

Empirical Test of Higher Order Beliefs in Chinese Stock Market

(Keynesian Beauty-Contest-Effect in Reality)

Sheng Yiran

School of Economics and Management Class 62

Advisor: Prof. Zhang, Lihong

Outline

Introduction

Literature Review and Related Works

The Model

Model Test

Conclusion and Summary

The Story

- ▶ Keynes described the action of rational agents in a market using an analogy based on a fictional newspaper contest, in which entrants are asked to choose a set of six faces from photographs of women that are the "most beautiful". Those who picked the most popular face are then eligible for a prize.
- ▶ Participants do not choose faces best of their judgment, rather they try to guess what's the average opinion is about which faces are the prettiest.

The Story: Getting Sophisticated

- ▶ "It is not a case of choosing those [faces] that, to the best of one's judgment, are really the prettiest, nor even those that average opinion genuinely thinks the prettiest. We have reached the third degree where we devote our intelligences to anticipating what average opinion expects the average opinion to be. And there are some, I believe who practice the fourth, fifth and higher degrees." (Keynes 1936)

The Story: Why is it important?

- ▶ Central to financial theories: how individuals form their expectations?
 - ▶ Traditional Approaches: the consensus expectation of future payoffs adjusted by risk preference.
 - ▶ Rational Expectation Models.
 - ▶ BCE: beliefs about other people's beliefs matters.
 - ▶ Higher order beliefs.
- ▶ Empirical inconsistency of Traditional REE models.
 - ▶ Excess price volatilities (Shiller , 1981) (LeCoy and Porter, 1981)
 - ▶ Assets Price Drift (Miller, 1977)
 - ▶ Herding in financial markets. (Chrimpf and Schmeling, 2009)

The Story: Why is it important?

- ▶ Central to financial theories: how individuals form their expectations?
 - ▶ Traditional Approaches: the consensus expectation of future payoffs adjusted by risk preference.
 - ▶ Rational Expectation Models.
 - ▶ BCE: beliefs about other people's beliefs matters.
 - ▶ Higher order beliefs.
- ▶ Empirical inconsistency of Traditional REE models.
 - ▶ Excess price volatilities (Shiller , 1981) (LeCoy and Porter, 1981)
 - ▶ Assets Price Drift (Miller, 1977)
 - ▶ Herding in financial markets. (Chrimpf and Schmeling, 2009)

The Story: Why is it important?

- ▶ Central to financial theories: how individuals form their expectations?
 - ▶ Traditional Approaches: the consensus expectation of future payoffs adjusted by risk preference.
 - ▶ Rational Expectation Models.
 - ▶ BCE: beliefs about other people's beliefs matters.
 - ▶ Higher order beliefs.
- ▶ Empirical inconsistency of Traditional REE models.
 - ▶ Excess price volatilities (Shiller , 1981) (LeCoy and Porter, 1981)
 - ▶ Assets Price Drift (Miller, 1977)
 - ▶ Herding in financial markets. (Chrimpf and Schmeling, 2009)

The Story: Why is it important?

- ▶ Central to financial theories: how individuals form their expectations?
 - ▶ Traditional Approaches: the consensus expectation of future payoffs adjusted by risk preference.
 - ▶ Rational Expectation Models.
 - ▶ BCE: beliefs about other people's beliefs matters.
 - ▶ Higher order beliefs.
- ▶ Empirical inconsistency of Traditional REE models.
 - ▶ Excess price volatilities (Shiller , 1981) (LeCoy and Porter, 1981)
 - ▶ Assets Price Drift (Miller, 1977)
 - ▶ Herding in financial markets. (Chrimpf and Schmeling, 2009)

The Story: Why is it important?

- ▶ Central to financial theories: how individuals form their expectations?
 - ▶ Traditional Approaches: the consensus expectation of future payoffs adjusted by risk preference.
 - ▶ Rational Expectation Models.
 - ▶ BCE: beliefs about other people's beliefs matters.
 - ▶ Higher order beliefs.
- ▶ Empirical inconsistency of Traditional REE models.
 - ▶ Excess price volatilities (Shiller , 1981) (LeCoy and Porter, 1981)
 - ▶ Assets Price Drift (Miller, 1977)
 - ▶ Herding in financial markets. (Chrimpf and Schmeling, 2009)

The Story: Why is it important?

- ▶ Central to financial theories: how individuals form their expectations?
 - ▶ Traditional Approaches: the consensus expectation of future payoffs adjusted by risk preference.
 - ▶ Rational Expectation Models.
 - ▶ BCE: beliefs about other people's beliefs matters.
 - ▶ Higher order beliefs.
- ▶ Empirical inconsistency of Traditional REE models.
 - ▶ Excess price volatilities (Shiller , 1981) (LeCoy and Porter, 1981)
 - ▶ Assets Price Drift (Miller, 1977)
 - ▶ Herding in financial markets. (Chrimpf and Schmeling, 2009)

The Story: Why is it important?

- ▶ Central to financial theories: how individuals form their expectations?
 - ▶ Traditional Approaches: the consensus expectation of future payoffs adjusted by risk preference.
 - ▶ Rational Expectation Models.
 - ▶ BCE: beliefs about other people's beliefs matters.
 - ▶ Higher order beliefs.
- ▶ Empirical inconsistency of Traditional REE models.
 - ▶ Excess price volatilities (Shiller , 1981) (LeCoy and Porter, 1981)
 - ▶ Assets Price Drift (Miller, 1977)
 - ▶ Herding in financial markets. (Chrimpf and Schmeling, 2009)

The Story: Why is it important?

- ▶ Central to financial theories: how individuals form their expectations?
 - ▶ Traditional Approaches: the consensus expectation of future payoffs adjusted by risk preference.
 - ▶ Rational Expectation Models.
 - ▶ BCE: beliefs about other people's beliefs matters.
 - ▶ Higher order beliefs.
- ▶ Empirical inconsistency of Traditional REE models.
 - ▶ Excess price volatilities (Shiller , 1981) (LeCoy and Porter, 1981)
 - ▶ Assets Price Drift (Miller, 1977)
 - ▶ Herding in financial markets. (Chrimpf and Schmeling, 2009)

The Story: Why is it important?

- ▶ Central to financial theories: how individuals form their expectations?
 - ▶ Traditional Approaches: the consensus expectation of future payoffs adjusted by risk preference.
 - ▶ Rational Expectation Models.
 - ▶ BCE: beliefs about other people's beliefs matters.
 - ▶ Higher order beliefs.
- ▶ Empirical inconsistency of Traditional REE models.
 - ▶ Excess price volatilities (Shiller , 1981) (LeCoy and Porter, 1981)
 - ▶ Assets Price Drift (Miller, 1977)
 - ▶ Herding in financial markets. (Chrimpf and Schmeling, 2009)

The Story: Why is it important?

- ▶ Central to financial theories: how individuals form their expectations?
 - ▶ Traditional Approaches: the consensus expectation of future payoffs adjusted by risk preference.
 - ▶ Rational Expectation Models.
 - ▶ BCE: beliefs about other people's beliefs matters.
 - ▶ Higher order beliefs.
- ▶ Empirical inconsistency of Traditional REE models.
 - ▶ Excess price volatilities (Shiller , 1981) (LeCoy and Porter, 1981)
 - ▶ Assets Price Drift (Miller, 1977)
 - ▶ Herding in financial markets. (Chrimpf and Schmeling, 2009)

Outline

Introduction

Literature Review and Related Works

The Model

Model Test

Conclusion and Summary

Literature Review and Related Works

- ▶ Why traditional theories consider HOB redundant?
 - ▶ Representative Agent Model: Equivalent Martingale Property.
 - ▶ Iterated Expectations under the equivalent probability measure.
- ▶ Is it the case?
 - ▶ Failed law of Iterated Expectations (Allen, Morris and Shin 2006)
 - ▶ Dynamic set-up and emphasizing short-run actions.
 - ▶ Heterogenous Beliefs and Agree to Disagree.(Miller, 1977)(Kremer, Banerjee, and Kaniel, 2009)

Literature Review and Related Works

- ▶ Why traditional theories consider HOB redundant?
 - ▶ Representative Agent Model: Equivalent Martingale Property.
 - ▶ Iterated Expectations under the equivalent probability measure.
- ▶ Is it the case?
 - ▶ Failed law of Iterated Expectations (Allen, Morris and Shin 2006)
 - ▶ Dynamic set-up and emphasizing short-run actions.
 - ▶ Heterogenous Beliefs and Agree to Disagree.(Miller, 1977)(Kremer, Banerjee, and Kaniel, 2009)

Literature Review and Related Works

- ▶ Why traditional theories consider HOB redundant?
 - ▶ Representative Agent Model: Equivalent Martingale Property.
 - ▶ Iterated Expectations under the equivalent probability measure.
- ▶ Is it the case?
 - ▶ Failed law of Iterated Expectations (Allen, Morris and Shin 2006)
 - ▶ Dynamic set-up and emphasizing short-run actions.
 - ▶ Heterogenous Beliefs and Agree to Disagree.(Miller, 1977)(Kremer, Banerjee, and Kaniel, 2009)

Literature Review and Related Works

- ▶ Why traditional theories consider HOB redundant?
 - ▶ Representative Agent Model: Equivalent Martingale Property.
 - ▶ Iterated Expectations under the equivalent probability measure.
- ▶ Is it the case?
 - ▶ Failed law of Iterated Expectations (Allen, Morris and Shin 2006)
 - ▶ Dynamic set-up and emphasizing short-run actions.
 - ▶ Heterogenous Beliefs and Agree to Disagree.(Miller, 1977)(Kremer, Banerjee, and Kaniel, 2009)

Literature Review and Related Works

- ▶ Why traditional theories consider HOB redundant?
 - ▶ Representative Agent Model: Equivalent Martingale Property.
 - ▶ Iterated Expectations under the equivalent probability measure.
- ▶ Is it the case?
 - ▶ Failed law of Iterated Expectations (Allen, Morris and Shin 2006)
 - ▶ Dynamic set-up and emphasizing short-run actions.
 - ▶ Heterogenous Beliefs and Agree to Disagree.(Miller, 1977)(Kremer, Banerjee, and Kaniel, 2009)

Literature Review and Related Works

- ▶ Why traditional theories consider HOB redundant?
 - ▶ Representative Agent Model: Equivalent Martingale Property.
 - ▶ Iterated Expectations under the equivalent probability measure.
- ▶ Is it the case?
 - ▶ Failed law of Iterated Expectations (Allen, Morris and Shin 2006)
 - ▶ Dynamic set-up and emphasizing short-run actions.
 - ▶ Heterogenous Beliefs and Agree to Disagree.(Miller, 1977)(Kremer, Banerjee, and Kaniel, 2009)

Literature Review and Related Works

- ▶ Why traditional theories consider HOB redundant?
 - ▶ Representative Agent Model: Equivalent Martingale Property.
 - ▶ Iterated Expectations under the equivalent probability measure.
- ▶ Is it the case?
 - ▶ Failed law of Iterated Expectations (Allen, Morris and Shin 2006)
 - ▶ Dynamic set-up and emphasizing short-run actions.
 - ▶ Heterogenous Beliefs and Agree to Disagree.(Miller, 1977)(Kremer, Banerjee, and Kaniel, 2009)

Differences between HOB and Heterogenous Beliefs

- ▶ Both type of models try to improve REE
- ▶ Both type of models allow investors to receive different signals w.r.p.t a future r.v.(eg, payoff or price)
- ▶ In Heterogenous Beliefs cases, individual investor conditions merely on this private signal to make decisions.
- ▶ In HOB cases, he or she also try to guess other investors' signals (eg, market average expectations).

Differences between HOB and Heterogenous Beliefs

- ▶ Both type of models try to improve REE
- ▶ Both type of models allow investors to receive different signals w.r.p.t a future r.v.(eg, payoff or price)
- ▶ In Heterogenous Beliefs cases, individual investor conditions merely on this private signal to make decisions.
- ▶ In HOB cases, he or she also try to guess other investors' signals (eg, market average expectations).

Differences between HOB and Heterogenous Beliefs

- ▶ Both type of models try to improve REE
- ▶ Both type of models allow investors to receive different signals w.r.p.t a future r.v.(eg, payoff or price)
- ▶ In Heterogenous Beliefs cases, individual investor conditions merely on this private signal to make decisions.
- ▶ In HOB cases, he or she also try to guess other investors' signals (eg, market average expectations).

Differences between HOB and Heterogenous Beliefs

- ▶ Both type of models try to improve REE
- ▶ Both type of models allow investors to receive different signals w.r.p.t a future r.v.(eg, payoff or price)
- ▶ In Heterogenous Beliefs cases, individual investor conditions merely on this private signal to make decisions.
- ▶ In HOB cases, he or she also try to guess other investors' signals (eg, market average expectations).

HOB models: general features

- ▶ Multi-period and dynamic. (Bacchetta and Wincoop, 2004)
- ▶ Overlapping generations of market participants. (Allen Morris and Shin, 2006) (Bacchetta and Wincoop, 2004) (Monnin, 2004) etc.
- ▶ Information structure: distinguish public and private signals. (Allen Morris and Shin, 2006) (Kremer, Banerjee, and Kaniel, 2009)

HOB models: general features

- ▶ Multi-period and dynamic. (Bacchetta and Wincoop, 2004)
- ▶ Overlapping generations of market participants. (Allen Morris and Shin, 2006) (Bacchetta and Wincoop, 2004) (Monnin, 2004) etc.
- ▶ Information structure: distinguish public and private signals. (Allen Morris and Shin, 2006) (Kremer, Banerjee, and Kaniel, 2009)

HOB models: general features

- ▶ Multi-period and dynamic. (Bacchetta and Wincoop, 2004)
- ▶ Overlapping generations of market participants. (Allen Morris and Shin, 2006) (Bacchetta and Wincoop, 2004) (Monnin, 2004) etc.
- ▶ Information structure: distinguish public and private signals. (Allen Morris and Shin, 2006) (Kremer, Banerjee, and Kaniel, 2009)

Current empirical evidence

- ▶ Rich literatures focus on heterogenous beliefs models.(Etemple and Murthy 1994) (Boswijk, Hommes and Manzan 2005)
- ▶ HOB models lack empirical validation.
 - ▶ (Schrimpf and Schmeling, 2009) adopts interviewed opinions from 350 institutional forecastors in German market. Confirms the link between HOB and herding.
 - ▶ (Monnin 2004) provides a generalized framework introducing HOB on expectations w.r.p.t real dividends growth rate. Adopts data in US market from 1871 to 2003, confirms the existance of HOB. *yet recieves certain critics
- ▶ Relevant empirical study in Chinese Market
 - ▶ (Song and Gao, 2003) states no evidence suggests REE in Chinese stock market.
 - ▶ (Wang and Zhao, 2006) confirms heterogenous beliefs exist and have strong impact stock prices in Chinese stock

Current empirical evidence

- ▶ Rich literatures focus on heterogenous beliefs models.(Etemple and Murthy 1994) (Boswijk, Hommes and Manzan 2005)
- ▶ HOB models lack empirical validation.
 - ▶ (Schrimpf and Schmeling, 2009) adopts interviewed opinions from 350 institutional forecastors in German market. Confirms the link between HOB and herding.
 - ▶ (Monnin 2004) provides a generalized framework introducing HOB on expectations w.r.p.t real dividends growth rate. Adopts data in US market from 1871 to 2003, confirms the existence of HOB. *yet receives certain critics
- ▶ Relevant empirical study in Chinese Market
 - ▶ (Song and Gao, 2003) states no evidence suggests REE in Chinese stock market.
 - ▶ (Wang and Zhao, 2006) confirms heterogenous beliefs exist and have strong impact stock prices in Chinese stock

Current empirical evidence

- ▶ Rich literatures focus on heterogenous beliefs models.(Etemple and Murthy 1994) (Boswijk, Hommes and Manzan 2005)
- ▶ HOB models lack empirical validation.
 - ▶ (Schrimpf and Schmeling, 2009) adopts interviewed opinions from 350 institutional forecastors in German market. Confirms the link between HOB and herding.
 - ▶ (Monnin 2004) provides a generalized framework introducing HOB on expectations w.r.p.t real dividends growth rate. Adopts data in US market from 1871 to 2003, confirms the existence of HOB. *yet recieves certain critics
- ▶ Relevant empirical study in Chinese Market
 - ▶ (Song and Gao, 2003) states no evidence suggests REE in Chinese stock market.
 - ▶ (Wang and Zhao, 2006) confirms heterogenous beliefs exist and have strong impact stock prices in Chinese stock

Current empirical evidence

- ▶ Rich literatures focus on heterogenous beliefs models.(Etemple and Murthy 1994) (Boswijk, Hommes and Manzan 2005)
- ▶ HOB models lack empirical validation.
 - ▶ (Schrimpf and Schmeling, 2009) adopts interviewed opinions from 350 institutional forecastors in German market. Confirms the link between HOB and herding.
 - ▶ (Monnin 2004) provides a generalized framework introducing HOB on expectations w.r.p.t real dividends growth rate. Adopts data in US market from 1871 to 2003, confirms the existance of HOB. *yet recieves certain critics
- ▶ Relevant empirical study in Chinese Market
 - ▶ (Song and Gao, 2003) states no evidence suggests REE in Chinese stock market.
 - ▶ (Wang and Zhao, 2006) confirms heterogenous beliefs exist and have strong impact stock prices in Chinese stock

Current empirical evidence

- ▶ Rich literatures focus on heterogenous beliefs models.(Etemple and Murthy 1994) (Boswijk, Hommes and Manzan 2005)
- ▶ HOB models lack empirical validation.
 - ▶ (Schrimpf and Schmeling, 2009) adopts interviewed opinions from 350 institutional forecastors in German market. Confirms the link between HOB and herding.
 - ▶ (Monnin 2004) provides a generalized framework introducing HOB on expectations w.r.p.t real dividends growth rate. Adopts data in US market from 1871 to 2003, confirms the existance of HOB. *yet recieves certain critics
- ▶ Relevant empirical study in Chinese Market
 - ▶ (Song and Gao, 2003) states no evidence suggests REE in Chinese stock market.
 - ▶ (Wang and Zhao, 2006) confirms heterogenous beliefs exist and have strong impact stock prices in Chinese stock

Current empirical evidence

- ▶ Rich literatures focus on heterogenous beliefs models.(Etemple and Murthy 1994) (Boswijk, Hommes and Manzan 2005)
- ▶ HOB models lack empirical validation.
 - ▶ (Schrimpf and Schmeling, 2009) adopts interviewed opinions from 350 institutional forecastors in German market. Confirms the link between HOB and herding.
 - ▶ (Monnin 2004) provides a generalized framework introducing HOB on expectations w.r.p.t real dividends growth rate. Adopts data in US market from 1871 to 2003, confirms the existance of HOB. *yet recieves certain critics
- ▶ Relevant empirical study in Chinese Market
 - ▶ (Song and Gao, 2003) states no evidence suggests REE in Chinese stock market.
 - ▶ (Wang and Zhao, 2006) confirms heterogenous beliefs exist and have strong impact stock prices in Chinese stock

Current empirical evidence

- ▶ Rich literatures focus on heterogenous beliefs models.(Etemple and Murthy 1994) (Boswijk, Hommes and Manzan 2005)
- ▶ HOB models lack empirical validation.
 - ▶ (Schrimpf and Schmeling, 2009) adopts interviewed opinions from 350 institutional forecastors in German market. Confirms the link between HOB and herding.
 - ▶ (Monnin 2004) provides a generalized framework introducing HOB on expectations w.r.p.t real dividends growth rate. Adopts data in US market from 1871 to 2003, confirms the existance of HOB. *yet recieves certain critics
- ▶ Relevant empirical study in Chinese Market
 - ▶ (Song and Gao, 2003) states no evidence suggests REE in Chinese stock market.
 - ▶ (Wang and Zhao, 2006) confirms heterogenous beliefs exist and have strong impact stock prices in Chinese stock

Outline

Introduction

Literature Review and Related Works

The Model

Model Test

Conclusion and Summary

Model Set-up

► Timeline

- T periods. Free trade happens at date 1, ... T-1.

► Assets

- one risk-free, r_f ; N risky assets with a price vector P_t at date t .

► Market Participants

- Overlapping generations of investors: at each date t a new generation enters the market with wealth endowed 1 and a continuation index $i \in (0, 1)$. They quit at date $t + 1$, selling all assets holdings and retire. Each investor has a CARA utility, with Risk-aversion θ .

Model Set-up

► Timeline

- T periods. Free trade happens at date 1, ... T-1.

► Assets

- one risk-free, r_f ; N risky assets with a price vector P_t at date t .

► Market Participants

- Overlapping generations of investors: at each date t a new generation enters the market with wealth endowed 1 and a continuation index $i \in (0, 1)$. They quit at date $t + 1$, selling all assets holdings and retire. Each investor has a CARA utility, with Risk-aversion θ .

Model Set-up

- ▶ Timeline
 - ▶ T periods. Free trade happens at date 1, ... T-1.
- ▶ Assets
 - ▶ one risk-free, r_f ; N risky assets with a price vector P_t at date t .
- ▶ Market Participants
 - ▶ Overlapping generations of investors: at each date t a new generation enters the market with wealth endowed 1 and a continuum index $i \in (0, 1)$. They quit at date $t + 1$, selling all assets holdings and retire. Each investor has a CARA utility, with Risk-aversion θ .

Model Set-up

- ▶ Timeline
 - ▶ T periods. Free trade happens at date 1, ... T-1.
- ▶ Assets
 - ▶ one risk-free, r_f ; N risky assets with a price vector P_t at date t .
- ▶ Market Participants
 - ▶ Overlapping generations of investors: at each date t a new generation enters the market with wealth endowed 1 and a continuum index $i \in (0, 1)$. They quit at date $t + 1$, selling all assets holdings and retire. Each investor has a CARA utility, with Risk-aversion θ .

Model Set-up

▶ Assets Supply

- ▶ As common in REE, the risky assets follow a zero-mean noise supply.

$$Z_t = \sum_{k=1}^t z_t$$

where vector

$$z_t \sim N(0, \Sigma_z)$$

▶ Information Structure

- ▶ Denote $\Theta_{i,t}$ the information set for individual i at date t .

▶ Some notations

- ▶ $E_{i,t}[\cdot] = E[\cdot | \Theta_{i,t}]$ and $\bar{E}_t = \int_0^1 E_{i,t} di$

Model Set-up

▶ Assets Supply

- ▶ As common in REE, the risky assets follow a zero-mean noise supply.

$$Z_t = \sum_{k=1}^t z_t$$

where vector

$$z_t \sim N(0, \Sigma_z)$$

▶ Information Structure

- ▶ Denote $\Theta_{i,t}$ the information set for individual i at date t .

▶ Some notations

- ▶ $E_{i,t}[\cdot] = E[\cdot | \Theta_{i,t}]$ and $\bar{E}_t = \int_0^1 E_{i,t} di$

Model Set-up

▶ Assets Supply

- ▶ As common in REE, the risky assets follow a zero-mean noise supply.

$$Z_t = \sum_{k=1}^t z_t$$

where vector

$$z_t \sim N(0, \Sigma_z)$$

▶ Information Structure

- ▶ Denote $\Theta_{i,t}$ the information set for individual i at date t .

▶ Some notations

- ▶ $E_{i,t}[\cdot] = E[\cdot | \Theta_{i,t}]$ and $\bar{E}_t = \int_0^1 E_{i,t} di$

Model Set-up

▶ Assets Supply

- ▶ As common in REE, the risky assets follow a zero-mean noise supply.

$$Z_t = \sum_{k=1}^t z_t$$

where vector

$$z_t \sim N(0, \Sigma_z)$$

▶ Information Structure

- ▶ Denote $\Theta_{i,t}$ the information set for individual i at date t .

▶ Some notations

- ▶ $E_{i,t}[\cdot] = E[\cdot | \Theta_{i,t}]$ and $\bar{E}_t = \int_0^1 E_{i,t} di$

Individual's Maximization Problem



$$X_{i,t} = \arg \max_x E_{i,t}[-\exp\{\theta \tilde{W}_i\}]$$

$$\tilde{W}_i = (1 - X_{i,t}^T P_t)(1 + r_f) + X_{i,t} P_{t+1}$$

- ▶ Under joint-normal payoff and CARA utility, we have the following solution:



$$X_{i,t} = \frac{1}{\theta} \Sigma_{P_{t+1}}^{-1} [E_{i,t}(P_{t+1}) - (1 + r_f) P_t]$$

Individual's Maximization Problem



$$X_{i,t} = \arg \max_x E_{i,t}[-\exp\{\theta \tilde{W}_i\}]$$

$$\tilde{W}_i = (1 - X_{i,t}^T P_t)(1 + r_f) + X_{i,t} P_{t+1}$$

- ▶ Under joint-normal payoff and CARA utility, we have the following solution:



$$X_{i,t} = \frac{1}{\theta} \Sigma_{P_{t+1}}^{-1} [E_{i,t}(P_{t+1}) - (1 + r_f) P_t]$$

Individual's Maximization Problem



$$X_{i,t} = \arg \max_x E_{i,t}[-\exp\{\theta \tilde{W}_i\}]$$

$$\tilde{W}_i = (1 - X_{i,t}^T P_t)(1 + r_f) + X_{i,t} P_{t+1}$$

- ▶ Under joint-normal payoff and CARA utility, we have the following solution:



$$X_{i,t} = \frac{1}{\theta} \Sigma_{P_{t+1}}^{-1} [E_{i,t}(P_{t+1}) - (1 + r_f) P_t]$$

Demand for Risky assets

- ▶ aggregate the above equation, derive the market demand of risky assets:

$$X_t = \frac{1}{\theta} \Sigma_{P_{t+1}}^{-1} [\bar{E}_t(P_{t+1}) - (1 + r_f) P_t]$$

- ▶ let demand be equal to supply, we have

$$Z_t = \frac{1}{\theta} \Sigma_{P_{t+1}}^{-1} [\bar{E}_t(P_{t+1}) - (1 + r_f) P_t]$$

- ▶ sloving P_t , where $\beta = (1 + r_f)^{-1}$

$$P_t = \beta \bar{E}_t(P_{t+1}) - \beta \theta Z_t \Sigma_{P_{t+1}}$$

Demand for Risky assets

- ▶ aggregate the above equation, derive the market demand of risky assets:

$$X_t = \frac{1}{\theta} \Sigma_{P_{t+1}}^{-1} [\bar{E}_t(P_{t+1}) - (1 + r_f) P_t]$$

- ▶ let demand be equal to supply, we have

$$Z_t = \frac{1}{\theta} \Sigma_{P_{t+1}}^{-1} [\bar{E}_t(P_{t+1}) - (1 + r_f) P_t]$$

- ▶ sloving P_t , where $\beta = (1 + r_f)^{-1}$

$$P_t = \beta \bar{E}_t(P_{t+1}) - \beta \theta Z_t \Sigma_{P_{t+1}}$$

Demand for Risky assets

- ▶ aggregate the above equation, derive the market demand of risky assets:

$$X_t = \frac{1}{\theta} \Sigma_{P_{t+1}}^{-1} [\bar{E}_t(P_{t+1}) - (1 + r_f) P_t]$$

- ▶ let demand be equal to supply, we have

$$Z_t = \frac{1}{\theta} \Sigma_{P_{t+1}}^{-1} [\bar{E}_t(P_{t+1}) - (1 + r_f) P_t]$$

- ▶ sloving P_t , where $\beta = (1 + r_f)^{-1}$

$$P_t = \beta \bar{E}_t(P_{t+1}) - \beta \theta Z_t \Sigma_{P_{t+1}}$$

Further assumptions with regards to information structure

- ▶ first we iterate the equation of P_t

$$P_t = \bar{E}^{T-t}[P_T] - \sum_{k=1}^{T-t} \beta^k [\theta Z_{t+k-1} \Sigma P_{t+k}]$$

where $\bar{E}_t^k[.] = \bar{E}_t \bar{E}_{t+1} \dots \bar{E}_{t+k-1}[.]$

- ▶ Note we need to understand the form of the iterated conditional expectation $\bar{E}_t^k[.]$ in order to calculate P_t 's. According to (Allen Morris and Shin, 2006) the direct iteration fails here.

Further assumptions with regards to information structure

- ▶ first we iterate the equation of P_t

$$P_t = \bar{E}^{T-t}[P_T] - \sum_{k=1}^{T-t} \beta^k [\theta Z_{t+k-1} \Sigma_{P_{t+k}}]$$

where $\bar{E}_t^k[.] = \bar{E}_t \bar{E}_{t+1} \dots \bar{E}_{t+k-1}[.]$

- ▶ Note we need to understand the form of the iterated conditional expectation $\bar{E}_t^k[.]$ in order to calculate P_t 's. According to (Allen Morris and Shin, 2006) the direct iteration fails here.

Further assumptions with regards to information structure

- ▶ each investor's information set $\Theta_{i,t}$ contains both the public information $\Omega_t = \{P_1, P_2, \dots, P_{t-1}\}$ and a private signal s_i w.r.p.t the end value of risky assets P_T .
- ▶ The aggregation of the private signals is unbiased,
 $k = T - t$.

$$\int_0^1 s_{t+k}^i di = P_{t+k}$$

- ▶ In addition we assume, when forming expectation, each investor put a weight λ upon his private signal and $1 - \lambda$ on the public best forecast (λ is constant across individuals and in time), i.e.:

$$E_t^i[P_{t+1}] = (1 - \lambda)P_{t+1}^* + \lambda s_{t+1}^i$$

Further assumptions with regards to information structure

- ▶ each investor's information set $\Theta_{i,t}$ contains both the public information $\Omega_t = \{P_1, P_2, \dots, P_{t-1}\}$ and a private signal s_i w.r.p.t the end value of risky assets P_T .
- ▶ The aggregation of the private signals is unbiased, $k = T - t$.

$$\int_0^1 s_{t+k}^i di = P_{t+k}$$

- ▶ In addition we assume, when forming expectation, each investor put a weight λ upon his private signal and $1 - \lambda$ on the public best forecast (λ is constant across individuals and in time), i.e.:

$$E_t^i[P_{t+1}] = (1 - \lambda)P_{t+1}^* + \lambda s_{t+1}^i$$

Further assumptions with regards to information structure

- ▶ each investor's information set $\Theta_{i,t}$ contains both the public information $\Omega_t = \{P_1, P_2, \dots, P_{t-1}\}$ and a private signal s_i w.r.p.t the end value of risky assets P_T .
- ▶ The aggregation of the private signals is unbiased, $k = T - t$.

$$\int_0^1 s_{t+k}^i di = P_{t+k}$$

- ▶ In addition we assume, when forming expectation, each investor put a weight λ upon his private signal and $1 - \lambda$ on the public best forecast (λ is constant across individuals and in time), i.e.:

$$E_t^i[P_{t+1}] = (1 - \lambda)P_{t+1}^* + \lambda s_{t+1}^i$$

Further assumptions with regards to information structure

- plug the equation into the previous results:

$$P_t = \bar{E}^{T-t}[P_T] - \sum_{k=1}^{T-t} \beta^k [\theta Z_{t+k-1} \Sigma_{P_{t+k}}]$$

$$E_t^i[P_{t+1}] = (1 - \lambda)P_{t+1}^* + \lambda s_{t+1}^i$$

- we get:

$$\bar{E}_t^k[P_{t+k}] = (1 - \lambda^k)P_{t+k}^* + \lambda^k P_{t+k}$$

$$P_t = (1 - \lambda^k) P_{t+k}^* + \lambda^k P_{t+k} - \sum_{k=1}^{T-t} \beta^k [\theta Z_{t+k-1} \Sigma_{P_{t+k}}]$$

Further assumptions with regards to information structure

- plug the equation into the previous results:

$$P_t = \bar{E}^{T-t}[P_T] - \sum_{k=1}^{T-t} \beta^k [\theta Z_{t+k-1} \Sigma_{P_{t+k}}]$$

$$E_t^i[P_{t+1}] = (1 - \lambda)P_{t+1}^* + \lambda s_{t+1}^i$$

- we get:

$$\bar{E}_t^k[P_{t+k}] = (1 - \lambda^k)P_{t+k}^* + \lambda^k P_{t+k}$$

$$P_t = \left(1 - \lambda^k\right) P_{t+k}^* + \lambda^k P_{t+k} - \sum_{k=1}^{T-t} \beta^k [\theta Z_{t+k-1} \Sigma_{P_{t+k}}]$$

Sum everything together

- Note the last factor in the previous equation is but a random noise, we have the final model in a standerlized HOB fashion:

$$P_t = \beta^k \left[\left(1 - \lambda^k \right) P_{t+k}^* + \lambda^k P_{t+k} \right] + u_t$$

Model Discussion

- ▶ vs. Heterogenous Beliefs models.
 - ▶ in Heterogenous Beliefs models, λ is set to 1.
 - ▶ $P_t = \beta^k P_{t+k} + u_t$
 - ▶ in Heterogenous Beliefs models, the unbiased average opinion does not hold.
 - ▶ an example by (Wang and Zhao, 2006),
$$P_t = \beta(P_{t+1}^* + n_1 E_{1,t}[\Delta_{t+1}] + n_2 E_{1,t}[\Delta_{t+1}])$$
- ▶ vs. (Monnin, 2004)
 - ▶ we focus on short-run HBO effect, while (Monnin, 2004) emphasize long-run HBO effects.
 - ▶ we place BCE directly upon prices rather than dividends.

Model Discussion

- ▶ vs. Heterogenous Beliefs models.
 - ▶ in Heterogenous Beliefs models, λ is set to 1.
 - ▶ $P_t = \beta^k P_{t+k} + u_t$
 - ▶ in Heterogenous Beliefs models, the unbiased average opinion does not hold.
 - ▶ an example by (Wang and Zhao, 2006),
$$P_t = \beta(P_{t+1}^* + n_1 E_{1,t}[\Delta_{t+1}] + n_2 E_{1,t}[\Delta_{t+1}])$$
- ▶ vs. (Monnin, 2004)
 - ▶ we focus on short-run HBO effect, while (Monnin, 2004) emphasize long-run HBO effects.
 - ▶ we place BCE directly upon prices rather than dividends.

Model Discussion

- ▶ vs. Heterogenous Beliefs models.
 - ▶ in Heterogenous Beliefs models, λ is set to 1.
 - ▶ $P_t = \beta^k P_{t+k} + u_t$
 - ▶ in Heterogenous Beliefs models, the unbiased average opinion does not hold.
 - ▶ an example by (Wang and Zhao, 2006),
$$P_t = \beta(P_{t+1}^* + n_1 E_{1,t}[\Delta_{t+1}] + n_2 E_{1,t}[\Delta_{t+1}])$$
- ▶ vs. (Monnin, 2004)
 - ▶ we focus on short-run HBO effect, while (Monnin, 2004) emphasize long-run HBO effects.
 - ▶ we place BCE directly upon prices rather than dividends.

Model Discussion

- ▶ vs. Heterogenous Beliefs models.
 - ▶ in Heterogenous Beliefs models, λ is set to 1.
 - ▶ $P_t = \beta^k P_{t+k} + u_t$
 - ▶ in Heterogenous Beliefs models, the unbiased average opinion does not hold.
 - ▶ an example by (Wang and Zhao, 2006),
$$P_t = \beta(P_{t+1}^* + n_1 E_{1,t}[\Delta_{t+1}] + n_2 E_{1,t}[\Delta_{t+1}])$$
- ▶ vs. (Monnin, 2004)
 - ▶ we focus on short-run HBO effect, while (Monnin, 2004) emphasize long-run HBO effects.
 - ▶ we place BCE directly upon prices rather than dividends.

Model Discussion

- ▶ vs. Heterogenous Beliefs models.
 - ▶ in Heterogenous Beliefs models, λ is set to 1.
 - ▶ $P_t = \beta^k P_{t+k} + u_t$
 - ▶ in Heterogenous Beliefs models, the unbiased average opinion does not hold.
 - ▶ an example by (Wang and Zhao, 2006),
$$P_t = \beta(P_{t+1}^* + n_1 E_{1,t}[\Delta_{t+1}] + n_2 E_{1,t}[\Delta_{t+1}])$$
- ▶ vs. (Monnin, 2004)
 - ▶ we focus on short-run HBO effect, while (Monnin, 2004) emphasize long-run HBO effects.
 - ▶ we place BCE directly upon prices rather than dividends.

Outline

Introduction

Literature Review and Related Works

The Model

Model Test

Conclusion and Summary

How to test the model?

- ▶ Two problems:
 - ▶ how to derive estimation of public forecasts P_{t+k}^*
 - ▶ require future prices as explanatory variables.
- ▶ Solutions:
 - ▶ use a AR(p)-ARCH(q) as the best public forecasts, small case $p_t = \log P_t$.

$$\Delta p_t = \alpha + \varphi_1 \Delta p_{t-1} + \dots + \varphi_p \Delta p_{t-p} + v_t$$

$$v_t^2 = \gamma + \phi_1 v_{t-1}^2 + \dots + \phi_q v_{t-q}^2 + w_t$$

- ▶ use ex post prices to estimate the model.

How to test the model?

- ▶ Two problems:
 - ▶ how to derive estimation of public forecasts P_{t+k}^*
 - ▶ require future prices as explanatory variables.
- ▶ Solutions:
 - ▶ use a AR(p)-ARCH(q) as the best public forecasts, small case $p_t = \log P_t$.

$$\Delta p_t = \alpha + \varphi_1 \Delta p_{t-1} + \dots + \varphi_p \Delta p_{t-p} + v_t$$

$$v_t^2 = \gamma + \phi_1 v_{t-1}^2 + \dots + \phi_q v_{t-q}^2 + w_t$$

- ▶ use ex post prices to estimate the model.

Data Description

- ▶ Source: <http://www.resset.cn>
 - ▶ Our model allows for multi-asset case, however, for simplicity we include only one risky asset in the test, the Shanghai Stock Exchange Index as a approximation. We use the monthly data and a time interval from 2000-05-03 to 2010-05-03.
 - ▶ We use the monthly risk-free interest rate given by www.resset.cn to calculate β .

Data Description

- ▶ Source: <http://www.resset.cn>
 - ▶ Our model allows for multi-asset case, however, for simplicity we include only one risky asset in the test, the Shanghai Stock Exchange Index as a approximation. We use the monthly data and a time interval from 2000-05-03 to 2010-05-03.
 - ▶ We use the monthly risk-free interest rate given by www.resset.cn to calculate β .

Step 1: Calculate the public best forecasts

- ▶ Conduct the ADF test for the AR(p)-ARCH(q) model.
 - ▶ Reject unit root null hypothesis at 5% significance level under AR(3)-ARCH(1)
 - ▶ ADF T-value Table.

Table: output

Variables	ADF T-value
Δp_{t-1}	-7.44***
Δp_{t-2}	-22.00***
Δp_{t-3}	-12.81***

- ▶ the regression model to calculate P_{t+k}^* :

$$\Delta p_t = -0.0429\Delta p_{t-1} - 0.1260\Delta p_{t-2} - 0.0738\Delta p_{t-3}$$

Step 1: Calculate the public best forecasts

- ▶ Conduct the ADF test for the AR(p)-ARCH(q) model.
 - ▶ Reject unit root null hypothesis at 5% significance level under AR(3)-ARCH(1)
 - ▶ ADF T-value Table.

Table: output

Variables	ADF T-value
Δp_{t-1}	-7.44***
Δp_{t-2}	-22.00***
Δp_{t-3}	-12.81***

- ▶ the regression model to calculate P_{t+k}^* :

$$\Delta p_t = -0.0429\Delta p_{t-1} - 0.1260\Delta p_{t-2} - 0.0738\Delta p_{t-3}$$

Step 1: Calculate the public best forecasts

- ▶ Conduct the ADF test for the AR(p)-ARCH(q) model.
 - ▶ Reject unit root null hypothesis at 5% significance level under AR(3)-ARCH(1)
 - ▶ ADF T-value Table.

Table: output

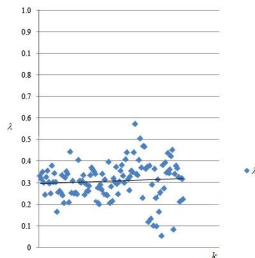
Variables	ADF T-value
Δp_{t-1}	-7.44***
Δp_{t-2}	-22.00***
Δp_{t-3}	-12.81***

- ▶ the regression model to calculate P_{t+k}^* :

$$\Delta p_t = -0.0429\Delta p_{t-1} - 0.1260\Delta p_{t-2} - 0.0738\Delta p_{t-3}$$

Step 2: Regression P_t with P_{t+k} and P_{t+k}^* for different k 's.

- ▶ Note $k = T - t$ represents the number of periods till the end period value of which investors try to guess.
- ▶ Plot the estimated value of λ which passed a 10% T-test in accordance to k .



Outline

Introduction

Literature Review and Related Works

The Model

Model Test

Conclusion and Summary

Conclusion

- ▶ From the above figure we discover the λ 's lies between 0.2 to 0.4. The implication here is that investors place 1-3 more times of weight on the public available information compared to their private signals.
- ▶ In addition, regress λ w.r.p.t k , we discover no strong link between the two variables. This is consistent with our assumption that λ is constant in time.

Conclusion

- ▶ From the above figure we discover the λ 's lies between 0.2 to 0.4. The implication here is that investors place 1-3 more times of weight on the public available information compared to their private signals.
- ▶ In addition, regress λ w.r.p.t k , we discover no strong link between the two variables. This is consistent with our assumption that λ is constant in time.

Summary

- ▶ In this article, we proposed a standard HOB model in line with (Allen, Morris and Shin, 2006) under assumptions common in HOB literatures. The main features are: first, our model is simple and testable without losing general HOB insights; second, our model high-lights short-run BCE on prices.
- ▶ Adopting monthly data from Shanghai Stock Exchange, we confirm the existence and magnitude of BCE in Chinese Stock Market, reaching a conclusion that Chinese investors place more weight on public information than private signals.

Summary

- ▶ In this article, we proposed a standard HOB model in line with (Allen, Morris and Shin, 2006) under assumptions common in HOB literatures. The main features are: first, our model is simple and testable without losing general HOB insights; second, our model highlights short-run BCE on prices.
- ▶ Adopting monthly data from Shanghai Stock Exchange, we confirm the existence and magnitude of BCE in Chinese Stock Market, reaching a conclusion that Chinese investors place more weight on public information than private signals.

Further Discussion

- ▶ Loosen some restrictive assumptions. For example, the constant λ ; though the data in this article supports constant λ in time, it is quite possible that the information weight can vary across individuals. Thus, individual specific λ 's may provide more through understanding of BCE in reality.
- ▶ For simplicity concerns, we only include one risky asset in market participants' portfolio. Even if this could give us a basic illustration about how HOB affects price, a detailed study incorporates multi-asset will render a more general picture.

Further Discussion

- ▶ Loosen some restrictive assumptions. For example, the constant λ ; though the data in this article supports constant λ in time, it is quite possible that the information weight can vary across individuals. Thus, individual specific λ 's may provide more through understanding of BCE in reality.
- ▶ For simplicity concerns, we only include one risky asset in market participants' portfolio. Even if this could give us a basic illustration about how HOB affects price, a detailed study incorporates multi-asset will render a more general picture.

Questions and Answers.

Thank you for listening!