# 1.

$$\frac{d(R^{-1}p)}{dR} = \lim_{\varphi \to 0} \frac{[R \cdot \exp(\varphi^\wedge)]^{-1} p - R^{-1}p}{\varphi}$$

右乘

$$= \lim_{\varphi \to 0} \frac{\exp(\varphi^\wedge)^{-1} \cdot R^{-1} \cdot p - R^{-1}p}{\varphi}$$

错误位置
进行改正

$$= \lim_{\varphi \to 0} \frac{(I + \varphi^\wedge)^{-1} R^{-1}p - R^{-1}p}{\varphi}$$

$$= \lim_{\varphi \to 0} \frac{(I - \varphi^\wedge) R^{-1}p - R^{-1}p}{\varphi}$$

$$= \lim_{\varphi \to 0} \frac{-\varphi^\wedge (R^{-1}p)}{\varphi}$$

$$= \lim_{\varphi \to 0} \frac{(R^{-1}p)^\wedge \varphi}{\varphi}$$

$$= (R^{-1}p)^\wedge$$

因为 $\exp(\varphi^\wedge) = R = (R^T)^{-1}$，则其等价于反向旋转．$\varphi = -\varphi$

所以 $\exp(\varphi^\wedge)^{-1} = \exp(-\varphi^\wedge)$

$$= \lim_{\varphi \to 0} \frac{\exp(-\varphi^\wedge) \cdot R^{-1} \cdot p - R^{-1} \cdot p}{\varphi}$$

$$= \lim_{\varphi \to 0} \frac{(I - \varphi^\wedge) R^{-1} \cdot p - R^{-1}p}{\varphi}$$

Taylor 展开

$$\exp((-\varphi)^\wedge) = \sum_{n=0}^{\infty} \frac{1}{n!} ((-\varphi)^\wedge)^n = I + (-\varphi)^\wedge + \cdots$$

$$= \lim_{\varphi \to 0} \frac{-\varphi^\wedge R^{-1} p}{\varphi}$$

$\approx I - \varphi^\wedge$

$$= \lim_{\varphi \to 0} \frac{(R^{-1} p)^\wedge \varphi}{\varphi}$$

$$= \boxed{(R^{-1} p)^\wedge}$$

改正后结果.

# Q1. 方法二，使用 Lie Algebre 求解.

$$\frac{d[\exp(\phi^\wedge)^{-1}\cdot p]}{d\phi} = \lim_{\Delta\phi\to 0}\frac{\exp((\phi+\varphi)^\wedge)^{-1}p - \exp(\phi^\wedge)^{-1}p}{\Delta\phi}$$

根据公式 $\exp((\phi+\Delta\phi)^\wedge) = \exp(\phi^\wedge)\cdot\exp((J_r\cdot\Delta\phi)^\wedge)$

有原式 $= \lim\limits_{\Delta\phi\to 0}\dfrac{[\exp(\phi^\wedge)\cdot\exp((J_r\cdot\Delta\phi)^\wedge)]^{-1}p - \exp(\phi^\wedge)^{-1}p}{\Delta\phi}$

拆开 $(a\cdot b)^{-1} = b^{-1}\cdot a^{-1}$ 形式

$$= \lim_{\Delta\phi\to 0}\frac{\exp((J_r\cdot\Delta\phi)^\wedge)^{-1}\cdot\exp(\phi^\wedge)^{-1}\cdot p - \exp(\phi^\wedge)^{-1}\cdot p}{\Delta\phi}$$

$$= \lim_{\Delta\phi\to 0}\frac{\underbrace{[\exp((J_r\cdot\Delta\phi)^\wedge)^{-1} - I]}\cdot\exp(\phi^\wedge)^{-1}\cdot p}{\Delta\phi}$$

现在分析 $\exp((J_r\cdot\Delta\phi)^\wedge)^{-1} - I$ 项

将逆转为负号

$$= \exp((-J_r\cdot\Delta\phi)^\wedge) - I$$

Taylor 展开

$$\approx I - (J_r\cdot\Delta\phi)^\wedge - I$$

$$= -(J_r\cdot\Delta\phi)^\wedge$$

则原式为

$$= \lim_{\Delta\phi\to 0} \frac{-(J_r\cdot\Delta\phi)^\wedge \cdot \exp(\phi^\wedge)^{-1}\cdot p}{\Delta\phi}$$

$$= \lim_{\Delta\phi\to 0}\frac{[\exp(\phi^\wedge)^{-1}\cdot p]^\wedge\cdot J_r\cdot\Delta\phi}{\Delta\phi}$$

$$= [\exp(\phi^\wedge)^{-1}\cdot p]^\wedge \cdot J_r$$

$$= (R^{-1}p)^\wedge\cdot J_r$$

# 2.

$$\frac{d\left[\ln(R_1 R_2^{-1})\right]^\vee}{dR_2} = \lim_{\varphi \to 0} \frac{\left[\ln\left(R_1 \cdot (R_2 \cdot \exp(\varphi^\wedge))^{-1}\right)\right]^\vee - \left[\ln(R_1 R_2^{-1})\right]^\vee}{d\varphi}$$

$$= \lim_{\varphi \to 0} \frac{\left[\ln(R_1 \cdot \exp^{-1}(\varphi^\wedge) \cdot R_2^{-1})\right]^\vee - \left[\ln(R_1 R_2^{-1})\right]^\vee}{d\varphi}$$

注: $\exp^{-1}(\varphi^\wedge) = \exp(-\varphi^\wedge)$

$$= \lim_{\varphi \to 0} \frac{\left[\ln(R_1 \cdot R_2^{-1} R_2 \exp(-\varphi^\wedge) \cdot R_2^{-1})\right]^\vee - \left[\ln(R_1 R_2^{-1})\right]^\vee}{d\varphi}$$

由 $R^T \exp(\phi^\wedge) R = \exp((R^T\phi)^\wedge)$

$$= \lim_{\varphi \to 0} \frac{\left[\ln(R_1 R_2^{-1} \exp(-(R_2\varphi)^\wedge))\right]^\vee - \left[\ln(R_1 R_2^{-1})\right]^\vee}{d\varphi}$$

$$= \lim_{\varphi \to 0} \frac{\ln(R_1 R_2^{-1})^\vee + J_r^{-1}(\ln(R_1 R_2^{-1})^\vee)(-R_2\varphi) - \ln(R_1 R_2^{-1})^\vee}{}$$

$$= -J_r^{-1}\left[\ln(R_1 R_2^{-1})\right]^\vee R_2$$

由 SLAM 4.28 公式

$$\left[\ln(\exp(\phi_1^\wedge) \cdot \exp(\phi_2^\wedge))\right]^\vee = J_r(\phi_1)^{-1} \cdot \phi_2 + \phi_1$$

$$\Downarrow$$

$$\left[\ln(R \cdot \exp(\varphi^\wedge))\right]^\vee = J_r^{-1}(\phi_1) \cdot \varphi + \left[\ln(R)\right]^\vee$$