Multiparameter Models

$\theta = (\theta_1, \theta_2)$

$\theta_1 \in \mathbb{R}^{d_1}$ parameter of interest

$\theta_2 \in \mathbb{R}^{d_2}$ nuisance parameters

e.g. $y_i \overset{i.i.d}{\sim} N(\mu, \sigma^2)$

μ: poi

σ^2: nuisance parameters

- General Strategy: joint posterior $p(\theta_1, \theta_2 \mid obs)$

$\rightarrow$ marginal $p(\theta_1 \mid obs) = \int_{\mathbb{R}^{d_2}} p(\theta_1, \theta_2 \mid obs)\, d\theta_2$

Recall: in single parameter models, when σ^2 is known

$\rightarrow$ inference for μ: Gaussian - Ga ... Conjugate model

Now: σ^2 is unknown,

but we still want to make inference for μ.

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prior: $p(\mu, \sigma^2) = p(\sigma^2)\, p(\mu \mid \sigma^2)$

$\overset{or}{=} p(\sigma^2)\, p(\mu)$ (we'll use this)

$p(\sigma^2) \propto \frac{1}{\sigma^2}$ non-informative prior

posterior (joint):

$$p(\mu, \sigma^2 \mid y_1, y_2, \dots, y_n) \propto p(\mu, \sigma^2)\, p(y_1, \dots, y_n \mid \mu, \sigma^2)$$

$$\propto 1/\sigma^2 \cdot \prod_{i=1}^{N} \left[\boxed{\frac{1}{\sigma}} \exp\left(-\frac{(y_i-\mu)^2}{2\sigma^2}\right) \right]$$

since σ is also unknown

$$= \sigma^{-n-2} \exp\left(-\frac{1}{2\sigma^2}\left[(n-1)S_n^2 + n(\bar{y}-\mu)^2\right]\right)$$

Check?

Sample variance

$$S_n^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i-\bar{y})^2$$

Sample mean