

One-dimensional Transient Heat Conduction in a semi-infinite Domain

by CTTC

1. Exercise

- Write a computer program (according to the specifications given in section 4) to solve the heat conduction equation in the situation described in section 2.
- Ensure that the code is correct
- Choose a suitable mesh.
- Run the simulation and submit us the files/information requested in section 3.
- We will check the code and the results:
 - If the code and results are correct, we will ask you to write a short report about the work made and then you passed the test.
 - If the code looks good but there are problems, such as not enough accuracy, wrong programming style, etc, we will help you to enhance it and we will give you more opportunities.
 - We will not accept candidates who have used software not developed by themselves OR have given their own codes to other persons.

Comments:

- You must write your own code, not use already available software.
- A normal PC is enough to do this exercise.
- This is a personal problem, you can not ask for help of other persons.
- Don't give this problem to other persons.
- If you have questions about how to do this exercise, we suggest you to read the chapters 1-4 of the book "Numerical Heat Transfer and Fluid Flow" [1].
- If you have a question about the exercise please ask us.

2. Problem definition

We are interested in the temporal evolution of the ground temperature distribution. A one-dimensional time-dependant heat conduction equation will be assumed to be valid to model the ground temperature (therefore, neglecting humidity changes or other aspects that may be actually relevant),

$$\rho c_p \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2} \quad (1)$$

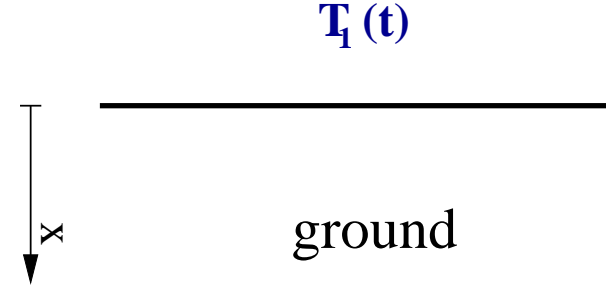


Figure 1: General schema of the proposed problem

The surface in contact with the outdoor ambient will be at $x = 0$ (see figure 1). The ground is assumed thick enough to be treated as a semi-infinite domain. The heat transfer coefficient h can be considered constant, but the temperature of the medium changes follows the law

$$T_1(t) = A_0 + A_1 \sin(\omega_1 t) + A_2 \sin(\omega_2 t) \quad (2)$$

where t is time in seconds and

$$\omega_1 = \frac{2\pi}{24 \cdot 3600} ; \quad \omega_2 = \frac{2\pi}{365 \cdot 24 \cdot 3600} \quad (3)$$

The initial temperature of the ground is $T = T_0$.

$\rho [Kg/m^3]$	$c_p [J/Kg]$	$k [W/mK]$	$h [W/m^2K]$	$T_0 [K]$	$A_0 [K]$	$A_1 [K]$	$A_2 [K]$
2300.00	700.00	1.60	8.50	19.00	19.00	5.70	9.10

Table 1: Physical parameters

3. Information requested

We want to obtain a description of the thermal behaviour of the medium according to the previous model. The student shall carry out all the simulations and present the plots that he/she considers necessary in order to describe the thermal behaviour of the ground. The plots should be presented with a short written description of the most relevant features. The student must find the mesh size, time step and domain length appropriated to obtain a good approximation of the correct solution.

Moreover, you must submit us the following.

- Your code (see section 4, for details), named "BNAME.c" where BNAME stands for your surname.
- A plot of the temperature at $x = 0.21m$ and $x = 1.00m$ for time between $t = 0s$ and $t = 1000 \cdot 24 \cdot 3600s$.

4. Code requirements

- You must write a C++, C or FORTRAN code that can be compiled in a Linux environment.
- You can not use libraries (such as linear algebra libraries, PDE solvers, etc) not developed by you.
- The code should be modular (split in subroutines), presented in a single file and compile with no errors.

- The code must run without any input parameter and produce the output file BNAME.dat, that must contain three columns with the following information:
 - Column 1: Time (in seconds).
 - Column 2: Temperature at position $x = 0.21m$ at the instant specified in column 1.
 - Column 3: Temperature at position $x = 1.00m$ at the instant specified in column 1.

5. Hints

The problem should be treated as one-dimensional. The ground is assumed thick enough to be treated as a semi-infinite domain. All physical properties are constant. The student must find the mesh size, timestep and domain length appropriated to obtain a good enough approximation. After code verification, show the influence of the different numerical parameters (mesh, time step, unsteady scheme, etc.) and physical parameters.

References

1. Suhas V. Patankar. *Numerical Heat Transfer and Fluid Flow*. Hemisphere Publishing Corporation, McGraw-Hill Book Company, 1980.