Methods of the track reconstruction

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Outline

- Introduction;
- Track reconstruction;
- Kalman Filter;
- Cellular Automaton;
- Hough Transform;
- Summary.

Introduction

- In the extreme density and temperature conditions realized during the relativistic heavy ion collisions, hundreds of charged particles are produced. In order to extract information about physics in the experiment, particle tracks should be efficiently reconstructed and their parameters should be determined with high precision. Reconstruction of events with high multiplicity is a rather challenging task.

- Track reconstruction is a very important part in the analysis of a high-energy physics experiment. The parameters of the tracks are used in higher level analyses, for example in vertex reconstruction.

Track reconstruction

- This task of track reconstruction is often divided into two different subtasks:
  - Track finding (pattern recognition);
  - Track fitting (parameter estimation).

- Track finding:
  starts out with a set of position measurements (provided by a tracking detector). The aim is to group these measurements together in subsets, where each subset containing measurements from one traveling charged particle.
- Track fitting:
  for each of the subsets provided by the track finder, finds the optimal estimate of a set of parameters uniquely describing the state of the particle.

Track reconstruction

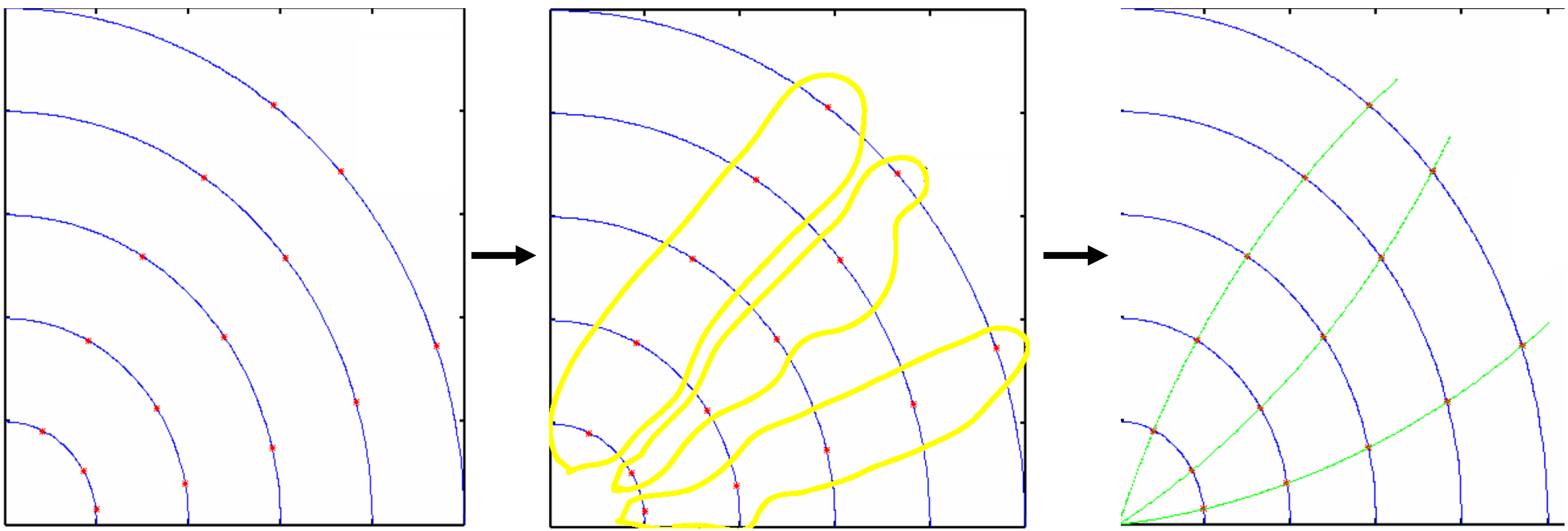
Measurements

Track finding

choosing which hits are
part of the track

Track fitting

finding the optimal estimate of
parameters of a subset



Ref.

1. A. Strandlie, 3D silicon detector workshop, Department of Physics, University of Oslo, September 2007

Track reconstruction algorithms (1)

The algorithms can be classified using several methods:

1. Combinatorial method:

   All possible combinations of the points are generated and checked against a track model. If the fit is good, the track candidate is accepted.

   - The simplest and most time requiring method → can only be used with very few points.
   - The number of combinatorial tracks in a detector with different layers is approximately proportional to N^l, where N is the number of tracks, equal to the number of points in one layer, and l is the number of layers.

     With 1000 tracks in 5 layers this gives 10^{15} combinations.

- The number of combinations can be reduced by using additional knowledge of the track geometry. This leads to the more appropriate tracking algorithms, the local and global methods.

Ref.

1. Tracking in the Silicon Tracker System of the CBM Experiment using Hough Transform. J.Gläß, C. Steinle, R. Männer, University of Mannheim, Computer Science V, D-68131 Mannheim, Germany

Track reconstruction algorithms (2)

2. Local methods:

Processes of track-finding are independent from each other.

- Some few points generate an initial track candidate;
- Using interpolation or extrapolation additional points are collected;
- When more points can be found the track candidate is assumed to be good, otherwise the track candidate is discarded.

- Local methods always contain unsuccessful track candidates, that’s why the same points have to be used in several combinations. Therefore the complexity in time rises faster than linear with the number of points. The processing depends on the ordering of the points and the initial track candidate.
- Ex: the Cellular Automaton

Track reconstruction algorithms (3)

3. Global methods:

All points are processed by the algorithm in the same way.

- Global methods can be seen as transformation.
- The algorithm produces a list of tracks or a list where tracks can be found easier than in the original data.

- The processing complexity of a global method in principle is proportional to the number of points within an event. In contrast to local methods, global methods do not depend on the order of the processing of the points.

- Ex: the Hough Transform

Kalman Filter

The Kalman filter is a recursive filter, which evaluates the state of a linear dynamic system using a set of inaccurate measurements with the errors distributed according to the Gauss distribution[1].

- Filter: The process of finding the “best estimate” from noisy data amounts to “filtering out” the noise. However a Kalman filter also doesn’t just clean up the data measurements, but also projects these measurements onto the state estimate;
- Recursive: so that new measurements can be processed as they arrive;
- Optimal: the Kalman filter minimizes the mean square error of the estimated parameters.

Ref.

1. R. Fruehwirth, Application of Kalman filtering to track and vertex fitting, Nuclear Instruments and Methods in Physics Research A 262. 1987. 444-450pp.
2. L.Kleeman Understanding and Applying Kalman Filtering

How works Kalman Filter

- The task is to estimate state vector X that is governed by the linear stochastic equation
$$X_k = A_{k-1} X_{k-1} + \nu_{k-1} \qquad k = 1, \ldots, N$$
where the matrix A_{k-1} relates the state at step $k-1$ to state k;
$\{\nu\}$ – the process noise (sequence of independent Gaussian variables (Ex: account for the influence of multiple scattering on the state vector) with the covariance matrix Q_k.

- The input information: a sequence of measurements $\{z\}$ described by a linear function of the vector X - measurement equation
$$z_k = H_k X_k + \eta_k \qquad k = 1, \ldots, N$$
where the matrix H_k relates the state to the measurement;
$\{\eta\}$ is a sequence of Gaussian random variables with the covariance matrix V_k.

How works Kalman Filter

- The main idea of the Kalman filter is that the optimal (in mean square sense) estimate $\hat{X}_k$ should be the sum of an extrapolated estimate $\tilde{X}_k$ and a weighted difference between the actual measurement z_k and the measurement prediction $H_k \tilde{X}_k$

$$\hat{X}_k = \tilde{X}_k + K_k (z_k - H_k \tilde{X}_k)$$

  where $\tilde{X}_k = A_{k-1} \hat{X}_{k-1}$

- The matrix K_k is called *the filter gain* and is chosen to minimize the sum of diagonal elements of the estimated error covariance matrix $\hat{P}_k$

$$\hat{P}_k = E[(X_k - \hat{X}_k)(X_k - \hat{X}_k)^T]$$

  where E denotes the mathematical expectation. The minimization leads to the following formula for K_k : $K_k = \dfrac{\tilde{P}_k H_k^T}{V_k - H_k \tilde{P}_k H_k^T}$

  where $\tilde{P}_k$ is an estimate extrapolated from the error covariance matrix

$$\tilde{P}_k = A_{k-1} \hat{P}_k A_{k-1}^T + Q_{k-1}$$

  The new minimized value of the error covariance matrix $\hat{P}_k$ is defined by the equation

$$\hat{P}_k = (I - K_k H_k) \tilde{P}_k$$

How works Kalman Filter (formulas)

The algorithm of the Kalman filter consists of two steps:

- Prediction (the estimation of the state vector at a "future" time) step defined by

$$\tilde{X}_k = A_{k-1}\hat{X}_{k-1}$$

$$\tilde{P}_k = A_{k-1}\hat{P}_k A_{k-1}^T + Q_{k-1}$$

- Filtering (the estimation of the "present" state vector, based upon all "past" measurements) step given by

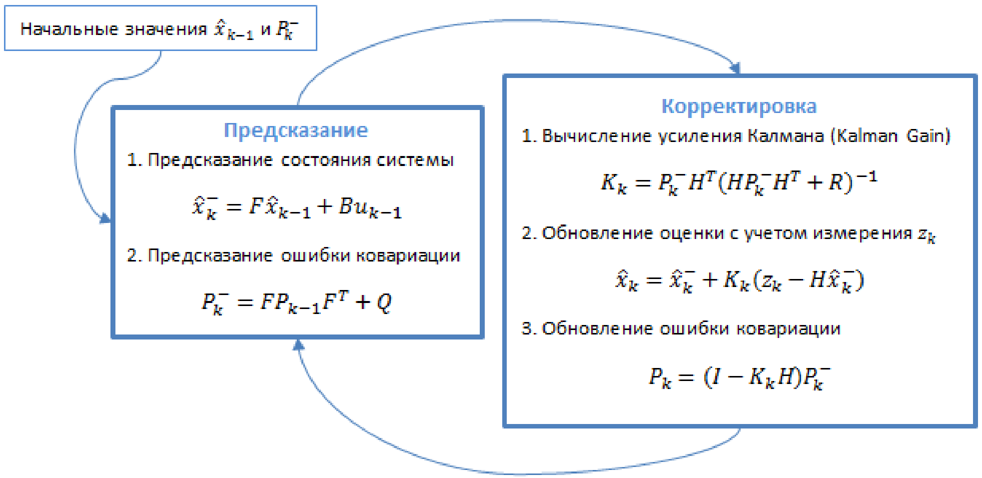
$$K_k = \frac{\tilde{P}_k H_k^T}{V_k - H_k \tilde{P}_k H_k^T}$$

$$\hat{X}_k = \tilde{X}_k + K_k(z_k - H_k \tilde{X}_k)$$

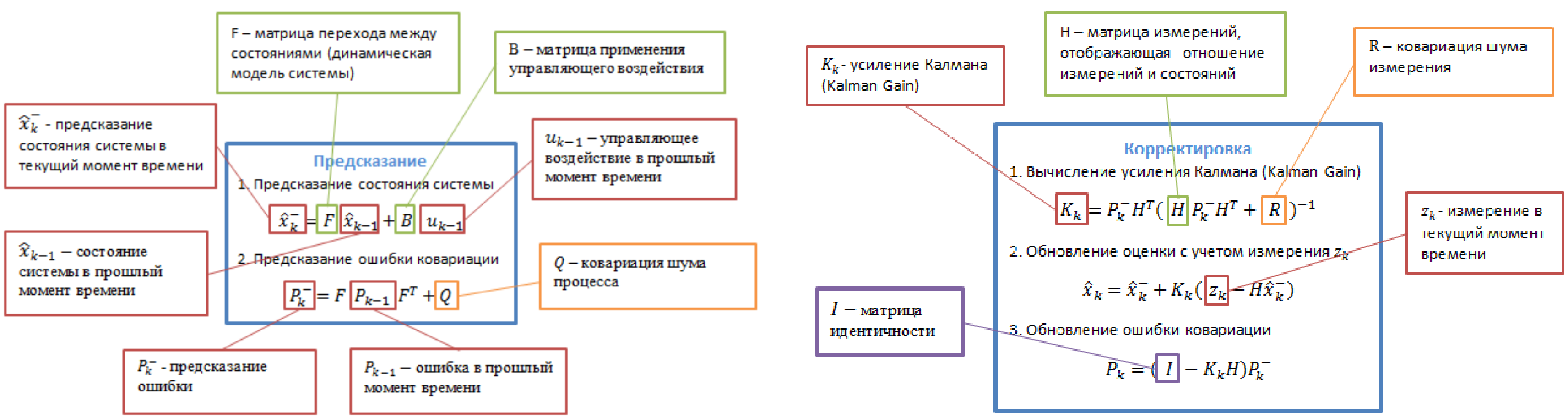
$$\hat{P}_k = (I - K_k H_k)\tilde{P}_k$$

Scheme of Kalman Filter

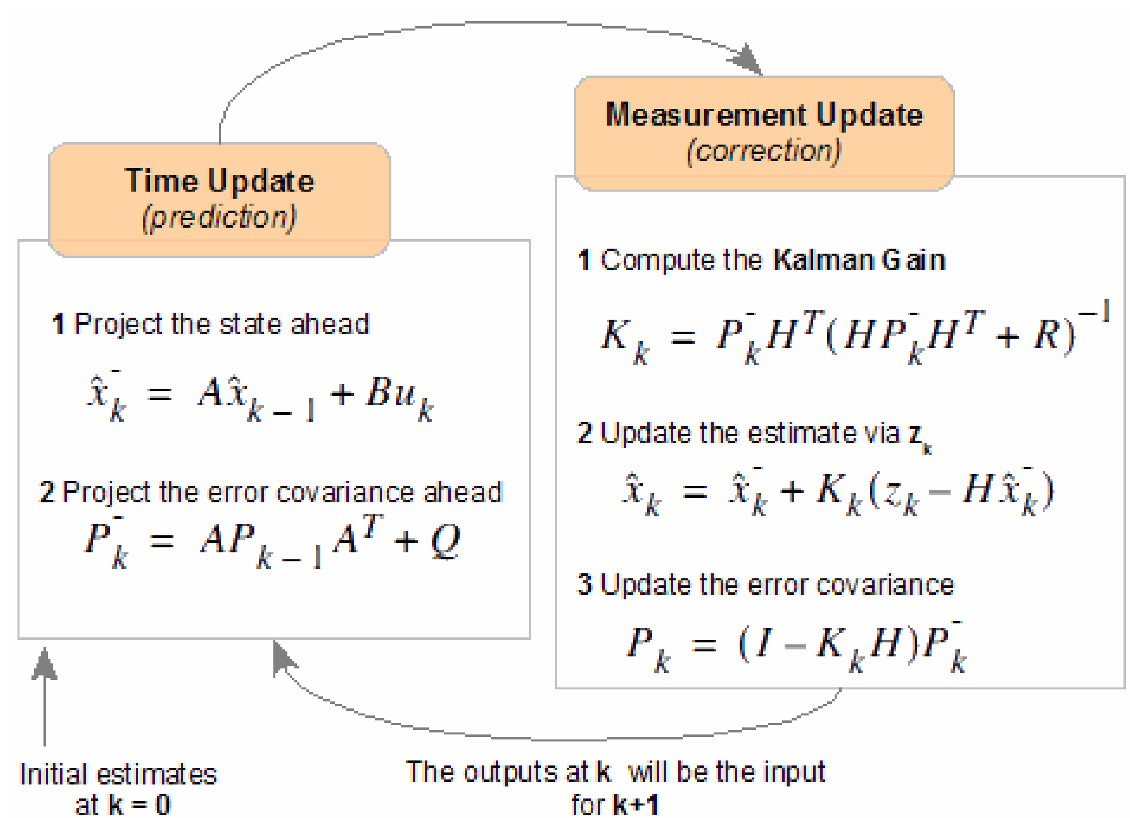
in multi-dimensional case



- Prediction
- Correction



Scheme of Kalman Filter



Kalman Filter

- The Kalman Filter has two functions:
  - track finding -using information about positions of detector hits it finds track candidates;
  - track fitting –track candidates are fitted to extract track parameters.

  could be used for track finding and fitting concurrently due to recursive nature of filter

Problems:

- The Kalman Filter is a local method in the sense that we can get a next hit in the track on one step, so if we get a fake hit, further part of the reconstructed track will be fake too and we can't make a difference between true and fake hits. So it will be better to use it together with other reconstruction methods (Ex: the Cellular Automation or the Hough Transform).
- In order to begin the Kalman filter procedure one needs so-called track seeds.

Seeds for Kalman Filter

- Connect any 2-3 hits as a seed , fit them and continue.
  All possible combinations of the points have to be checked.
  → as the result, many combinations → requires much time.
- So at least some rational criterion for selection should be used
  - SUSi (Searching Utility for the Silicon) [1]
  - HOLMES [2]
  - L2Sili [3]
  - OSCAR [4]

  - Cellular Automation

  - Hough Transform

Ref.
1. J. Rieling, Ph.D. Thesis, Ruprecht-Karls-Universit.at, Heidelberg, 1997.
2. M. Schmelling, HERA-B Note 99-086, 1999.
3 S. Schmidt, Ph.D. Thesis, University of Copenhagen, 2000.
4. U. Schwanke, Ph.D. Thesis, Humboldt-Universit.at zu Berlin, 2000.

Cellular Automaton

- As one of the possibilities of getting seeds for the Kalman Filter one can take the Cellular Automaton method.

- The cellular automaton is a dynamic method, it evolves in a discrete space consisting of cells: Each cell can take several states.
- The laws of evolution are local, i.e., the dynamics of the system is determined by an unchanged set of rules (for example a table) that relates the new state of a cell to the states of its nearest neighbors.
- The update of the states of the cells is done simultaneously at discrete time instants.

Ref.

1. I. Kisel, Event reconstruction in the CBM experiment, Nuclear Instruments and Methods in Physics Research A 566, 2006, 85–88pp

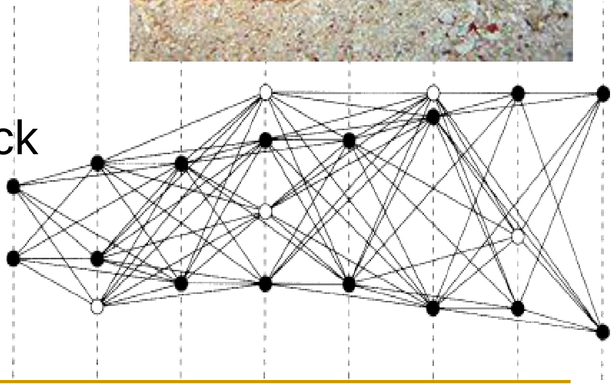
Cellular Automaton

Thus, to define a cellular automaton one has to define:

1. a cell and a set of its states (Ex: 0 and 1);
2. neighbors each cell can communicate with;
3. rules of evolution;
4. time evolution (Ex: simultaneous update of the states at given time instants).

There are two kinds of cellular automata:

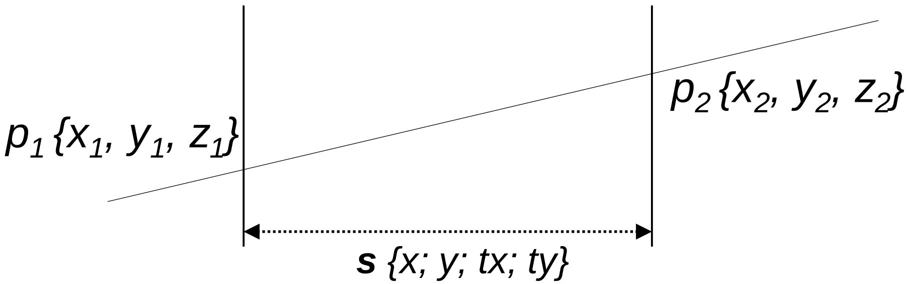
- using space-points as cell units (similar to the game "Life");
- segment-based algorithms which use short track segments as cell units.



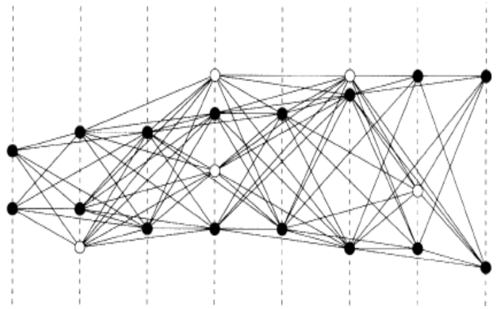
Cellular Automaton (Segment model)

- Each track in the detector is represented as a sequence of straight-line track segments.
- A segment *s* is a straight line with parameters *{x; y; tx; ty}* that connects two space-points (p_1; p_2) with (z_1; z_2) ($z_1 < z_2$) Each segment s is completely defined by its two space-points.

according a cellular automaton in silicon in the HERA-B vertex detector[1]



- A segment connects space-points
  - in neighboring layers of the detector;
  - (a segment can skip one layer).



Ref.

1. I. Abt, D. Emeliyanov, I. Kisel, S. Masciocchi, CATS: a cellular automaton for tracking in silicon for the HERA-B vertex detector, Nuclear Instruments and Methods in Physics Research A 489. 2002. 389-405pp

Algorithm of the Cellular Automaton

- The task is to find an optimal segment sequence U {$s_1, s_2, \ldots s_N$} that
  - maximizes track length by maximizing the number of segments N composing the track;
  - maximizes track smoothness by minimizing the sum of breaking angles between neighboring segments in the sequence.
- Index of such optimization: $$J(U) = N - \gamma \sum_{i=1}^{N-1} \frac{\varphi(s_{i+1}, s_i)}{z_{i+1,right} - z_{i,left}} \rightarrow \max_U \quad (*)$$
  - N - the number of segments
  - γ – a weight coefficient
  - breaking angle $$\varphi(s_i, s_{i+1}) = \sqrt{(t_{x,i+1} - t_{x,i})^2 + (t_{y,i+1} - t_{y,i})^2}$$

1. A cell is defined as a track segment *s*
2. Two segments are considered neighbors if
   - They share a common space-point $p_{i,right} = p_{i+1,left} \quad i = 1, \ldots, N-1$
   - Their breaking angle is less then *Φ* $$\varphi(s_1, s_j) \le \Phi, \forall s_j : p_{1,left} = p_{j,right}$$ $$\varphi(s_N, s_j) \le \Phi, \forall s_j : p_{N,right} = p_{j,left}$$

$\Phi = 2.5^\circ; \gamma = 0.1 m / \Phi$

Ref.

1. I. Abt, D. Emeliyanov, I. Kisel, S. Masciocchi, CATS: a cellular automaton for tracking in silicon for the HERA-B vertex detector, Nuclear Instruments and Methods in Physics Research A 489. 2002. 389-405pp

Algorithm of the Cellular Automaton

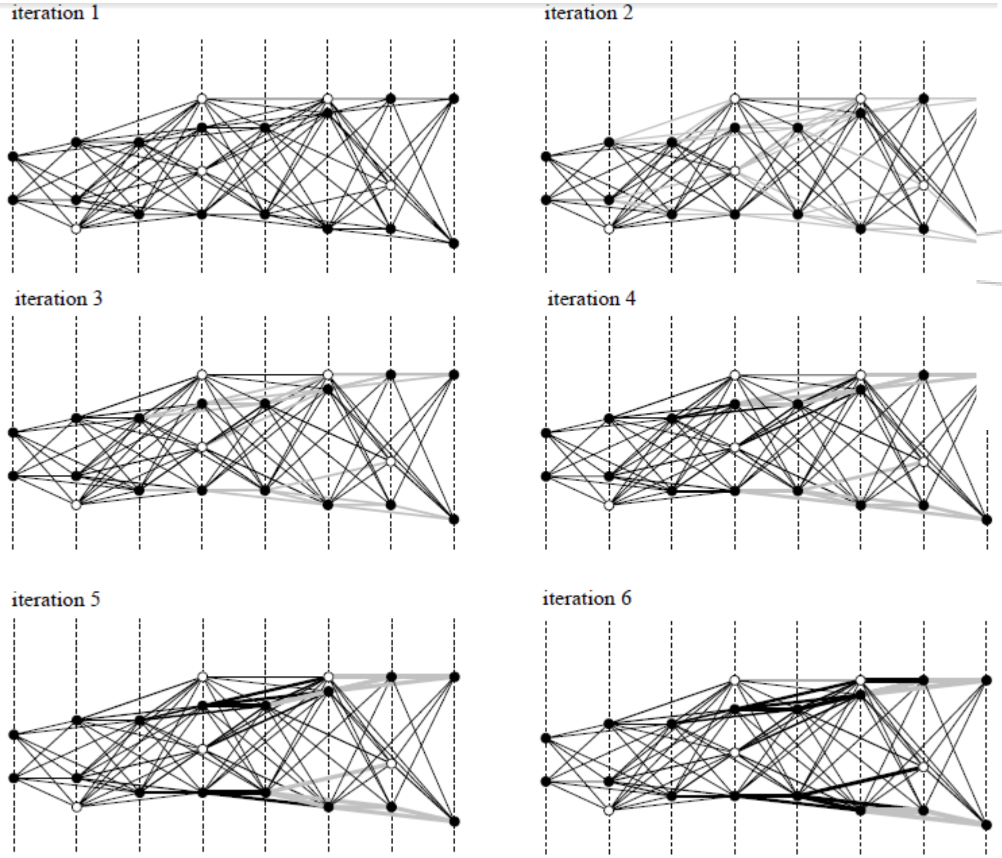
3. The evolution process is divided into
   - forward evolution when the automaton iteratively updates the states of all cells having neighbors;
   - backward pass when the automaton collects optimal sequences starting from cells with the highest states.

4. The rules of evolution

- During the forward evolution the automaton takes each cell and looks for its leftward neighbors. If there is such a neighbor and its state is equal to the cell's state, the cell's state will be increased by one.
- After completing a loop over all the cells, their previous states are simultaneously replaced by the increased ones.
  This process is iteratively repeated until there are no neighboring cells with the same states.
- At the end of the forward evolution, the state of each cell is equal to the length of an optimizing sequence that can be traced leftwards starting from this cell. Thus, the final cell state is equal to the first term in criterion ($*$).
- Thus the cellular automaton iteratively finds local maxima of the integer part of criterion ($*$).

- The backward pass starts with investigating the set of cells that have the highest state. These cells - as the first segments of track candidates.
- For each cell the automaton looking through the cell's leftward neighbors for the best (in the sense of the second term in ($*$)) prolongation.
- To be a prolongating one a cell must have a state lower by unity.
  If such a cell is found → assigns it to the track candidate, looks for its leftward neighbors and so on.
- The candidate tracing stops when a segment with the state of unity is assigned to the candidate.
- The baseline algorithm marks all segments assigned to a track as used and
  the automaton starts with another cell that has the highest state and is not marked.

Forward evolution and backward pass of the Cellular Automaton



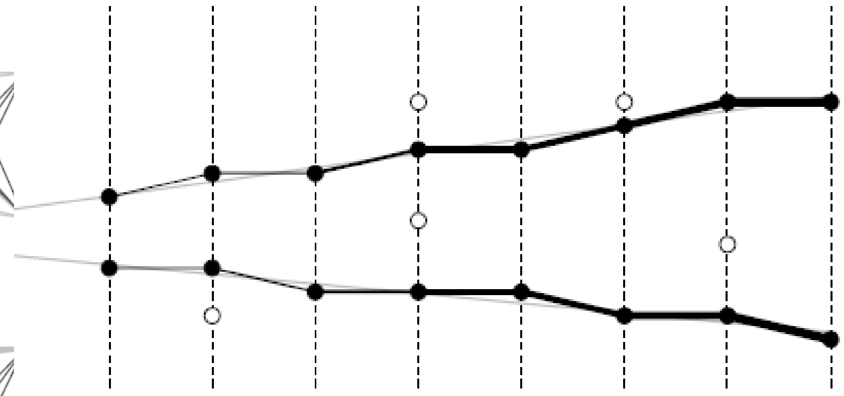
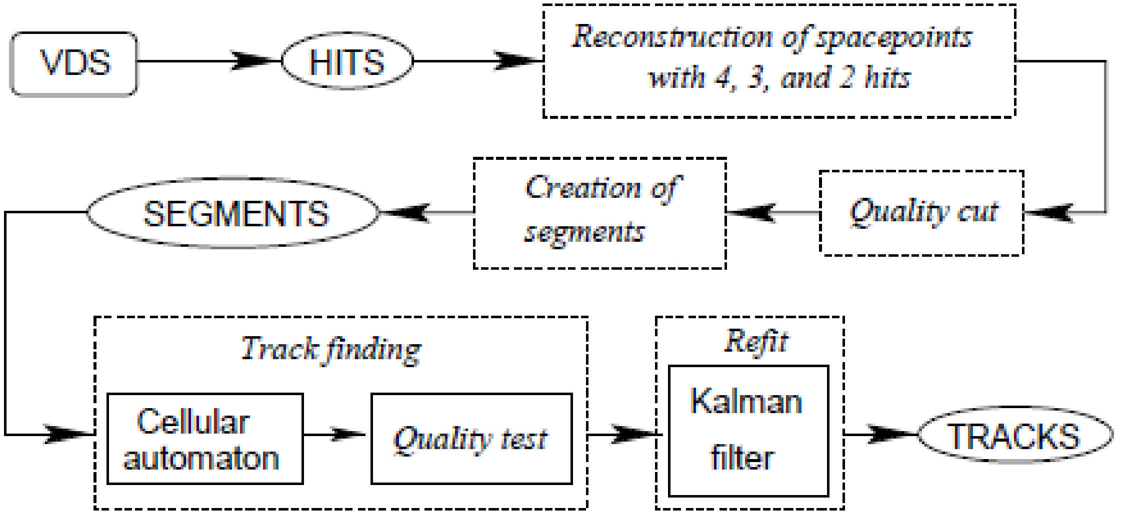


fig.1 the width of a segment represents its current state, i.e. segments with state 1 are shown by single lines, the width of the lines showing segments with state 2 is doubled and so on. Grey lines represent segments whose states are changed during an iteration.

fig.2 simulated tracks are shown by gray lines

Scheme of the track reconstruction chain implemented in CATS



Track fitting

- The task of the track fitting is

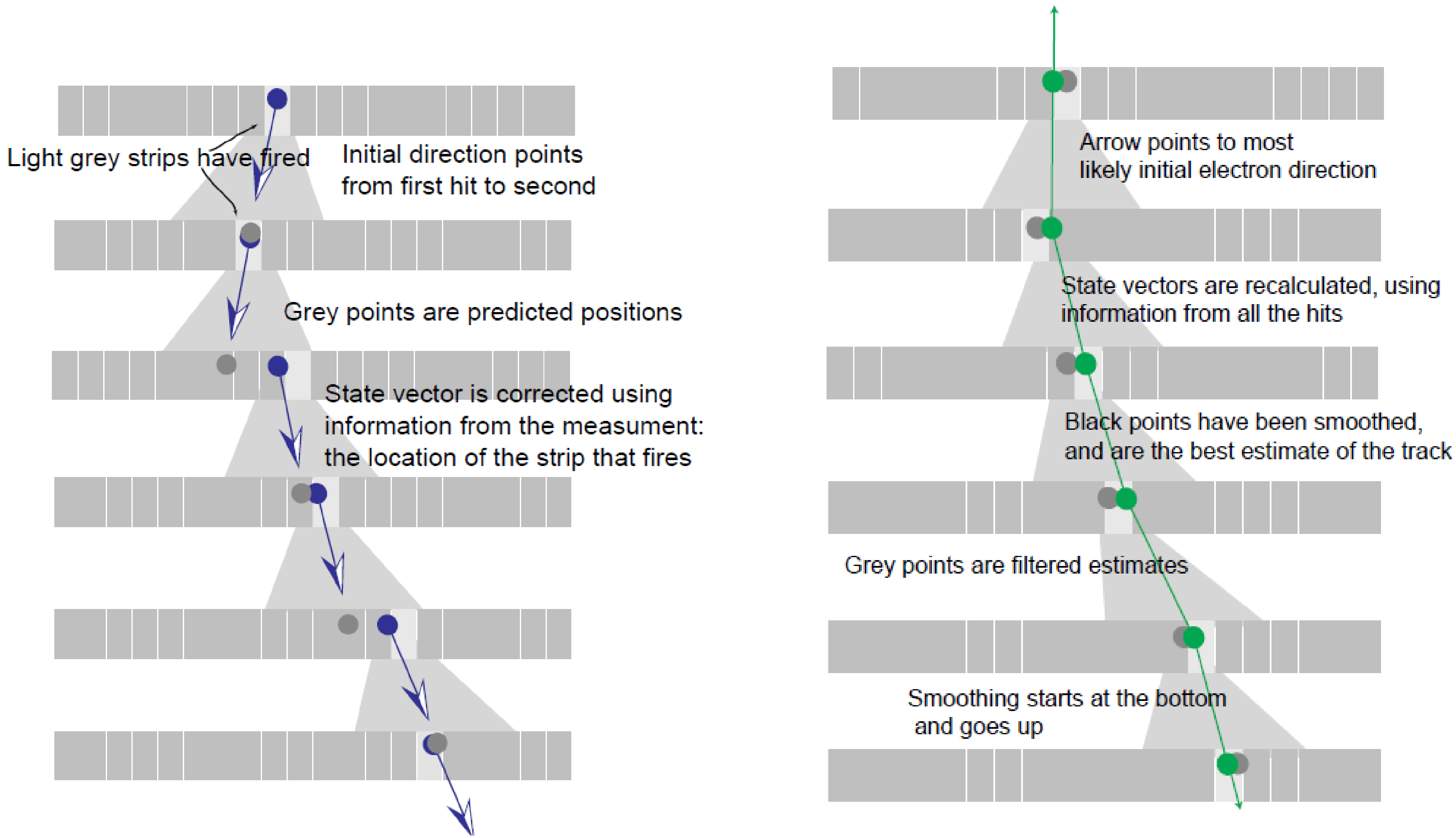
  for each of the subsets containing measurements from one traveling charged particle provided by the track finder,
  finding the optimal estimate of a set of parameters.

- Realization of the least-square method.
- Kalman Smoothing is the estimation of the state vector at some time in the "past" based on all measurements taken up to the "present" time.
  - easy to obtain optimal estimates anywhere along the track;
  - enables efficient outlier rejection, as track parameter predictions are calculated from all other measurements in a track.

Ref.

1. R. Fruehwirth, Application of Kalman filtering to track and vertex fitting, Nuclear Instruments and Methods in Physics Research A 262. 1987. 444-450pp.

Kalman finding and fitting process



Ref.

1. Jones B., Tompkins B. A Physicist's Guide to Kalman Filters //World Wide Web, Mar. 1998.

Hough Transform

- The Hough transform is a global method.

  It uses a parametric description of a track by a set of parameters. Once the track model and detector measurement model are given, all hits in the detector can be projected into the track parameter space.
- The Hough Transform is based on the voting procedure on best matching value for some parameter describing the feature. This voting procedure is carried out in a parameter space, from which object candidates are obtained as local maxima in a so-called accumulator space.
- The Hough transform converts the coordinates of the detector hits of particle tracks into the space of track parameters.

Ref.

1. P.Hough, Proc. Conf. High Energy Acc. Instr., CERN, 1959

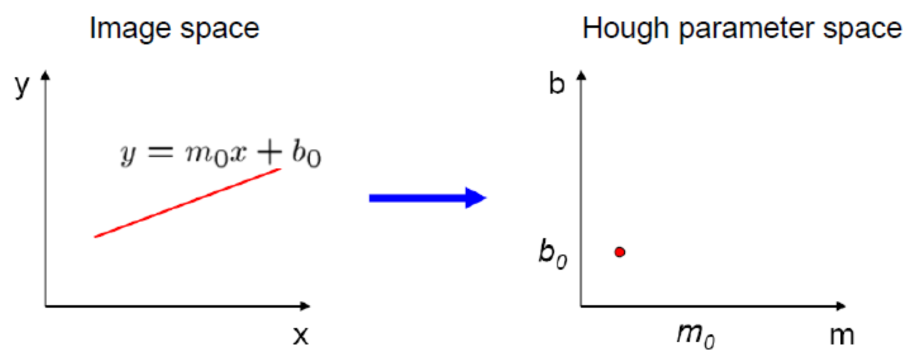
Hough Transform

- The classical Hough transform was concerned with the identification of lines in the image, but later the Hough transform has been extended to identifying positions of arbitrary shapes, most commonly circles or ellipses.

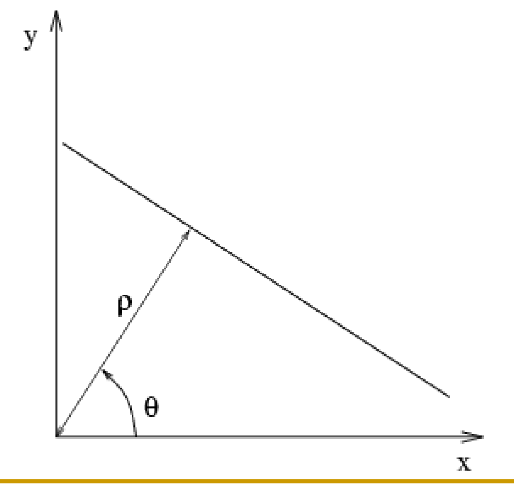
- The algorithm:
  - Converting the coordinates into the Hough space;
  - Accumulating sets of bins in histogram for each of the measurements in segment;
  - Measurements that are lying along straight lines show up as peaks in histogram, so peaks assumed to correspond “real” track.

Hough Transform for a strait line

- The simplest case of Hough transform is detecting straight lines. In general, the straight line $y = kx + b$ can be represented as a point (k, b) in the parameter space. Thus, $(x,y) \rightarrow (k,b)$



- However, vertical lines pose a problem. They would give rise to unbounded values of the slope parameter k. Thus it is used another form: r = xcosθ + ysinθ where r is the distance from the origin to the closest point on the straight line, and θ is the angle between the x axis and the line connecting the origin with that closest point. (we don't need it)



Hough Transform for a circle

- *A circle* $(x-a)^2+(y-b)^2 = r^2$ can be represented as a point *(a, b, r)* in the 3-dimentional parameter space.

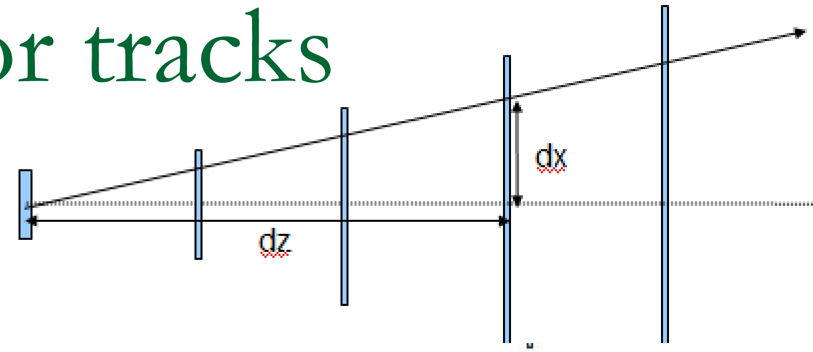
  Thus, $(x,y) \rightarrow (a,b,r)$

- In principle, the transform method extends to arbitrary curves. We need only pick a convenient parameterization for the family of curves of interest and then proceed in the similar way.

Ref.

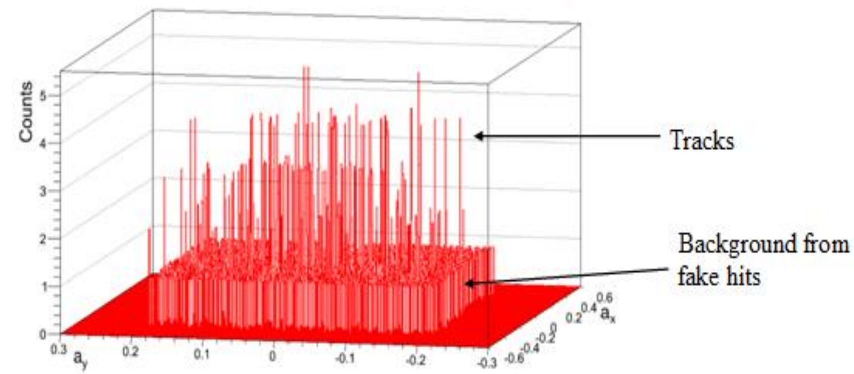
1. A. Strandlie, 3D silicon detector workshop, Department of Physics, University of Oslo, September 2007

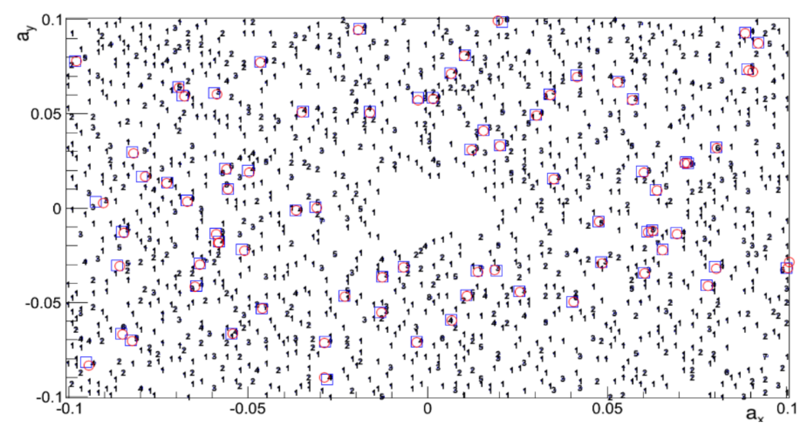
Hough Transform for tracks



- Transform each hits x, y positions to a_x, a_y space (HT space) where a_x and a_y refers to x and y-slopes of the anticipated track associated with a given hit.
- The most “popular” candidate tracks assumed to be “real” track.

  →
- Perform clusterization in the HT space.
- <u>Extract</u> tracks from clusters by fitting all possible combinations of hits laying on different stations using linear regression (weighted or none-weighted). Chi2 cut is used to select aligned combinations of hits → reconstructed tracks.
- <u>Cleaning</u>: Usually, the above procedure leads to creation of same fake tracks. The subset of tracks from a single cluster is cleaned in such a way that for tracks sharing same hit on first station the one with the smallest chi2 is selected.



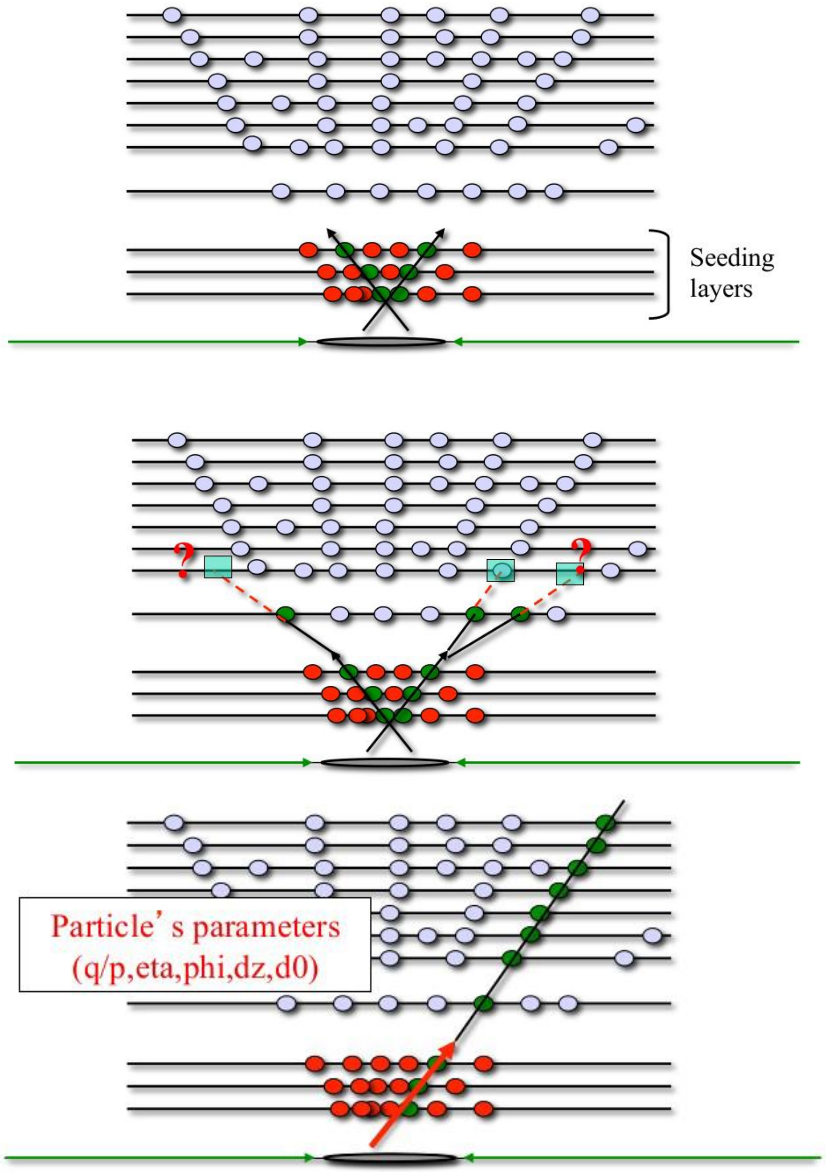


Ref.

1. D.T.Larsen, VD: simulation, tracking, performance. Paris, 26.05.15

Track reconstruction (with KF)

- Seed generation:
  - Provides initial track candidates with 2 or 3 hits;
  - Gives the initial estimation of the trajectory parameters and uncertainties.
- Track finding:
  - using current parameters to determine possible intersected layers;
  - clustering: listing compatible hits group;
  - updating trajectory parameters at each layer.
- Track fitting:
  - Kalman filter + smoother.
- Track selection:
  - Reducing the number of reconstructed tracks not associated to a charged particle (fake rate).



Ref.

1. E. Brondolin, CMS experiment recent and planned trracking improvements, CTD 2015

Summary

- The Kalman filter is a recursive filter, which evaluates the state of a linear dynamic system using a set of inaccurate measurements with the errors distributed.
- The Kalman filter has two functions:
  1. track finding - using information about positions of detector hits it finds track candidates;
  2. track fitting – track candidates are fitted to extract track parameters.
- In order to begin the Kalman filter procedure one needs so-called track seeds, and one of the possibilities is Combinatorial method. Another one is Cellular Automaton method. Cellular automaton is a dynamic method, it evolves in a discrete space consisting of cells. In application to tracking system, connected pairs of detector hits on two consecutive layers can be taken as track segments (cells).
- Being local and parallel cellular automata avoid exhaustive combinatorial searches. Since cellular automata operate with highly structured information, the amount of data to be processed in the course of the track search is significantly reduced. Usually cellular automata employ a very simple track model which leads to utmost computational simplicity and a fast algorithm.
- So the task of track reconstruction with Cellular Automaton consists of three main steps:
  1. formation of tracks-segments;
  2. construction of tracks-candidates;
  3. track fitting.

- The Hough transform is a global method. It uses a parametric description of a track by a set of parameters. Using the track model and given detector measurement model, converts the coordinates of the detector hits of particle tracks into the space of track parameters. In this case the track recognition becomes a search for local maxima corresponding to tracks.

Thank you for your attention!

Good Tracking!