

5. 伴随.

① 证明: $\forall a \in \mathbb{R}^3, Ra^{\wedge} R^T = (Ra)^{\wedge}$

$$\begin{aligned}(Ra)^{\wedge} \cdot v &= (Ra) \times v \\&= (Ra) \times (RR^{-1}v) \\&= R[a \times (R^{-1}v)] \\&= Ra^{\wedge} R^{-1}v\end{aligned}$$

$$\Rightarrow (Ra)^{\wedge} = Ra^{\wedge} R^{-1}$$

因为 R 是正交矩阵

$$(Ra)^{\wedge} = Ra^{\wedge} R^{-1} = Ra^{\wedge} R^T$$

② 证明伴随.

$$\perp p^{\wedge} = \theta a^{\wedge}$$

$$\begin{aligned}R \exp(\theta a^{\wedge}) R^T &= R(\cos \theta I + (1 - \cos \theta) \underline{a} \underline{a}^T + \sin \theta \underline{a}^{\wedge}) R^T \\&= \cos \theta I + (1 - \cos \theta) \underline{R} \underline{a} (\underline{R} \underline{a})^T + \sin \theta \underline{R} \underline{a}^{\wedge} \underline{R}^T \\&= \cos \theta I + (1 - \cos \theta) \underline{R} \underline{a} (\underline{R} \underline{a})^T + \sin \theta (Ra)^{\wedge} \\&= \exp((Ra)^{\wedge})\end{aligned}$$

证毕