Bayes Analysis:

modeling ☆

$$\overset{\text{posterior}}{P(A|B)} = \frac{\overset{\text{prior}}{P(A)}\,\boxed{\overset{\text{likelihood}}{P(B|A)}}}{P(B)}$$

$P(B)$? evidence (marginal likelihood)

computation

θ: unobserved (can't see)

y: observed datasets / known quantities. (can see)

- estimate unknown (parameters) from known (data)

  quantify uncertainty from.

- induction. 循环 by several data.

  second round

  $$P(A|B) = P(A)\,P(B|A) / P(B)$$

  1° 2° 3° 4°

  start of second round, $P(A) = P(A|B)$ from last round

Steps of Bayesian analysis

Joint probability dis'

↓

posterior dis'

↓

evaluating...

y_i observation

θ probability of survival

$y_i \overset{iid}{\sim} p(\cdot|\theta),\ 1\le i\le n$, $\theta \sim p(\cdot)$ prior

likelihood model

$$p(\theta|y) = \frac{p(\theta)\,p(y|\theta)}{p(y)}$$

$p(y) \to \sum_\theta p(\theta)\,p(y|\theta)$

$$p(\theta|y) \propto p(\theta)\,p(y|\theta)$$

e.g. hemophilia 血友病 . ♂ > ♀

XX $\quad$ X^AY $\quad$ XY

50% XX^A $\quad$ XX^A.

50% XX

possibilities $\begin{cases} \theta=1 & \text{carrier} \\ \theta=0 & \text{not} \end{cases}$

prior distribution: $p(\theta=1)=p(\theta=0)=\frac{1}{2}$

$\frac{1}{2}\ X\underline{X}$ $\qquad \frac{1}{2}\ X\underline{X}^A$ $\qquad \theta$: 带.

prior. $P(\theta) = \frac{1}{2}$

$$P(A_{带} \mid 都是无) = \frac{\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}}{\frac{1}{2} \times \frac{1}{4} + \frac{1}{2} \times 1} = \frac{1}{5}$$

Another solution?

$$\frac{P(X^AX^a \mid y)}{\underbrace{P(X^AX^A \mid y)}_{\substack{\text{relative probability} \\ \text{(a trick)}}}} = \frac{P(X^AX^a) \cdot P(y \mid X^AX^a) / \cancel{P(y)}}{P(X^AX^A) \cdot P(y \mid X^AX^A) / \cancel{P(y)}} = \frac{P(y \mid X^AX^a)}{P(y \mid X^AX^A)} = \frac{\frac{1}{2} \times \frac{1}{2}}{1}$$

$$= \frac{1}{4}$$

Since $P(X^AX^a \mid y) + P(X^AX^A \mid y) = 1$,

thus $P(X^AX^a \mid y) = \frac{1}{5}$

$P(X^AX^A \mid y) = \frac{4}{5}$

$$P(\theta \mid y_1 = y_2 = y_3 = 0) = \frac{\frac{1}{2} \times \frac{1}{8}}{\frac{1}{16} + \frac{1}{2} \times 1}$$

if $\theta = \frac{1}{2}$

$= \ldots$

if $\theta = \frac{1}{5}$ $= \frac{\frac{1}{5} \times \frac{1}{8}}{\frac{1}{40} + \frac{4}{5} \times 1}$

② Relative :

$$\frac{P(X^A X^a)\, P(y \mid X^A X^a)}{P(X^A X^A)\, P(y \mid X^A X^A)} = \frac{\frac{1}{5}\ \frac{1}{8}}{1} \times \frac{1}{4}$$

$=$

$\frac{1}{5}, \frac{4}{5}$ P 10