ESCOLA TÈCNICA SUPERIOR D'ENGINYERIES
INDUSTRIAL I AERONÀUTICA DE TERRASSA

Grau en Enginyeria en
Tecnologies Aeroespacials
Propulsió, curs 2019 – 2020

PROJECT

Team-Work (maximum 4 people)
Deadline: June 24th 2020

Description

Consider a launch vehicle with an initial mass of 540 t, consisting of two stages, the first one equipped with a rocket with the following features:

- Combustor pressure $p_c = 100$ atm
- Combustor temperature $T_c = 3500$ K
- Propellant gas properties $\gamma = 1.25$; $MW = 16$ g mol^{-1}
- Propellant mass flow $\dot{m} = 2000$ kg s^{-1}
- Burning time $t_b = 60$ s.

The drag reference cross sectional area of the vehicle is $S_{ref} = 25$ m^2 and the drag coefficient C_D is a function of the flight Mach number, as indicated in Annex 1.

For a vertical ascent, assuming ISA atmosphere, the following is requested:

1. Plot the altitude and velocity vs. time, for area ratios $A_e/A_t = 20$, 40, 60, 80, neglecting the aerodynamic drag.
2. Repeat the computations and plots including the aerodynamic drag.
3. Accounting for the above results (of question 2), indicate what of the above area ratios is the best choice (obviate the effects on weight).

Remarks:

- Equations for thrust and drag

$$F = C_F p_c A_t \qquad C_F = C_{F_v} - \frac{p_a}{p_c}\frac{A_e}{A_t}$$
$$D = \frac{1}{2}\rho V^2 S_{ref} C_D$$

- Use ISA equations according to the formulas for each interval.
- The leapfrog numerical integration scheme (Annex 2) is optional, and needn't be used if the student is familiar with other integration schemes.

ANNEXES

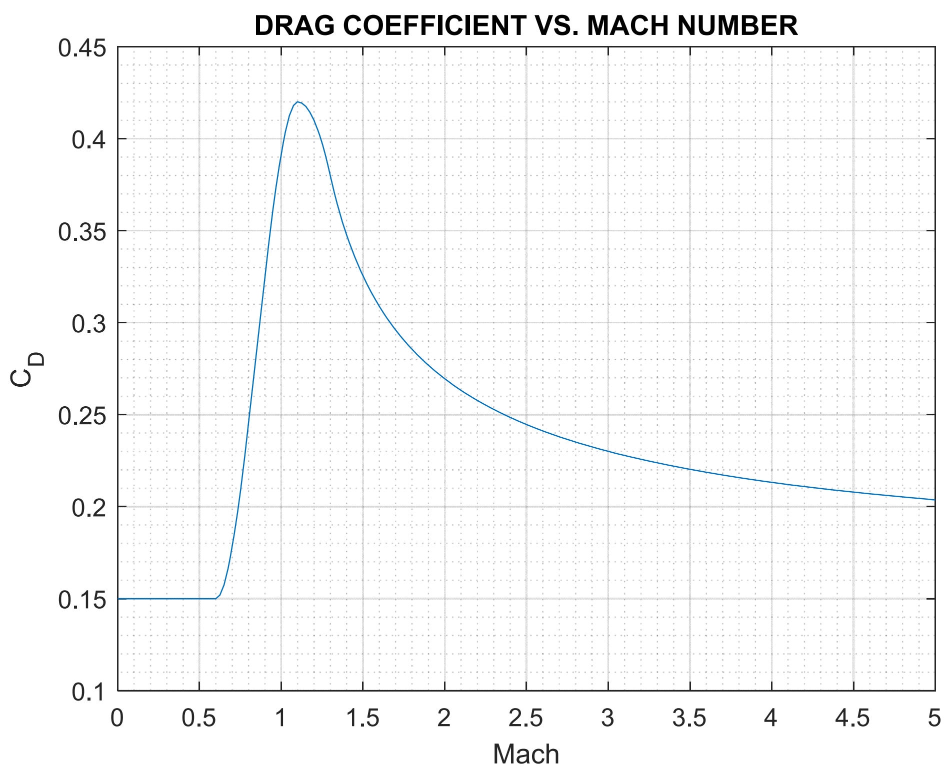
Annex 1. Drag coefficient

$M \leq 0.6 \rightarrow C_D = 0.15$

$0.6 < M \leq 1.1 \rightarrow C_D = -4.32M^3 + 11.016M^2 - 8.5536M + 2.24952$

$1.1 < M \leq 1.3 \rightarrow C_D = -M^2 + 2.2M - 0.79$

$1.3 < M \leq 5.0 \rightarrow C_D = 0.16769 + \dfrac{0.17636}{\sqrt{M^2 - 1}}$



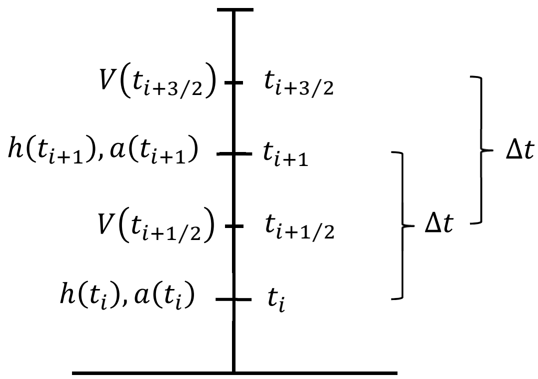
Annex 2. Leapfrog numerical integration

O.D.E. system:

$$\frac{dh}{dt} = V$$

$$\frac{dV}{dt} = \frac{F - D}{m(t)} - g = a$$

Initial conditions: $h(0) = 0, V(0) = 0$



Discretization (with $t_i = i\Delta t$):

$$h(t_{i+1}) = h(t_i) + V(t_{i+1/2})\Delta t$$

$$V(t_{i+3/2}) = V(t_{i+1/2}) + a(t_{i+1})\Delta t$$

For $i = 0$

$$V(t_{1/2}) = V_0 + a(t_0)\frac{\Delta t}{2} = a(t_0)\frac{\Delta t}{2}$$

For $i \geq 1$, the computation of $a(t_i)$ involves $V(t_i)$, and aiming at 2nd order accuracy, this can be obtained in two steps (predictor-corrector),

$$V_{pred}(t_i) = V(t_{i-1/2}) + a(t_{i-1/2})\frac{\Delta t}{2} \;\; \rightarrow \;\; a_{pred}(t_i) = \frac{F(t_i) - D_{pred}(t_i)}{m(t_i)} - g$$

$$V_{corr}(t_{i+1/2}) = V(t_{i-1/2}) + a_{pred}(t_i)\Delta t$$

$$V(t_i) = \frac{1}{2}\left[V(t_{i-1/2}) + V_{corr}(t_{i+1/2})\right] \;\; \rightarrow \;\; a(t_i) = \frac{F(t_i) - D(t_i)}{m(t_i)} - g$$

$$V(t_{i+1/2}) = V(t_{i-1/2}) + a(t_i)\Delta t$$