Homework 1

Question 1

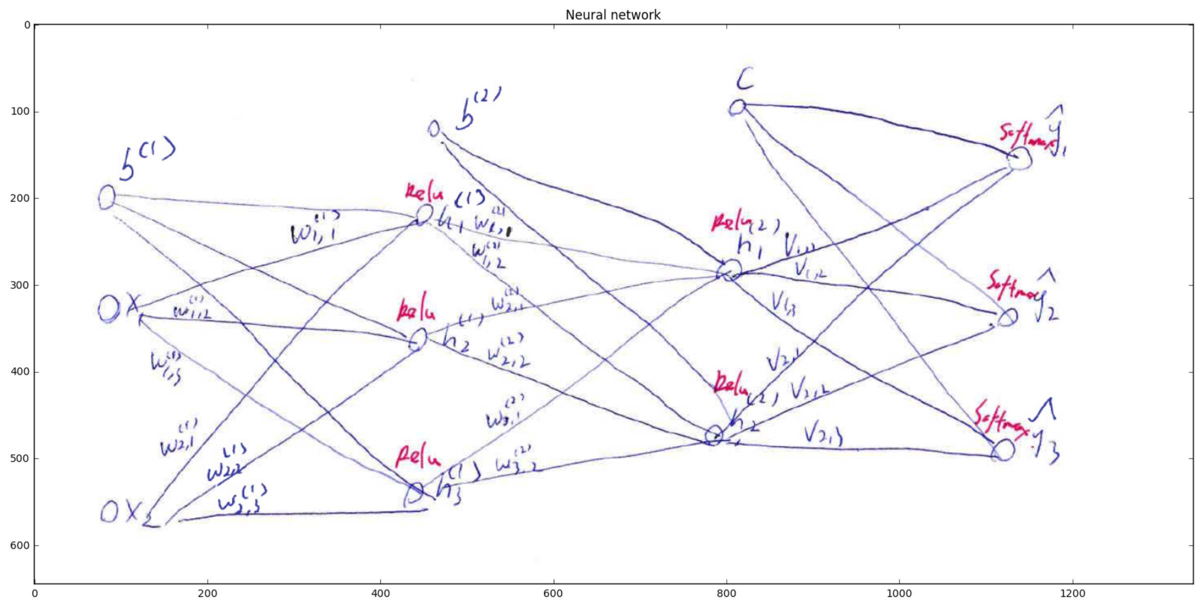
Part a ¶

In [1]:

```
import matplotlib.pyplot as plt
import matplotlib.image as mpimg
%matplotlib inline
img=mpimg.imread("C:/Users/yipinl/590/Capture.PNG")
plt.figure(figsize = (20,15))
plt.imshow(img),plt.title('Neural network')
```

Out[1]:

```
(<matplotlib.image.AxesImage at 0x25567b60f28>,
 <matplotlib.text.Text at 0x2556780b208>)
```



Part b

Accodinng to this graph, the

(1) The first layer

$$h_1^{(1)} = Relu(a_1^{(1)}) = Relu(x_1 W_{1,1}^{(1)} + x_2 W_{2,1}^{(1)} + b^{(1)})$$

$$h_2^{(1)} = Relu(a_2^{(1)}) = Relu(x_1 W_{1,2}^{(1)} + x_2 W_{2,2}^{(1)} + b^{(1)})$$

$$h_3^{(1)} = Relu(a_3^{(1)}) = Relu(x_1 W_{1,3}^{(1)} + x_2 W_{2,3}^{(1)} + b^{(1)})$$

(2) The second layer:

$$h_1^{(2)} = Relu(a_1^{(2)}) = Relu(h_1^{(1)} W_{1,1}^{(2)} + h_2^{(1)} W_{2,1}^{(2)} + h_3^{(1)} W_{3,1}^{(2)} + b^{(2)})$$

$$h_2^{(2)} = Relu(a_2^{(2)}) = Relu(h_1^{(1)} W_{1,2}^{(2)} + h_2^{(1)} W_{2,2}^{(2)} + h_3^{(1)} W_{3,2}^{(2)} + b^{(2)})$$

(3) The output:

$$\hat{y}_1 = Softmax(h_1^{(2)} V_{1,1} + h_1^{(2)} V_{2,1} + c)$$

$$\hat{y}_2 = Softmax(h_1^{(2)} V_{1,2} + h_1^{(2)} V_{2,2} + c)$$

$$\hat{y}_2 = Softmax(h_1^{(2)} V_{1,3} + h_1^{(2)} V_{2,3} + c)$$

Part c

In [2]:
```
import numpy as np
import pandas as pd
import matplotlib.cm as cm
from __future__ import division
import random
```

In [3]:
```
#######################
#   Helper functions  #
#######################
# Linear activation
def a(x,w,b):
    a_out = x.dot(w) + b
    return a_out

sigmoid = lambda x: 1/(1+np.exp(-x))
relu = np.vectorize(lambda x: np.fmax(0,x))
softmax = lambda x: np.exp(x)/(np.exp(x).sum(axis=1, keepdims=True))

# Logistic unit
def logistic(x,w,b):
    s = sigmoid(a(x,w,b))
    y = np.round(s)
    return np.array([y,s]).T
```

In [4]:
```
def ff_nn_2_ReLu(x, w_1, w_2, v, b_1, b_2):
    '''
    A simple 2 layer neural network with relu activation and binary output.
    '''
    h1 = relu(a(x,w_1,b_1))
    h2 = relu(a(h1,w_2,b_2))
    o = softmax(a(h2,v,c))

    return np.round(o,3)
```

Part d

In [5]:
```
w_1 = np.array([[1,0],[-1,0],[0,0.5]])
w_2 = np.array([[1,0,0],[-1,-1,0]])
v = np.array([[1,1],[0,0],[-1,-1]])
x = np.array([[1,1,1],[-1,-1,1]])
b_1 = np.array([0,0,1])
b_2 = np.array([1,-1])
c = np.array([1,0,0])

print(ff_nn_2_ReLu(x.T, w_1.T, w_2.T, v.T, b_1, b_2))
```

```
[[ 0.946  0.047  0.006]
 [ 0.946  0.047  0.006]
 [ 0.946  0.047  0.006]]
```

Question 2

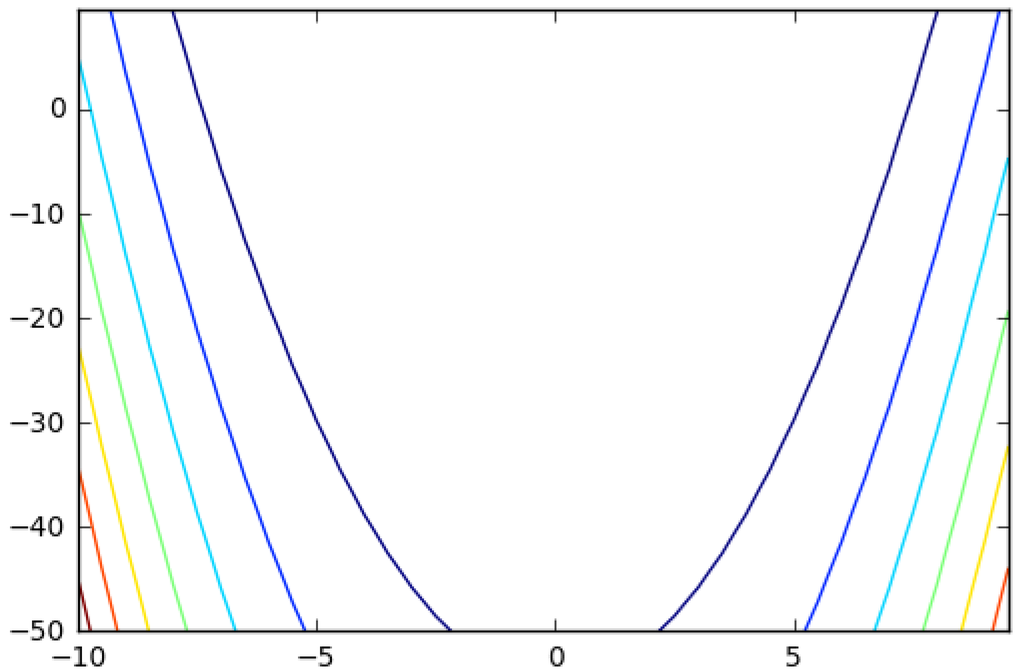
Part a

$$\partial_x f(x,y) = -2(1-x) + 100 \times 2(y-x^2)(-2x) = 400x^3 - 400xy + 2x - 2$$

$$\partial_y f(x,y) = 100 \times x(y-x^2) = 200y - 200x^2$$

Part b

In [6]:
```
delta = 0.5
x = np.arange(-10.0,10.0, delta)
y = np.arange(-50, 10.0, delta)
X, Y = np.meshgrid(x, y)
Z = (1-X)**2 + 100*(Y - X**2)**2
fig, ax = plt.subplots()
CS = ax.contour(X, Y, Z)
```



Part c

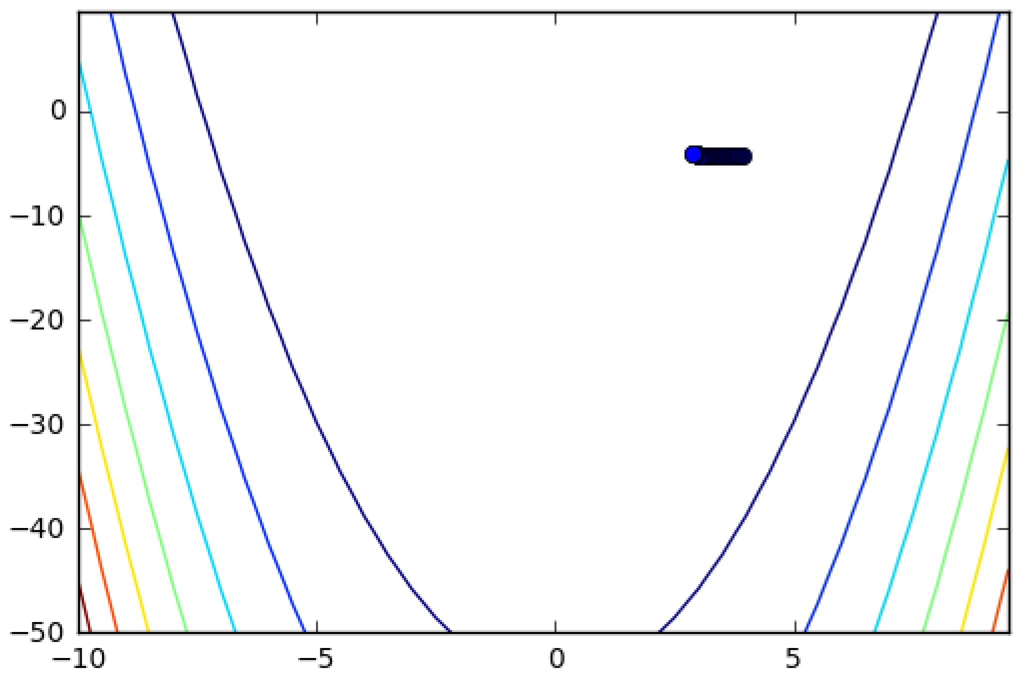
In [7]:
```
# --- Defining gradient ----
def grad_f(vector):
    x, y = vector
    df_dx = -2+2*x-400*x*y+400*np.power(x,3)
    df_dy = 200*y-200*np.power(x,2)
    return np.array([df_dx, df_dy])
```

In [8]:
```
# --- Grad Descent ----
def grad_descent(starting_point=None, iterations=50, learning_rate=0.0001):
    if starting_point:
        point = starting_point
    else:
        point = np.random.uniform(-10,10,size=2)
    trajectory = [point]

    for i in range(iterations):
        grad = grad_f(point)
        point = point - learning_rate * grad
        trajectory.append(point)
    return np.array(trajectory)
```

Case 1: Learning rate = 0.000001

In [9]:
```
# --- Visualize Trajectory ---
np.random.seed(123)
traj = grad_descent(iterations=50, learning_rate=10**-6)

fig, ax = plt.subplots()
CS = ax.contour(X, Y, Z)
x= traj[:,0]
y= traj[:,1]
plt.plot(x,y,'-o')
```

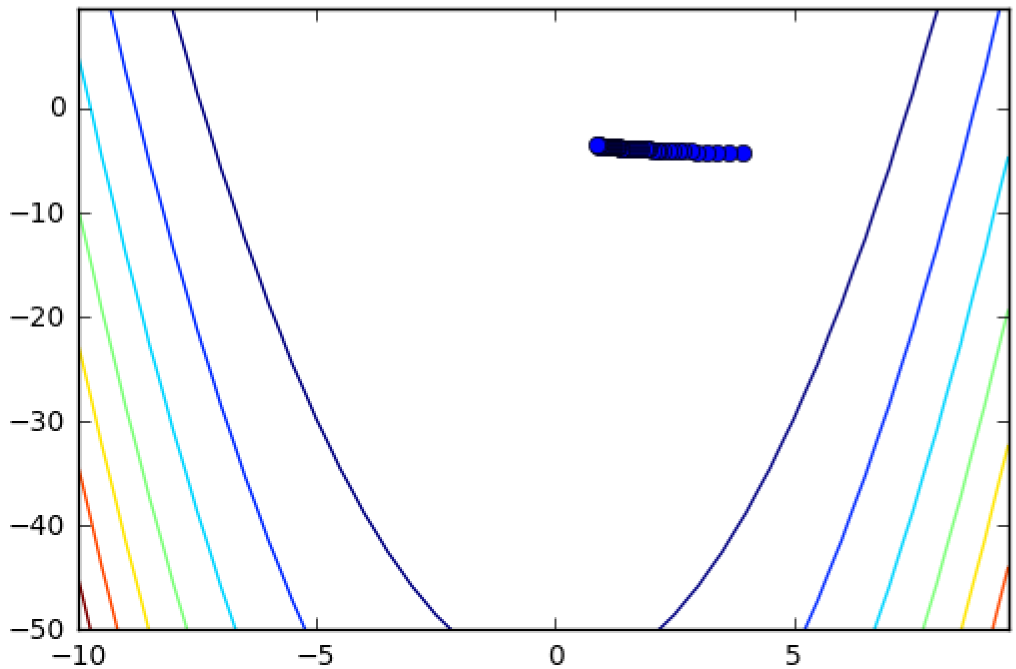
Out[9]: [<matplotlib.lines.Line2D at 0x2556a8581d0>]



Case 2: Learning rate = 0.00001

In [10]:
```
# --- Visualize Trajectory ---
np.random.seed(123)
traj = grad_descent(iterations=50, learning_rate=10**-5)

fig, ax = plt.subplots()
CS = ax.contour(X, Y, Z)
x= traj[:,0]
y= traj[:,1]
plt.plot(x,y,'-o')
```

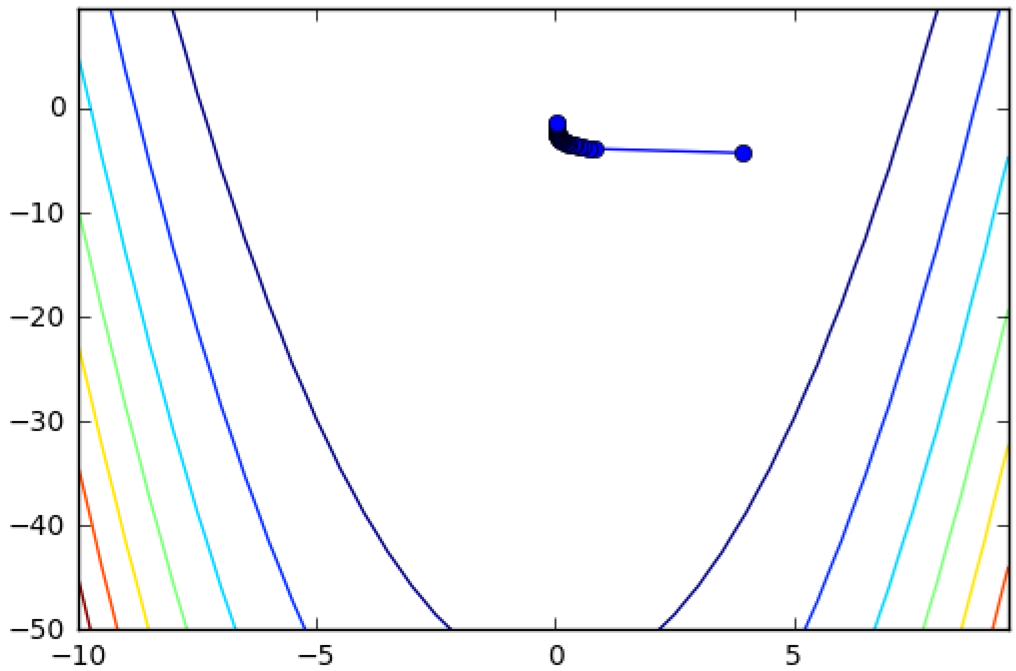
Out[10]: [<matplotlib.lines.Line2D at 0x2556a8e7c88>]



Case 3: Learning rate = 0.0001

In [11]:
```
# --- Visualize Trajectory ---
np.random.seed(123)
traj = grad_descent(iterations=50, learning_rate=0.0001)

fig, ax = plt.subplots()
CS = ax.contour(X, Y, Z)
x= traj[:,0]
y= traj[:,1]
plt.plot(x,y,'-o')
```

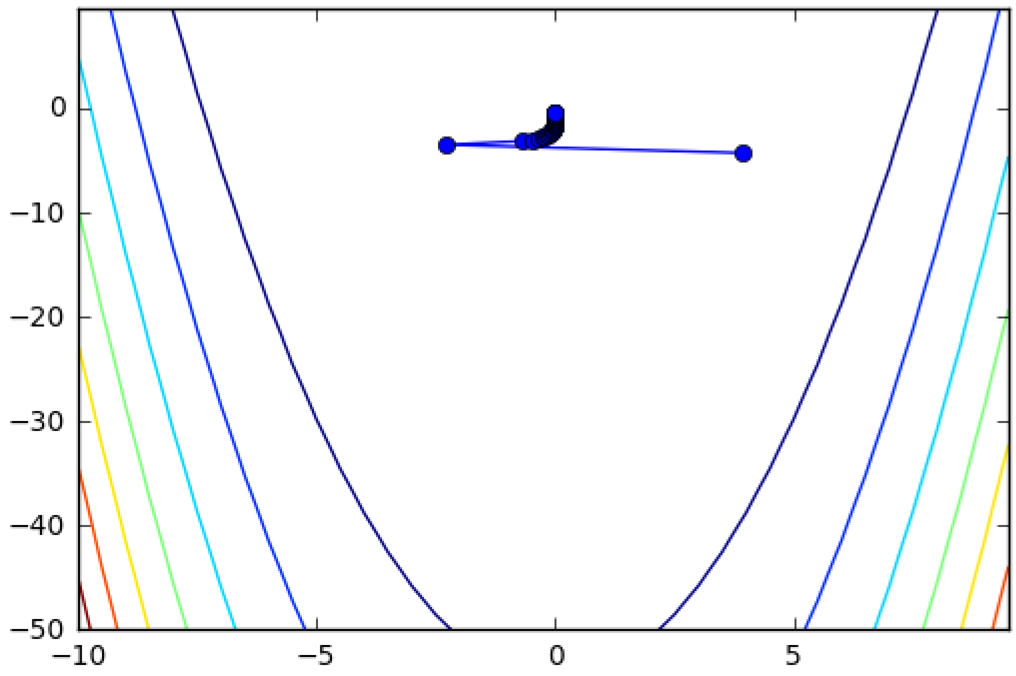
Out[11]:
```
[<matplotlib.lines.Line2D at 0x2556a8f94a8>]
```



Case 4: Learning rate = 0.0002

In [12]:
```
# --- Visualize Trajectory ---
np.random.seed(123)
traj = grad_descent(iterations=50, learning_rate=0.0002)

fig, ax = plt.subplots()
CS = ax.contour(X, Y, Z)
x= traj[:,0]
y= traj[:,1]
plt.plot(x,y,'-o')
```

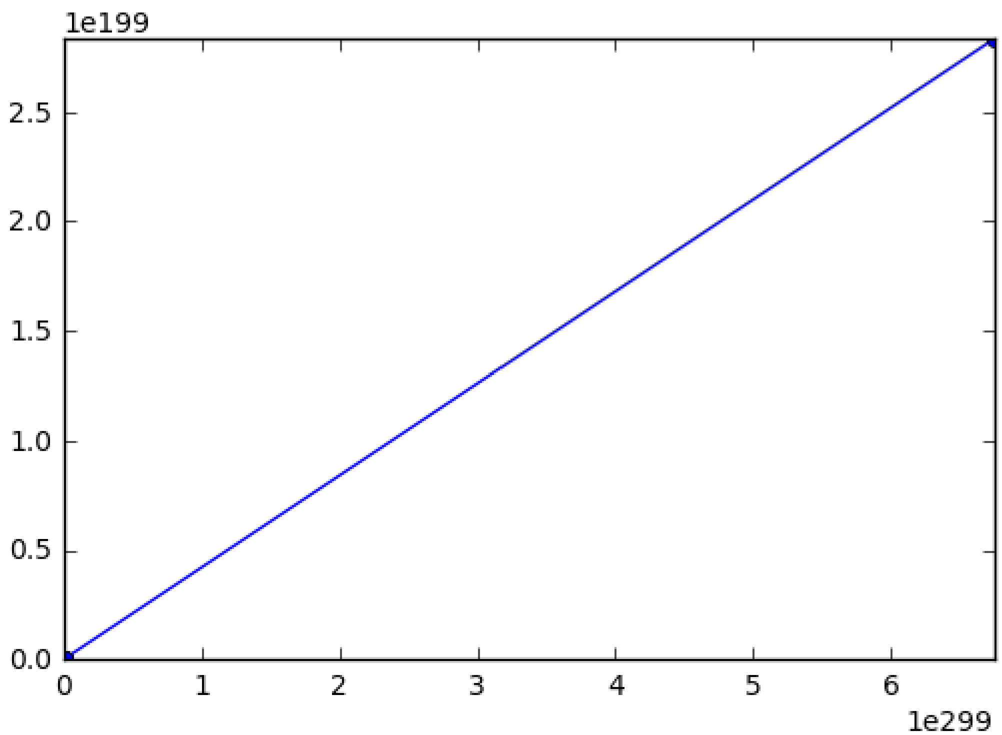
Out[12]: [<matplotlib.lines.Line2D at 0x2556a96eef0>]



Case 5: Learning rate = 0.001

In [13]:
```
# --- Visualize Trajectory ---
np.random.seed(123)
traj = grad_descent(iterations=50, learning_rate=0.001)

fig, ax = plt.subplots()
CS = ax.contour(X, Y, Z)
x= traj[:,0]
y= traj[:,1]
plt.plot(x,y,'-o')
```

Out[13]:
```
[<matplotlib.lines.Line2D at 0x2556aa74e80>]
```



Summary:

In the case of learning rate is 0.000001 or 0.00001, the learning rate is too small to make the function reach the local minimizer. In the case of learning rate is 0.001, the learning rate is too large that the function cannot converge. Hence, in these cases, the learning rate 0.0001 is the best.

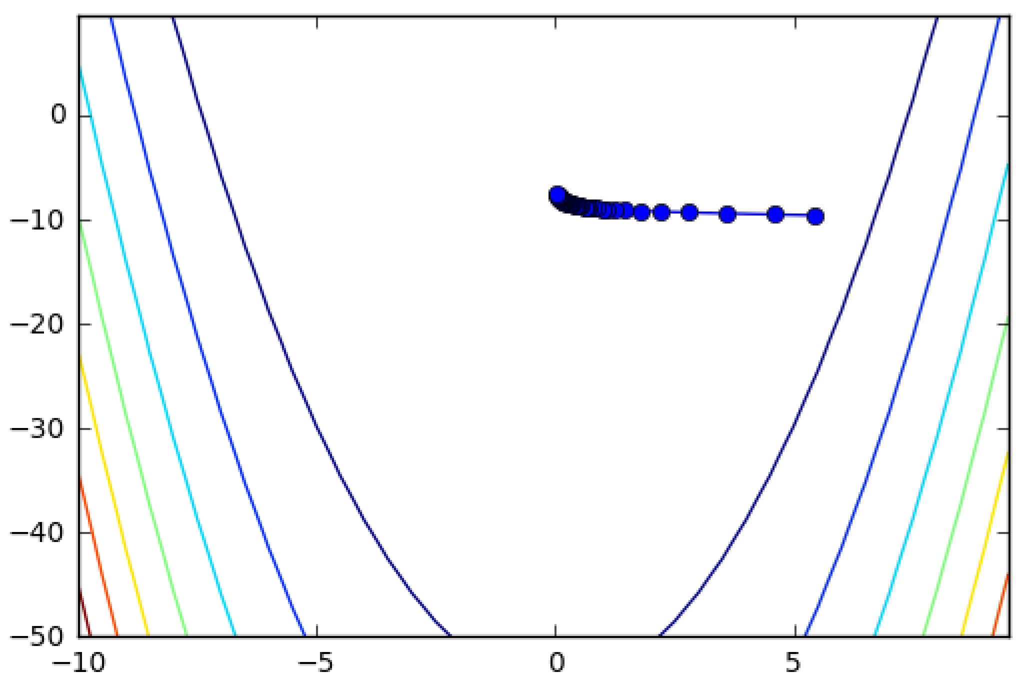
Part d

In [14]:
```
def grad_descent_with_momentum(starting_point=None, iterations=50, alpha=.9, e
psilon=0.0001):
    if starting_point:
        point = starting_point
    else:
        point = np.random.uniform(-10,10,size=2)
    trajectory = [point]
    v = np.zeros(point.size)

    for i in range(iterations):
        grad = grad_f(point)
        v = alpha*v + epsilon*grad
        point = point - v
        trajectory.append(point)
    return np.array(trajectory)
```

Case 1: Alpha = 0.5, Epsilon = 0.00001

In [15]:
```
# --- Visualizing trajectory --
np.random.seed(10)
traj = grad_descent_with_momentum(iterations=50, epsilon=0.00001, alpha=.5)

fig, ax = plt.subplots()
CS = ax.contour(X, Y, Z)
x= traj[:,0]
y= traj[:,1]
plt.plot(x,y,'-o')
```

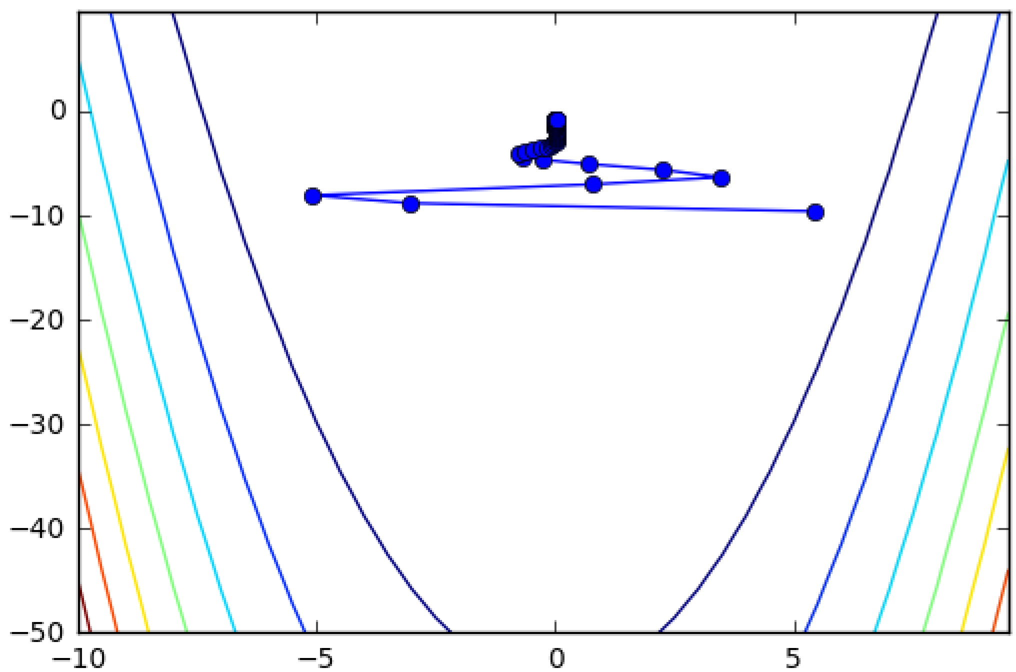
Out[15]:
```
[<matplotlib.lines.Line2D at 0x2556a8cbba8>]
```



Case 2: Alpha = 0.5, Epsilon = 0.0001

In [16]:
```
# --- Visualizing trajectory --
np.random.seed(10)
traj = grad_descent_with_momentum(iterations=50, epsilon=0.0001, alpha=.5)

fig, ax = plt.subplots()
CS = ax.contour(X, Y, Z)
x= traj[:,0]
y= traj[:,1]
plt.plot(x,y,'-o')
```

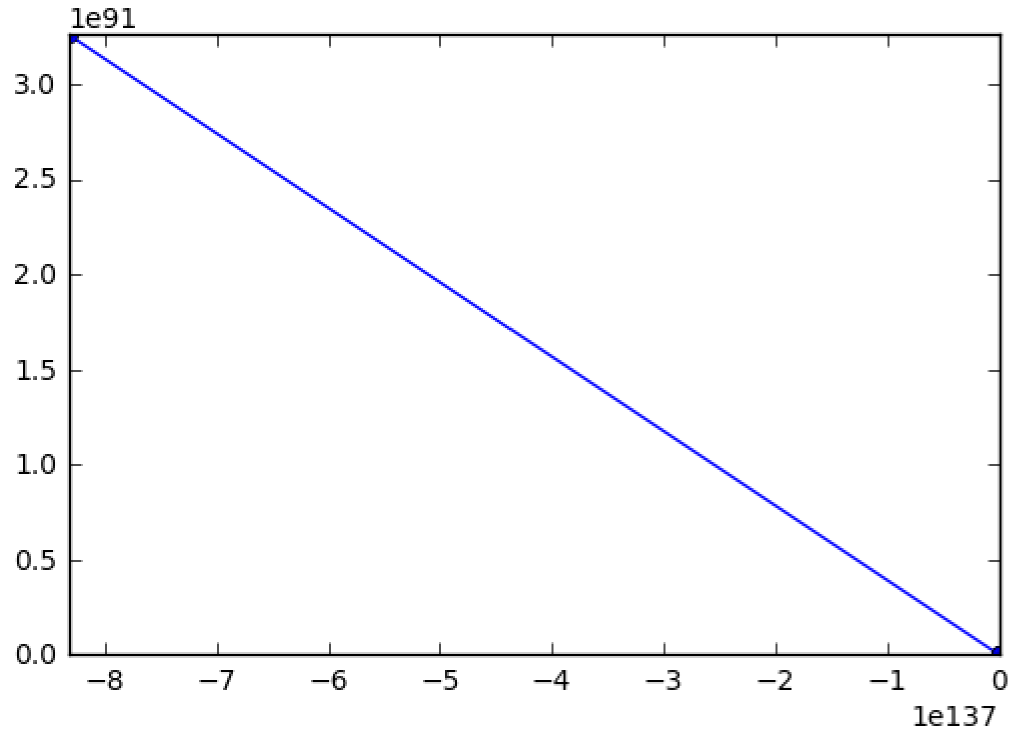
Out[16]:
```
[<matplotlib.lines.Line2D at 0x2556a89f198>]
```



Case 3: Alpha = 0.5, Epsilon = 0.001

In [17]:
```
# --- Visualizing trajectory --
np.random.seed(10)
traj = grad_descent_with_momentum(iterations=50, epsilon=0.001, alpha=.5)

fig, ax = plt.subplots()
CS = ax.contour(X, Y, Z)
x= traj[:,0]
y= traj[:,1]
plt.plot(x,y,'-o')
```

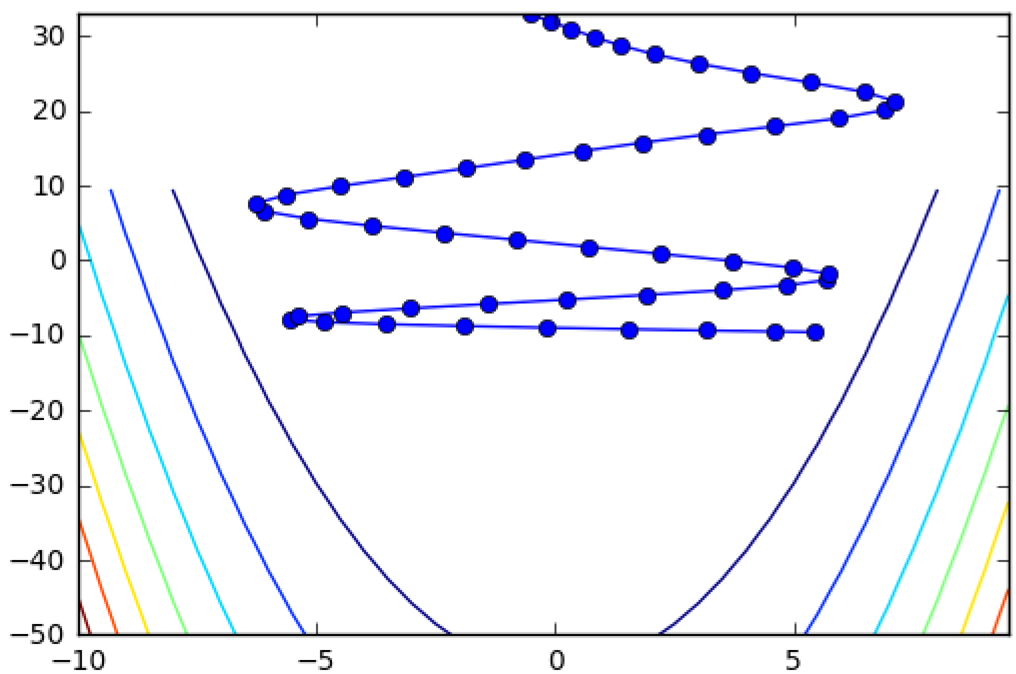
Out[17]: [<matplotlib.lines.Line2D at 0x2556aac9470>]



Case 4: Alpha = 1, Epsilon = 0.00001

In [18]:
```
# --- Visualizing trajectory --
np.random.seed(10)
traj = grad_descent_with_momentum(iterations=50, epsilon=0.00001, alpha=1)

fig, ax = plt.subplots()
CS = ax.contour(X, Y, Z)
x= traj[:,0]
y= traj[:,1]
plt.plot(x,y,'-o')
```

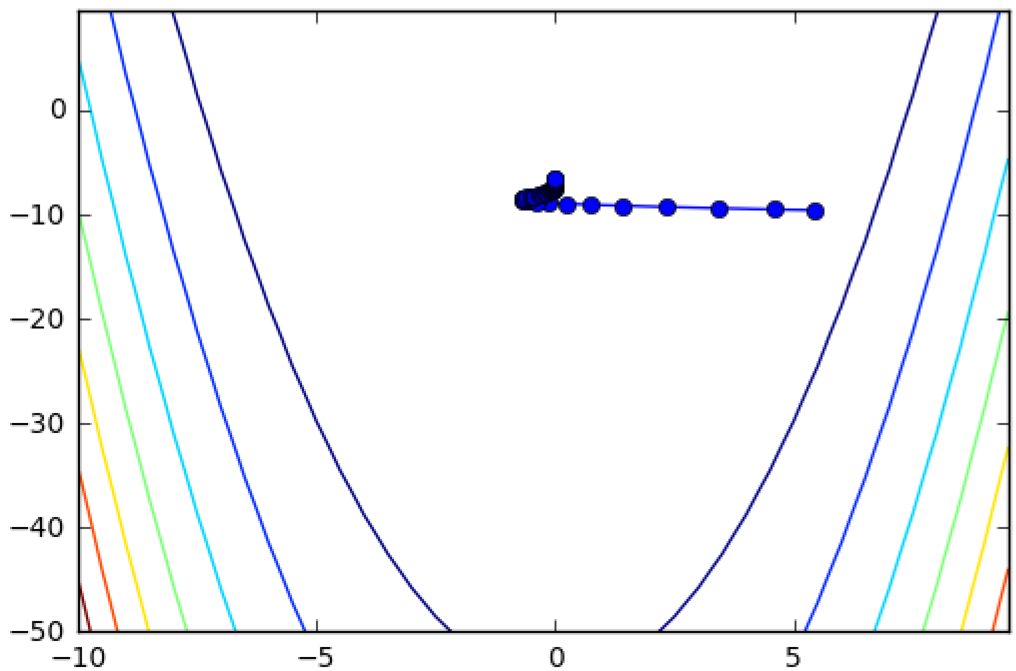
Out[18]: [<matplotlib.lines.Line2D at 0x2556ab31780>]



Case 5: Alpha = 0.7, Epsilon = 0.00001

In [19]:
```
# --- Visualizing trajectory --
np.random.seed(10)
traj = grad_descent_with_momentum(iterations=50, epsilon=0.00001, alpha=0.7)

fig, ax = plt.subplots()
CS = ax.contour(X, Y, Z)
x= traj[:,0]
y= traj[:,1]
plt.plot(x,y,'-o')
```

Out[19]: [<matplotlib.lines.Line2D at 0x2556aa2abe0>]



Summary:

When we fix alpha to be 0.5, if the epsilon is 0.001, the function cannot converge. If the epsilon is epsilon=0.0001, then we can show the function oscillates around the minimizer. When the epsilon is 0.00001, the function converges when it does not swing around the minimizer. Hence, it is the best value in this case.

Second, we fix epsilon to be 0.00001. When the alpha is 1, the function oscillates around the minimizer back and forth. When the alpha is 0.7, it swings a little. Hence, in this case, alpha = 0.5 is the best solution.

Question 3

Part a

$$L(y,\hat{y}) = -(y_1\log(\hat{y}_1) + y_2 log(\hat{y}_2) + y_3 log(\hat{y}_3))$$

$$\frac{\partial L_s}{\partial V_{k,s}} = \frac{\partial L_s}{\partial \hat{y}_s}\frac{\partial \hat{y}_s}{\partial a_s^{(3)}}\frac{\partial a_s^{(3)}}{\partial V_{k,s}} = \frac{y_s-\hat{y}_s}{\hat{y}_s(1-\hat{y}_s)}\hat{y}_s(1-\hat{y}_s)h_k^{(2)} = (\prod(1 = True\ class) - \hat{y}_s)h_k^{(2)}$$

$$\frac{\partial L_s}{\partial c} = \frac{\partial L_s}{\partial \hat{y}_s}\frac{\partial \hat{y}_s}{\partial a_s^{(3)}}\frac{\partial a_s^{(3)}}{\partial c} = (\prod(1 = True\ class) - \hat{y}_s)$$

$$\frac{\partial L_s}{\partial W_{j,k}^{(2)}} = \frac{\partial L_s}{\partial \hat{y}_s}\frac{\partial \hat{y}_s}{\partial a_s^{(3)}}\frac{\partial a_s^{(3)}}{\partial h_k^{(2)}}\frac{\partial h_k^{(2)}}{\partial W_{j,k}^{(2)}} = (\prod(1 = True\ class) - \hat{y}_s)V_{k,s}\prod(a_k^{(2)} > 0)h_j^{(1)}$$

$$\frac{\partial L_s}{\partial b^{(2)}} = \frac{\partial L_s}{\partial \hat{y}_s}\frac{\partial \hat{y}_s}{\partial a_s^{(3)}}\sum_k \frac{\partial a_s^{(3)}}{\partial h_k^{(2)}}\frac{\partial h_k^{(2)}}{\partial b^{(2)}} = (\prod(1 = True\ class) - \hat{y}_s)\sum_k V_{k,s}\prod(a_k^{(2)} > 0)$$

$$\frac{\partial L_s}{\partial W_{i,j}^{(1)}} = \frac{\partial L_s}{\partial \hat{y}_s}\frac{\partial \hat{y}_s}{\partial a_s^{(3)}}\sum_k \frac{\partial a_s^{(3)}}{\partial h_k^{(2)}}\frac{\partial h_k^{(2)}}{\partial a_k^{(2)}}\frac{\partial a_k^{(2)}}{\partial h_j^{(1)}}\frac{\partial h_j^{(1)}}{\partial a_j^{(1)}}\frac{\partial a_j^{(1)}}{\partial W_{i,j}^{(1)}} = (\prod(1 = True\ class) - \hat{y}_s)\sum_k V_{k,s}\prod(a_k^{(2)} > 0)W_{j,k}^{(2)}\prod$$

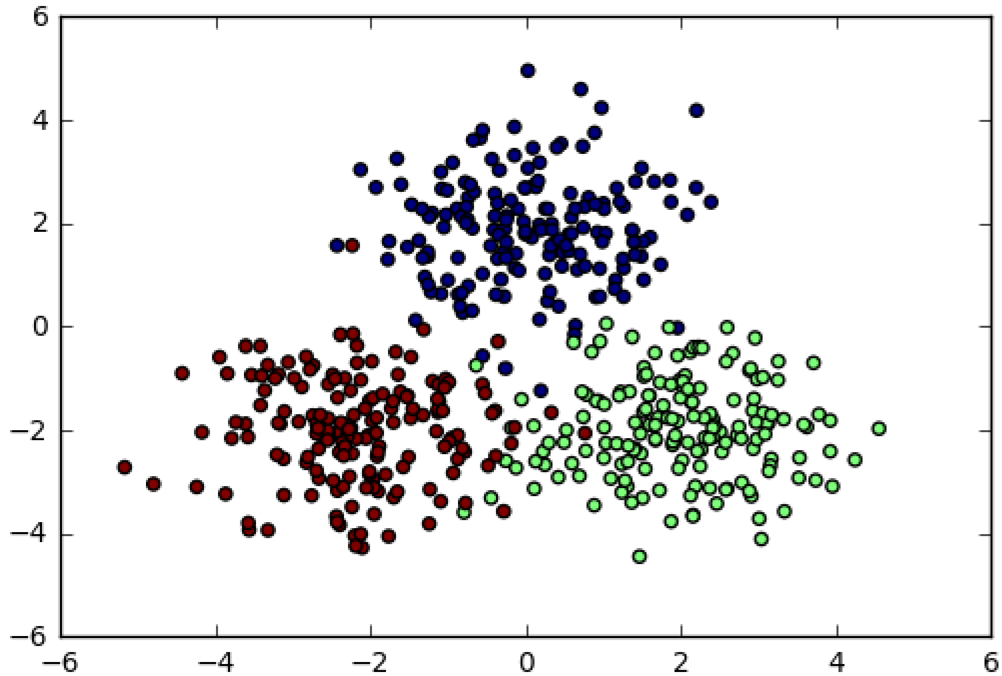
$$\frac{\partial L_s}{\partial b^{(1)}} = \frac{\partial L_s}{\partial \hat{y}_s}\frac{\partial \hat{y}_s}{\partial a_s^{(3)}}\sum_k \frac{\partial a_s^{(3)}}{\partial h_k^{(2)}}\frac{\partial h_k^{(2)}}{\partial a_k^{(2)}}\sum_j \frac{\partial a_k^{(2)}}{\partial h_j^{(1)}}\frac{\partial h_j^{(1)}}{\partial a_j^{(1)}}\frac{\partial a_j^{(1)}}{\partial b^{(1)}} = (\prod(1 = True\ class) - \hat{y}_s)\sum_k V_{k,s}\prod(a_k^{(2)} > 0)\sum_j W$$

Part b

In [20]:
```
def grad_f(X,h1,h2,Y,Y_hat,parameters):
    W1,b1,W2,b2,V,c = parameters
    dw1 = X.T.dot(((Y_hat - Y).dot(V.T)*(h2 > 0)).dot(W2.T)*(h1>0))
    db1 = (((Y_hat - Y).dot(V.T)*(h2 > 0)).dot(W2.T)*(h1>0)).sum(axis = 0)
    dw2 = h1.T.dot(((Y_hat - Y).dot(V.T)*(h2 > 0)))
    db2 = ((Y_hat - Y).dot(V.T)*(h2 > 0)).sum(axis = 0)
    dV = h2.T.dot((Y_hat - Y))
    dc = (Y_hat - Y).sum(axis = 0)

    return dw1, db1, dw2,  db2, dV, dc
```

Part c

In [21]:
```
#%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
#   Generate some training    #
#      data from a GMM        #
###############################
def gen_gmm_data(n = 999, plot=False):
    # Fixing seed for repeatability
    np.random.seed(123)

    # Parameters of a normal distribuion
    mean_1 = [0, 2] ; mean_2 = [2, -2] ; mean_3 = [-2, -2]
    mean = [mean_1, mean_2, mean_3] ; cov = [[1, 0], [0, 1]]

    # Setting up the class probabilities
    n_samples = n
    pr_class_1 = pr_class_2 = pr_class_3 = 1/3.0
    n_class = (n_samples * np.array([pr_class_1,pr_class_2, pr_class_3])).asty
pe(int)

    # Generate sample data
    for i in range(3):
        x1,x2 = np.random.multivariate_normal(mean[i], cov, n_class[i]).T
        if (i==0):
            xs = np.array([x1,x2])
            cl = np.array([n_class[i]*[i]])
        else:
            xs_new = np.array([x1,x2])
            cl_new = np.array([n_class[i]*[i]])
            xs = np.concatenate((xs, xs_new), axis = 1)
            cl = np.concatenate((cl, cl_new), axis = 1)

    # Plot?
    if plot:
        plt.scatter(xs[:1,:],xs[1:,:], c = cl)

    # One hot encoding classes
    y = pd.Series(cl[0].tolist())
    y = pd.get_dummies(y).as_matrix()

    # Normalizing data (prevents overflow errors)
    mu = xs.mean(axis = 1)
    std = xs.std(axis = 1)
    xs = (xs.T - mu) / std

    return xs, y, cl
#%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
#  Generate data for network    #
#################################
X, Y, cl = gen_gmm_data(n = 500,plot = True)
```



Part d

In [25]:
```
def predict(Y_hat):
    return np.argmax(Y_hat, axis=1)

def error_rate(Y_hat, cl):
    prediction = predict(Y_hat)
    return np.mean(prediction != cl)

def cost(Y_hat, Y):
    tot = Y * np.log(Y_hat)
    return -tot.sum()
```

In [26]:
```
####################################
#   2 - Hidden Layer ReLU Network   #
####################################
def forward(X,parameters):
    # Unpacking parameters
    W1,b1,W2,b2,V,c = parameters

    # Forward pass
    a1 = X.dot(W1) + b1
    H1 = relu(a1)
    a2 = H1.dot(W2) + b2
    H2 = relu(a2)
    a3 = H2.dot(V) + c
    Y_hat = softmax(a3)
    return H1,H2,Y_hat
```

In [27]:
```
#%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
#   Parameter Update: Momentum + Regularization   #
###################################################
def parameter_update(parameters,  grads,
                     momentum_params = [0,0,0,0],
                     lr = 1, reg = 0, alpha = 0):
    # Unpacking parameters
    W1,b1,W2,b2,V,c = parameters
    dW1,db1,dW2,db2,dV,dc = grads
    vW1,vb1,vW2,vb2,vV,vc = momentum_params

    # Momentum update
    vW1  = alpha * vW1 -  lr * (dW1 + reg*W1)
    vb1 = alpha * vb1 - lr * (db1 + reg*b1)
    vW2  = alpha * vW2 -  lr * (dW2 + reg*W2)
    vb2 = alpha * vb2 - lr * (db2 + reg*b2)
    vV  = alpha * vV -  lr * (dV + reg*V)
    vc = alpha * vc - lr * (dc + reg*c)
    momentum_params = [vW1,vb1,vW2,vb2,vV,vc]

    # Parameter updates
    W1  = W1  + vW1
    b1 = b1 + vb1
    W2  = W2  + vW2
    b2 = b2 + vb2
    V  = V  - lr*dV
    c = c - lr*dc
    parameters =[W1,b1,W2,b2,V,c]

    return parameters, momentum_params
```

In [28]:
```
# %%%%%%%%%%%%%%%%%%%%%%%%%%%%%
#      Building the model     #
###############################
def run_model(X, Y, cl,
              iterations = 1000,
              regularization_include = False,
              momentum_include = False):

    ###################################
    #   Initial values for network    #
    ###################################
    # Intialize weights
    np.random.seed(123)
    W1 = np.random.randn(6).reshape(3,2)
    b1 = 0
    W2= np.random.randn(6).reshape(2,3)
    b2 = 0
    V = np.random.randn(6).reshape(3,2)
    c = 0
    parameters = [W1.T,b1,W2.T,b2,V.T,c]

    # Hyperparameters
    lr = 0.0001 # learning rate
    reg = 0.01 * regularization_include

    # Momentum parameters
    alpha = 0.9 * momentum_include
    vV = 0
    vb2 = 0
    vW1 = 0
    vb1 = 0
    vW2 = 0
    vc = 0
    momentum_params = [vW1,vb1,vW2,vb2,vV,vc]

    # Place holder for losses
    losses = []
    errors = []

    ###################
    #   Run the model #
    ###################
    for i in range(0,iterations):
        # -- Forward propoagation --
        h1,h2,Y_hat = forward(X,parameters)

        # -- Backward propagation --
        # Gradient calculation
        grads_in = grad_f(X,h1,h2,Y,Y_hat,parameters)
        # Parameter update
        new_params, new_mom_param = parameter_update(parameters, grads_in,
                              momentum_params, alpha = alpha,
                              lr = lr, reg = reg)

        # -- Updating values --
        h1,h2,Y_hat = forward(X,new_params)
```

```
        parameters = new_params
        momentum_params = new_mom_param

        # Prediction and Error rate
        errs_i = error_rate(Y_hat, cl) ; errors.append(errs_i)
        loss_i = cost(Y_hat, Y); losses.append(loss_i)
        if ((i % 50) == 0):
            print(
            '''
            ---- Iteration {i} ----
            Error rate : {er}
            Loss: {loss}
            '''.format(i= i, er = errs_i, loss = loss_i))
    return losses,errors
```

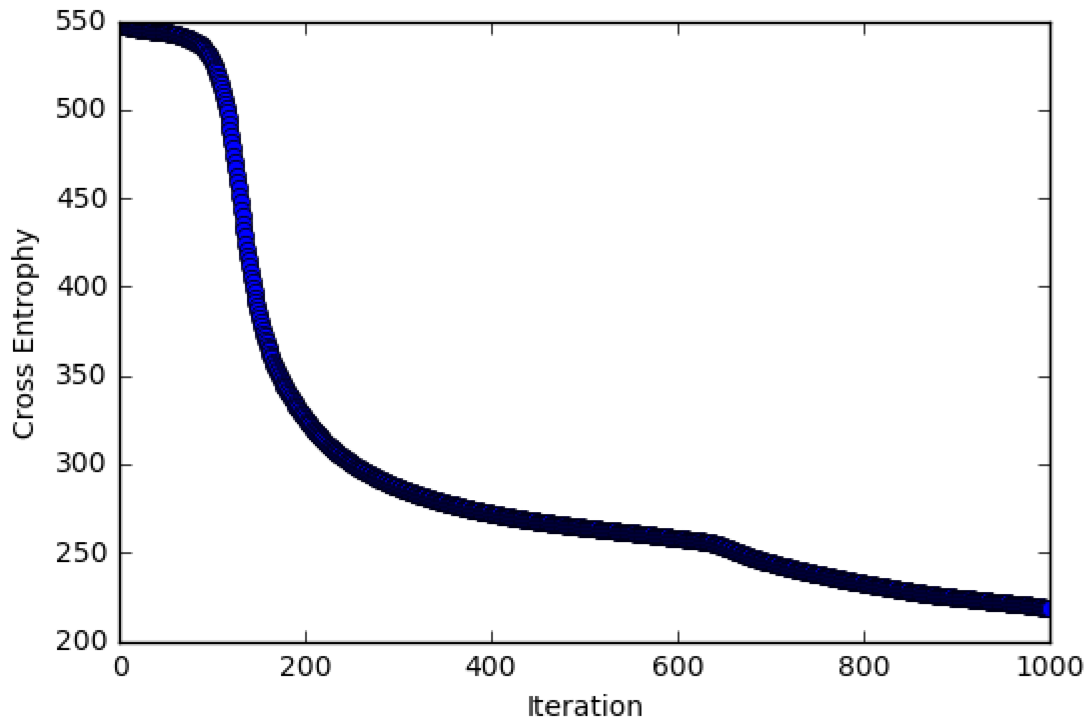
In [29]:
```
losses,errors = run_model(X,Y,cl, iterations = 1000,
                regularization_include = False,
                momentum_include = False)
```

```
---- Iteration 0 ----
Error rate : 0.6726907630522089
Loss: 548.7705600123807


---- Iteration 50 ----
Error rate : 0.6666666666666666
Loss: 543.5232720356951


---- Iteration 100 ----
Error rate : 0.6465863453815262
Loss: 527.1995540767311


---- Iteration 150 ----
Error rate : 0.3453815261044177
Loss: 384.309338047535


---- Iteration 200 ----
Error rate : 0.321285140562249
Loss: 325.51905381849633


---- Iteration 250 ----
Error rate : 0.2891566265060241
Loss: 300.2216633608075


---- Iteration 300 ----
Error rate : 0.28313253012048195
Loss: 286.75911337051434


---- Iteration 350 ----
Error rate : 0.28112449799196787
Loss: 278.1486305692009


---- Iteration 400 ----
Error rate : 0.27710843373493976
Loss: 272.07470991164035


---- Iteration 450 ----
Error rate : 0.2751004016064257
Loss: 267.59194679499393


---- Iteration 500 ----
Error rate : 0.27309236947791166
Loss: 264.1535781001806


---- Iteration 550 ----
Error rate : 0.27309236947791166
```

```
Loss: 261.21026193681723


---- Iteration 600 ----
Error rate : 0.2570281124497992
Loss: 258.113346282331


---- Iteration 650 ----
Error rate : 0.22690763052208834
Loss: 253.07710516925917


---- Iteration 700 ----
Error rate : 0.18473895582329317
Loss: 244.1878518969607


---- Iteration 750 ----
Error rate : 0.1746987951807229
Loss: 237.81966219101628


---- Iteration 800 ----
Error rate : 0.16666666666666666
Loss: 232.63167364250842


---- Iteration 850 ----
Error rate : 0.1606425702811245
Loss: 228.06573546524916


---- Iteration 900 ----
Error rate : 0.1566265060240964
Loss: 224.55512650440684


---- Iteration 950 ----
Error rate : 0.1566265060240964
Loss: 221.7783080302591
```

In [30]:
```
plt.plot(losses,'o')
plt.ylabel("Cross Entrophy")
plt.xlabel("Iteration")
```

Out[30]: <matplotlib.text.Text at 0x2556acebb70>



Part e

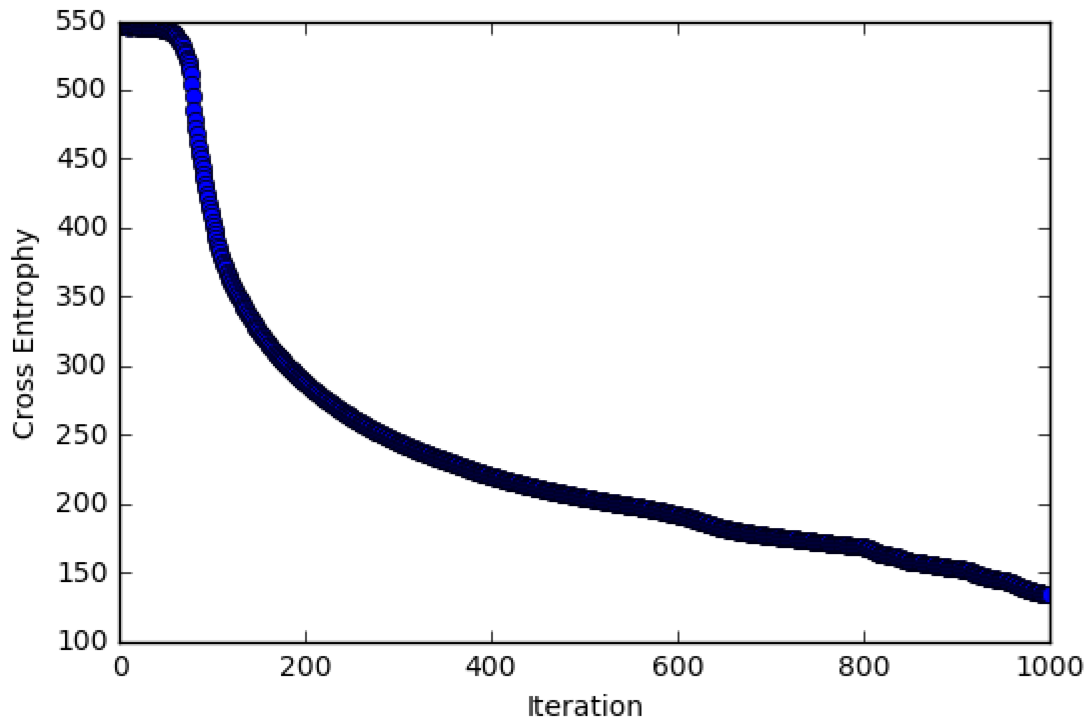
In [31]:
```
losses,errors = run_model(X,Y,cl, iterations = 1000,
                regularization_include = True,
                momentum_include = True)
```

```
---- Iteration 0 ----
Error rate : 0.6726907630522089
Loss: 548.7705347656686


---- Iteration 50 ----
Error rate : 0.6666666666666666
Loss: 543.4736304339127


---- Iteration 100 ----
Error rate : 0.37349397590361444
Loss: 404.67805662988127


---- Iteration 150 ----
Error rate : 0.2791164658634538
Loss: 324.8675658551154


---- Iteration 200 ----
Error rate : 0.2248995983935743
Loss: 287.4350946793522


---- Iteration 250 ----
Error rate : 0.1706827309236948
Loss: 262.3423827471351


---- Iteration 300 ----
Error rate : 0.14457831325301204
Loss: 244.41749453961773


---- Iteration 350 ----
Error rate : 0.13453815261044177
Loss: 231.1819930715339


---- Iteration 400 ----
Error rate : 0.12248995983935743
Loss: 219.84231264457847


---- Iteration 450 ----
Error rate : 0.11044176706827309
Loss: 211.0746633305445


---- Iteration 500 ----
Error rate : 0.10240963855421686
Loss: 204.13850363907494


---- Iteration 550 ----
Error rate : 0.09036144578313253
```

```
Loss: 198.2229019248392


---- Iteration 600 ----
Error rate : 0.09036144578313253
Loss: 191.5846169411418


---- Iteration 650 ----
Error rate : 0.08032128514056225
Loss: 181.70798614241897


---- Iteration 700 ----
Error rate : 0.0823293172690763
Loss: 176.28770293463523


---- Iteration 750 ----
Error rate : 0.0823293172690763
Loss: 172.15947591533467


---- Iteration 800 ----
Error rate : 0.07630522088353414
Loss: 168.25112396319537


---- Iteration 850 ----
Error rate : 0.06827309236947791
Loss: 157.8333662512769


---- Iteration 900 ----
Error rate : 0.06827309236947791
Loss: 152.60298929609502


---- Iteration 950 ----
Error rate : 0.06626506024096386
Loss: 144.05242654552166
```

In [32]:
```
plt.plot(losses,'o')
plt.ylabel("Cross Entrophy")
plt.xlabel("Iteration")
```

Out[32]: <matplotlib.text.Text at 0x2556ad63e48>



It is obvious that using Momentum can make the algorithm to converge more efficiently. In the 1000th iteration, using Momentum can give us a Cross Entropy that is less than 150, while the Cross Entropy is greater than 200 without Momentum.