

1. 左微分

$$k(t) = \exp(\phi_0^1 t)$$

$$\begin{aligned} \frac{\partial k p}{\partial \phi} &= \lim_{\phi \rightarrow 0} \frac{\exp(\phi^1) \exp(\phi^1) p - \exp(\phi^1) p}{\phi} \\ &\approx \lim_{\phi \rightarrow 0} \frac{(1 + \phi^1) \exp(\phi^1) p - \exp(\phi^1) p}{\phi} \\ &= \lim_{\phi \rightarrow 0} \frac{\phi^1 k p}{\phi} \\ &= \lim_{\phi \rightarrow 0} \frac{-(k p)^1 \phi}{\phi} \\ &= -(k p)^1 \end{aligned}$$

右微分

$$\begin{aligned} \frac{\partial (k p)}{\partial \phi} &= \lim_{\phi \rightarrow 0} \frac{\exp(\phi^1) \exp(\phi^1) p - \exp(\phi^1) p}{\phi} \\ &\approx \lim_{\phi \rightarrow 0} \frac{\exp(\phi^1) (1 + \phi^1) p - \exp(\phi^1) p}{\phi} \\ &= \lim_{\phi \rightarrow 0} \frac{\exp(\phi^1) \phi^1 p}{\phi} \\ &= \lim_{\phi \rightarrow 0} \frac{-\exp(\phi^1) p^1 \phi}{\phi} \\ &= -k p^1 \end{aligned}$$

2. (8) 左扰动

$$\frac{\partial \ln(R_1 R_2)^V}{\partial R_1} \approx \frac{\partial \ln[\exp(\phi_1^1) \exp(\phi_2^1)]^V}{\partial R_1}$$

$$\approx \frac{\partial [J_{\ell}(\phi_2)^{-1} \phi_1 + \phi_2]}{\partial \phi_1}$$

$$\approx J_{\ell}(\phi_2)^{-1}$$

其中 $J_{\ell} = J = \frac{\sin \phi_2}{\phi_2} \mathbf{I} + (1 - \frac{\sin \phi_2}{\phi_2}) a a^T + \frac{1 - \cos \phi_2}{\phi_2} a^1$

(9) 右扰动

$$\frac{\partial \ln(R_1 R_2)^V}{\partial R_2} \approx \frac{\partial [J_{rc}(\phi_1)^{-1} \phi_2 + \phi_1]}{\partial \phi_2}$$

$$\approx J_{rc}(\phi_1)^{-1}$$

$$\approx J_{\ell}(-\phi_1)^{-1}$$