$$\begin{cases} x_k = x_{k-1} + v_k + w_k \\ y_k = x_k + n_k \end{cases} \qquad w_k \sim \mathcal{N}(0, Q) \qquad n_k \sim \mathcal{N}(0, R)$$

1. $\underline{e} = \underline{z} - \underline{H}\,\underline{x}$

$\mathbb{R}^6 \quad \mathbb{R}^6 \quad \mathbb{R}^{6\times 4} \quad \mathbb{R}^4$

$$\begin{cases} v_k = x_k - x_{k-1} - w_k \\ y_k = x_k + n_k. \end{cases}$$

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ y_1 \\ y_2 \\ y_3 \end{bmatrix} = \underbrace{\begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{=:H} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} -w_1 \\ -w_2 \\ -w_3 \\ +n_1 \\ +n_2 \\ +n_3 \end{bmatrix}$$

2. $$W = \begin{bmatrix} Q & & & & & \\ & Q & & & & \\ & & Q & & & \\ & & & R & & \\ & & & & R & \\ & & & & & R \end{bmatrix}$$

3. 求解

$$\underset{4\times1}{\underline{x}} = \underset{\underline{x}}{\operatorname{argmin}} \underbrace{\frac{1}{2}(\underline{z}-\underline{H}\underline{x})^T W^{-1}(\underline{z}-\underline{H}\underline{x})}_{f(\underline{x})}$$

1×6　6×6　6×1　　1×1

$$\underset{4\times1}{\frac{df(\underline{x})}{d\underline{x}}} = -\underline{H}^T W^{-1}(\underline{z}-\underline{H}\underline{x})$$

$$= \underline{H}^T W^{-1}(\underline{H}\underline{x}-\underline{z}) \doteq \underline{0}$$

$$\downarrow$$

$$\underline{H}^T W^{-1}\underline{H}\underline{x} = \underline{H}^T W^{-1}\underline{z}$$

$$\underline{x} = (H^T W^{-1} H)^{-1} H^T W^{-1} z$$

为唯一解.