COMPUTATIONAL AEROSPACE ENGINEERING

ASSIGNMENT 4

Students: IVÁN SERMANOUKIAN MOLINA, YI QIANG JI ZHANG

Professor: JOAQUÍN ALBERTO HERNÁNDEZ ORTEGA

Aerospace Engineering
The School of Industrial, Aerospace and Audiovisual Engineering of Terrassa
Polytechnic University of Catalonia

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Contents

1 Tasks

1. Adapt the matlab program *mainELASTOSTATIC.m* (developed in assignment 3) so that it can also compute the mass matrix of the studied mesh (see video Vid01_mass.avi).

- Add in the input data file the nodal densities of the body under study ($\rho = 2.7\text{g/cm}^3$):
- The function computing the mass matrix is to be invoked from function SolveElasFE.m (hint: follow the steps made for computing the stiffness matrix).

2. Develop a matlab program able to calculate the **natural frequencies and modes** of a given structure (see video Vid02_ modes.avi).

- The input data need for running this program is to be generated by the modified script *mainELASTOSTATIC.m*. More specifically, the required data are
  - Stiffness matrix: $\mathbf{K} \in \mathbb{R}^{\mathbf{3n_{pt}} \times \mathbf{3n_{pt}}}$
  - Mass matrix: $\mathbf{M} \in \mathbb{R}^{\mathbf{3n_{pt}} \times \mathbf{3n_{pt}}}$
  - List of unrestricted degrees of freedom ($\mathbf{l}$)

  For post-processing purposes (with GID), the following variables are also needed:
  - Coordinate matrix: $COOR$
  - Connectivity matrix: CN
  - Type of element: $TypeElement$
  - Position of the Gauss points: $posgp$
  - Name of mesh file generated by gid (for instance, "malla1.msh")

  The other input of the program is the number of modes to be analyzed.
- For plotting the deformed shapes associated to each node, use the file: GidPostProcessModes.m (note: copy all the files contained in folder NEWFUNCTIONS in the folder in which you have the program of Assignment 3.)
- In order to avoid running the script "mainELASTOSTATIC.m" each time you need the stiffness and mass matrices, store these and the other required variables in a binary matlab file ".mat" (by using the *save* function). The information saved in the corresponding binary file can be recovered by just employing the *load* function.
- Assess the performance of the developed code by computing the natural frequencies and modes of the boxed beam studied in assignment 3 .

3. Apply the program to calculate the dynamic response of the cantilever boxed beam studied in Assignment 3, Part 1, (using MESH 4) (see video Vid03_damp.avi).

In particular, analyze the dynamic response when the initial displacement is:

- The one caused by the distributed load on the surface $y = y_{max}$
- The one caused by the torque applied on the free end of the cantilever.

4. Examine which are the natural modes that are more dominant in each case —by plotting the initial amplitude of the temporal response for each mode.

5. The damping ratios $\bar{\xi}_i$ for each natural frequency should be an input of the program.

6. The interval of time to be studied should be m times the maximum natural period of the system $\left(T_1 = \frac{2\pi}{\omega_1}\right)$. Choose, for instance, $m = 40$.

7. Include in the modal approximation the first $n = 25$ modes.
8. To postprocess the computed solution, use the function

$$\boxed{\text{GidPostProcessDynamic(COOR, CN, TypeElement, DISP, NAME\_INPUT\_DATA, posgp, NameFileMesh, t);}}$$

where

- $t \Rightarrow nstep \times 1$ vector containing the discretization of the time interval under study (use 500 steps).
- $DISP$: $3n_{pt} \times nstep$ matrix formed by the nodal displacement solutions at each time step.
- NAME_INPUT_DATA: Name for identifying the corresponding results file (for instance "DYN-sol")

9. Generate videos using GID of the simulations corresponding to the case of $\bar{\xi}_i = 0.01$ for $i = 1, 2...n$ (for the two initial conditions). Attach a LINK to these videos (you can place the videos in Google Drive, for instance) in the report. Alternatively, you can generate a sequence of snapshots showing the temporal evolution of the motion of the structure.

2 Results

2.1 Undamped free vibration

An undamped free vibration analysis has been performed in order to compute both natural frequencies and vibration modes of the cantilever boxed beam.

Mode	**Natural freq.** ω_n	**Mode**	**Natural freq.** ω_n
1	12.01997	**14**	411.59268
2	12.01997	**15**	413.81163
3	67.20993	**16**	413.81432
4	67.21006	**17**	416.53752
5	71.58085	**18**	436.38765
6	127.18878	**19**	483.60922
7	165.46565	**20**	508.21020
8	165.46623	**21**	551.65740
9	215.24790	**22**	551.6617
10	283.46241	**23**	563.42142
11	283.46384	**24**	583.60405
12	360.44095	**25**	593.62623
13	381.40383		

Table 1 Natural frequencies for each mode shape

2.2 Theoretical estimation of the natural frequencies and modes

The theoretical expression to calculate the order of magnitude of the cantilever box beam's natural frequencies is given by [1]. This expression gives us a hint on the approximate order of magnitude of the results obtained in MATLAB.

$$\omega_n = \alpha_n^2 \sqrt{\frac{EI}{\rho A L^4}} \tag{1}$$

where the vibration mode is set by the coefficient $\alpha_n = \{1.875,\ 4.694,\ 7.885\}$ for $i = \{1,\ 2,\ 3\}$, respectively.

Besides, the following physical parameters were considered: $E = 70000\text{MPa}$, $\rho = 2700\,\text{kg/m}^3$, $L = 2\,\text{m}$ and $A = h_y \cdot h_z - (h_y - 2e) \cdot (h_z - 2e) = 0.04\,\text{m}^2$.

As regards the beamed box inertia, this was already computed in *Assignment 3*, being its value such that:

$$I_z = \underbrace{\left[\frac{1}{12} \cdot (h_y)^3 \cdot h_z\right]}_{\text{Outer square}} - \underbrace{\left[\frac{1}{12} \cdot (h_y - 2e)^3 \cdot (h_z - 2e)\right]}_{\text{Inner square}} \tag{2}$$

Replacing the values,

$$I_z = \underbrace{\left[\frac{1}{12} \cdot (0.25)^3 \cdot 0.25\right]}_{\text{Outer square}} - \underbrace{\left[\frac{1}{12} \cdot (0.25 - 2 \cdot 0.05)^3 \cdot (0.25 - 2 \cdot 0.05)\right]}_{\text{Inner square}} = 2.83333 \cdot 10^{-4}\ \text{m}^4 \tag{3}$$

where the direction of the inertia is indicated by the diagram of the figure below:

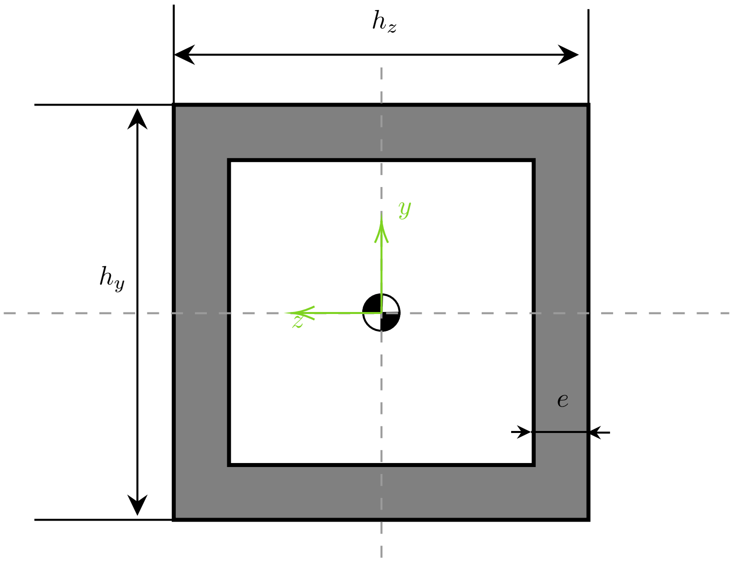


Figure 1 Cross section view

Hence, the first, second and third natural frequency calculated with equation (1) are shown below:

$$\omega_1 = 377.1083 \text{ rad/s}$$

$$\omega_2 = 2360.5304 \text{ rad/s}$$

$$\omega_3 = 6610.2184 \text{ rad/s}$$

Notice that the value differs with the order of magnitude with respect the one computed with MATLAB. The order of magnitude of this frequencies differs from the latter one, we believe that the problem may reside due to a possible misuse of the estimated frequency equations.

2.3 Mode shapes analysis

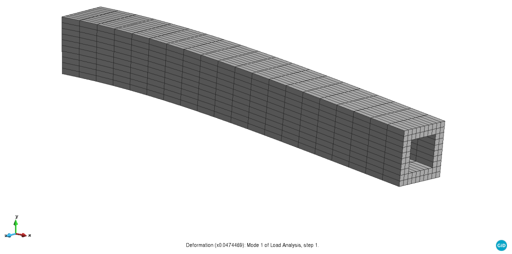
2.3.1 General description

Next, the different mode shapes of the beam is presented in the Figure 2 and 3. The first, second and third modes presents a how the structure bends. Notice how mode shape 5 considers the torsional rotation around x axis. Finally, the sixth mode presents the pure traction and pure compression of the beam.

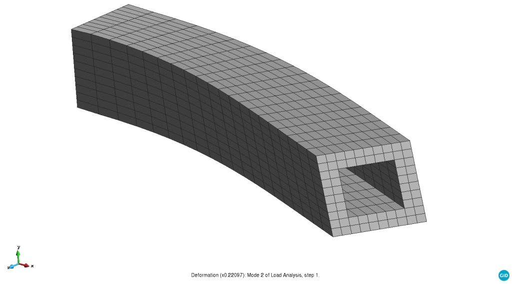
Eventually, for even higher modes, the motion of the structure gains more complexity. Those modes presents unintuitive variations and oscillations. Nevertheless, these modes tend to have few impact to the beam, since their amplitude overall is almost negligible. That said, the most important ones that can induce structure's collapse are clearly the first modes. It is simple to check the results (frequencies and modes) obtained by the stiffness matrix, mass matrix by checking the following product:

$$\text{MODES}' \cdot M_{LL} \cdot \text{MODES} = I \tag{4}$$

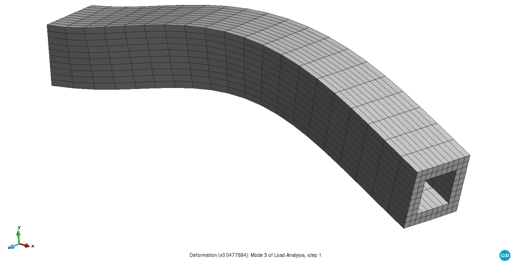
If the above expression is computed, we obtain the identity matrix I, thereby, validating the results.



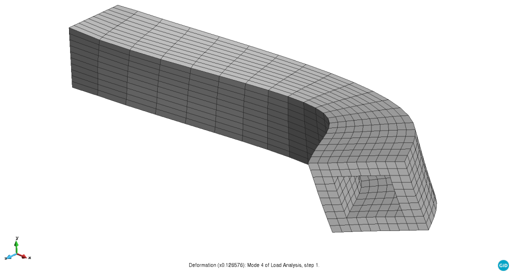
(a) 1st mode.



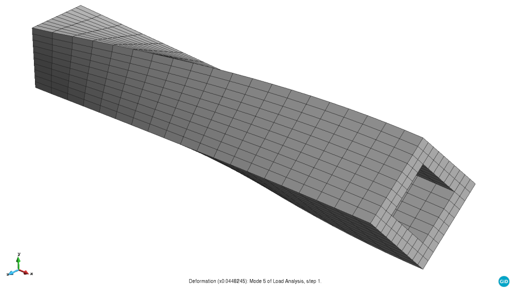
(b) 2nd mode.



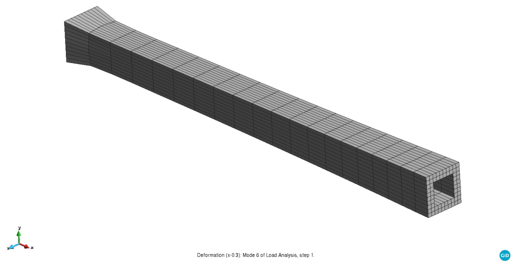
(c) 3rd mode.



(d) 4th mode.

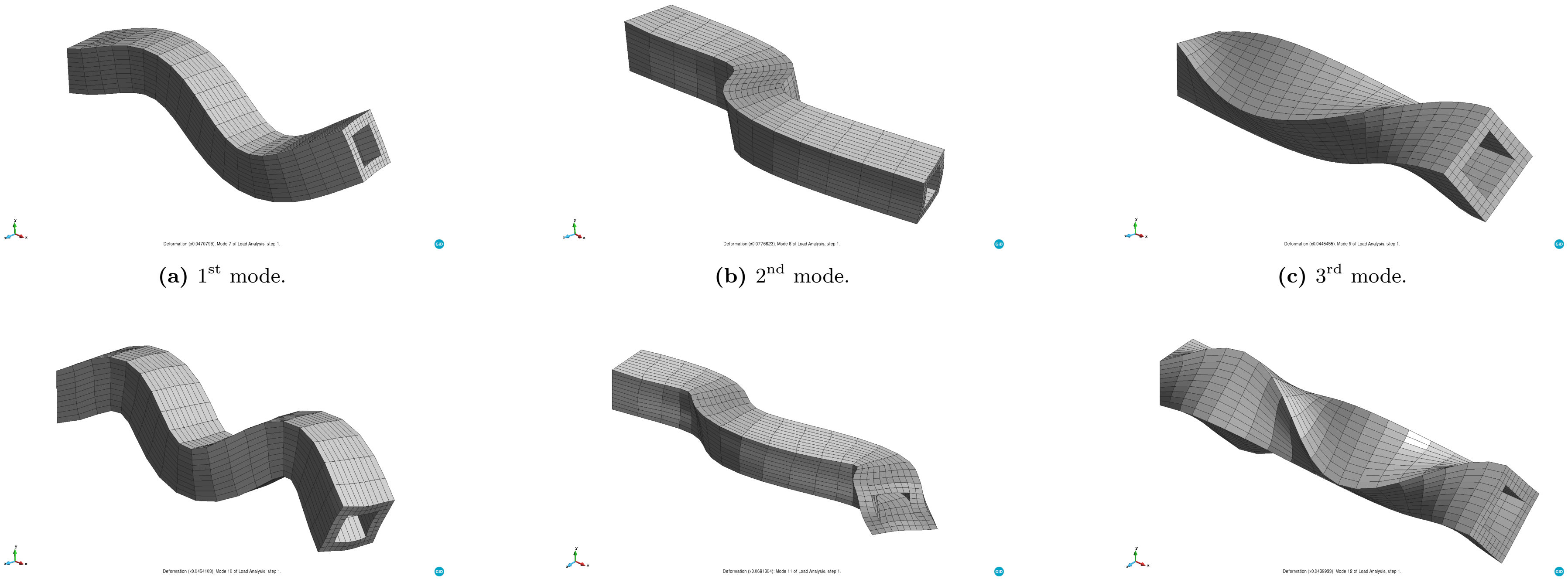


(e) 5th mode.



(f) 6th mode.

Figure 2 Vibration modes



(a) 1st mode.
(b) 2nd mode.
(c) 3rd mode.
(d) 4th mode.
(e) 5th mode.
(f) 6th mode.

Figure 3 Vibration modes

2.3.2 Mode shape dominance analysis comparison with distributed force

As shown in the previous section, the different modes shapes were presented showing their oscillation movement. However, not all mode shapes have the same weight in the structure. Consequently, the first mode shapes are the ones that has most amplitude. Take a look at Figure 4, the amplitude of the first mode shape is almost double the size of the second mode, and beyond the third one, the contribution is rather small or negligible.

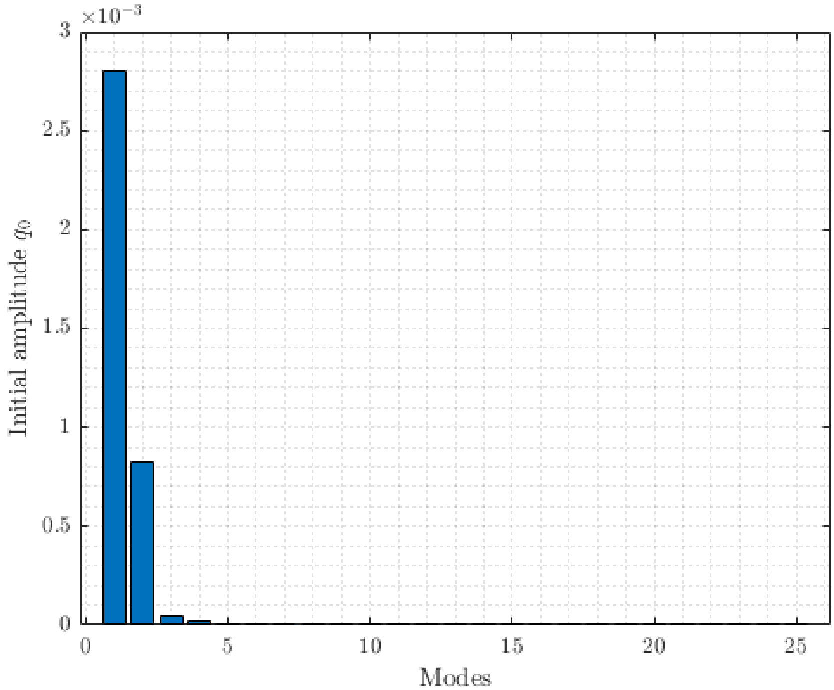


Figure 4 Contribution in amplitude of each mode shape in the distributed load case

The above results clearly affirms the oscillation we got to see in the GID analysis, the bending movement of the first mode is the one that impacts most the structure's integrity. This is very logical since the first mode motion coincides with deformation caused by the effect of the distributed force's direction.

2.3.3 Mode shape dominance analysis comparison with torsional force

Next, studying the torsional case, unlike the first scenario, the fifth mode is the one that has the greatest effect. This is explained as the 5th mode is related to the torsional excitation. Additionally, other torsional related modes are activated as well such as mode 9, 12 and 20.

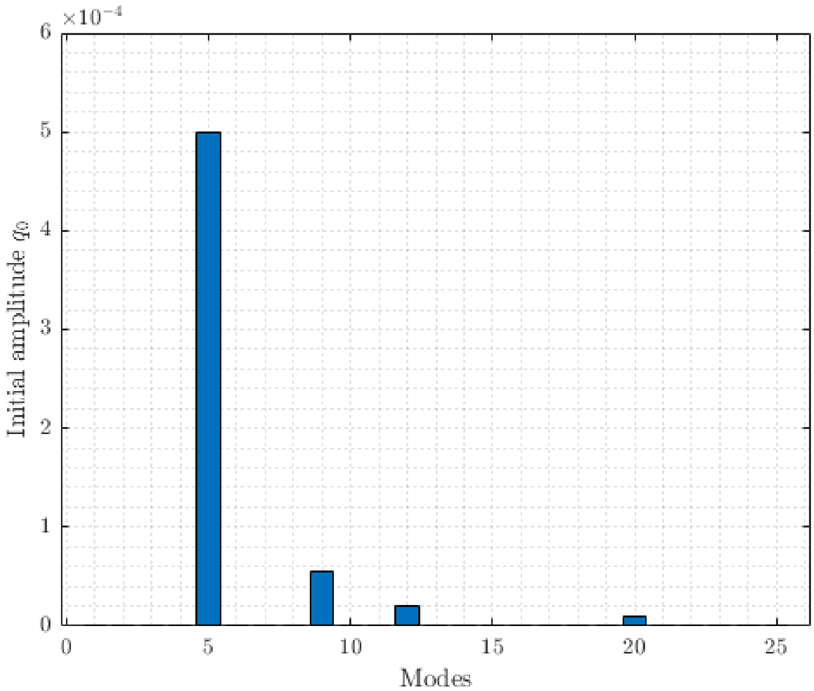


Figure 5 Contribution in amplitude of each mode shape in the torsional case

3 Study of the dynamic response of the beam

Continuing with the analysis, the following section intends to analyse the temporary response of the system with the node 1 initial position. This node is presented below:

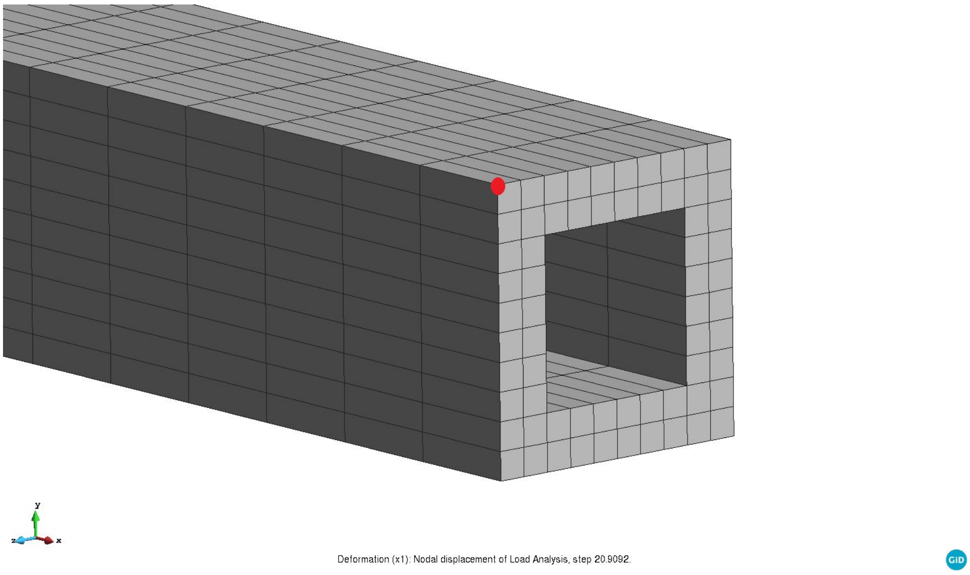


Figure 6 Position of node 1

3.1 Distributed load case

This first dynamic response analysis, we proceed to analyse the displacements caused by the distributed load. In order to get a proper analysis, the

The response points are extracted from the results achieved in Assignment

First, the response of the system when departing from the displacements caused by a distributed load. The whole deformed configuration is going to be extracted from the results achieved in Assignment 3. Owing to the subsequent temporal study of the vertical displacement of two specific nodes, their departure values are detailed in Table 2. This will provide a continuity reference between both assignments.

Node	Vertical displacement y [m]	Damping
1	-0.012247	Yes

Table 2 Initial displacements with a distributed load.

3.1.1 Undamped free vibration

The temporary response of the undamped case shows a sinusoidal displacement along the time. Since there is not any mechanism that reduces diminishes the displacement, its amplitude does not change and the amplitude remains constant. This movement is produced since a force was applied and the force energy stored in a potential form is later translated to a kinetic energy.

Furthermore, notice that there is no transitional phase and just after the first period, the displacements are stationary by entering directly into the steady oscillation.

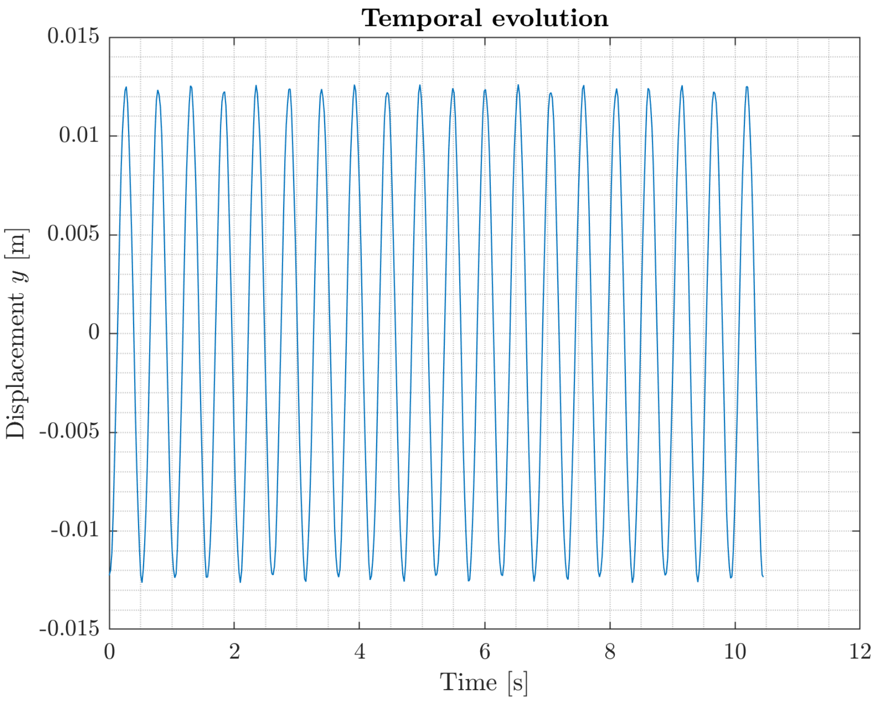


Figure 7 Temporal evolution

Video animation available at: Google Drive

3.1.2 Damped free vibration

Moving on to the damped case, we-ll proceed to analyze the first node since it is the one that presents the major displacements as it is a free end node.

The behaviour of the temporary evolution is now different from the prior case. Now, the first difference that strikes to the observer is the decrease of the amplitude with time caused by the presence of damping. It is noticeable how the amplitude decreases exponentially with respect to time, caused by the addition of the damping environment such as friction, thermal dissipation, etc.

The resultant plots are shown below:

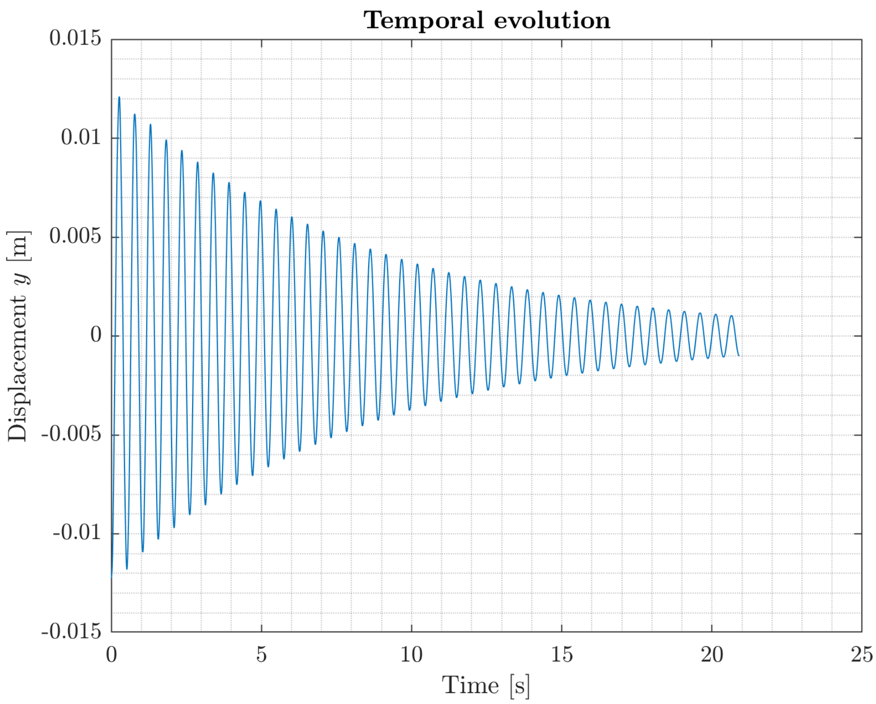


Figure 8 Temporal evolution

Video animation available at: Google Drive

3.2 Torque case

Following with the analysis, the cantilever's dynamic behaviour with an additional torque has been studied. This torque is added in the free $M_x = 100kNm$. Additionally, as shown in assignment 3, the addition of 4 point forces was fair, although using more points have greater precision.

Since the mode shape that is activated the most is the number 5, the study time is defined as $t = m{\cdot}T_5$, being $m = 20$.

It must be noted that the initial displacements of the selected nodes for the corresponding undamped and damped cases, data which is obtained from Assignment 3, are indicated in Table 3. These values are useful to assure the continuity between Assignment 3 and 4.

Node	**Vertical displacement** y [m]	**Damping**
1	-0.001967	Yes

Table 3 Initial displacements for a case of torque load.

3.2.1 Undamped free vibration

Analogously, the studied node is the same as the one from previous analysis in the distributed load case. Observe how the initial displacement is coincident with the prior one for the same node.

Now considering the vertical displacement, it is shown that the dynamic response is major in terms of frequency. The frequency is over 5 times than the mode 1 of the distributed load case. Also, the amplitude of the vertical displacement is differs from the distributed load case. The undamped free vibration of the distributed load case has on order of magnitude higher that the distributed load case. This is due to the fact that the initial amplitude of the 5th vibration mode provides a light contribution to the final displacement value.

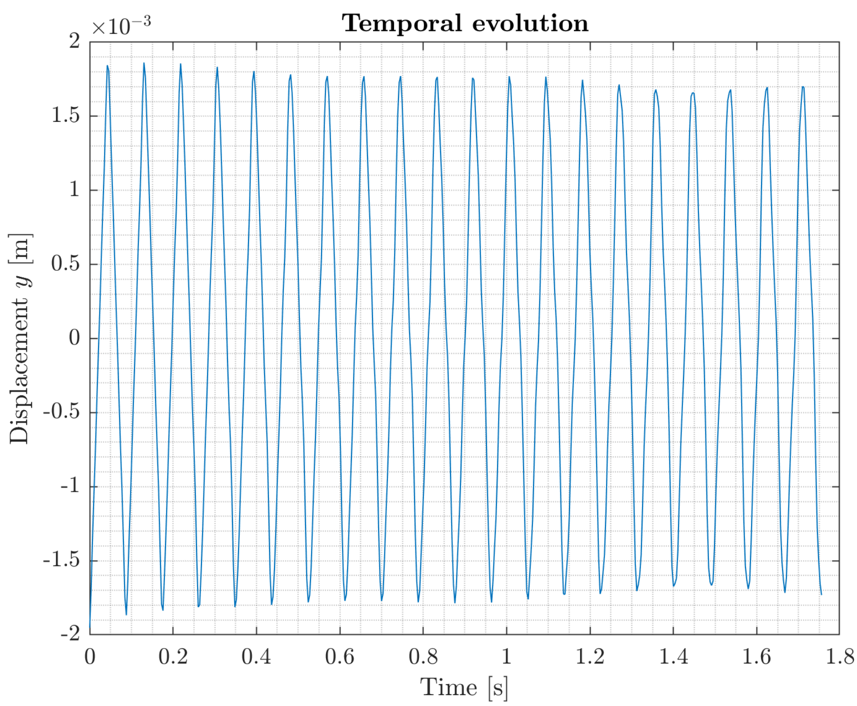


Figure 9 Temporal evolution

Video animation available at: Google Drive

3.2.2 Damped free vibration

Finally, as regards the damped study, the considered node is the number 1. Another time, the initial vertical displacement corresponds with the one provided in the table. Comparing the case of damped torsion and undamped torsion, it can be seen that the amplitudes are higher for $t \approx 0$. Then the amplitude rapidly decreases with time and decays following a negative exponential function.

One interesting parameter is that for the distributed load case the amplitude decreased 1/3 of the departing value in approximately 10 seconds whereas the torsion case, this was achieved in a much shorter amount of time. This phenomena ows to the fact that the negative exponent of the natural frequency of the node is proportional to the natural frequency of each mode. Since mode 5 has a greater value of natural frequency it translates to a faster dissipation rate.

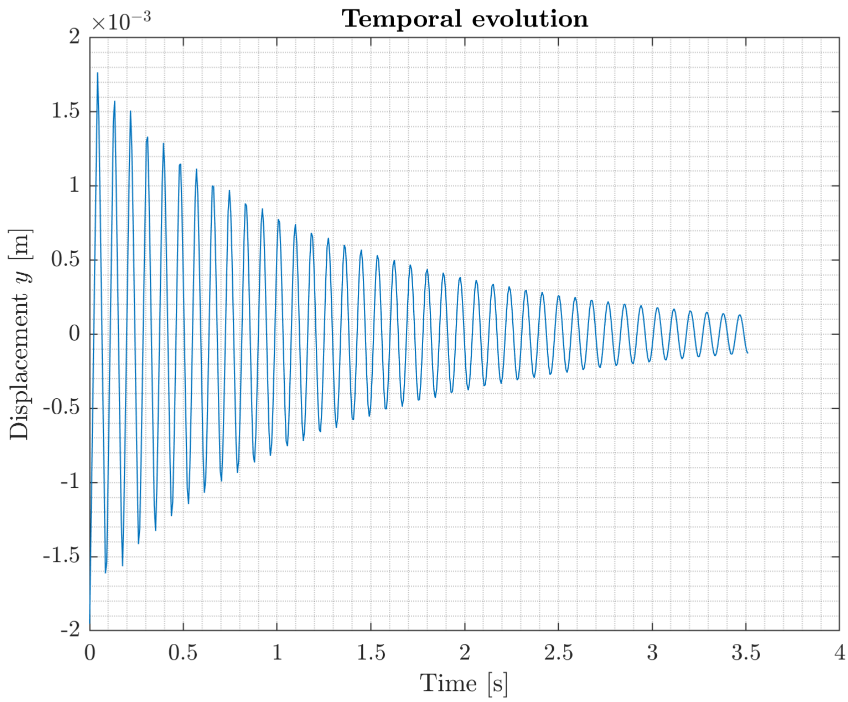


Figure 10 Temporal evolution

Video animation available at: Google Drive

References

[1] Virtual Amrita Laboratories Universalizing Education. Free Vibration of a Cantilever Beam (Continuous System). URL: https://vlab.amrita.edu/?sub=3%5C&brch=175&sim=1080%5C&cnt=1.

[2] Hernández, Joaquín. "Classical Linear Elastodynamics". In: 1st ed. UPC, 2020, pp. 1–39.