1. Find a nonzero vector orthogonal to the plane through the points P, Q, and R; find the area of triangle PQR.

$$P(-6, 2, 4), \quad Q(-8, 2, 6), \quad R(-9, 4, 3).$$

   Solution. We start by finding two vectors representing the two edges of the triangle first, say, $\overrightarrow{PQ}$ and $\overrightarrow{PR}$.

$$\overrightarrow{PQ} = \langle -2, 0, 2 \rangle$$
$$\overrightarrow{PR} = \langle -3, 2, -1 \rangle$$

   Now we take the cross product to find a vector orthogonal to the plane.

$$\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & 0 & 2 \\ -3 & 2 & -1 \end{vmatrix}$$
$$= \left\langle \begin{vmatrix} 0 & 2 \\ 2 & -1 \end{vmatrix}, -\begin{vmatrix} -2 & 2 \\ -3 & -1 \end{vmatrix}, \begin{vmatrix} -2 & 0 \\ -3 & 2 \end{vmatrix} \right\rangle$$
$$= \langle -4, 8, -4 \rangle.$$

   The area of the triangle is given by

$$\frac{1}{2}|\overrightarrow{PQ}| \cdot |\overrightarrow{PR}| \cdot \sin \angle QPR = \frac{1}{2}|\overrightarrow{PQ} \times \overrightarrow{PR}|$$
$$= \frac{1}{2}\sqrt{(-4)^2 + 8^2 + (-4)^2}$$
$$= 2\sqrt{6}.$$

2. *True / False?* For any vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$,

$$(\mathbf{a} \cdot \mathbf{b})\mathbf{c} \text{ and } (\mathbf{a} \cdot \mathbf{c})\mathbf{b}$$

   may not be parallel but will always have the same magnitude by associativity.

   Solution. False. They generally do not have the same magnitude.

$$|(\mathbf{a} \cdot \mathbf{b})\mathbf{c}| = |\mathbf{a}| \cdot |\mathbf{b}|| \cdot |\mathbf{c}| \cos \theta_{\mathbf{a}, \mathbf{b}},$$

   while

$$|(\mathbf{a} \cdot \mathbf{c})\mathbf{b}| = |\mathbf{a}| \cdot |\mathbf{b}|| \cdot |\mathbf{c}| \cos \theta_{\mathbf{a}, \mathbf{c}}.$$

Here $\theta_{\mathbf{a},\mathbf{b}}$ is the angle between $\mathbf{a}$ and $\mathbf{b}$. Similarly $\theta_{\mathbf{a},\mathbf{c}}$ is defined. Since these two angles do not need to be the same, the magnitudes above do not need to be the same. There is no associativity with both scalar multiplication and dot product in the mix.

3. *True / False?* Suppose A and B are two planes that intersect at a line ℓ. To find the angle between A and B, we can follow the recipe here:

   - first, select a point C on line ℓ;
   - then, select lines ℓ_A and ℓ_B through C and orthogonal to ℓ, on A and B;
   - finally, find the angle between lines ℓ_A and ℓ_B.

   Solution. The plane containing ℓ_A and the normal vector to A is orthogonal to ℓ and passes through C. Similarly, the plane containing ℓ_B and the normal vector to B is orthogonal to ℓ and passes through C. In other words, ℓ_A, ℓ_B, and the normal vectors to A and B are all on the same plane. Since ℓ_A and the normal vector to A are orthogonal, and ℓ_B and the normal vector to B, the angle between ℓ_A and ℓ_B is the same as that between the two normal vectors.