

# 伸展树学习文档（备忘录）

简化自一个ppt

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## Thoughts

- Balanced BSTs aren't necessarily optimal! Goal: Construct a binary search tree  $T^*$  that minimizes the total expected access time.
- Challenge: Can we construct an optimal BST without knowing the access probabilities in advance?
- After looking up an element, repeatedly rotate that element with its parent until it becomes the root.
- Problem: Rotating an element  $x$  to the root significantly "helps"  $x$ , but "hurts" the rest of the tree. Most of the nodes on the access path to  $x$  have depth that increases or is unchanged.
- Splay: Rotates an element to the root of the tree, but does so in a way that's more "fair" to other nodes in the tree.

## Splay

3 cases: zigzig zigzag zig

## Why Splaying works?

Claim: After doing a splay at  $x$ , the average depth of any nodes on the access path to  $x$  is halved. Intuitively, splaying  $x$  benefits nodes near  $x$ , not just  $x$  itself. This "altruism" will ensure that splays are efficient. Each rotation done only slightly penalizes each other part of the tree (say, adding +1 or +2 depth). But Each splay rapidly cuts down the height of each node on the access path.

## Lookups Operations

To do a lookup in a splay tree: • Search for that item as usual. • If it's found, splay it up to the root. • Otherwise, splay the last-visited node to the root.

## Insertion&Delete Operations

To insert a node into a splay tree: • Insert the node as usual. • Splay it up to the root. To delete a key  $k$  from the tree: • Splay  $k$  to the root. • Delete  $k$ . • Join the two resulting subtrees.

## Join&Split Operations

To join two trees  $T_1$  and  $T_2$ , where all keys in  $T_1$  are less than the keys in  $T_2$ : • Splay the max element of  $T_1$  to the root. • Make  $T_2$  a right child of  $T_1$ . To split  $T$  at a key  $k$ : • Splay the successor of  $k$  up to the root. • Cut the link from the root to its left child.

**All of these operations require amortized time  $O(\log n)$ .**

## Compared with RedBlackTree

- No need to store any kind of balance information. • Only three rules to memorize.