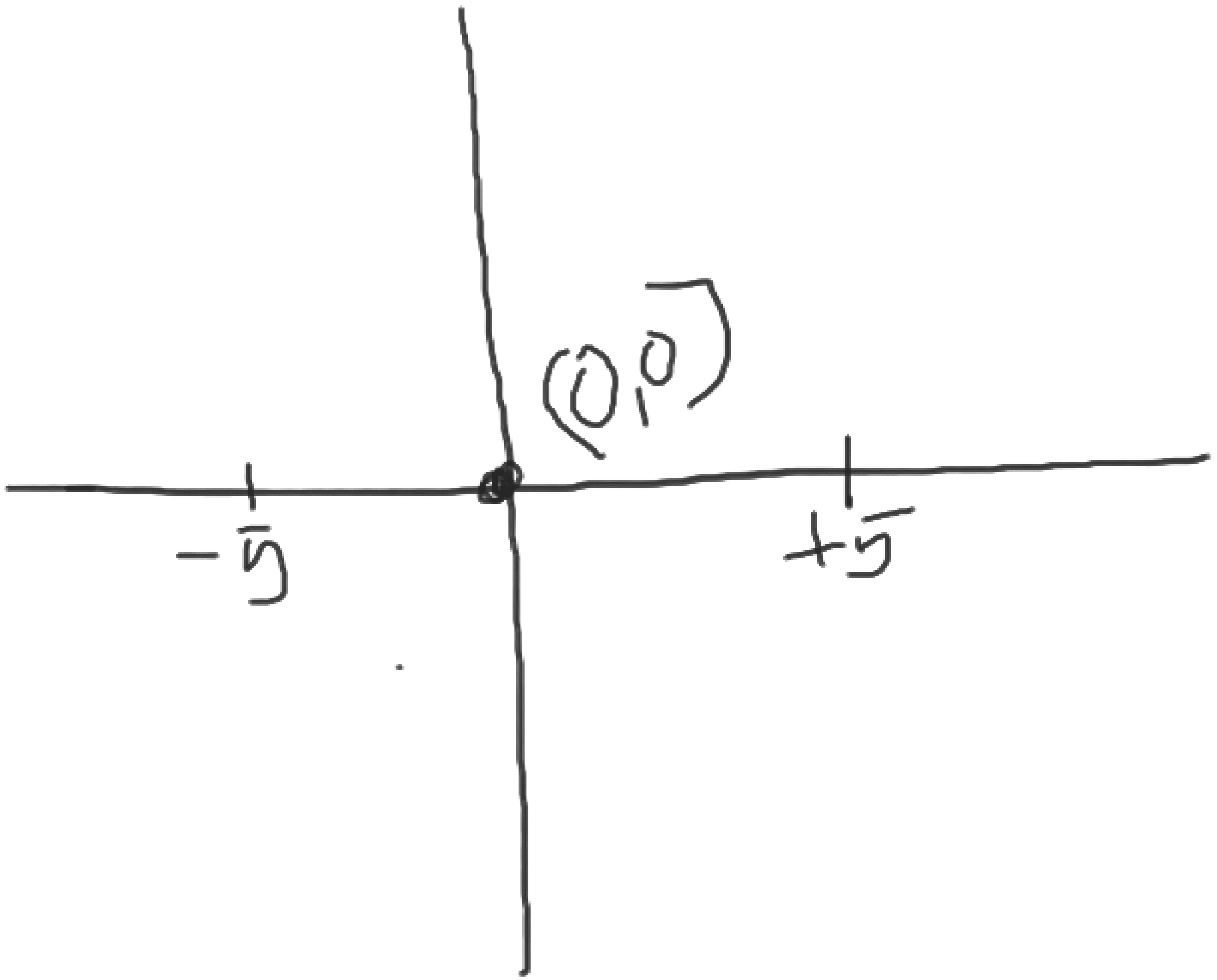
$V_5 = 18$

$V_3 = 10$

PG_1

PG_2

PG_3



$$Var = \frac{\sum (x_i - \bar{x})^2}{N}$$

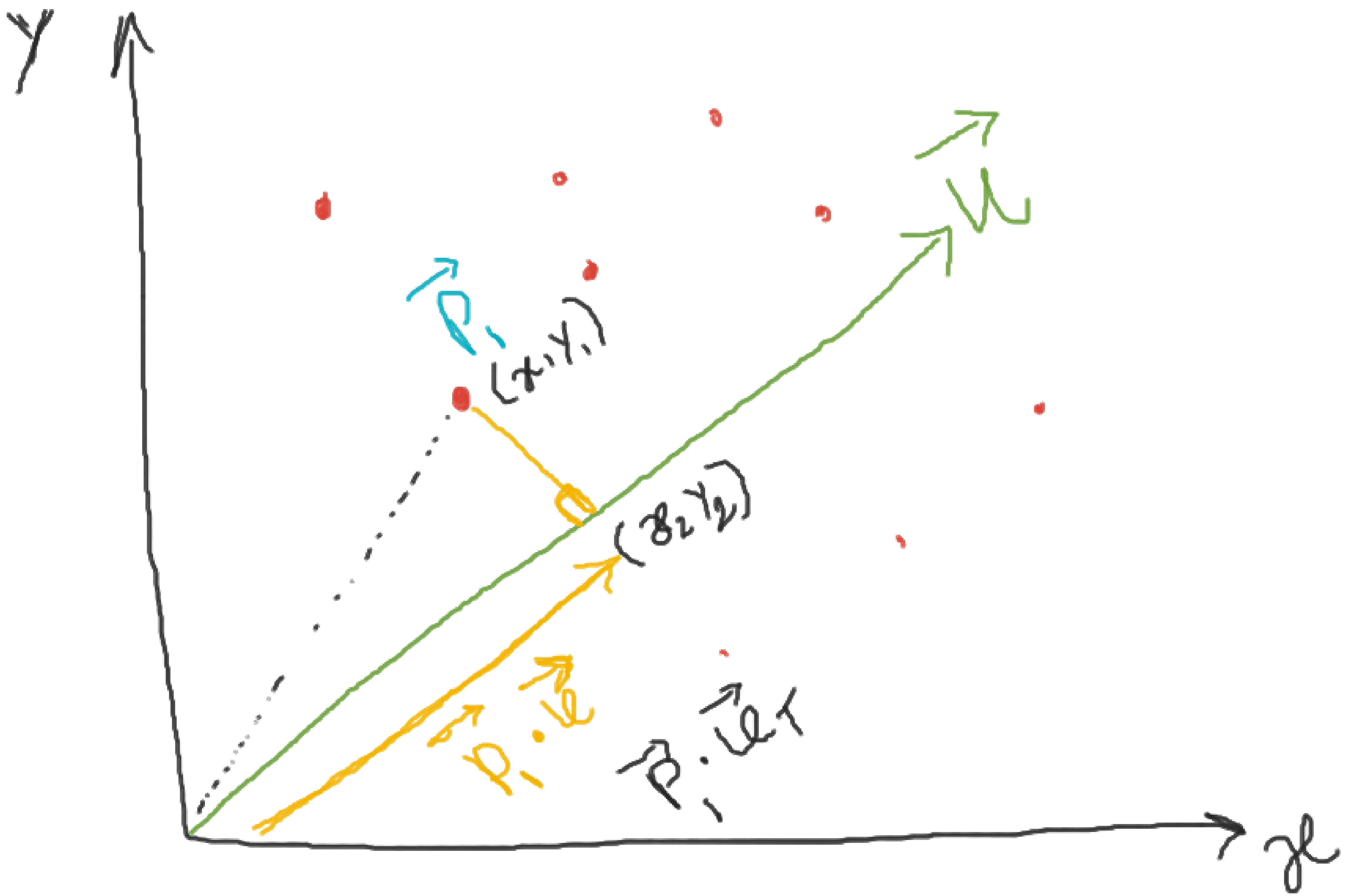
mean = 0

$$= \frac{25 + 0 + 25}{3}$$

$$= \frac{50}{3}$$

$$= 16.66$$

Variance $\propto$ spread of data



Y
x
$\vec{u}$
$\vec{p}_1$
$(x_1 y_1)$
$(x_2 y_2)$
$\vec{p}_1 \cdot \vec{u}$
$\vec{p}_1 \cdot \vec{u}^T$

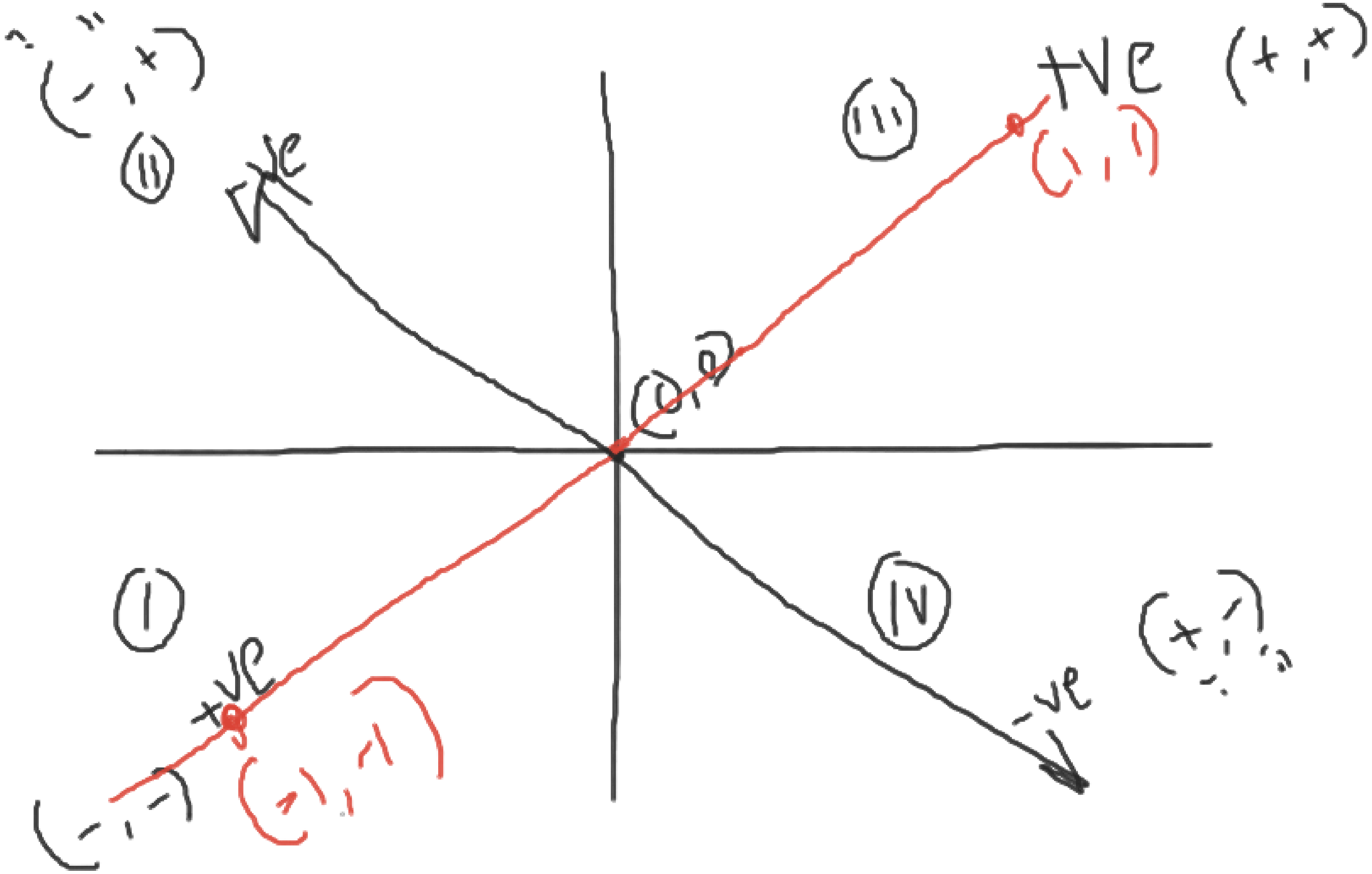
$$d_1 = \frac{\vec{p}_1 \cdot \vec{u}}{|\vec{u}|} = \frac{\vec{p}_1 \cdot \vec{u}}{1} = \vec{p}_1 \cdot \vec{u}$$

$$\vec{p}_1 \cdot \vec{u} = \underset{1\times 2}{\begin{bmatrix} x_1 & y_1 \end{bmatrix}} \cdot \underset{1\times 2}{\begin{bmatrix} x_2 & y_2 \end{bmatrix}}$$

$$\vec{p}_1 \cdot \vec{u}_T = \begin{bmatrix} x_1 & y_1 \end{bmatrix} \cdot \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} x_1 x_2 + y_1 y_2 \end{bmatrix}$$

$$\vec{P_1} \cdot \vec{U_t} = \left[x_1 x_2 + y_1 y_2 \right] \rightarrow \text{Scaler}$$

Matrix



$\bar{x}=0, \bar{y}=0$

$$= \frac{(-1)(1)+0+(1)(-1)}{3}$$

$$= -1-1/3 = -2/3$$

$$= \frac{(-1)(-1)+0+(1)(1)}{3}$$

$$= 2/3$$

$$\boxed{Cov = \frac{\sum (x_i-\bar{x})\cdot(y_i-\bar{y})}{N}}$$

$$\mathrm{Cov} = \frac{\sum (x_i - \bar{x}) \cdot (y_i - \bar{y})}{N}$$

$$\mathrm{Cov} = -\infty \text{ to } +\infty$$

$$\begin{array}{c|cc} & x_1 & x_2 \\ \hline x_1 & \mathrm{cov}(x_1 x_1) & \mathrm{cov}(x_1 x_2) \\ x_2 & \mathrm{cov}(x_2 x_1) & \mathrm{cov}(x_2 x_2) \end{array}$$

Direction

$$Var : \frac{\sum (x_i - \bar{x})^2}{N}$$

$$Cov = \frac{\sum (x_i - \bar{x}) \cdot (y_i - \bar{y})}{N}$$

$$= \frac{\sum (x_i - \bar{x})(x_i - \bar{x})}{N}$$

$$= \frac{\sum (x_i - \bar{x})^2}{N}$$

Var//

$$Ax = \lambda x$$

Eigen Values

$$Ax - \lambda x = 0$$

$$(A - \lambda)\underline{\underline{x}} = 0$$

$$[A - \lambda I] = 0$$

$$A = \begin{bmatrix} 2 & 3 \\ 4 & 3 \end{bmatrix}, \quad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A - \lambda I = 0$$

$$|A - \lambda I| = \begin{bmatrix} 2 & 3 \\ 4 & 3 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 3 \\ 4 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

$$= \begin{bmatrix} 2-\lambda & 3-0 \\ 4-0 & 3-\lambda \end{bmatrix}$$

$$A - \lambda I = \begin{bmatrix} 2-\lambda & 3 \\ 4 & 3-\lambda \end{bmatrix}$$

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} a_{11} a_{22} - a_{12} a_{21} \end{bmatrix}$$