1.

设：$p=(w, x, y, z)$

$q=(d, a, b, c)$

则：$q^{-1} = \frac{1}{\|q\|}(d, -a, -b, -c)$

$$
\begin{aligned}
p' &= q p q^{-1} \\
&= q^{+} (q^{-1})^{\oplus} p \\
&= \begin{pmatrix} d & -c & b & a \\ c & d & -a & b \\ -b & a & d & c \\ -a & -b & -c & d \end{pmatrix} \frac{1}{\|q\|} \begin{pmatrix} d & c & -b & -a \\ -c & d & a & -b \\ b & -a & d & -c \\ a & b & c & d \end{pmatrix} \begin{pmatrix} w \\ x \\ y \\ z \end{pmatrix} \\
&= \frac{1}{\|q\|} \begin{pmatrix} d & -c & b & a \\ & & & \\ & & & \\ & & & \end{pmatrix} \begin{pmatrix} dw + cx - by - az \\ -cw + dx + ay - bz \\ bw - ax + dy - cz \\ aw + bx + cy + dz \end{pmatrix}
\end{aligned}
$$

$$
\begin{aligned}
\text{第一行} &= d^2 w + \cancel{cdx} - \cancel{bdy} - \cancel{adz} \\
&\quad + c^2 w - \cancel{cdx} - \cancel{acy} + \cancel{bcz} \\
&\quad + b^2 w - \cancel{abx} + \cancel{bdy} - \cancel{bcz} \\
&\quad + a^2 w + \cancel{abx} + \cancel{acy} + \cancel{adz} \\
&= (a^2 + b^2 + c^2 + d^2) w
\end{aligned}
$$

因为 p 是点，$w=0$. $\Rightarrow$ 首行 $=0$.

$\Rightarrow$ p' 实部为 0