Optional exercise: Nonlinear 1D equilibrium

A bar of length L and varying cross-sectional area

$$A(x) = A_0 \left(1 + 2\frac{x}{L}(\frac{x}{L} - 1)\right) \tag{0.4}$$

where $A_0 > 0$, is fixed at one end while the other end is subjected to a linearly increasing displacement $u_L(t) = u_m \frac{t}{T}$, where $u_m > 0$ and $t \in [0, T]$, $T > 0$ being the interval of time to analyze —this interval is considered sufficiently large so as to ignore inertial effects. On the other hand, the material of the bar obeys the following (exponential) constitutive equation (relation between stress σ and strain ε)

$$\sigma = \sigma_0 \left(1 - e^{-\frac{E}{\sigma_0}\varepsilon}\right). \tag{0.5}$$

Using the Finite Element method, find the displacement solution $u = u(x, t)$ as a function of $t \in [0, T]$ and $x \in [0, X]$. To this end, follow the steps outlined below.

1. Formulate the strong form of the boundary value problem for the 1D equilibrium of a bar with varying cross-sectional area.
2. Determine the weak form of the problem formulated above.
3. Formulate the corresponding matrix equations.
4. Solve the resulting system of nonlinear equations by means of a *Newton-Raphson algorithm.*

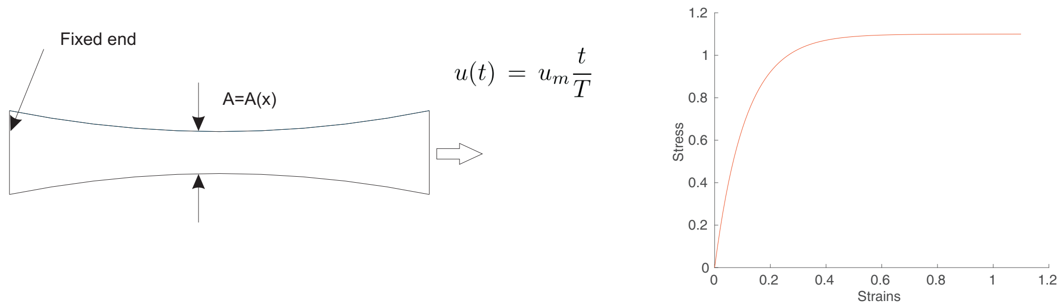

Figure 1 *Geometry and constitutive equation*

5. Check that the solution converges upon increasing number of elements and time steps.

DATA: $E = 10\ MPa$; $\sigma_0 = 1\ MPa$; $L = 1$ m; $A_0 = 10^{-2}\ m^2$; $u_m = 0.2L$