2a

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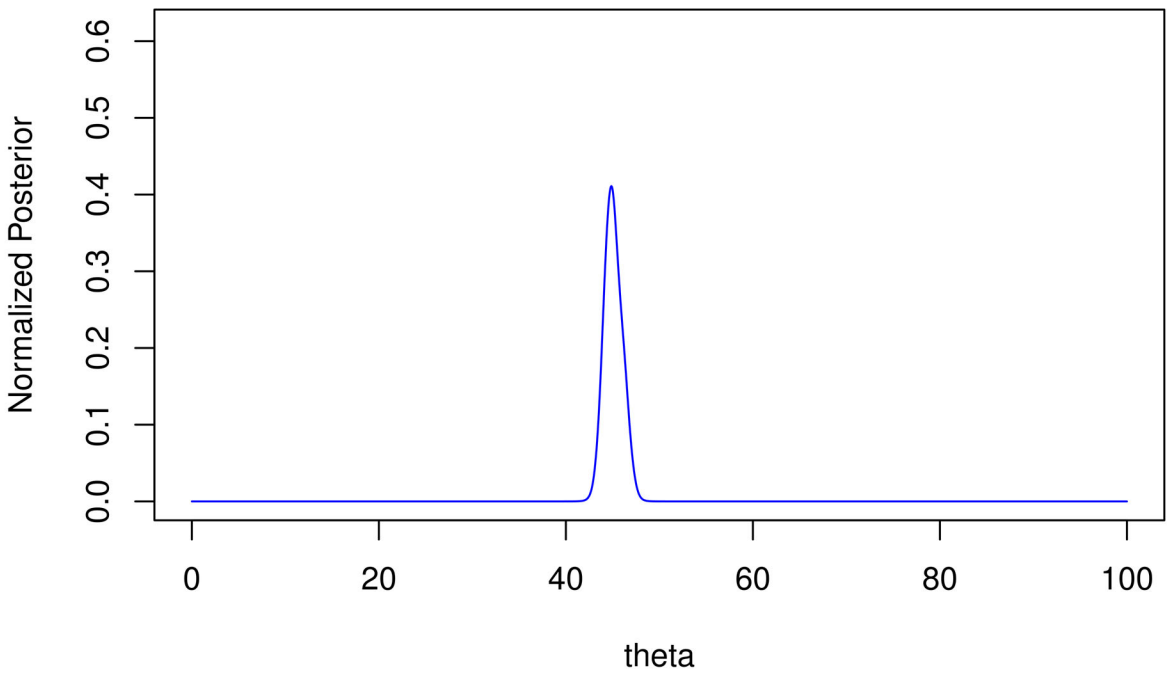
1/16/2021

In this problem, we want to compute nonconjugate single-parameter model: suppose $y_1, ..., y_5$ are independent samples from a Cauchy distribution with unknown center θ and known scale 1, i.e. it follows the following form

$$p(y_i|\theta) \approx \frac{1}{1 + (y_i - \theta)^2)}$$

a. Compute the unnormalized posterior

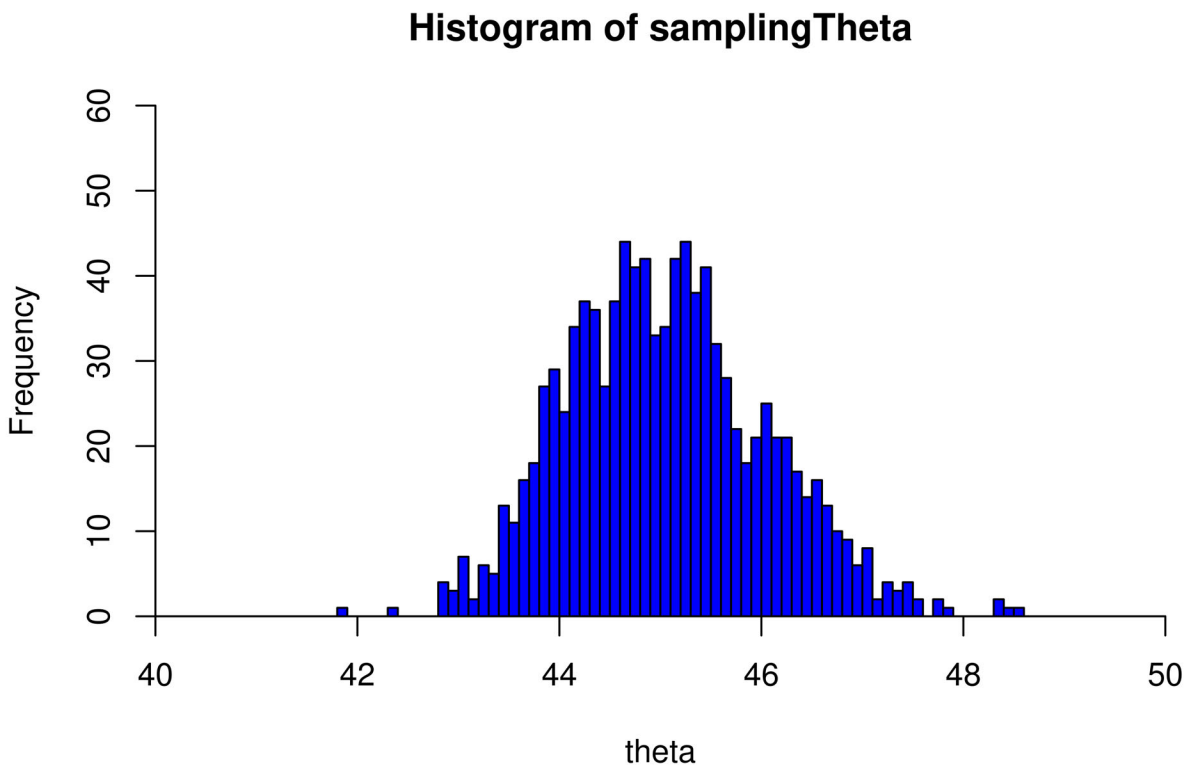
```
# sampling distribution
sampleDistribution = function(y, theta) {
  d0 = NULL
  for (i in 1:length(theta)) { d0 = c(d0, prod(dcauchy(y, theta[i]))) }
  return(d0)
} # finished function definition

# data
y = c(43, 44, 45, 46.5, 47.5)
step = 0.01
theta = seq(0, 100, step)

# unnormalized f
unnormalizedDistribution = sampleDistribution(y, theta)

# normalized f
normalizedDistribution = unnormalizedDistribution / (step*sum(unnormalizedDistribution))

# plot
plot(
  theta,
  normalizedDistribution,
  ylim = c(0, 1.5*max(normalizedDistribution)),
  type = "l", xlab = "theta", ylab = "Normalized Posterior",
  col = "blue"
)
```



b. Sample 1000 draws

Sample 1000 draws of θ from the posterior density and plot a histogram of the draws.

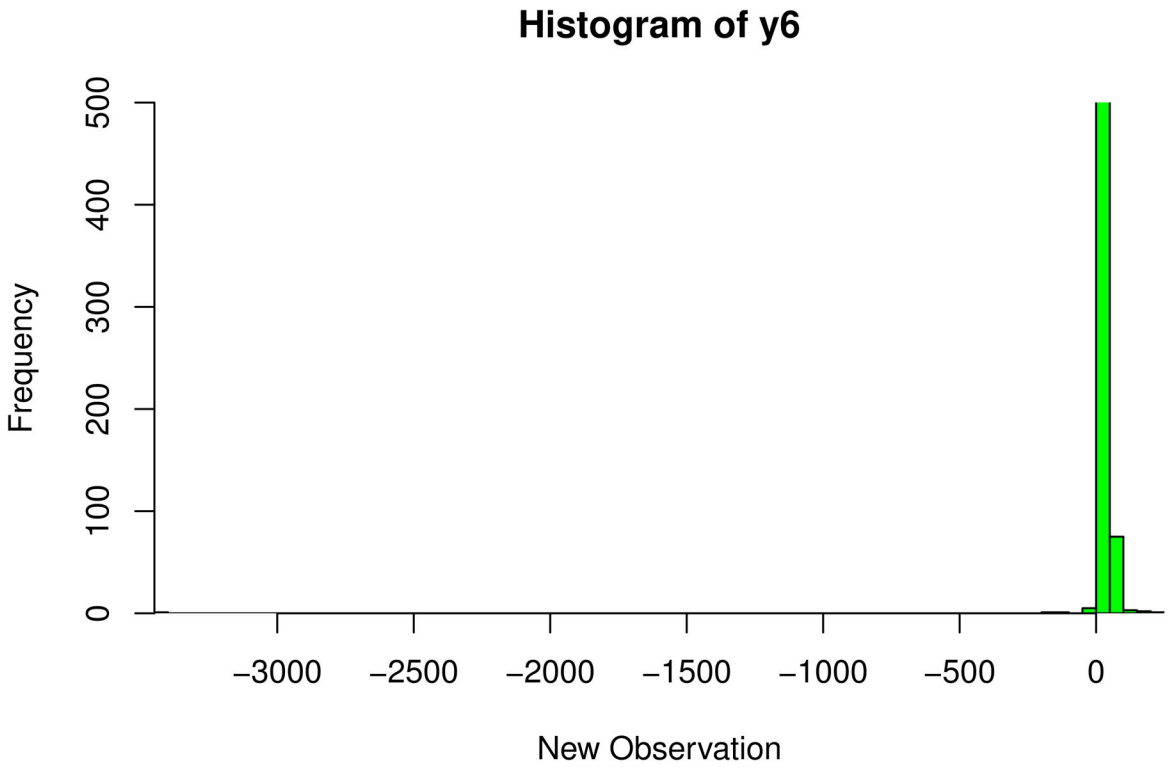
```r
# sampling
samplingTheta = sample(theta, 1000, step*normalizedDistribution, replace = TRUE)

# histogram
hist(
  samplingTheta,
  xlab = "theta",
  breaks = seq(40, 50, 0.1),
  xaxs = "i", yaxs = "i",
  ylim = c(0, 60), col = "blue"
)
```



c. Use 1000 samples

Use the 1000 samples of θ to obtain 1000 samples from the predictive distribution of a future observation, y_6, and plot a histogram of the predictive draws.

```
y6 = rcauchy(length(samplingTheta), samplingTheta, 1)
hist(
  y6,
  xlab = "New Observation", nclass = 100,
  xaxs = "i", yaxs = "i",
  ylim = c(0, 500), col = "green" )
```


Histogram of y6
Frequency
0
100
200
300
400
500
−3000
−2500
−2000
−1500
−1000
−500
0
New Observation