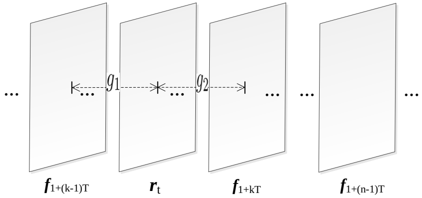
Subsampling and Prediction in DMRI

We obtain the prediction frame at time t by:

$$\mathbf{r}_t = \frac{g_2}{g_1 + g_2}\mathbf{f}_{1+(k-1)T} + \frac{g_1}{g_1 + g_2}\mathbf{f}_{1+kT}$$

where g_1 is the temporal distance between $\mathbf{f}_{1+(k-1)T}$ and $\mathbf{r}_t$, and g_2 is the temporal distance between $\mathbf{f}_{1+kT}$ and $\mathbf{r}_t$. and $1 + (k-1)T \leqslant t \leqslant kT$.



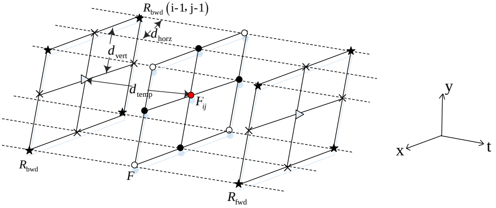
T is the period of reference frames

We obtain reference frames with high sample ratio and other locations with low sample density

$\mathbf{f}_{1+(k-1)T}$, $\mathbf{f}_{1+kT}$ and $\mathbf{f}_{1+(n-1)T}$ denote the recovered first frame in the k^{th}, $(k+1)^{\text{th}}$ and n^{th} periods, respectively

3-dimensional Omni-directional Total Variation

The 26-neighbor structure of the pixel $\mathbf{F}_{ij}$ with different distances.



We denote the distance between two pixels along horizontal (x-axis), vertical (y-axis) and temporal (t-axis) dimensions are d_{horz}, d_{vert} and d_{temp} respectively.

where $\mathbf{R}_{\text{bwd}}$ and $\mathbf{R}_{\text{fwd}}$ are the backward and forward reference frames respectively. We denote the neighbor pixels of different directions by subscript $\bullet$, $\circ$, $\triangleright$, $\star$, $\times$.

3-dimensional Omni-directional Total Variation

$$\|[\mathbf{R}_{\mathsf{bwd}}, \mathbf{F}, \mathbf{R}_{\mathsf{fwd}}]\|_{\mathsf{3OTV}} = \sum_{i=1}^{M}\sum_{j=1}^{N}\{\sum_{k=1}^{26} v_k(\mathbf{F}_{ij} - \mathbf{F}_k)^2\}^{\frac{1}{2}}$$

$$= \sum_{i=1}^{M}\sum_{j=1}^{N}\{\sum_{k_1=1}^{4} v_{k_1}(\mathbf{F}_{ij} - \mathbf{F}_{k_1})^2 + \sum_{k_2=1}^{4} v_{k_2}(\mathbf{F}_{ij} - \mathbf{F}_{k_2})^2$$

$$+ \sum_{k_3=1}^{2} v_{k_3}(\mathbf{F}_{ij} - \mathbf{F}_{k_3})^2 + \sum_{k_4=1}^{8} v_{k_4}(\mathbf{F}_{ij} - \mathbf{F}_{k_4})^2 + \sum_{k_5=1}^{8} v_{k_5}(\mathbf{F}_{ij} - \mathbf{F}_{k_5})^2\}^{\frac{1}{2}}$$

positive weights v_k that tune the gradients along different directions can be expressed :

$$v_k = \frac{1}{\sqrt{d_{\mathsf{horz}}^2 + d_{\mathsf{vert}}^2 + (\eta d_{\mathsf{temp}})^2}}$$

where η is used to make the distances in spatial and temporal domains with the same scale. Similar to 2OTV we have

$$\|[\mathbf{R}_{\mathsf{bwd}}, \mathbf{F}_t, \mathbf{R}_{\mathsf{fwd}}]\|_{\mathsf{3OTV}} = \|[\mathbf{G}_{\mathsf{3D}}^1\underline{\mathbf{f}}, \mathbf{G}_{\mathsf{3D}}^2\underline{\mathbf{f}}, \cdots, \mathbf{G}_{\mathsf{3D}}^{26}\underline{\mathbf{f}}]\|_{2,1}$$

$\underline{\mathbf{f}} = [\mathbf{f}_t^{\mathsf{T}}, \mathbf{r}_{\mathsf{bwd}}^{\mathsf{T}}, \mathbf{r}_{\mathsf{fwd}}^{\mathsf{T}}]^{\mathsf{T}}$ and $\mathbf{G}_{\mathsf{3D}}^1$, $\mathbf{G}_{\mathsf{3D}}^2$, ..., $\mathbf{G}_{\mathsf{3D}}^{26}$ stand for the $K \times 3K$ weighted finite difference matrices.

Recovery model based on ODTV

optimization model :

$$\min_{\mathbf{u}_t} \frac{1}{2}\|\mathbf{E}_t(\mathbf{r}_t + \mathbf{u}_t) - \mathbf{b}_t\|_2^2 + \lambda_1\|\mathbf{u}_t\|_{\text{2OTV}} + \lambda_2\|[\mathbf{r}_{\text{bwd}}, \mathbf{r}_t + \mathbf{u}_t, \mathbf{r}_{\text{fwd}}]\|_{\text{3OTV}}$$

where $\mathbf{E}_t$ is the undersampling operator and $\mathbf{B}_t$ is the measurement data. λ_1 and λ_2 are data consistency parameters.

$\mathbf{r}_t = \text{vec}(\mathbf{R}_t)$ is the prediction frame, $\mathbf{u}_t = \text{vec}(\mathbf{U}_t)$ is the residual frame. $\mathbf{r}_{\text{fwd}}$ are vectorization forms of matrices $\mathbf{R}_{\text{bwd}}$ and $\mathbf{R}_{\text{fwd}}$ respectively.

The frame to be reconstructed

$$\mathbf{f}_t = \mathbf{r}_t + \mathbf{u}_t$$

Solution: IRLS

We can have obtain $\mathbf{u}$ by

$$\mathbf{u}_t^{k+1} = \arg\min_{\mathbf{u}_t} \frac{1}{2}\|\mathbf{E}_t(\mathbf{u}_t + \mathbf{r}_t) - \mathbf{b}_t\|_2^2$$

$$+\frac{\lambda_1}{2}(\sum_{i=1}^{8} \mathbf{u}_t^{\mathsf{H}}(\mathbf{G}_{\mathsf{2D}}^i)^{\mathsf{H}}\mathbf{W}_1^k\mathbf{G}_{\mathsf{2D}}^i\mathbf{u}_t + \mathrm{Tr}((\mathbf{W}_1^k)^{-1}))$$

$$+\frac{\lambda_2}{2}(\sum_{i=1}^{26} \underline{\mathbf{f}}^{\mathsf{H}}(\mathbf{G}_{\mathsf{3D}}^i)^{\mathsf{H}}\mathbf{W}_2^k\mathbf{G}_{\mathsf{3D}}^i\underline{\mathbf{f}} + \mathrm{Tr}((\mathbf{W}_2^k)^{-1}))$$

where $\mathbf{W}_1^k$ and $\mathbf{W}_2^k$ are all $K \times K$ diagonal matrices at the k-th step, and both of them are weighted matrices for updating iteration. The diagonal entries of $\mathbf{W}_1^k$ and $\mathbf{W}_2^k$ can be obtained by:

$$(w_1^k)_i = \frac{1}{\|[\mathbf{G}_{\mathsf{2D}}^1\mathbf{u}^k, \mathbf{G}_{\mathsf{2D}}^2\mathbf{u}^k, \cdots, \mathbf{G}_{\mathsf{2D}}^8\mathbf{u}^k]_i\|_2}$$

$$(w_2^k)_i = \frac{1}{\|[\mathbf{G}_{\mathsf{3D}}^1\underline{\mathbf{f}}, \mathbf{G}_{\mathsf{3D}}^2\underline{\mathbf{f}}, \cdots, \mathbf{G}_{\mathsf{3D}}^{26}\underline{\mathbf{f}}]_i\|_2}$$

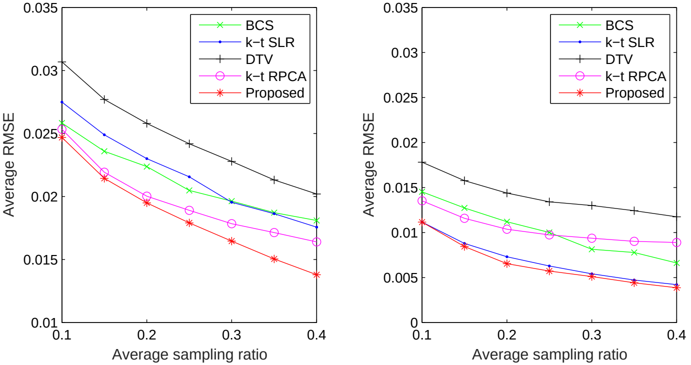
Experimental results of the reconstruction

RMSE is adopted as the metric for result quantitative evaluation:

$$\text{RMSE}\left(\mathbf{f}_t{}^0, \hat{\mathbf{f}}_t\right) = \sqrt{\|\mathbf{f}_t{}^0 - \hat{\mathbf{f}}_t\|_2^2 / K}$$

$\mathbf{f}_t{}^0$ and $\hat{\mathbf{f}}_t$ represent the full-sampling image and reconstructed image of the t-th frame respectively.

Average reconstruction accuracy comparisons at different sampling ratio



Reconstruction accuracy comparisons at 25% sampling ratio