Mission Analysis Executive Report for a Mars Remote Sensing Observation Mission

Mathematics and Programming for Astrodynamics and Trajectory Design

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Abstract

The following project is an implementation of a trajectory design problem for a Mars Remote Sensing Observation Mission. The code used in this report introduces a methodology to solve Lambert's problem by first guessing the Lambert arc using a Minimum Energy Transfer trajectory. Subsequently, the spacecraft is shot and propagated to Mars. As this initial guess may not be the correct one since Mars' position is not static, hence, a differential corrector (shooting method) will be implemented by means of a State Transition Matrix. Then, a Continuation Method will be implemented to improve the robustness of the code in general. Finally, the code will output ΔV and mass pork chop plots. The output of this project is to provide a solid launch window for the mission.

Keywords: Lambert problem, Minimum Energy Transfer, Continuation Method, Astrodynamics, Patched comics

I. INTRODUCTION

The Lambert problem intends to determine the orbital trajectory between two objects by means of a specific time of flight. The goal of this study is to determine the optimal trajectory between Earth and Mars in terms of the minimum ΔV required for a transfer between the two planets and provide a suitable launch window.

The requirements for this mission are listed below:

REQ- **Mission Requirements**

REQ-01. Mission shall be launched between 01/01/2024 and 31/12/2025.
REQ-02. Launcher is a Falcon 9 Heavy.
REQ-03. Orbiter's mass is 1250 kg.
REQ-04. Propulsion system Isp is 310s.
REQ-05. Targeted final orbit is a 400 km altitude circular orbit.

II. METHODOLOGY

A. *Overview*

The following section aims to detail the procedure followed to explain the code developed (see 1).

The code begins by setting up the workspace and defining necessary parameters such as the spacecraft mass, specific impulse, gravitational constants of the celestial bodies, and other constants. Then, the departure and arrival dates are defined and converted to Julian Epoch for convenience.

Next, the script creates a grid of departure and arrival times between the Earth and Mars for a given range of time or Time of Flight (TOF). The code then calculates the necessary delta-V for a spacecraft to transfer between Earth and Mars at each departure and arrival time using a Minimum Energy Transfer trajectory, propagated and corrected using a differential corrector and finally enhanced by means of a continuation method.

The script finally computes the total ΔV for the spacecraft travelling from Earth to Mars and solved for both short ($t_m = +1$) and long ($t_m = -1$) transfer methods. Eventually, the script automatically selects the better of the two paths and plots the pork chop plot for the total ΔV required to perform the transfer and the dry mass pork chop plot at arrival.

B. *Algorithm*

First, the main constants are defined, namely, the spacecraft's performance parameters and also the Earth's, Sun's and Mars' astronomical constants. Subsequently, the potential departure dates are defined, from 01/01/2024 to 31/12/2025 giving a time span of 2 years. Notice, how the TOF (Time of Flight) is also defined here with an average duration of 215 days. Once the TOF is defined, it is now possible to compute the departure and arrival days grid times.

With all the variables defined, the following task is to create a double-nested loop to compute the both ΔV and dry masses for each pair of departure and arrival date and plot them into a Pork Chop plot for clear visualisation.

`EphSS_car.m` will return the ephemerides of Earth and Mars for the departure and arrival date.

The Minimum Energy Transfer (MET) will be used as a first guess in an iterative process to determine the trajectory between the two endpoints [1].

The Initial Value Problem for position and velocity as a function of time is what the MET will try to find. To do so, Lagrange Coefficients will be used to find the solution to the orbital motion.

`MinETranfer.m` shows how the minimum energy transfer is done. This minimum energy transfer is performed using an orbit with the minimum orbital size since the mechanical energy ϵ only depends on the semi-major axis a. The problem with this transfer is that, in reality, the

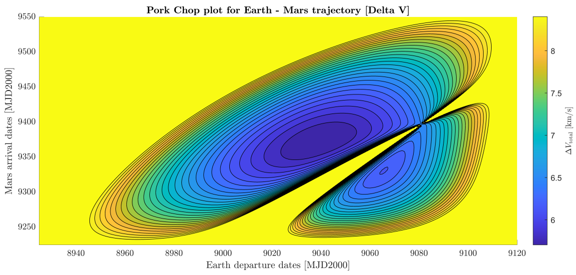

Figure 1: Total ΔV pork chop plot for Earth-Mars transfer. Source: Own.

spacecraft's final position state and Mars's final position state are not the same and deviate.

To tackle that problem, a Differential Corrector (or Shooting Method) is added. Ultimately, a State Transition Matrix (STM) relates the variations in the initial state to variations in the final state [`STM_Lambert.m`]. It is a sensitivity matrix that aims to correct and reach the final initial position by correcting the initial position or velocity. In this project, the correction is only performed on the initial velocity. Hence, the objective of the differential corrector is to determine which modification of $\vec{r}(t_0)$ and $\vec{v}(t_0)$ has to be applied to reach a desired state $\vec{r}(t_f)$ and $\vec{v}(t_f)$. To do so, a first-order Taylor expansion is performed. This is an iterative process, meaning it has to go through several iterations to reach the desired solution. Besides, the STM calculated is a closed-form solution.

`FGKepler_dt.m` is a function that given one position and one velocity and the TOF will compute where the position of the spacecraft $\vec{r}_2$ will be and its velocity. It also solves the Eccentric anomaly ΔE from Kepler's equation using MATLAB's `fzero` function.

Then, a continuation method is implemented to increase the robustness of the script. This continuation method relies on giving weightings for the desired TOF and the initial TOF. This way, even if the initial guess is far away from the result it will provide intermediate solutions that will be increasingly closer to the desired solution.

Finally, the ΔV costs are estimated using patched comics (in this case link conics as the assumption made here is that the radius of the Sphere Of Influence (SOI) is 0). Simultaneously, the final dry mass of the spacecraft to Mars is also plotted as a pork chop diagram.

The algorithm of the script is detailed in 1.

III. RESULTS

Let's now discuss the resultant plots obtained. Eligibility criteria require looking for not only a launch window that has associated the lowest total ΔV and the maximum insertion mass at arrival but also flexible enough to satisfy the requirements, with a launch window span of at least 2 weeks for margin.

Figure 1 illustrates the total ΔV that the spacecraft has to perform to reach Mars. Notice how the minimum ΔV is located in the long path lobe is the one that actually enables the least amount of ΔV. Inside the darkest lobe from the longest path, ΔV ranges from 5.8 km/s in a launch window of approximately 30 days from 15/09/2024

Algorithm 1 Script's algorithm

Calculate astronomical parameters
Initialise departure and arrival dates grids
Declare launcher performance variables
for all departure dates **do**
  for all arrival dates **do**
    if $\Delta t_{\text{target}} \neq \Delta t_{\text{minE}}$ **then**
      Compute $\vec{r}_2$ `LambertArc.m`
      Calculate the TOF
      Compute $\vec{r}_1 = \vec{r}_{\text{minE}}$
      for N iterations **do**
        Calculate λ parameter
        Get weighted Δt
        while error>tolerance and i<N **do**
          Compute $\vec{r}_2$ `FGKepler_dt.m`
          if $\|\vec{r}_{2\text{target}} - \vec{r}_2 < \varepsilon\|$ **then**
            $i = i + 1$
          else if Compute STM `STM_Lambert.m` **then**
            Apply correction to $\vec{r}_1$
            Compute new $\vec{r}_2$
          end if
        end while
      end for
    end if
    Compute the ΔV for $t_m = \pm 1$
    Compute the final dry mass
  end for
end for
Plot results

to 15/10/2024 and arriving to Mars between 11/08/2025 (331 days) and 13/09/2025 (334 days). On the other hand, regarding the short transfer method, the lowest ΔV is about 6.9% higher compared to the long t_m with a $\Delta V = 6.2$ km/s. Additionally, as seen from the diagram, another major issue with the short path is the narrow 3-day launch window associated with that ΔV. Nonetheless, increasing the $\Delta V = 6.30$ km/s helps to extend the launch dates to around 16 days, however, the arrival dates to Mars are also more drifted apart, namely, departing on 19/10/2024 to 04/11/2024 with a separation of 47 days and arriving to Mars on 27/06/2025 (252 days) to 13/08/2025 (283 days).

In terms of maximum insertion mass at arrival, Figure 2 gives an idea of the maximum amount of insertion mass that will be available for our mission. Notice how the minimum dry mass at arrival shall be at least 1250 kg as it is the orbiter's dry mass. Consequently, the real available insertion mass pork chop plot can be filtered for values higher than that (see Figure 3). Besides, another insight that may have been drawn from this diagram is that a Hohmann transfer is also not available as the final insertion mass at arrival would be less than 1250 kg. Recall also that Hohmann transfers assume circular and coplanar orbits.

The results from the valid insertion mass diagram (Figure 3) are exceedingly surprising. The observed difference between short and long transfer methods differs vastly. To begin with, the maximum amount of insertion mass that the F9 launcher is able to put into Mars orbit is

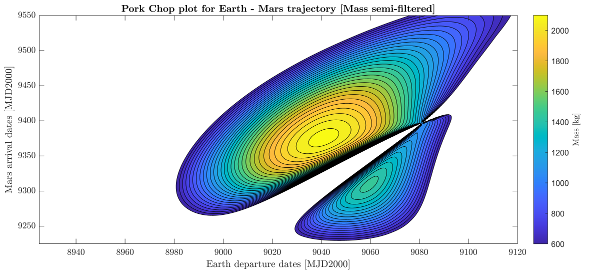

Figure 2: Pork chop plot for the maximum insertion mass of the spacecraft at arrival to Mars. Source: Own.

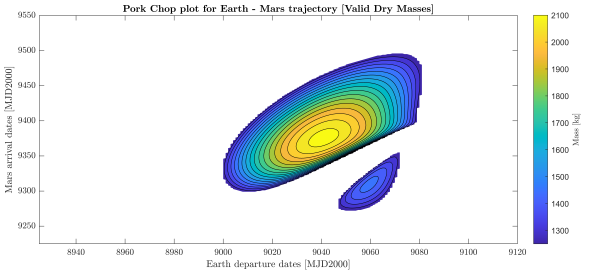

Figure 3: Filtered Pork chop plot for the maximum insertion mass of the spacecraft at arrival to Mars. Source: Own.

around $m_{\max} = 2100$ kg providing 850 kg for instruments and propellant. Conversely, the short path is capable of providing only $m_{\max} = 1460$ kg which means that only 210 kg is free for other subsystems.

It is also important to bear in mind the launch windows that both t_m enable. The long transfer method corresponding to the $m_{\max} = 2100$ kg can be first launched on 27/09/2024 up until 07/10/2024 and arriving on Mars 26/08/2025 (334 days) to 23/09/2025 (348 days). By contrast, the short path is only 8 days long to achieve the aforementioned maximum insertion mass of 1460 kg. By reducing the available dry mass to 1410 kg will have an effect of an increase in the launch window to around 13 days, departing between 14/10/2024 to 27/10/2024 and arriving on Mars between 09/06/2025 (239 days) and 19/07/2025 (265 days).

As seen from the previous results, several trade-offs can be extracted depending on the goal of the mission:

- Lowest ΔV
- Fastest arrival
- Minimum v_∞ at arrival
- Maximum insertion mass at arrival
- Best overall

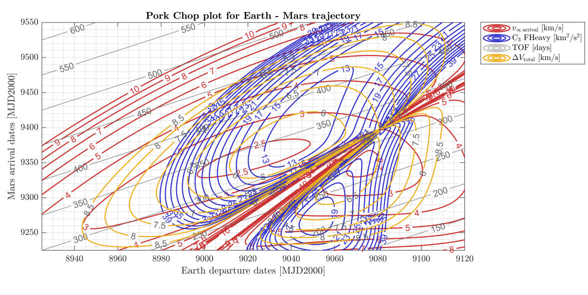

Figure 4: Overall Pork Chop plot. Source: Own.

To assess the overall best option, in an ideal scenario, it would be beneficial to decrease all three factors and allow the maximum mass. However, as TOF and propellant consumption is usually inversely related, some trade-offs have to be made. For this case, it is advisable to travel using the long transfer method as it accommodates the maximum payload and a wider launch window as the TOF is not constrained. Thereupon, the optimal option between lowest ΔV, maximum insertion mass at arrival and a safe v_∞ at arrival shall be traded off. An option that validates all mission requirements, reduces the propellant and allows the largest payload possible for the spacecraft is to launch between 22/09/2024 to 06/10/2024 (15 days) and arrive on Mars between 17/08/2025 (330 days) and 18/08/2025 (317 days). This launch window allows for a maximum dry mass at the arrival of $m_{\max} = 2060$ kg, meaning that 40% of the spacecraft's mass can be used for payload, subsystems and propellant. Moreover, the required total $\Delta V = 5.75$ km/s approximately which is one of the lowest accessible options. The associated C3 parameter of the F9 Heavy to this configuration is also one of the lowest available, with a $C3 \approx 13$ km^2/s^2. Furthermore, taking a look at the arrival excess velocity in Figure 4, the fact that it arrives with an excess velocity of $v_\infty \approx 2.6$ km/ sec is significant considering that the spacecraft would not require an excessive amount of propellant to inject to the operational orbit.

Also, the best option for each scenario has been analysed in Table I:

Table I: Launch window scenarios. Source: Own.

Scenario	Departure window	Arrival window	ΔV [km/s]	$m_{\max}$ [kg]
Lowest ΔV	23/09/2024 06/10/2024	27/08/2025 27/08/2025	5.73	2089
Maximum mass at arrival	29/09/2024 07/10/2024	31/08/2025 01/09/2025	5.74	2100
Best overall	22/09/2024 04/10/2024	17/08/2025 18/08/2025	5.75	2060

IV. CONCLUSIONS

This report has performed a mission analysis for a Mars Remote Sensing Observation Mission. The aim of this report is to provide a valid launch window for the mission. A script is developed in MATLAB to solve this trajectory problem using a Minimum Energy Transfer trajectory, propagated with a Differential Corrector and consolidated using a Continuation Method. The output of the script are the total ΔV Pork Chop plot, the maximum dry mass Pork Chop plot as well as a combined Pork Chop plot. Several scenarios were assessed and their respective launch window was given. Overall a highly advisable launch window is between 22/09/2024 to 04/10/2024.

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