

# Regression Discontinuity Design: Extensions

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# Agenda

- ▶ Local Randomization Approach
- ▶ Fuzzy RD
- ▶ GRD
- ▶ Difference-in-discontinuities
- ▶ PCRD

## Recap: Continuity-based Approach

- ▶ Assume regression functions are continuous to obtain

$$\tau_{SRD} = \mathbb{E}[Y_i(1) - Y_i(0) | X_i = c] = \lim_{x \downarrow c} \mathbb{E}[Y_i | X_i = x] - \lim_{x \uparrow c} \mathbb{E}[Y_i | X_i = x]$$

- ▶ Approximates regression function and relies on continuity assumptions.
- ▶ Requires: choosing weights, bandwidth and polynomial order.
- ▶ Alternative: local randomization approach

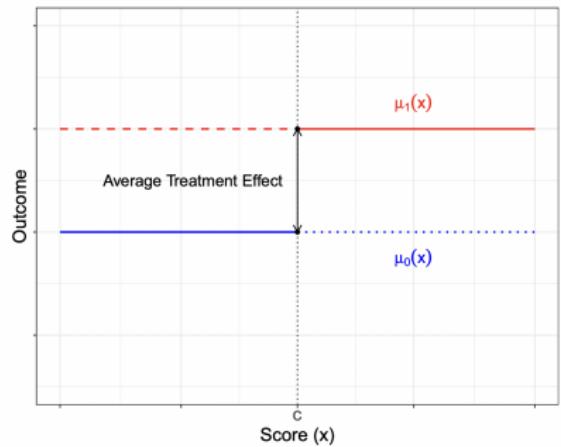
## Analogies with experiment

- ▶ Lee (2008): RD design can be as credible as a randomized experiment for units very near cutoff
- ▶ Imagine that score depends on each unit's unobservables characteristics and choices
- ▶ If the two following conditions hold:
  - ▶ there is a random chance element to score that unit receives
  - ▶ probability of this random “error” doesn’t change abruptly at cutoff
- ▶ Then the RD design can be seen as an experiment:
  - ▶ units barely above the cutoff as-if randomly assigned to treatment
  - ▶ units barely above the cutoff as-if randomly assigned to control
- ▶ This fails if individuals have ability to exactly control their score

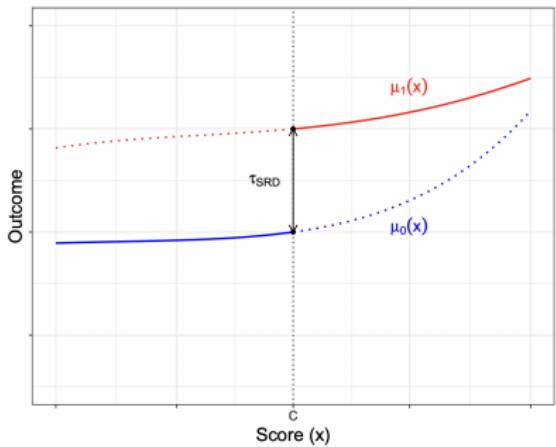
## Analogies with experiment

- ▶ Consider an RD Design where:
  - ▶ treatment is assigned based on score exceeding cutoff
  - ▶ units lack ability to manipulate score (continuity holds)
- ▶ Crucial distinction:
  - ▶ Experiment → no need to make assumptions about shape of the average potential outcomes
  - ▶ RD design → inferences depend crucially on assumptions regarding functional form of regression functions
- ▶ Any experiment can be recast as an RD design where
  - ▶ score is a uniform random variable
  - ▶ cutoff chosen to ensure a given probability of treatment
  - ▶ Ex: each student assigned uniform random number between 0 and 100, scholarship given to students whose score is above 50

# Experiment vs. RD design

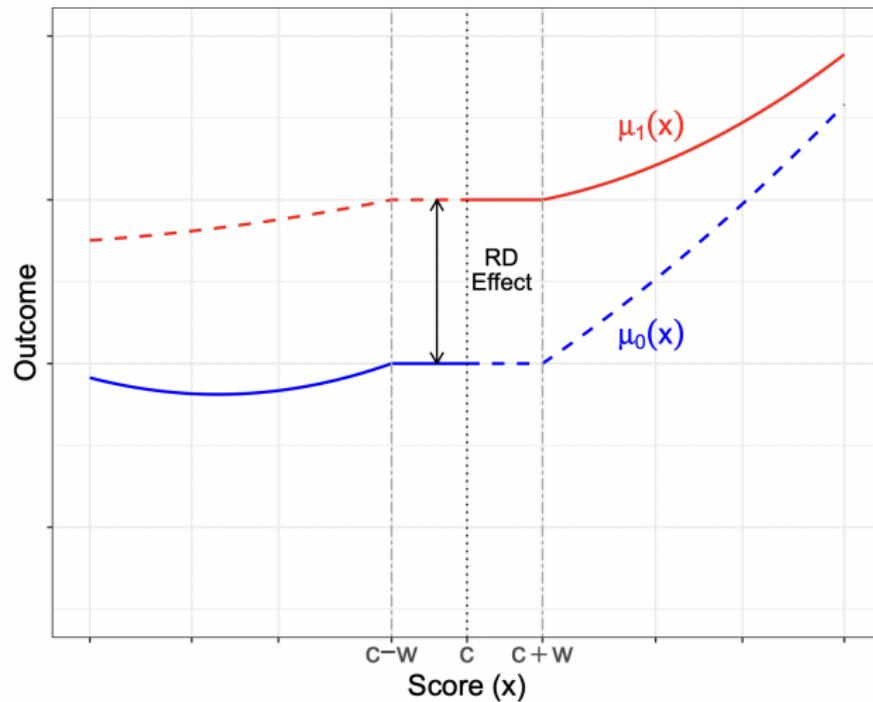


(a) Randomized Experiment



(b) RD Design

# If as-if random interpretation is true: Local Randomization RD



## Local Randomization Approach to RD Design

- ▶ Gives an alternative that can be used as a robustness check.
- ▶ Key assumption: exists window  $W = [-w, w]$  around cutoff ( $-w < c < w$ ) where (assuming random potential outcomes)  $T_i$  independent of  $(Y_i(0), Y_i(1))$  (for all  $X_i \in W$ )
- ▶ Thus, inside  $W_0$  subjects are as-if randomly assigned to either side of cutoff
  - ▶ The distribution of running variable same for all units inside  $W_0$
  - ▶ Potential outcomes in  $W_0$  depend on running variable only through threshold indicators within  $W_0$
- ▶ Stronger than Continuity-Based Approach → Relevant population functions are not only continuous at  $x_0$ , but also completely unaffected by the running variable in  $W_0$

## Local Randomization Approach to RD Design

- ▶ In window  $W_0$ , subjects randomly assigned to either side of cutoff:
  - ▶ Window  $W_0$
  - ▶ Assignment mechanism
- ▶ If assignment mechanism and  $W_0$  are known, RD becomes an experiment in  $W_0$
- ▶ If few units inside  $W_0$ , adopt a Fisherian setup: potential outcomes are fixed, only randomness is in the assignment of subjects

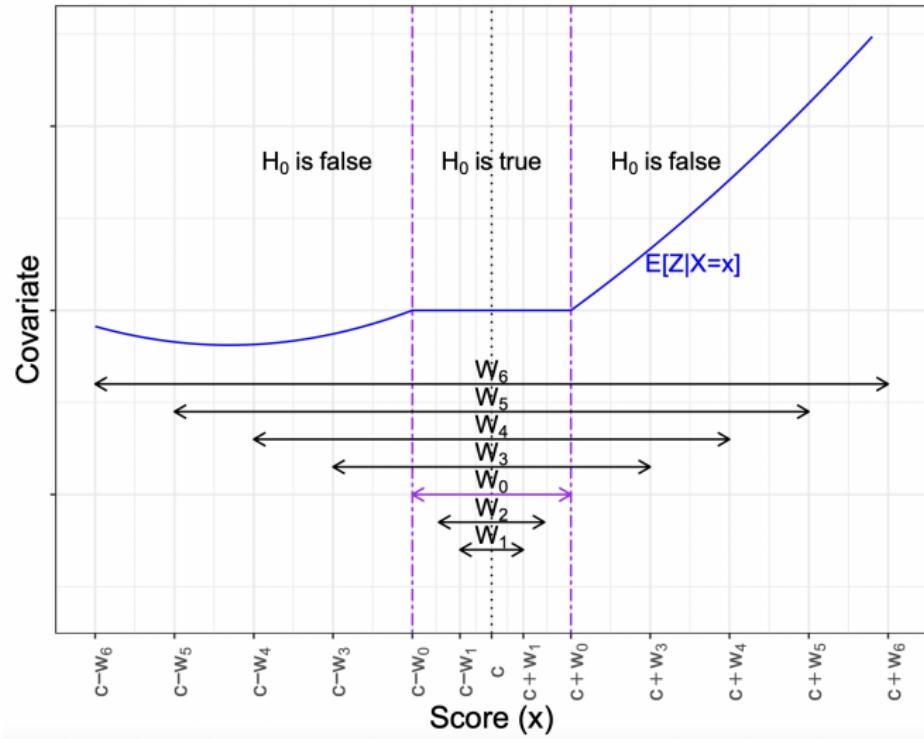
## Local Randomization Approach Using Fisherian Methods

- ▶ Approach has two steps:
  - ▶ Step 1: Choose window around cutoff where randomization holds
  - ▶ Step 2: Apply randomization inference tools, given a hypothesized treatment assignment, within  $W_0$

## Step 1: Choose the window $W_0$

- ▶ How to choose window?
  - ▶ Use balance tests on pre-determined/exogenous covariates.
  - ▶ Very intuitive, easy to implement.

# Window Selector Based on Covariate Balance in Locally Random RD



## Step 2: Use Randomization Inference Tools within $W_0$

- ▶ Under this framework, we can treat observations within the window  $W_0$  as if generated by a randomized experiment
- ▶ One possible randomization mechanism:
  - ▶  $T_i$  is Bernoulli with parameter  $\pi$ : for all vectors  $t$  in  $\Omega_{W_0}$ ,

$$\Pr(T_{W_0} = t) = \pi^{t'1}(1 - \pi)^{(1-t)'1}$$

- ▶ Since  $\pi$  is unknown, we estimate it  $\hat{\pi} = \frac{T'_{W_0}}{n_{W_0}}$
- ▶ Another possible randomization mechanism:
  - ▶ Fix number of treated units within the window at  $m_{W_0}$ , which leads to

$$\Pr(T_{W_0} = t) = \frac{1}{\binom{n_{W_0}}{m_{W_0}}} \text{ for all } t \in \Omega_{W_0}$$

## Step 2: Use Randomization Inference Tools within $W_0$

- ▶ Given local random assumption, can test sharp null hypothesis of no treatment effect for any  $i$
- ▶ Under this hypothesis, observed outcomes are fixed regardless of realization of  $T_{W_0}$ :  $y_i(t) = y_i$  for all  $i$  within  $W_0$  and for all  $t \in \Omega_{W_0}$
- ▶ Thus, the distribution of any test statistic  $Q(T_{W_0}, Y_{W_0})$  is known, since it depends only on the known distribution of  $T_{W_0}$
- ▶ One-sided significance level:

$$\Pr(Q(T_{W_0}, Y_{W_0}) \geq Q(t_{W_0}, Y_{W_0})) = \sum_{t \in \Omega_W} \mathbb{1}(Q(t, Y_{W_0}) \geq Q(t_{W_0}, Y_{W_0})) \Pr(T_{W_0} = t)$$

- ▶ Different test statistics may be used

## Hypothetical Randomization Distribution with Five Units

## Empirical Illustration: Inc incumbency Advantage (CFT, 2015, JCI)

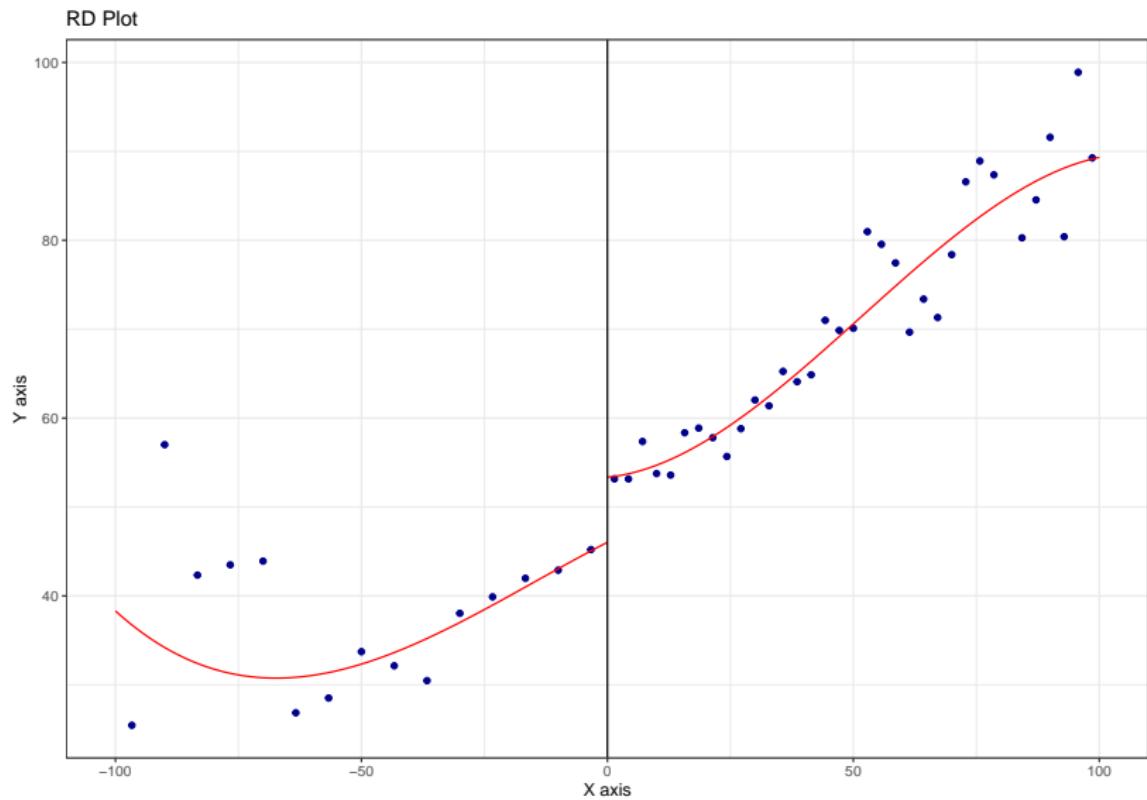
- ▶ Problem: incumbency advantage (U.S. senate).
- ▶ Data:
  - ▶  $Y_i$  = election outcome at  $t + 1$ .
  - ▶  $T_i$  = whether party wins election at  $t$ .
  - ▶  $X_i$  = margin of victory at  $t$  ( $c = 0$ ).
  - ▶  $Z_i$  = covariates (*demvoteshlag1*, *demvoteshlag2*, *dopen*, etc.).
- ▶ Potential outcomes:
  - ▶  $Y_i(0)$  = election outcome at  $t + 1$  if had not been incumbent.
  - ▶  $Y_i(1)$  = election outcome at  $t + 1$  if had been incumbent.
- ▶ Causal Inference:
  - ▶  $Y_i(0) \neq Y_i | T_i = 0$
  - ▶  $Y_i(1) \neq Y_i | T_i = 1$

# Empirical Illustration: Incumbency Advantage (CFT, 2015, JCI)

```
library(rdlocrand)
library(rdrobust)
data <- read.csv("rdlocrand_senate.csv")
X <- cbind(data$presdemvoteshlag1,
            data$population/1000000, data$demvoteshlag1,
            data$demvoteshlag2, data$demwinprv1,
            data$demwinprv2, data$dopen,
            data$dmidterm, data$dpresdem)
colnames(X) <- c("DemPres Vote", "Population",
                 "DemSen Vote t-1", "DemSen Vote t-2",
                 "DemSen Win t-1", "DemSen Win t-2",
                 "Open", "Midterm", "DemPres")
R <- data$demmv
Y <- data$demvoteshfor2
D <- as.numeric(R>=0)
```

# Simple RD plot

```
rdplot(Y,R,p=3)
```



# Randomization inference using selected window

```
out <- rdrandinf(Y, R, wl = -2.5, wr = 2.5, seed = 50)
```

```
##  
## Selected window = [-2.5;2.5]  
##  
## Running randomization-based test...  
## Randomization-based test complete.  
##  
##  
## Number of obs      =        1297  
## Order of poly     =          0  
## Kernel type       =      uniform  
## Reps              =        1000  
## Window            = set by user  
## H0:                 tau =        0.000  
## Randomization     = fixed margins  
##  
## Cutoff c =    0.000  Left of c  Right of c  
##           Number of obs      595      702  
##   Eff. number of obs      63       57  
##   Mean of outcome      44.068    53.235  
##   S.d. of outcome       10.627    8.289  
##           Window       -2.500     2.500  
##  
## ======  
##                         Finite sample          Large sample  
##-----  
##      Statistic      T      P>|T|      P>|T|  Power vs d =  5.313  
## ======  
##      Diff. in means  9.167      0.000      0.000          0.866  
## ======
```

# Binomial randomization mechanism

```
bern_prob <- numeric(length(R))
bern_prob[abs(R) > 2.5] <- NA
bern_prob[abs(R) <= 2.5] <- 1/2
out <- rrandinf(Y, R, wl = -2.5, wr = 2.5, seed = 50, bernoulli = bern_prob)
```

```
##
## Selected window = [-2.5;2.5]
##
## Running randomization-based test...
## Randomization-based test complete.
##
##
## Number of obs      =          120
## Order of poly     =          0
## Kernel type       =      uniform
## Reps              =         1000
## Window            = set by user
## H0:           tau =        0.000
## Randomization    =    Bernoulli
##
## Cutoff c = 0.000  Left of c  Right of c
##   Number of obs    63        57
##   Eff. number of obs 63        57
##   Mean of outcome  44.068    53.235
##   S.d. of outcome  10.627    8.289
##   Window          -2.500    2.500
##
## =====
##                                     Finite sample          Large sample
##                                     -----
##   Statistic      T    P>|T|    P>|T|  Power vs d =  5.313
## =====
##   Diff. in means 9.167  0.000   0.000        0.866
## =====
```

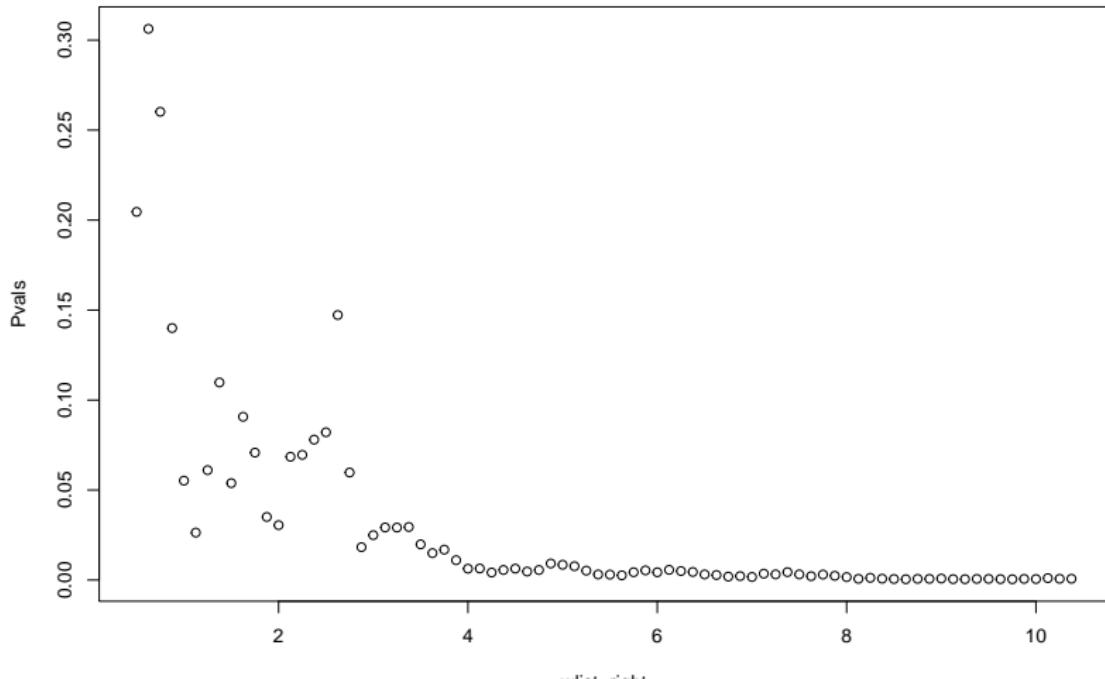
# Selecting the Window

```
## Mass points detected in running variable
## You may use wmasspoints option for constructing windows at each mass point
##
##
## Window selection for RD under local randomization
##
## Number of obs      =      1390
## Order of poly      =          0
## Kernel type        =      uniform
## Reps               =      1000
## Testing method     =    rdrandinf
## Balance test       =   diffmeans
##
## Cutoff c =  0.000  Left of c  Right of c
##           Number of obs      640      750
##           1st percentile      7        7
##           5th percentile     32       37
##           10th percentile    64       75
##           20th percentile   127      149
##
## =====
##      Window          p-value      Var. name  Bin.test  Obs<c  Obs>=c
## =====
## -0.5287  0.5287  0.186 DemSen.Vote.t.2  0.327      10      16
## -0.5907  0.5907  0.404      Open  0.362      12      18
## -0.6934  0.6934  0.464      Open  0.311      14      21
## -0.7652  0.7652  0.241      Open  0.154      15      25
## -0.9694  0.9694  0.076      Open  0.135      17      28
## -1.0800  1.0800  0.034      Open  0.119      19      31
## -1.1834  1.1834  0.097      Open  0.134      21      33
## -1.2960  1.2960  0.115      Open  0.245      25      35
## -1.3289  1.3289  0.225 Midterm  0.382      28      36
## -1.4174  1.4174  0.126 Midterm  0.396      30      38
## =====
## Recommended window is [-0.7652;0.7652] with 40 observations (15 below, 25 above).
```

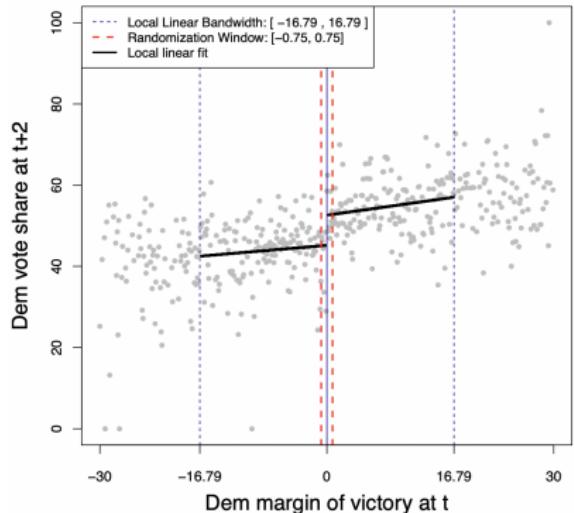
# P-value for Different Windows

```
tmp <- rdwinselect(R,X,wmin=.5,wstep=.125,approx=TRUE,nwin=80,quietly=TRUE,plot=TRUE)
```

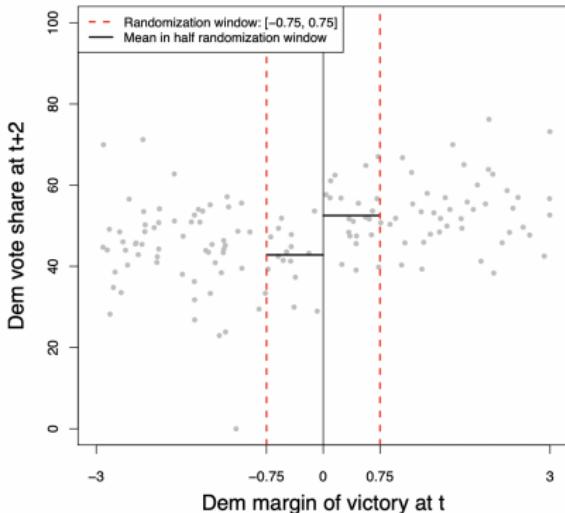
```
## Mass points detected in running variable  
## You may use wmasspoints option for constructing windows at each mass point
```



# Continuity-Based vs Local Randomization Analysis, CFT



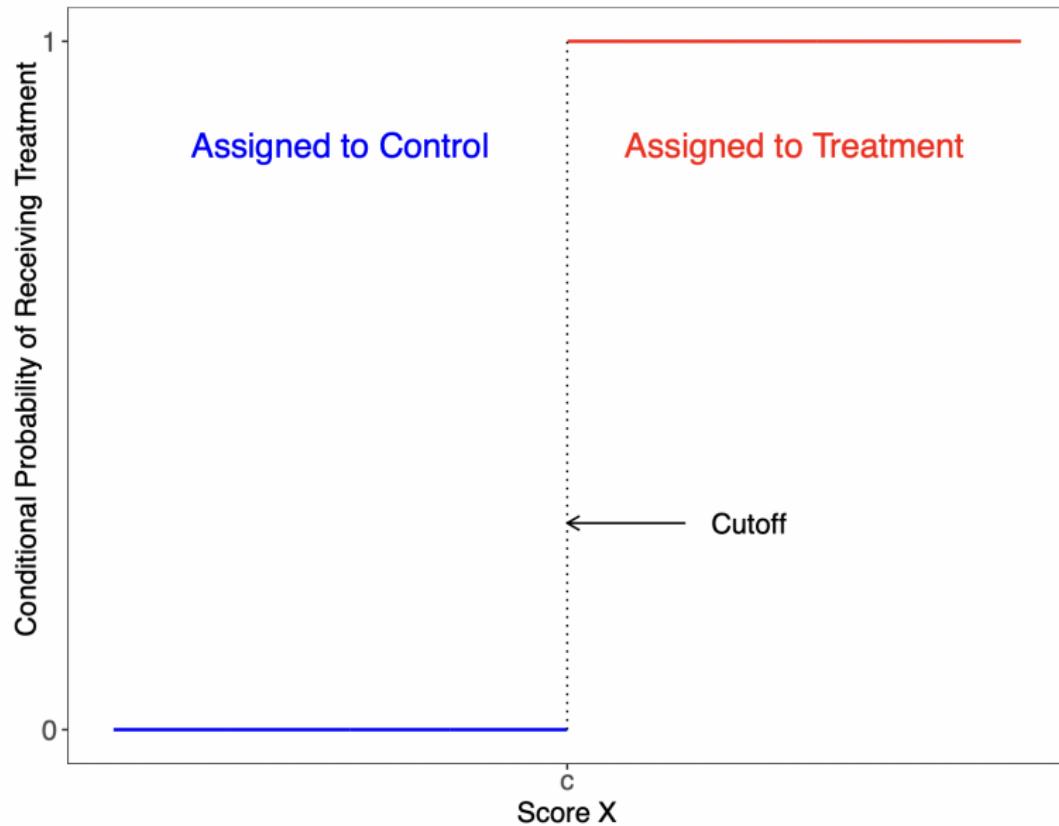
(a) Continuity-Based Analysis



(b) Local Randomization Analysis

# Fuzzy RD

## Treatment Assignment in (Sharp) RD Design



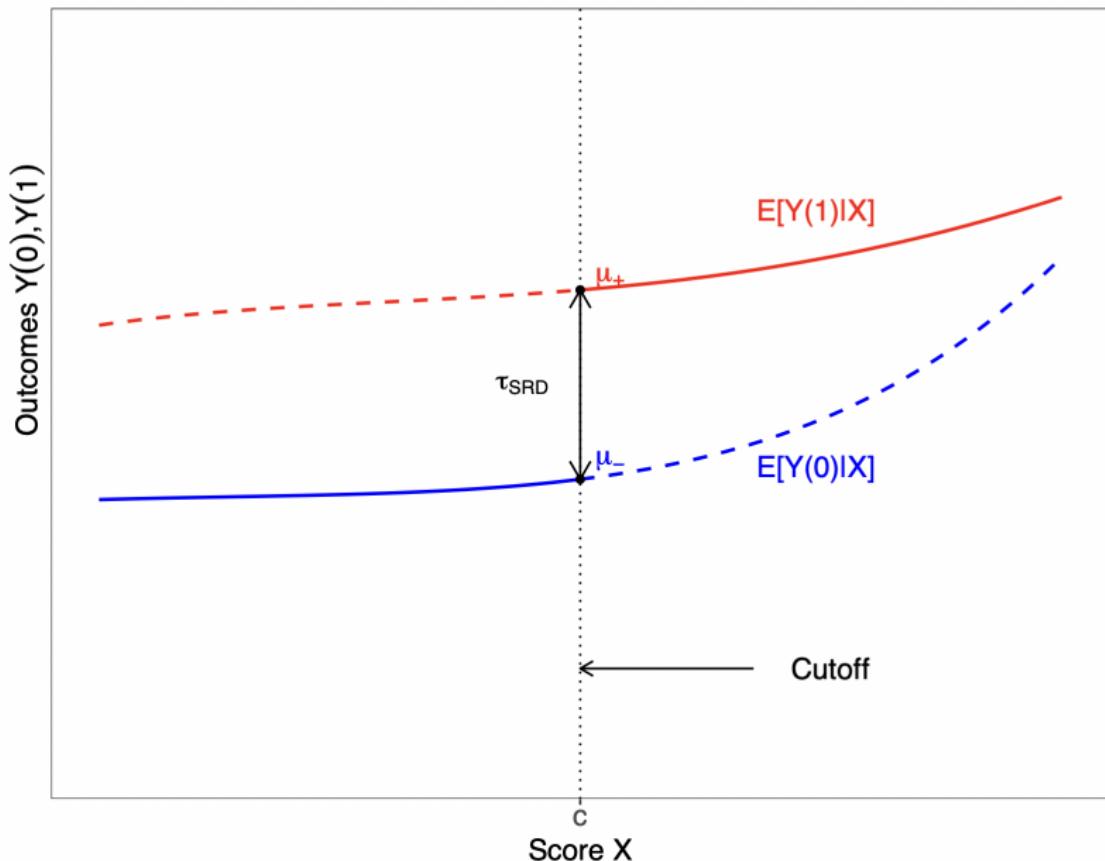
## Sharp Regression Discontinuity Design

- ▶  $n$  units, indexed by  $i = 1, 2, \dots, n$
- ▶ Unit's score is  $X_i$ , treatment is  $T_i = 1(X_i \geq c)$
- ▶ Each unit has two potential outcomes:
  - ▶  $Y_i(1)$ : outcome that would be observed if  $i$  received treatment
  - ▶  $Y_i(0)$ : outcome that would be observed if  $i$  received control
- ▶ The *observed* outcome is

$$Y_i = \begin{cases} Y_i(0) & \text{if } X_i < c, \\ Y_i(1) & \text{if } X_i \geq c. \end{cases}$$

- ▶ Fundamental problem of causal inference: only observe  $Y_i(0)$  for units below cutoff and only observe  $Y_i(1)$  for units above cutoff

# RD Treatment Effect in Sharp RD Design



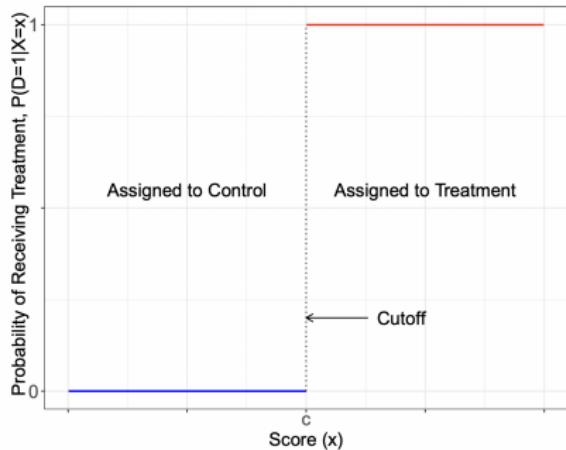
## Fuzzy RD Design

- ▶ Imperfect compliance:
  - ▶ Probability of treatment changes at  $c$ , but not necessarily from 0 to 1
  - ▶ Some units with score above  $c$  may decide not to take up treatment
  - ▶ Example: voting eligibility at 18
- ▶  $T_i$  is treatment assigned,  $D_i$  is treatment taken
- ▶ Now for some units  $T_i \neq D_i$
- ▶ Treatment taken has two potential values,  $D_i(1)$  and  $D_i(0)$ , and observed treatment taken is  
$$D_i = T_i \cdot D_i(1) + (1 - T_i) \cdot D_i(0)$$
- ▶ Four potential outcomes instead of two:

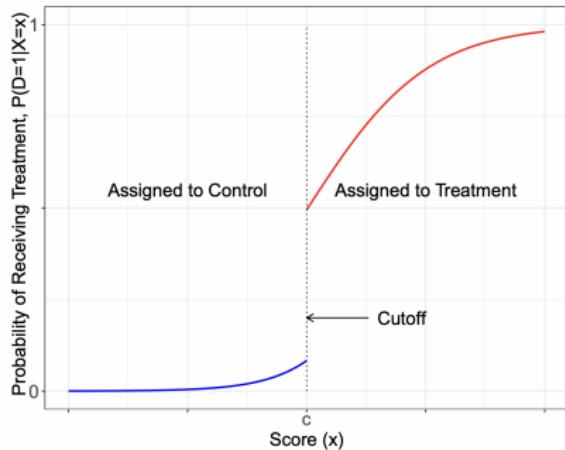
$$Y_i(1, D_i(1)) = D_i(1) Y_i(1, 1) + (1 - D_i(1)) Y_i(1, 0)$$

$$Y_i(0, D_i(0)) = D_i(0) Y_i(0, 1) + (1 - D_i(0)) Y_i(0, 0).$$

# Conditional Probability of Receiving Treatment Sharp vs. Fuzzy RD Designs



(a) Sharp RD



(b) Fuzzy RD

## Fuzzy RD Design

- ▶ Interest in both the effect of being assigned to treatment (i.e., the effect of  $T$ ) and the effect of actually receiving treatment (i.e., the effect of  $D$ )
- ▶ Since treatment assignment cannot be changed, compliance with the assignment is always perfect. Thus, analysis of the effect of  $T$  follows a Sharp RD design
- ▶ In contrast, the study of the effect of  $D$  requires modifications and additional assumptions

## Fuzzy RD Design: Continuity-based parameters

- ▶ The Sharp RD estimator of the effect of  $T_i$  on  $Y_i$  consistently estimates the quantity

$$\begin{aligned}\tau_Y &:= \lim_{x \downarrow c} \mathbb{E}[Y_i | X_i = x] - \lim_{x \uparrow c} \mathbb{E}[Y_i | X_i = x] \\ &= \lim_{x \downarrow c} \mathbb{E}[Y_i(1, D_i(1)) | X_i = x] - \lim_{x \uparrow c} \mathbb{E}[Y_i(0, D_i(0)) | X_i = x]\end{aligned}$$

where the equality follows from the more general definition of the observed outcome as

$Y_i = T_i Y_i(1, D_i(1)) + (1 - T_i) Y_i(0, D_i(0))$ , and thus requires no special assumptions.

## Fuzzy RD Design: Intention-to-treat

- ▶ Assuming continuity of  $\mathbb{E}[Y_i(1, D_i(1))|X_i = x]$  and  $\mathbb{E}[Y_i(0, D_i(0))|X_i = x]$ , seen as functions of  $x$ , at the cutoff  $c$ , we have

$$\tau_Y = \tau_{ITT}, \quad \tau_{ITT} := \mathbb{E}[Y_i(1, D_i(1)) - Y_i(0, D_i(0))|X_i = c],$$

and thus estimated jump in the average observed outcome at the cutoff recovers the average effect of  $T$  on  $Y$  at  $c$ .

- ▶  $\tau_{ITT}$  is usually called average “intention-to-treat” effect, and it captures effect (at the cutoff) of being assigned to treatment
- ▶ This parameter is different from Sharp RD parameter  $\tau_{SRD}$  under perfect compliance,

$$\tau_{SRD} = \mathbb{E}[Y_i(1) - Y_i(0)|X_i = c]$$

## Fuzzy RD Design: Intention-to-treat

- ▶ Perfect compliance is a particular case where
  - ▶  $\mathbb{P}[D_i(0) = 0 | X_i = x] = 1$  for  $x < c$  and  
 $\mathbb{P}[D_i(1) = 1 | X_i = x] = 1$  for  $x \geq c$
  - ▶  $D_i = T_i = 1(X_i \geq c)$
  - ▶  $Y_i(1, 1) := Y_i(1)$  and  $Y_i(0, 0) := Y_i(0)$
- ▶ Thus, when compliance is perfect, the RD ITT effect of the treatment assignment on the outcome is equivalent to the Sharp RD effect of the treatment received:

$$\tau_{ITT} = \mathbb{E}[Y_i(1) - Y_i(0) | X_i = c]$$

- ▶ But when some units are non-compliers,  $\tau_{ITT}$  captures the effect of the treatment assignment, which will be in general different from the effect of actually receiving the treatment

## Fuzzy RD Design: First Stage

- ▶ Fuzzy analysis includes study of how the RD assignment rule affects the probability of receiving the treatment.
- ▶ Treating  $D_i$  as the outcome, a Sharp RD strategy estimates

$$\tau_D := \lim_{x \downarrow c} \mathbb{E}[D_i | X_i = x] - \lim_{x \uparrow c} \mathbb{E}[D_i | X_i = x]$$

- ▶ Since  $D_i$  is binary,  $\tau_D$  captures the difference in the probability of receiving the treatment between units assigned to treatment vs. assigned to control, at the cutoff.

## Fuzzy RD Design: First Stage

- ▶ Assuming continuity at  $c$  of  $\mathbb{E}[D_i(1)|X_i = x]$  and  $\mathbb{E}[D_i(0)|X_i = x]$ , seen as functions of  $x$ , we have

$$\tau_D = \tau_{FS}, \quad \tau_{FS} := \mathbb{E}[D_i(1) - D_i(0)|X_i = c]$$

and thus can interpret  $\tau_D$  as the causal effect of  $T_i$  on  $D_i$ .

- ▶  $\tau_{FS}$  captures the effect of assigning the treatment on receiving the treatment for units with scores near or at the cutoff, usually called “first-stage” effect.

## Fuzzy RD Design: Estimation of FS and ITT effects

- ▶ Since both  $\tau_{FS}$  and  $\tau_{ITT}$  are Sharp RD parameters, analysis follows standard continuity-based Sharp RD methods, using  $X_i$  as running variable,  $T_i = 1(X_i \geq c)$  as treatment of interest, and  $D_i$  and  $Y_i$  as outcomes:

$$\hat{\tau}_{ITT} = \lim_{x \downarrow c} \hat{\mathbb{E}}[Y_i | X_i = x] - \lim_{x \uparrow c} \hat{\mathbb{E}}[Y_i | X_i = x]$$

$$\hat{\tau}_{FS} = \lim_{x \downarrow c} \hat{\mathbb{E}}[D_i | X_i = x] - \lim_{x \uparrow c} \hat{\mathbb{E}}[D_i | X_i = x],$$

with bandwidth selection and inference methods as discussed before.

## Fuzzy RD Design: Effect of Actual Treatment

- When interest is on the effect of the treatment received, it is common to focus on

$$\tau_{FRD} := \frac{\tau_Y}{\tau_D}$$

We call  $\tau_{FRD}$  the “fuzzy RD parameter.”

- (Under the augmented continuity conditions for ITT effects,  $\tau_{FRD} = \frac{\tau_{ITT}}{\tau_{FS}}$ . This interpretation of the Fuzzy RD parameter as ratio of two ITT effects is analogous to result in IV literature. Below we do not assume that these conditions hold.)

## Fuzzy RD Design: Effect of Actual Treatment

Explore conditions under which  $\tau_{FRD}$  can be directly interpreted as the average treatment effect of the treatment for some subpopulations.

- ▶ Non-zero first stage:  $\tau_{FS}$  must be nonzero—ideally, well-separated from zero: Moving above/below the cutoff must induce some units to actually take the treatment.
- ▶ Exclusion Restriction: the treatment assignment must affect the potential outcomes and potential treatments only via the treatment received, but not directly:  $\mathbb{E}[Y_i(T_i, 0)|X_i = x]$  and  $\mathbb{E}[Y_i(T_i, 1)|X_i = x]$  must be continuous in  $x$  at  $c$ .
- ▶ Compliance Restriction: many possibilities, including
  - ▶ Local independence: potential outcomes independent of potential treatments near the cutoff (Hahn, Todd, and van der Klaauw, 2001).
  - ▶ Monotonicity: there are no units who receive the opposite treatment to the one they are assigned near the cutoff (i.e., no “defiers”).

## Important Issues for Implementation of Fuzzy RD analysis

- ▶ Falsification: density test and covariates effects should focus on intention-to-treat effects.
- ▶ Bandwidth Selection: two bandwidths if focus on ITT and FS effects, single bandwidth if focus on Fuzzy RD effect.
- ▶ Weak Assignment: Avoid analyzing Fuzzy RD effects when the RD assignment rule has weak effect on the adoption of the treatment.

## Empirical Example

### Upstream and Downstream Impacts of College Merit-Based Financial Aid for Low-Income Students: Ser Pilo Paga in Colombia<sup>†</sup>

By JULIANA LONDOÑO-VÉLEZ, CATHERINE RODRÍGUEZ, AND FABIO SÁNCHEZ\*

*How does financial aid affect postsecondary enrollment, college choice, and student composition? We present new evidence based on a large-scale program available to high-achieving, low-income students for attending high-quality colleges in Colombia. RD estimates show financial aid eligibility raised immediate enrollment by 56.5 to 86.5 percent, depending on the complier population. This rise, driven by matriculation at private, high-quality colleges, closed the SES enrollment gap among high achievers. Moreover, a DID approach suggests enrollment of aid-ineligible students also improved because college supply expanded in response to heightened demand. With ability stratification largely replacing SES stratification, diversity increased 46 percent at private, high-quality colleges. (JEL I22, I23, I24, I26, J24, O15)*

## Empirical Example

- ▶ Study by Londoño-Vélez, Rodríguez, and Sánchez (AEJ, 2020) on the effects Ser Pilo Paga (SPP), a governmental program in Colombia that funds full tuition to attend higher education institutions (HEIs).
- ▶ To be eligible, students must score in the top 9 percent of scores in the national high school exit exam (“SABER 11” score), and must come from a household with wealth index below a region-specific threshold (“SISBEN” wealth score).
- ▶ Focus on students who took the SABER 11 test in the fall of 2014.
- ▶ Transform this two-dimensional RD design into one-dimensional design: only students whose SABER 11 score is above the cutoff.
- ▶ Score: difference between student's SISBEN wealth index and respective cutoff.
- ▶ Cutoff: normalized to zero.
- ▶ Treatment assignment (T): an indicator equal to one if score below zero. Treatment received (D): indicator equal to one if

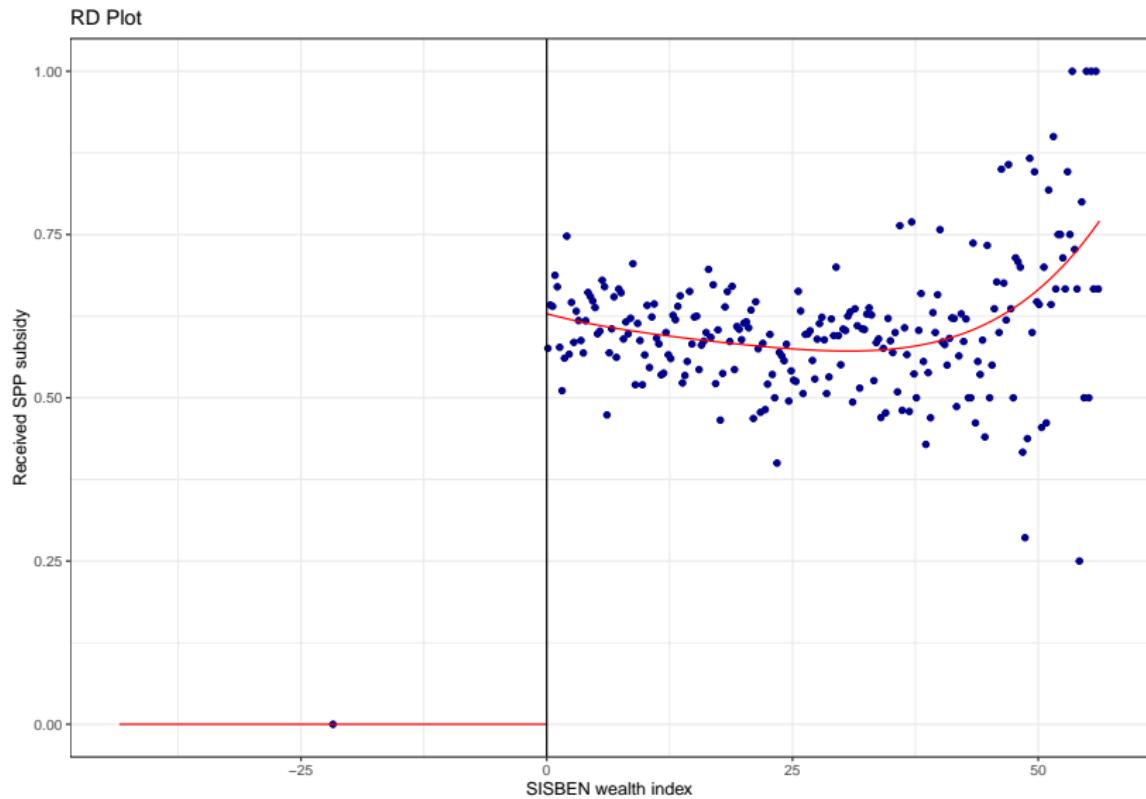
# Empirical Example

```
library(haven)
data = read_dta("spp.dta")
data=data[data$running_saber11>=0,]
data = data[!is.na(data$running_sisben),]
X = data$running_sisben
D = data$beneficiary_spp
Y1 = data$spadies_any
Y2 = data$spadies_hq
out = rdrobust(D,X) # First Stage: T on D
summary(out)
```

```
## Sharp RD estimates using local polynomial regression.
##
## Number of Obs.          23132
## BW type                 mserd
## Kernel                  Triangular
## VCE method               NN
##
## Number of Obs.          7709      15423
## Eff. Number of Obs.     6600      7466
## Order est. (p)          1         1
## Order bias (q)          2         2
## BW est. (h)              18.511    18.511
## BW bias (b)              28.994    28.994
## rho (h/b)                0.638     0.638
## Unique Obs.             3644      9327
##
## =====
##           Method   Coef. Std. Err.      z   P>|z|    [ 95% C.I. ]
## =====
##   Conventional   0.625    0.012   51.592   0.000  [0.601 , 0.649]
##   Robust        -        -     43.115   0.000  [0.595 , 0.652]
## =====
```

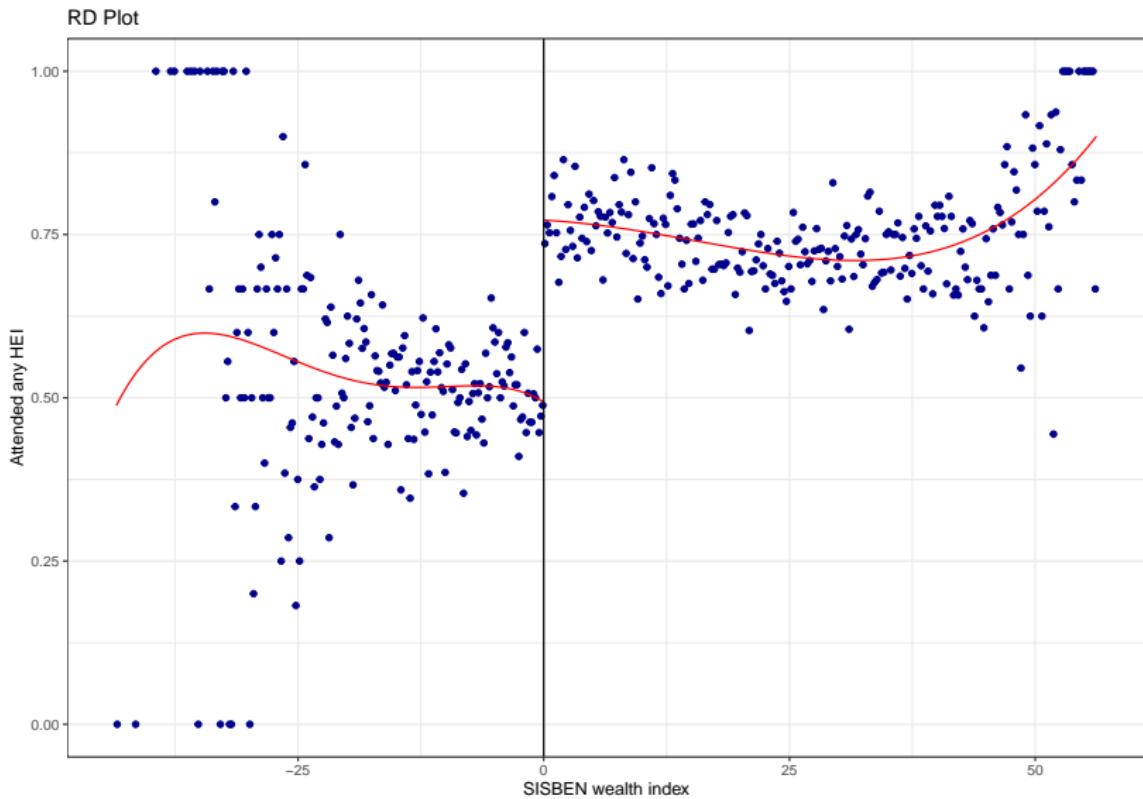
# Sharp RD Plot

```
## [1] "Mass points detected in the running variable."  
## [1] "Warning: not enough variability in the outcome variable below the threshold"
```



# ITT: T on Y

```
## [1] "Mass points detected in the running variable."
```



# ITT: T on Y

```
## Sharp RD estimates using local polynomial regression.
##
## Number of Obs.          23132
## BW type                 mserd
## Kernel                  Triangular
## VCE method               NN
##
## Number of Obs.          7709      15423
## Eff. Number of Obs.     3877      3908
## Order est. (p)          1          1
## Order bias (q)          2          2
## BW est. (h)              9.041     9.041
## BW bias (b)             14.404    14.404
## rho (h/b)                0.628     0.628
## Unique Obs.             3644      9327
##
## =====
##           Method   Coef. Std. Err.      z   P>|z|   [ 95% C.I. ]
## =====
##   Conventional   0.269    0.023   11.709   0.000   [0.224 , 0.314]
##   Robust        -         -    10.047   0.000   [0.221 , 0.328]
## =====
```

## Student Attended any high education institution (HEI) immediately after receiving subsidy

```
# Fuzzy effect
fout = rdrobust(Y1, X, fuzzy = D)
h = fout$bws[1,1]
b = fout$bws[2,1]

out = rdrobust(Y1, X, h = h, b=b)
itt = out$Estimate[1]

out = rdrobust(D, X, h=h, b=b)
fs = out$Estimate[1]

itt/fs

## [1] 0.4344912
```

# Student Attended any high education institution (HEI) immediately after receiving subsidy

```
## Fuzzy RD estimates using local polynomial regression.  
##  
## Number of Obs.          23132  
## BW type                 mserd  
## Kernel                  Triangular  
## VCE method               NN  
##  
## Number of Obs.          7709      15423  
## Eff. Number of Obs.     3877      3908  
## Order est. (p)          1          1  
## Order bias (q)          2          2  
## BW est. (h)              9.041     9.041  
## BW bias (b)             14.404    14.404  
## rho (h/b)               0.628     0.628  
## Unique Obs.             3644      9327  
##  
## First-stage estimates.  
##  
## ======  
##      Method   Coef. Std. Err.      z   P>|z|   [ 95% C.I. ]  
## ======  
##  Conventional   0.619   0.017   35.857   0.000   [0.585 , 0.653]  
##  Robust        -       -      29.885   0.000   [0.575 , 0.656]  
## ======  
##  
## Treatment effect estimates.  
##  
## ======  
##      Method   Coef. Std. Err.      z   P>|z|   [ 95% C.I. ]  
## ======  
##  Conventional   0.434   0.034   12.773   0.000   [0.368 , 0.501]  
##  Robust        -       -      11.026   0.000   [0.366 , 0.524]
```

## Geographic Regression Discontinuity

**GRD** is a design in which a geographic or administrative boundary splits units into treated and control areas and analysts make the case that the division into treated and control areas occurs in an as-if random fashion.

# Empirical Example #1

*Econometrica*, Vol. 78, No. 6 (November, 2010), 1863–1903

## THE PERSISTENT EFFECTS OF PERU'S MINING MITA

BY MELISSA DELL<sup>1</sup>

This study utilizes regression discontinuity to examine the long-run impacts of the *mita*, an extensive forced mining labor system in effect in Peru and Bolivia between 1573 and 1812. Results indicate that a *mita* effect lowers household consumption by around 25% and increases the prevalence of stunted growth in children by around 6 percentage points in subjected districts today. Using data from the Spanish Empire and Peruvian Republic to trace channels of institutional persistence, I show that the *mita*'s influence has persisted through its impacts on land tenure and public goods provision. *Mita* districts historically had fewer large landowners and lower educational attainment. Today, they are less integrated into road networks and their residents are substantially more likely to be subsistence farmers.

KEYWORDS: Forced labor, land tenure, public goods.

# Empirical Example #1

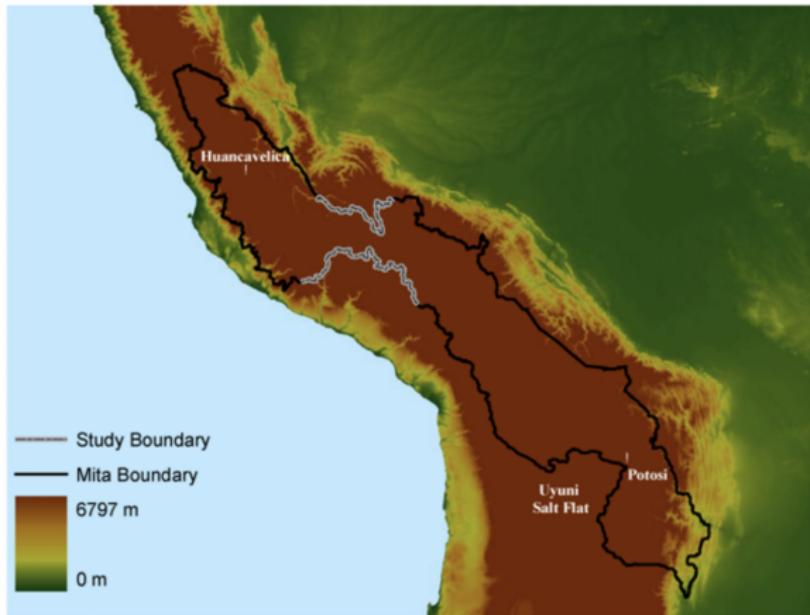


FIGURE 1.—The *mita* boundary is in black and the study boundary in light gray. Districts falling inside the contiguous area formed by the *mita* boundary contributed to the *mita*. Elevation is shown in the background.

## Empirical Example #2

# The Political Legacy of Violence: The Long-Term Impact of Stalin's Repression in Ukraine

---

**Arturas Rozenas**, New York University

**Sebastian Schutte**, University of Konstanz

**Yuri Zhukov**, University of Michigan

Political scientists have long been interested in how indiscriminate violence affects the behavior of its victims, yet most research has focused on short-term military consequences rather than long-term political effects. We argue that large-scale violence can have an intergenerational impact on political preferences. Communities more exposed to indiscriminate violence in the past will—in the future—oppose political forces they associate with the perpetrators of that violence. We document evidence for this claim with archival data on Soviet state violence in western Ukraine, where Stalin's security services suppressed a nationalist insurgency by deporting over 250,000 people to Siberia. Using two causal identification strategies, we show that communities subjected to a greater intensity of deportation in the 1940s are now significantly less likely to vote for “pro-Russian” parties. These findings show that indiscriminate violence systematically reduces long-term political support for the perpetrator.

# Empirical Example #2

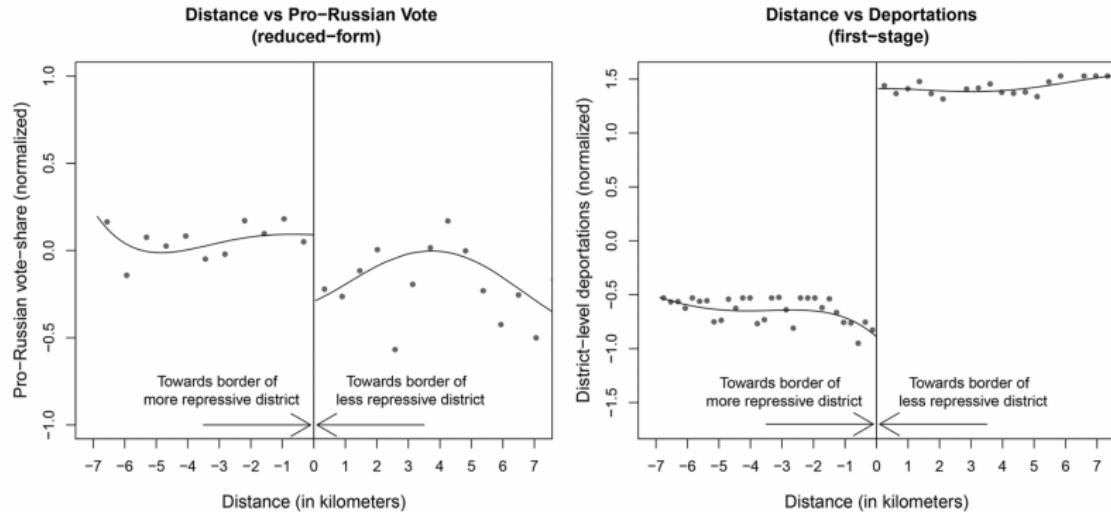
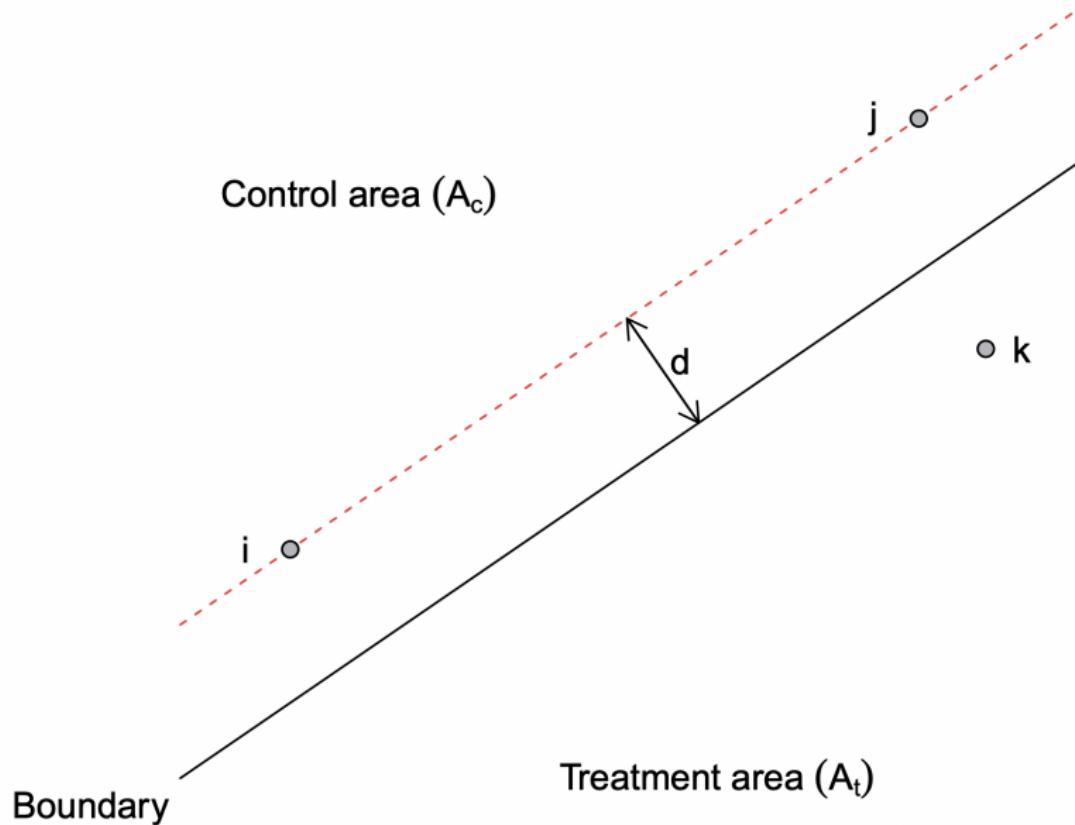


Figure 3. Reduced-form relationships between the instrument (distance from the contiguous district with more repression), deportations, and pro-Russian vote.

## Peculiarities of GRD (Keele and Titiunik, 2015)

1. *Compound* treatments — multiple treatments that affect the outcome of interest simultaneously;
2. Different measures of distance from the cutoffs may require different identification assumptions and affect fundamentally the interpretation of the results;
3. Spatial variation in treatment effects can be mapped to specific locations, which can be used to detect geographic areas where the identification assumptions are more (or less) likely to hold.

## Failure of one-dimensional distance to single out individual boundary points



## Recommendations for practice

- ▶ **Data:**
  - ▶ researchers need to collect geographic data (addresses, latitude and longitude, or other geographic information that can be used for geocoding) along with more traditional covariates;
  - ▶ qualitative research on the history of the border and conditions around it will often prove useful to justify various assumptions.
- ▶ **Falsification tests:**
  - ▶ researchers should at least rule out nonzero treatment effects on predetermined covariates, which can be easily implemented using covariates as outcomes in the estimation for each boundary point.
- ▶ **Isolating the treatment:**
  - ▶ restrict the analysis to areas around the border where other important geographically defined institutional units are kept constant on either side of border.

## Do Fiscal Rules Matter?<sup>†</sup>

By VERONICA GREMBI, TOMMASO NANNICINI, AND UGO TROIANO\*

*Fiscal rules are laws aimed at reducing the incentive to accumulate debt, and many countries adopt them to discipline local governments. Yet, their effectiveness is disputed because of commitment and enforcement problems. We study their impact applying a quasi-experimental design in Italy. In 1999, the central government imposed fiscal rules on municipal governments, and in 2001 relaxed them below 5,000 inhabitants. We exploit the before/after and discontinuous policy variation, and show that relaxing fiscal rules increases deficits and lowers taxes. The effect is larger if the mayor can be reelected, the number of parties is higher, and voters are older. (JEL E62, H71, H72, H74, R51)*

## Difference-in-discontinuities

- ▶ Grembi et al. (2016) investigate the effect of the fiscal policy on municipality outcomes;
- ▶ In 2001, fiscal rules were relaxed for municipalities with below 5,000 inhabitants in Italy;
- ▶ Though below/above 5,000 seems a standard regression discontinuity cutoff, there is another policy that the mayor's salary sharply changes at the 5,000 inhabitants threshold;
- ▶ In this context, before/after 2001 forms comparable groups to cancel out the confounding effect of the mayor's salary treatment.

## Estimation in Diff-in-disc design

The diff-in-disc estimator can be implemented by estimating the boundary points of four regression functions of  $Y_{it}$  on  $P_{it}$ : two on both sides of  $P_c$ , both before and after  $t_0$ . We apply a local linear regression, following Gelman and Imbens (2014).<sup>23</sup> The method consists in fitting linear regression functions to the observations distributed within a distance  $h$  on either side of  $P_c$ , both before and after  $t_0$ . Formally, we restrict the sample to cities in the interval  $P_{it} \in [P_c - h, P_c + h]$  and estimate the model

$$(1) \quad Y_{it} = \delta_0 + \delta_1 P_{it}^* + S_i(\gamma_0 + \gamma_1 P_{it}^*) + T_t[\alpha_0 \\ + \alpha_1 P_{it}^* + S_i(\beta_0 + \beta_1 P_{it}^*)] + \xi_{it},$$

where  $S_i$  is a dummy for cities below 5,000 capturing treatment status,  $T_t$  an indicator for the posttreatment period, and  $P_{it}^* = P_{it} - P_c$  the normalized population size. Standard errors are clustered at the city level. The coefficient  $\beta_0$  is the diff-in-disc estimator and identifies the treatment effect of relaxing fiscal rules, as the treatment is  $R_{it} = S_i \cdot T_t$ . We present the robustness of our results to multiple bandwidths  $h$ , optimally computed first following the algorithm developed by Calonico, Cattaneo, and Titiunik (2014a, b), and then implementing the cross-validation method proposed by Ludwig and Miller (2007).<sup>24</sup>

## Bonus: PCRD

- ▶ Marshall (2022) studies RD designs where  $D_i$  = some characteristic of winning politician
  - ▶ E.g. the effect of having a female politician in office
  - ▶ Sample restricted to close races between a woman and a man
- ▶ This is an unusual setup:
  - ▶ Standard RDD: effect of winning (e.g. on candidate longevity)  
→ observe outcomes for both winners and losers
  - ▶ Here: effect of being female *conditional on winning* → only observe outcomes for winners (or the district)

## Bonus: PCRD

How do we interpret the estimand?

- ▶ Standard issue: being female is an attribute not a cause — a bundle of characteristics
- ▶ Additional issue in RDDs: consider a characteristic  $W_i$  uncorrelated with  $D_i$  among all candidates
  - ▶ Suppose both  $D_i$  and  $W_i$  affect vote shares
  - ▶ Then among close races,  $D_i$  and  $W_i$  will be correlated — “compensating differential”

Is this a bias or a different interpretation/mechanism?

- ▶ Marshall (2022) argues for bias: it's not the effect of changing  $D_i$  while holding other characteristics fixed