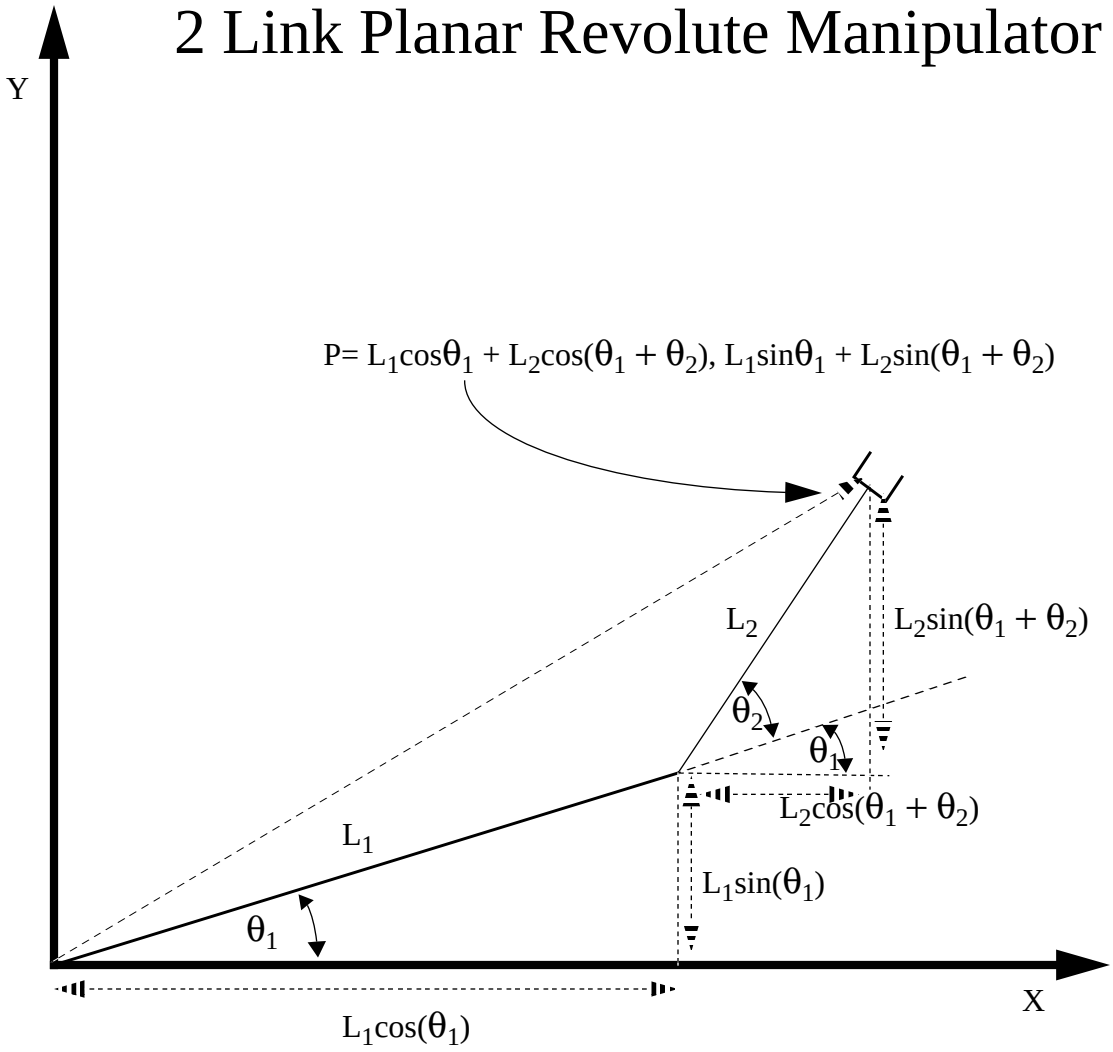
2 Link Planar Revolute Manipulator



$$\overbrace{\begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}^{\mathrm{ROT}(Z,\theta_1)} \overbrace{\begin{bmatrix} 1 & 0 & L_1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}^{\mathrm{TRANS}(X,\,L_1)} \overbrace{\begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 \\ \sin\theta_2 & \cos\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}}^{\mathrm{ROT}(Z,\theta_2)} \overbrace{\begin{bmatrix} 1 & 0 & L_2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}^{\mathrm{TRANS}(X,L_2)} \equiv$$

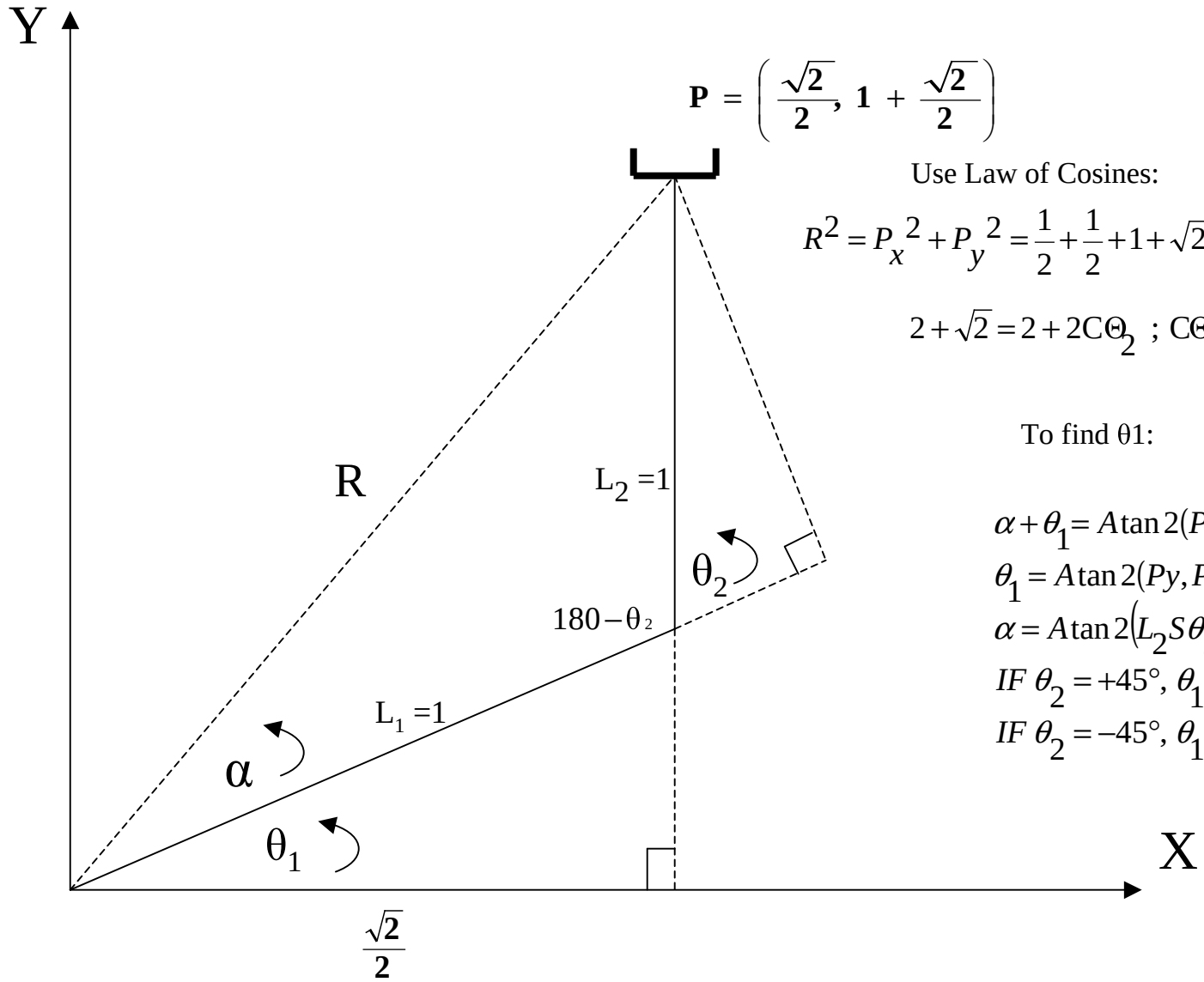
$$\begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & L_1 \\ \sin\theta_2 & \cos\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & L_2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \equiv$$

$$\begin{bmatrix} \cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2 & -\sin\theta_2\cos\theta_1 - \sin\theta_1\cos\theta_2 & L_1\cos\theta_1 \\ \sin\theta_1\cos\theta_2 + \sin\theta_2\cos\theta_1 & -\sin\theta_1\sin\theta_2 + \cos\theta_1\cos\theta_2 & L_1\sin\theta_1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & L_2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \equiv$$

$$\begin{bmatrix} \cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2 & -\sin\theta_2\cos\theta_1 - \sin\theta_1\cos\theta_2 & L_2(\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2) + L_1\cos\theta_1 \\ \sin\theta_1\cos\theta_2 + \sin\theta_2\cos\theta_1 & -\sin\theta_1\sin\theta_2 + \cos\theta_1\cos\theta_2 & L_2(\sin\theta_1\cos\theta_2 + \sin\theta_2\cos\theta_1) + L_1\sin\theta_1 \\ 0 & 0 & 1 \end{bmatrix} \equiv$$

$$\begin{bmatrix} \cos(\theta_1+\theta_2) & -\sin(\theta_1+\theta_2) & L_2\cos(\theta_1+\theta_2) + L_1\cos\theta_1 \\ \sin(\theta_1+\theta_2) & \cos(\theta_1+\theta_2) & L_2\sin(\theta_1+\theta_2) + L_1\sin\theta_1 \\ 0 & 0 & 1 \end{bmatrix}$$

Planar 2-Link Manipulator Inverse Kinematics



$$\mathbf{P} = \left(\frac{\sqrt{2}}{2}, \mathbf{1} + \frac{\sqrt{2}}{2} \right)$$

Use Law of Cosines:

$$R^2 = P_x{}^2 + P_y{}^2 = \frac{1}{2} + \frac{1}{2} + 1 + \sqrt{2} = 1 + 1 - 2C\left(180 - \theta_2\right)$$

$$2 + \sqrt{2} = 2 + 2C\Theta_2 \;;\; C\Theta_2 = \frac{\sqrt{2}}{2} \;;\; \Theta_2 = \pm 45^\circ$$

To find θ1:

$$\alpha + \theta_1 = A\tan 2(Py, Px)$$

$$\theta_1 = A\tan 2(Py, Px) - \alpha$$

$$\alpha = A\tan 2\left(L_2 S\theta_2, L_1 + L_2 C\theta_2\right)$$

$$IF\ \theta_2 = +45^\circ,\ \theta_1 = +45^\circ$$

$$IF\ \theta_2 = -45^\circ,\ \theta_1 = +90^\circ$$