Ecological Modeling of Blue Whale Migration Using the Stochastic Spatial Approach

Yunhe Xu
MATH113B Mathematical Modeling in Biology II
University of California, Irvine

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Abstract
In this study, a stochastic spatial model was used to analyze the migration patterns of blue whales along the West Coast of North America. The model aimed to evaluate how abiotic (environmental) and biotic factors influence blue whale migration and foraging behaviors. Previous papers indicate a close relationship with Sea Surface Temperature and Krill density, while recognizing the individual decision differences during route selection. This paper tried to explain some technical details and further discovery in simulating individual whale migration paths, as well as evaluating the variability with seasonal fluctuations. Overall, the prediction in blue whale migration behavior is crucial as an indicator of possible environmental changes, and it is also significant in maritime and whale-watching industries to better promote human-whale coordination.

Contents

1 Introduction

On February 16, 2025, I went on a whale-watching trip in San Diego, and I was lucky enough to observe the first Northbound traveling whale of this year, which was way earlier than usual. This experience sparked my interest in learning more about the whale migration patterns of West Coast America. Blue whales are observed to migrate northwards from April to June and southwards from October to December. Along the West Coast of America, blue whales are believed to spend winters off of Mexico and Central America. They likely feed during summer off the U.S. West Coast and, to a lesser extent, in the Gulf of Alaska and central North Pacific waters.[6]

The main drive of the blue whale migration is the spatiotemporal variation in resources[2]. Internally, each individual blue whale processes[5] local information available to it, including environmental cues like temperature and tracking of resource availability (krill density). Blue whales feed almost exclusively on krills (Euphausiids)[6], which are present in copious amounts near the Arctic Ocean. In the California Current System, krills are often distributed near the shores, with higher density seen in higher latitudes[4].

2 Methodology

The model is a semi-stochastic process aimed to incorporate two of blue whale's distinct migration states[3], transiting and foraging. It calculated separately how abiotic (sea surface temperature) and biotic (krill density) determined the probability of the whale to transit to a different state or remain in the same state as before. The importance of Sea Surface Temperature and Krill Density is weighed against seasonal and spatial changes. The model outputs hypothetical routes that individual whale would take during their migration.

2.1 Transit-Foraging Model

The Transit-Forage Model was derived from the discovery of an 2009 paper[3]. Blue whales are found to have 2 main states during their migration, namely, transit and forage. The two states are based on the movement differences. In transit state, the whales are heading towards their destinations, so they would have smaller turning angles and higher speeds. In their foraging states, blue whales would chase for the hotspots of krills, so they would demonstrate larger turning angles and lower speeds. The transition matrix of the stochastic process is modeled as follows

$$\begin{aligned}
\tau(X,\rho,\theta) &= \begin{pmatrix} \mathcal{S}_1 \to \mathcal{S}_1 & \mathcal{S}_1 \to \mathcal{S}_2 \\ \mathcal{S}_2 \to \mathcal{S}_1 & \mathcal{S}_2 \to \mathcal{S}_2 \end{pmatrix} \\
&= \begin{pmatrix} P(s_t = 1 | s_{t-1} = 1; X,\theta,\rho) & P(s_t = 2 | s_{t-1} = 1; X,\theta,\rho) \\ P(s_t = 1 | s_{t-1} = 2; X,\theta,\rho) & P(s_t = 2 | s_{t-1} = 2; X,\theta,\rho) \end{pmatrix} \\
&= \begin{pmatrix} 1 - P_t & P_t \\ 1 - P_t & P_t \end{pmatrix}. \qquad (1)
\end{aligned}$$

where $\mathcal{S}_1$ stands for transiting state and $\mathcal{S}_2$ for foraging state.

As discussed, the transition probability is mainly affected by two variables, sea surface temperature (SST) and krill density. Weighing factors are assigned to the two variables to derive the final probability of foraging, P_t, shown in the following equation

$$P_t = \frac{1}{\mathcal{Z}} \left[w_1 P_E(s_t = 2 | s_{t-1} = \{1,2\}; X, \theta) + w_2 P_K(s_t = 2 | s_{t-1} = \{1,2\}; \theta, \rho) \right]$$

where

$$z = w_1 + w_2$$

as a normalizing factor for the two weights assigned to the two states.

The SST transition probability is the simple linear response function

$$P_E(s_t = 2 | s_{t-1} = \{1,2\}; X, \theta) = \text{logit}^{-1}[\alpha_1 + \alpha_2(X - X_*)]$$

that selects for colder temperatures. Krill transition probabilities are set by the logistic selection function

$$P_K(s_t = 2 | s_{t-1} = \{1,2\}; \theta, \rho) = \text{logit}^{-1}[\beta(\rho - \rho_*)],$$

and this probability would increase as krill density increases.

2.2 North-South Model

Since the Northbound and Southbound migrations of blue whale occur in different times of the year, it is important to take into account seasonal effects on migration. The North-South Model adds seasonal effect on the basis of Transit-Foraging Model. Additionally, blue whale's desire to forage is also different. During Southbound traveling, they come from the northern region with an abundance of food, while during Northbound traveling, they just finished reproduction and were in fasting states. Therefore, foraging conditions are set to be stricter for Southbound migration. This gives us a 4-state transition probability matrix. states $\mathcal{S}_1$ and $\mathcal{S}_2$ describing transit and forage during northward migration and states $\mathcal{S}_3$ and $\mathcal{S}_4$ for southward migration.

The individual foraging rate averaged over the previous 10 days are used as a measurement for the likelihood of foraging from $\mathcal{S}_1$ and $\mathcal{S}_2$ to $\mathcal{S}_3$. Low foraging rates increase the probability of transitioning to southward migration, and the time-average provided sufficient opportunity for agents to travel between and seek out new krill patches. Figure 1 provides a clearer illustration of the structures of the two models.

2.3 Stochastic Spatial Model Analysis

The stochastic process of whale movement randomizes two variables, step lengths and turning angles. Step lengths are positive real numbers in units of meters and follow gamma distributions with probability density function

$$f_{SL}(x) = \left(\frac{x^{a-1}}{s^a \Gamma(a)} \right) \exp(-x/s), \quad x > 0$$

where $\Gamma(a)$ is the gamma function. Turning angles are sampled from a von Mises distribution with probability density function given by

$$f_{TA}(x) = \frac{1}{2\pi I_0(\kappa)} \exp(\kappa \cos(x - \mu)), \quad 0 < x < 2\pi,$$

where $I_0(\kappa)$ is the Bessel function of order 0.

The parameters of the model is derived mainly from Bailey et al. (2009)[3].

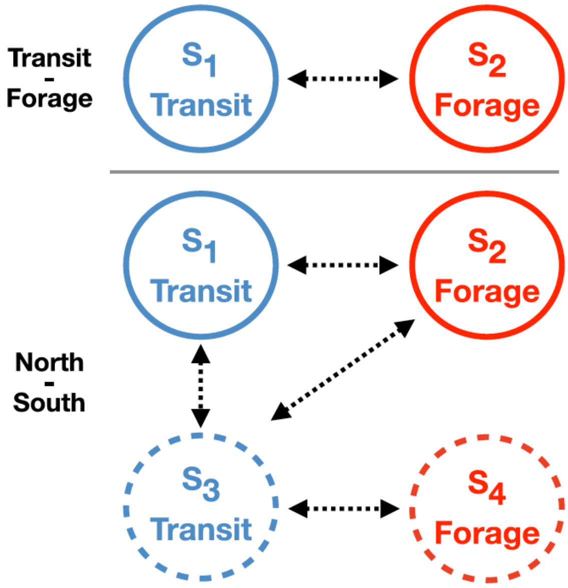

Figure 1: Stochastic processes of Transit-Forage and North-South models

	Parameter	Value
State Transition	α_1	-1
Probabilities	α_2	-0.2 (°C)$^{-1}$
	X_*	16 °C
	β	5
States $\mathcal{S}_1$ and $\mathcal{S}_2$	ρ_*	0.3
States $\mathcal{S}_3$ and $\mathcal{S}_4$	ρ_*	0.6
Transit Movement	a	2.96†
Distributions	s	7,495.9† m
	μ	0† radians
	κ	10†
Forage Movement	a	1.17†
Distributions	s	5,376.6† m
	μ	π† radians
	κ	3†

3 Implementation

My focus of this study is to simulate the movement patterns of individual blue whales using the spatial stochastic model on a world map to create a better visualization of the migration paths.

3.1 Environment Initialization

To better simulate the two main factors affecting blue whale migration, SST and krill density, I have searched NOAA sea surface temperature data and krill density monitoring data by Farallon Institute.

3.1.1 Sea Surface Temperature

The SST data is synthesized using the formula

$$T = 25 - 0.2(l - 20)$$

where the sea surface temperature T decreases as latitude l increases. A gaussian filter is added to simulate the California current system effect on SST, where SST is observed to be lower near the shore[1]. Here is the code used to general the SST data, taking into account the base formula, coastal effect and seasonal effect.

```
def generate_sst_data():
    sst_base = 25 - 0.2 * (LAT - 20)
    random_variation = np.random.normal(0, 0.8, size=sst_base.shape)
    smooth_variation = gaussian_filter(random_variation, sigma=5.0)
    coast_distance = np.abs(LON + 124)
    coastal_effect = -2 * np.exp(-0.5 * coast_distance)
    seasonal_effect = 2 * np.sin(np.pi * LAT / 60)
    sst = sst_base + smooth_variation + coastal_effect + seasonal_effect
    return sst
```

Figure 2 shows the resultant SST data distribution on North America map.

3.1.2 Krill Density

The krill density is observed to be distributed in patches, or hotspots, along the coastline. The hotspot density is higher as latitude increases, and they are present in minimal density in the breeding grounds of the blue whales. The krill density data follows an exponential decay of both the density of hotspot numbers and each hotspot density. The krill density distribution is generated from the following steps:

- Identify the coastline from `cartopy landmask` (illustrated in Section 3.2)
- Generate hotspots along coastline, following exponential decay where

$$f(x) = \lambda e^{-\lambda x}, \quad x \geq 0$$

- Set lower bound of latitude to be 20. Minimal krill density when latitude < 20

The simulated data is shown in Figure 3.

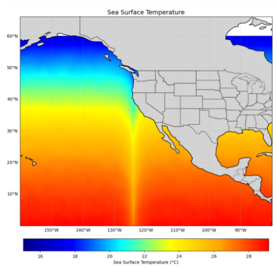

Figure 2: SST Simulation

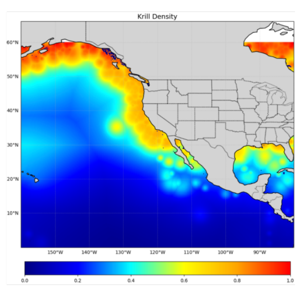

Figure 3: Krill Density Simulation

3.2 Whale Movement Simulation

3.2.1 Dealing with Irregular Boundary

Using the Python package `cartopy`, I was able to access the coordinate information from the map and create simulated plots of SST and krill density. However, a problem that arises from implementing data onto real world map is that the coastline is curled and random, making it hard to simulate the boundary conditions. A solution is to create a `landmask` using `cfeature` of `cartopy` package.

```
from shapely.prepared import prep
def create_land_mask():
    land_mask = np.zeros((LAT_SIZE, LON_SIZE), dtype=bool)
    land_feature = cfeature.NaturalEarthFeature('physical', 'land', '10m')
    points = np.vstack([LON.flatten(), LAT.flatten()]).T
    result = np.zeros(points.shape[0], dtype=bool)
    for geom in land_feature.geometries():
        prepared_geom = prep(geom)
        batch_size = 10000
        for i in range(0, len(points), batch_size):
            batch_points = points[i:i+batch_size]
            for j, point in enumerate(batch_points):
                if prepared_geom.contains(Point(point[0], point[1])):
                    result[i+j] = True
    land_mask = result.reshape(LAT_SIZE, LON_SIZE)
    return land_mask

land_mask = create_land_mask()
```

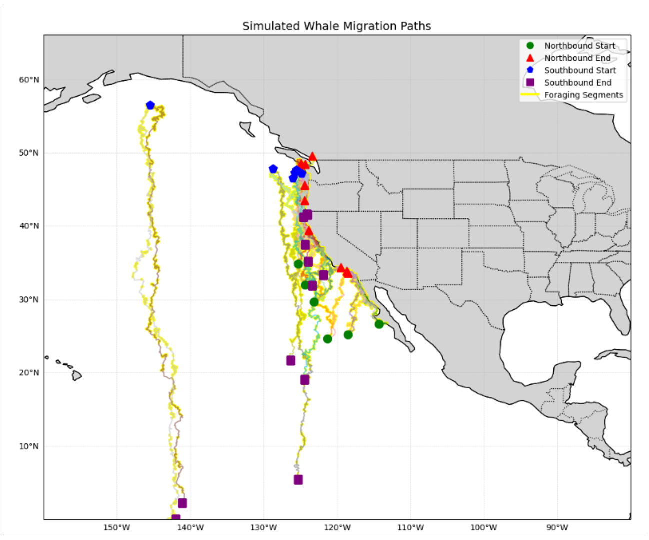

Figure 4: Stochastic Simulation of 23 whales

3.2.2 Whale Initialization

Overall, 23 whales are simulated, with 11 Northbound whales and 12 Southbound whales. Since the coastline in their feeding and breeding ground are hard to navigate with the map (in fact, a lot of whales are able to navigate within the Gulf of California but the gulf structure is programming unfriendly), we shall assume that the whales have memories of their routes and randomly select their starting position based on observation. Northbound whale starting points are randomly selected along Baja California to Southern California State. South bound whale starting points are randomly selected near Washington State and to some extent, Gulf of Alaska.

The simulation results are shown in Figure 4.

3.3 Model Evaluation

3.3.1 Observations

It is observed that across individuals, over the same period of time, they would end up in different latitudes. Intuitively, they would travel with different speeds, where foraging creates more variation in their paths.

3.3.2 Evaluative Metrics

The evaluative matrix provides a sum of the krill density encountered while in a foraging state.

$$\Omega_w = \sum_{t=1}^{N} \rho(Y_{w,t}) 1_{s_{w,t}=2}(S_{w,t}).$$

From the plot, we learn that Southbound whales indeed spend less time foraging, corresponding to the biological contexts.

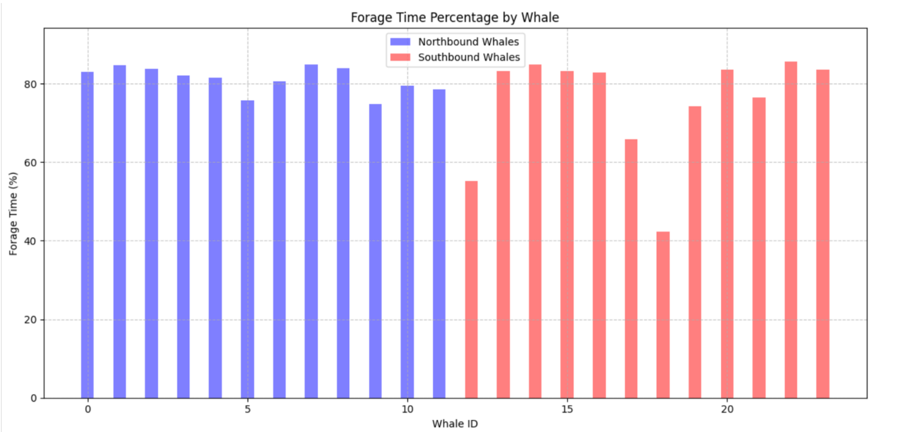

Figure 5: Forage Time Percentage by Whale

3.3.3 Whale Foraging Hotspots

Figure 6 shows the whale foraging hotspots plotted in histogram. It is observed that the histogram distribution highly correlates with the synthesized data of krill density. This shows that the correlation of krill to migration paths is high.

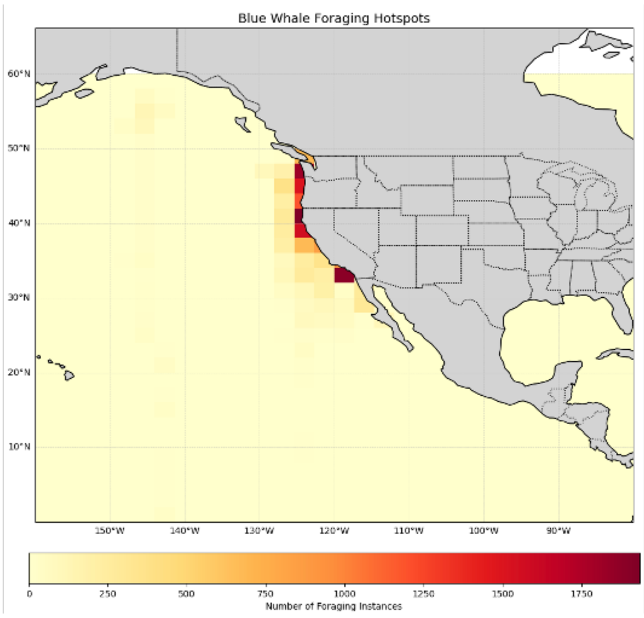

Figure 6: Foraging Hotspots Histogram

4 Conclusion

In this study, I applied a stochastic spatial model to analyze blue whale migration patterns. It was challenging to recreate the formulas and test them on a world map, but I found it meaningful to better visualize the migration stochastic paths and gain more contextual clues. A future area of improvement is to simulate the whole cycle of North and South bound whales so as to visualize the transition between $\mathcal{S}_1$, $\mathcal{S}_2$ to $\mathcal{S}_3$, $\mathcal{S}_4$.

References

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