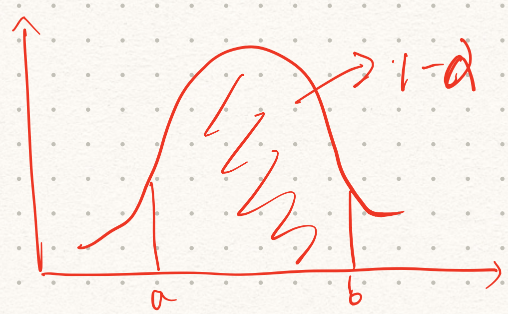
posterior $p(\theta|y) = \text{Beta}(y+1, n-y+1)$

$\rightarrow$ mean $= \frac{y+1}{n+2}$, median $\approx \frac{y+2/3}{n+4/3}$, mode $= \frac{y}{n}$

standard deviation $= \sqrt{\frac{(y+1)(n-y+1)}{(n+2)^2(n+3)}}$

posterior interval: $(100(1-\alpha)\%)$: $[a,b]$ s.t. $\int_a^b \frac{\theta^y(1-\theta)^{n-y}}{B(y+1,n-y+1)} = 1-\alpha$



Confidence interval vs. posterior interval:

$\hat{\theta} \pm 1.96\sqrt{\frac{\hat{\theta}(1-\hat{\theta})}{n}}$ $[a,b]$

for $p(\theta|y) \propto p(\theta)p(y|\theta) \propto \text{Beta}$

beta binomial

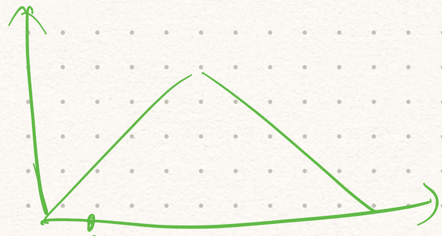
Use sampling: e.g. rnorm(1000) $\rightarrow$ histogram...

use a sequence of $\theta_1, \ldots, \theta_n$

$E[\theta|y] = \int \theta\, p(\theta|y)\, d\theta \approx \sum_{i=1}^{M} \theta_i / M$

$Var[\theta|y] \rightarrow$ sample variance

95% : quartile : R code



at every given point, know $p(\theta|y)$:

Grid Approx :

Cut

for each point ⇒ posterior distribution

→ resample according to weights..

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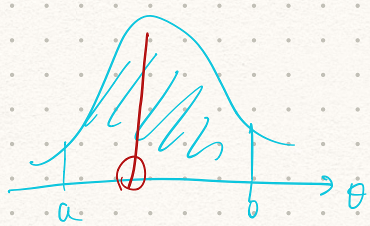
conjugate prior

• inverse variance — precision

posterior predictive distribution :

$$p(\tilde{y}|y)=\int p(\tilde{y}|\theta)\,p(\theta|y)\,d\theta.$$



Recall $E[\tilde{y}|\theta]=\theta$, $Var[\tilde{y}|\theta]=\sigma^2$

$$E[\tilde{y}|y] = E[E(\tilde{y}|\theta, y)|y] = E[\theta|y] = \mu_1$$

$$Var[\tilde{y}|y] = E[Var(\tilde{y}|\theta, y)|y]$$

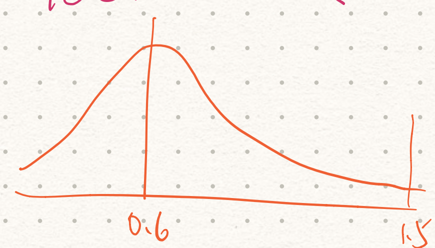
Asthma example

$y \sim \text{Poisson}(x\theta)$, $x = 2$

true . . . . rate

→ prior: $\approx 0.6, \leq 1.5$ per 100,000 人

$\theta \sim \text{Gamma}(3, 5)$



observation: $y = 3$, $x = 2$

(die) (2 year)

posterior: $\theta \sim \text{Gamma}(6, 7)$: (Poisson × Gamma = Gamma)

if observe 30 deathes

$y = 30$, $x = 10$

→ $\text{Gamma}(33, 25)$

Cancer example:

likelihood: $y_j \sim \text{Poisson}(\underset{\text{10 year}}{10}\, n_j \theta_j)$

informative prior:

Gamma(20, 430,000)

prior mean $\frac{\alpha}{\beta} = 4.65 \times 10^{-5}$,

sd $\frac{\sqrt{\alpha}}{\beta}$