

PROBLEM 1.1

Proof. By the definition of total variation distance, we have

$$\sum_{s_t} |p_{\pi_\theta}(s_t) - p_{\pi^*}(s_t)| = 2 \times d_{TV}(p_{\pi_\theta}, p_{\pi^*})$$

Let M_i denotes the event the learned policy π_θ makes a mistake at step i and makes no mistake in $i-1$ steps. Let E_t denotes the event the learned policy π_θ makes at least one mistake in t steps. It follows that

$$Pr(E_t) = Pr(\bigcup_{i=0 \dots t} (M_i)) \leq \bigcup_{i=0 \dots t} Pr(M_i) \leq \bigcup_{i=0 \dots T} Pr(M_i) \leq T\varepsilon$$

By the coupling lemma, the distance of state distributions at time t is bounded by the probability of the two trajectories have diverged by that time:

$$d_{TV}(p_{\pi_\theta}, p_{\pi^*}) \leq Pr(E_t) \leq T\varepsilon$$

Hence

$$\sum_{s_t} |p_{\pi_\theta}(s_t) - p_{\pi^*}(s_t)| \leq 2T\varepsilon$$

as we desired. \square

PROBLEM 1.2.A

Proof. Let S denotes the entire state set .

$$\begin{aligned} |J(\pi^*) - J(\pi_\theta)| &= |E_{p_{\pi^*}(s_T)} r(s_T) - E_{p_{\pi_\theta}(s_T)} r(s_T)| \\ &= |\sum_{s_T \in S} (p_{\pi^*}(s_T) - p_{\pi_\theta}(s_T)) \times r(s_T)| \\ &\leq \max(r(s_T)) \times |p_{\pi^*}(s_t) - p_{\pi_\theta}(s_t)| \end{aligned}$$

Recalled that $p_{\pi^*}(s_t) - p_{\pi_\theta}(s_t) \leq 2T\varepsilon$. It follows that $|J(\pi^*) - J(\pi_\theta)| \leq R_{max} \times 2T\varepsilon$.

Hence

$$J(\pi^*) - J(\pi_\theta) = \mathcal{O}(T\varepsilon)$$

as we desired. \square

PROBLEM 1.2.B

Proof.

$$\begin{aligned} |J(\pi^*) - J(\pi_\theta)| &= |\sum_{t=1}^T \sum (r(s_t) \times (p_{\pi^*}(s_t) - p_{\pi_\theta}(s_t)))| \\ &\leq \sum_{t=1}^T R_{max} |p_{\pi^*}(s_t) - p_{\pi_\theta}(s_t)| \\ &\leq T \times R_{max} \times 2T\varepsilon \end{aligned}$$

Hence

$$J(\pi^*) - J(\pi_\theta) = \mathcal{O}(T^2\varepsilon)$$

as we desired. \square

PROBLEM 2

No Problem.

PROBLEM 3.1

Both **Ant-v4** and **Hopper-v4** are train with n-layers=2&net-size=64&eval-batch-size=10000 and others kept default. Their performance ratios compared to expert are as follows.

environment	avg.ret	std.ret	avg.ret.exp	perf ratio
Ant-v4	4430	780.6	4682	94.62%
Hopper-v4	1098	7.242	3718	29.53%

Table 1: **Ant-v4** vs. **Hopper-v4**

PROBLEM 3.2

Varing **training batch size** from 100 to 1000 with step size 100, we get the performance vs. batch_size graph[1] of environment **Hopper-v4**.

train_batch_size	avg. ret	std. ret	avg. ret. exp	perf ratio
100	1099	12.6	3718	29.56%
200	1199	26.82	3718	32.25%
300	869.3	34.59	3718	23.38%
400	1221	21.77	3718	32.84%
500	1349	149.1	3718	36.28%
600	1727	381.1	3718	46.45%
700	1308	54.33	3718	35.18%
800	1505	302.5	3718	40.48%
900	1501	439.6	3718	40.37%
1000	1333	87.06	3718	35.85%

Table 2: Behavioral Cloning Performance Varying Training Batch Size On Hopper-v4

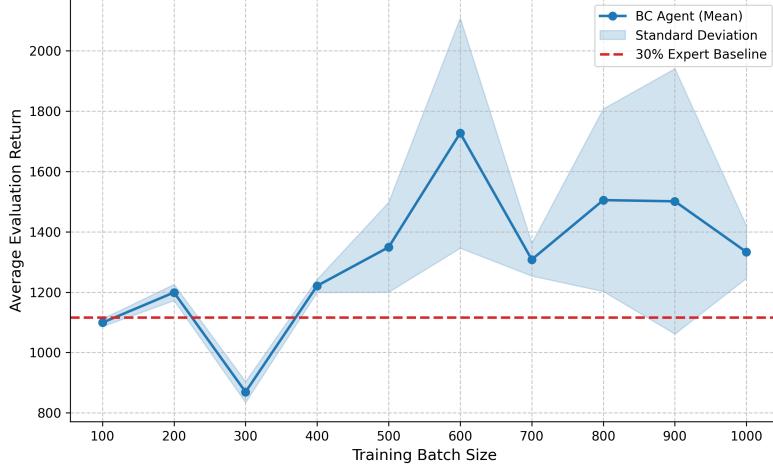
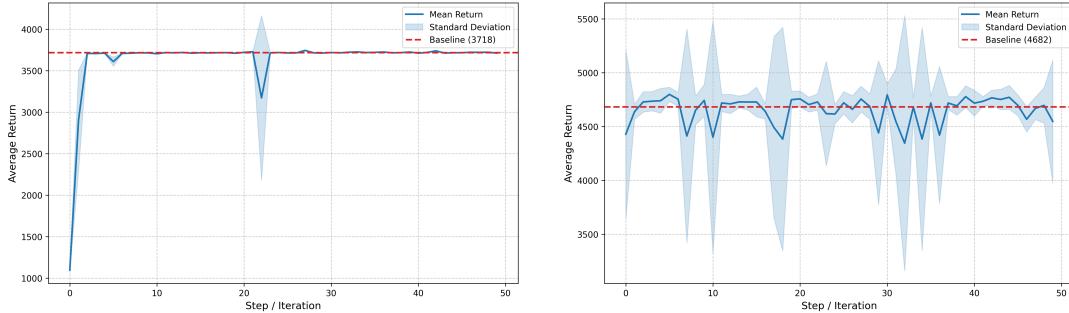


Figure 1: Mean Return vs. Training Batch Size On Hopper-v4
 (size=64&n_layers=2&eval_batch_size=10000)

PROBLEM 4.1



(a) Mean Return vs. Training Step On Hopper-v4
 (size=64&n_layers=2&eval_batch_size=10000)

(b) Mean Return vs. Training Step On Ant-v4
 (size=64&n_layers=2&eval_batch_size=10000)

PROBLEM 5.1

Proof. Let $C_{t,n}$ denote $C(\tilde{\pi}^n)$ with horizon T . $C_{0,n} = 0 \leq A(0, n) = 0$; $C_{T,0} = C(\pi^*) \leq A(T, 0) = 0$.

Assume that for all $t + n \leq k$ we have $C_{t,n} \leq A(t, n)$. For $t_1 + n_1 = k + 1$, consider $\tilde{\pi}^n = S_{X_n}(\hat{\pi}^n, \tilde{\pi}^{n-1})$ where $X_n \sim Geom(1 - \alpha)$:

1. $X_n = 0(Pr = 1 - \alpha)$: The policy immediately switch to $\tilde{\pi}^{n-1}$ with $C_{t,n-1}$. By our assumption, the cost is

$$\mathbb{E}[C_{t,n}|X_n = 0] = (1 - \alpha) \times A(t, n - 1)$$

2. $X_n \geq 1(Pr = \alpha)$: The policy gives action to the first step with ε probability of failing.

If the policy fails the first step, the whole trajectory's error is up to

$$\mathbb{E}[C_{t,n}|X_n \geq 1, \text{fails on step 1}] \leq \alpha\varepsilon T$$

If the policy succeeds on the first step, it matches the expert's action at step 1. For the remaining $t-1$ steps, the policy becomes $S^{X_n-1}(\hat{\pi}^n, \tilde{\pi}^{n-1})$ because the memoryless property of Geometric Distribution: the Distribution of $X_n - 1$ is identical to X_n given $X_n \geq 1$. So the cost is

$$\mathbb{E}[C_{t,n}|X_n \geq 1, \text{succeeds on step 1}] \leq \alpha \times (1 - \varepsilon)A(t - 1, n)$$

Adding up all the costs we got:

$$C_{t,n} \leq \alpha\varepsilon T + \alpha(1 - \varepsilon)A(t - 1, n) + (1 - \alpha)A(t, n - 1) = A(t, n)$$

Let T equals to t , we concluded:

$$C(\tilde{\pi}^n) \leq A(T, n)$$

□

PROBLEM 5.2

Proof. We proof by induction.

1. Basic conditions:

1. $t = 0$: $A(0, n) = 0 \leq 0 \times n\alpha\varepsilon$
2. $n = 0$: $A(t, 0) = 0 \leq 0 \times t\alpha\varepsilon$

2. Assumption:

For any $t + n \leq k$, $A(t, n) \leq Tn\alpha\varepsilon$.

3. Induction:

For any $t + n \leq k + 1$,

$$\begin{aligned} C_{t,n} &\leq A(t, n) \\ &= \alpha\varepsilon t + \alpha(1 - \varepsilon)A(t - 1, n) + (1 - \alpha)A(t, n - 1) \end{aligned}$$

Substituting assumption:

$$\begin{aligned} A(t, n) &\leq \alpha\varepsilon t + \alpha(1 - \varepsilon)(t - 1)n\alpha\varepsilon + (1 - \alpha)t(n - 1)\alpha\varepsilon \\ &= \alpha\varepsilon \times (t + n\alpha(t - 1 - \varepsilon t + \varepsilon) + t(n - 1 - \alpha n + \alpha)) \\ &= \alpha\varepsilon \times (t + tn\alpha - n\alpha - tn\alpha\varepsilon + n\alpha\varepsilon + tn - t - tn\alpha + t\alpha) \\ &= \alpha\varepsilon \times (n\alpha\varepsilon - n\alpha - tn\alpha\varepsilon + tn + t\alpha) \end{aligned}$$

Ignore the scale term $\alpha\varepsilon$, we get

$$\begin{aligned} & n\alpha\varepsilon - n\alpha - tn\alpha\varepsilon + tn + t\alpha \\ & = tn + \alpha \times ((t-n) + (1-t)n\varepsilon) \end{aligned}$$

In switchDagger, we let $n \geq t \geq 1$. And then $t-n \leq 0, 1-t \leq 0$. Hence, we conclude:

$$C(\tilde{\pi}^n) = C_{T,n} \leq A(T, n) \leq Tn\alpha\varepsilon$$

as we desired.

□

PROBLEM 5.3

Proof. First, for any policy π , $C(\pi) \leq \sum_{t=1}^T \max \mathbb{E}_{s_t \sim p_\pi} \Pr[\pi(s_t) \neq \pi^*(s_t)] \leq T$. π^n is policy that transfer control from $\tilde{\pi}^n$ to expert policy π^* at step X^* . If $X^* \geq T$, $\pi^n = \tilde{\pi}^n$. Hence,

$$\begin{aligned} C(\pi^n) - C(\tilde{\pi}^n) & \leq \Pr[X^* \leq T] \times T \\ & = e^{\frac{-n}{(1-\alpha)T}} \times T \end{aligned}$$

It follows that

$$C(\pi^n) \leq C(\tilde{\pi}^n) + e^{\frac{-n}{(1-\alpha)T}} T$$

as we desired.

□

PROBLEM 5.4

In summary we get the upper bound of policy:

$$C(\pi^N) \leq TN\alpha\varepsilon + e^{\frac{-n}{(1-\alpha)T}} T$$

Let $\alpha = 1/T$ and $N = T \log(1/\varepsilon)$. Substituting into the formula:

$$C(\pi^N) \leq T\varepsilon \log(1/\varepsilon) + T\varepsilon^{T/(T-1)} = \mathcal{O}(T\varepsilon(\log(1/\varepsilon) + 1))$$

Therefore:

$$C(\pi^N) = \mathcal{O}(T\varepsilon \log(1/\varepsilon))$$

as we desired.