Problem Set 1

February 13, 2023

Submit you solutions to pengyi0606@icloud.com by 22:59, Feb. 28th.

For this week's problem set, you may not skip any step in your proofs.

Problem 1. *Prove that, for any sets A, B, C, $A \subseteq B$, the following three statements are equivalent:*

1. $A \subseteq B$
2. $A \cap B = A$
3. $A \cup B = B$

Hints: This problem asks you to show that 1 iff 2 iff 3. You can show this by proving that 1 implies 2 and that 2 implies 3 and that 3 implies 1.

Problem 2. *Prove that, for any sets A and B, $A = B$ if and only if $\mathcal{P}(A) = \mathcal{P}(B)$.*

Problem 3. *Give an example where $\bigcap A \cap \bigcap B \neq \bigcap(A \cap B)$ for $A \neq \emptyset$, $B \neq \emptyset$, and $A \cap B \neq \emptyset$.*[1] *Then show that $\bigcap A \cap \bigcap B \subseteq \bigcap(A \cap B)$ for any sets A and B such that $A \neq \emptyset$, $B \neq \emptyset$, and $A \cap B \neq \emptyset$.*

Problem 4. *Prove that, for any sets A and B, $\bigcup A \cup \bigcup B = \bigcup(A \cup B)$.*

Problem 5 (De Morgan's Laws)**.** *Let $A \subseteq C$ and $B \subseteq C$. Prove that*

1. $C - (A \cup B) = (C - A) \cap (C - B)$
2. $C - (A \cap B) = (C - A) \cup (C - B)$

[1] You may wonder why we require A, B, and their intersection to be non-empty. A simple answer is that given the definition of arbitrary intersections, $\bigcap \emptyset$ turns out to be something "the universe of sets" (why?), i.e., the collection of all sets. But "the universe of sets" is not a set itself. So in this course we leave $\bigcap \emptyset$ as undefined.