Assignment 3 – Beams

Consider the spar of a typical aircraft, modelled as a 1D beam with a Young Modulus E = 71 GPa and the I-shaped cross-section area depicted in Figure 1-a. The aircraft flies in static equilibrium in the vertical direction, so the spar must support the weight of the overall plane and an aerodynamic load distributed along the wingspan as shown in Figure 1-b.

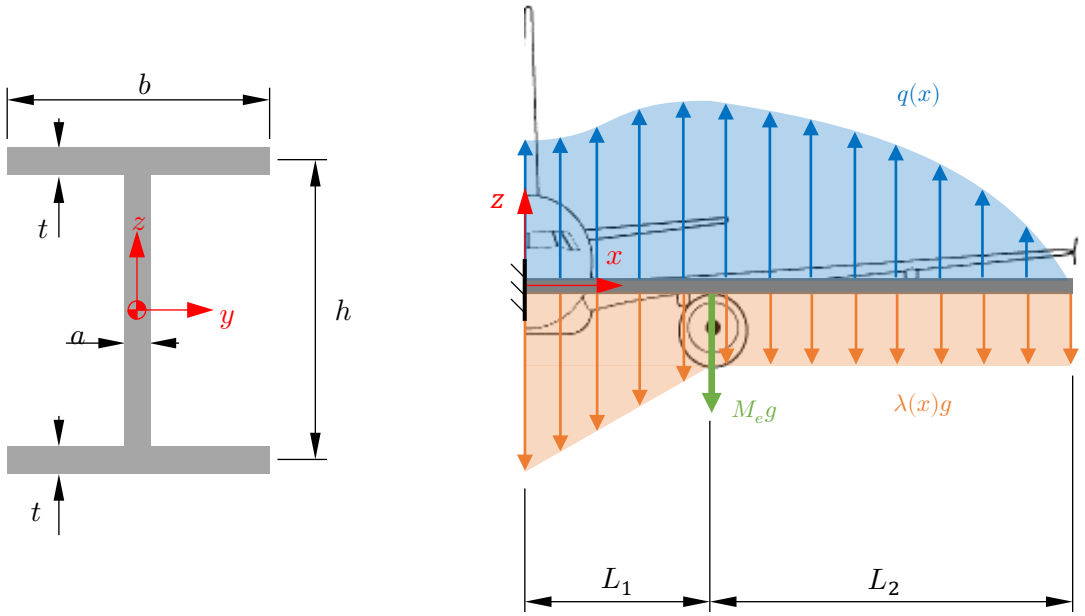


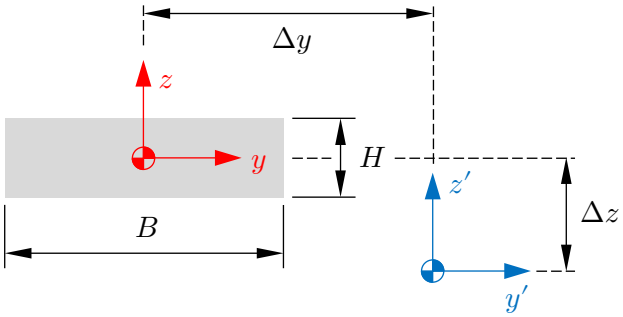
Figure 1. Problem setting: (a) Geometrical parameters of the spar's cross-section. (b) Load distribution on the spar.

Questions:

1. Compute the cross-section inertia for $a = 30$ mm, $b = 105$ mm, $h = 900$ mm and $t = 6$ mm. Hint:

$$I_z = \frac{1}{12}B^3H \qquad I_{z'} = I_z + BH\Delta y^2$$

$$I_y = \frac{1}{12}H^3B \qquad I_{y'} = I_y + BH\Delta z^2$$



2. Compute the parameter ℓ that makes the whole structure to be in static equilibrium considering:

   - Aerodynamic force density distribution:

$$q(x) = \begin{cases} \ell\left[0.85 - 0.15\cos\left(\pi\dfrac{x}{L_1}\right)\right] & \text{for } x < L_1 \\ -\ell\dfrac{(L_1 - L_2 - x)(L_1 + L_2 - x)}{L_2^2} & \text{for } x \geq L_1 \end{cases} \quad \frac{\text{N}}{\text{m}}$$

   - Mass density distribution:

$$\lambda(x) = \begin{cases} \dfrac{3M}{2L_1^2}(L_1 - x) + \dfrac{M}{4(L_1 + L_2)} & \text{for } x < L_1 \\ \dfrac{M}{4(L_1 + L_2)} & \text{for } x \geq L_1 \end{cases} \quad \frac{\text{kg}}{\text{m}}$$

   with $L_1 = 6$ m, $L_2 = 12$ m, and $M = 36000$ kg, $M_e = 2360$ kg (A-320). Consider the gravity acceleration 9.81 m/s^2.

3. Implement a MATLAB® code to numerically compute the displacement, rotation, shear force and bending moment distributions along the spar for the given condition and analyse the convergence of the numerical solution to the "analytical solution" (solution with the finest discretization) evaluating the relative error of spar deflection at the wing tip for various number of elements considered for the discretization:

$$\epsilon_r(n_{el}) = \frac{\left|\hat{u}_{y,\text{num}}^{\text{tip}}(n_{el}) - u_{y,*}^{\text{tip}}\right|}{u_{y,*}^{\text{tip}}}$$

   Consider two subdomains $x \in [0, L_1]$ and $x \in [L_1, L_1 + L_2]$ discretized with n_{el} elements each part. Solve for the following set of $n_{el} = \{2,4,8,16,32,64,128\}$.

The assignment can be done in groups of maximum 2 people. Only one of the members must submit a compressed (**.zip**) file to Atenea containing the following:

- All MATLAB® script files used for part 3 of the assignment. There must be one executable script file, which must be named '**main_03**'.
- A report including:
  - Names of the group members
  - For question 1 and 2:
    - Requested results.
  - For question 3:
    - Plots of the displacements, rotations, shear force and bending moments for the numerical solutions with the given loading conditions for several number of elements (use the provided functions to obtain the plots).
    - Plot of the relative error ϵ_r vs. the number of elements, n_{el} in logarithmic scale.

Note 1: The report can be written in Catalan, Spanish or English and both technical and presentation aspects will be considered in the grading.