Problem 1 (Markov Chains)

a) (a) Transition matrix is

$$P = \begin{bmatrix} 0 & 0.5 & 0 & 0 \\ 0.5 & 0 & 0 & 0 \\ 0 & 0.5 & 1 & 0 \\ 0.5 & 0 & 0 & 1 \end{bmatrix}$$

If $\Pi = [\pi_1 \pi_2 \pi_3 \pi_4]^T$ satisfies $\Pi = P\Pi$,
then $\pi_1 = \pi_2 = 0$ and $\pi_3, \pi_4 \in [0, 1]$ with $\pi_3 + \pi_4 = 1$. Therefore, there exist many (non-unique) invariant distributions.

(b) Transition matrix is

$$P = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

If $\Pi = [\pi_1 \pi_2 \pi_3]^T$ satisfies $\Pi = P\Pi$,
then $\pi_1 = \pi_2 = \pi_3 = 1/3$ Therefore, there is a unique invariant distribution.

b) Since the question states that "unlucky streak will be followed by luck", the outcome of a current state depends on past outcomes. Therefore cannot be a i.i.d. model. Also since it depends not just on the previous state but on the past few states (unlucky "streak" increases the chance), it also cannot be a markov model. Therefore, it can only constitute a hidden markov model.