Trajectory tracking for skid-steering mobile robots using sliding-mode control

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***Abstract*—In this paper, we tackle the problem of trajectory tracking for Skid-Steering mobile robots. Two strategies of kinematic modeling of the Skid-Steering mobile robot are considered. The controllers are designed based on Sliding Mode scheme to asymptotically stabilize the tracking errors in surge, sway and heading for different reference trajectories. The study on circular trajectory and on a straight line shows that the tracking is achieved even with large initial tracking errors and bounded disturbances. Comparison between the two strategies is presented based on the performance and of the efficiency of the tracking.**

I. Introduction

Motion planning for nonholonomic systems has been an area of active research for more than a decade. In general, the study of the motion control can be divided into two main problems: the stabilization problem and the tracking problem [1]. The stabilization concentrates on getting the system to a fixed point in its workspace, while the tracking focuses on controlling the system to follow a desired trajectory or path. Moreover, the tracking problem can be further divided into two groups. The first one is "trajectory tracking", where the system must reach a desired position at a specific time. The second one is "path following"which focuses on the geometric displacement between the system pose and the desired path regardless of the time.

Steering tracked vehicles has certain unique features which make it quite different from the case of wheeled vehicles. Consequently, it requires special treatment. Several track configurations for steering have been proposed such as: articulated steering, curved track steering and skid steering. The latter is the most widely used due to it's simplicity and mechanical robustness [2]. One kind of Skid-steering vehicles is composed of two rear driving wheels connected to frontal ones via chains. The principle of skid steering is based on controlling the relative velocities of both tracks in a similar way to the control of differential wheeled vehicles. However, there are two strategies of modeling these vehicles. First, they could be seen as pure rolling differential drive vehicles, and therefore, they are considered as nonholonomic systems. The second strategy respects the real model of the skid-steering vehicles. A key aspect of vehicle control is the kinematic model proposed for the vehicle in order to design the controller. However, the main difference between the two strategies of modeling is the sideway velocity which should be taken into account in the case of skid-steering due to the slippage.

In this paper, we focus on trajectory tracking problem for skid steering mobile robots. The two strategies of modeling are considered and a continuous-time model of the robot has been used for the controller design in each strategy. A continuous time variable structure control technique based on sliding mode scheme has been employed to achieve the trajectory tracking and the comparison between the two methods is presented.

This paper is organized as follows: In Section II, the kinematic models of the two strategies are presented. A sliding mode control strategy based on the kinematics is proposed in Section III. Section IV is dedicated to simulation results while experimental results are detailed in Section V.

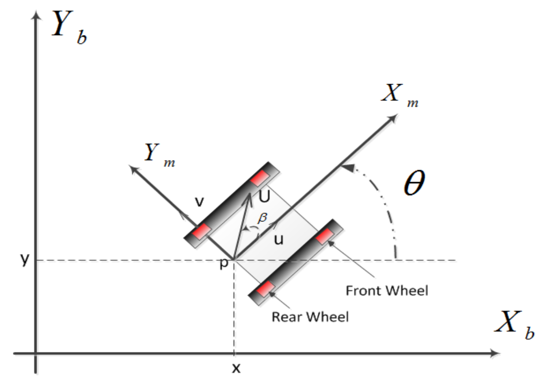

Fig. 1. Robot Cartesian Coordinate System

II. KINEMATIC MODEL OF DIFFERENTIAL DRIVE MOBILE ROBOT

In this study, we consider two strategies for modelling a skid steering mobile robot. The first one models the skid steering robot as a nonholonomic system with a differential drive. A model of the differential drive mobile robot is shown in Figure (1). The vehicle pose is described by the position (x, y) of the midpoint between the two driving wheels and by the orientation angle θ.

We denote:

- $\{X_m, Y_m\}$ moving frame.
- $\{X_b, Y_b\}$ base frame.

$q = (x, y, \theta)^T$ is the robot posture in base frame.

The rotation matrix that express the orientation of the moving frame with respect to the base frame is $R(\theta)$:

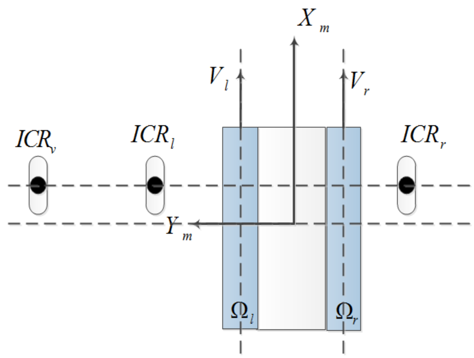

Fig. 2. Skid-Steering Robot ICRs

$$R(\theta) = \begin{pmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{1}$$

The simplest model of a nonholonomic WMR is the unicycle, which corresponds to a single wheel rolling on the plane [3]. A kinematic model is thus:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos(\theta) & 0 \\ \sin(\theta) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ r \end{bmatrix} \tag{2}$$

Where u is surge velocity (m/s) and r is yaw velocity (rad/s).

The second strategy of modelling takes into account the sideway velocity to express the overall kinematic model. Therefore, the resulted kinematic model is:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ v \\ r \end{bmatrix} \tag{3}$$

Where v is the sway velocity (m/s).

Moreover, the kinematic model of a skid-steering robot can be expressed according to the Equation (3) in terms of surge, sway and yaw velocities which can be calculated from the relations [4]:

$$\begin{aligned} u &= \tfrac{V_r+V_l}{2} - \tfrac{V_r-V_l}{y_{ICRr}-y_{ICRl}} \tfrac{y_{ICRr}+y_{ICRl}}{2} \\ v &= \tfrac{V_r-V_l}{y_{ICRr}-y_{ICRl}} x_{ICRv} \\ r &= \tfrac{V_r-V_l}{y_{ICRr}-y_{ICRl}} \end{aligned} \tag{4}$$

Where $y_{ICRl}, y_{ICRr}, x_{ICRv}$ are the coordinates of the instantaneous centers of rotation $(ICRs)$ on the 2-D ground plane. These $ICRs$ represent the position of equivalent differential drive ideal wheel contact points, as illustrated in Figure (2). V_l, V_r are the linear velocities of the left and right tracks of the robot.
However, the instantaneous center of rotation (ICR_v) is defined as the point in the horizontal plane where the motion of the vehicle can be represented by a rotation without translation.

III. Design of Sliding-Mode Controller

In general, tracking of a time-varying reference trajectory $\eta_d(t) = [x_d(t), y_d(t), \theta_d(t)]^T$ is achieved by minimizing the tracking error $e = \eta(t) - \eta_d(t)$, where $\eta(t) = [x(t), y(t), \theta(t)]^T$.
According to [5], the tracking error can be decomposed in a vehicle parallel (VP) reference frame to:

$$e = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = R(\theta)(\eta - \eta_d) \tag{5}$$

By expanding (5), we get:

$$\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} (x - x_d)\cos(\theta) + (y - y_d)\sin(\theta) \\ -(x - x_d)\sin(\theta) + (y - y_d)\cos(\theta) \\ \theta - \theta_d \end{bmatrix} \tag{6}$$

Based on this result, we can derive the following sliding mode controllers according to the strategy of modeling.

A. First Strategy

Here, the kinematic model is presented according to the equation (2). We choose two sliding surfaces as follows:

$$\begin{aligned} s_1 &= e_1 \\ s_2 &= e_3 + \arctan(u_d e_2) \end{aligned} \tag{7}$$

Where u_d is the desired surge velocity. To force the error to slide on the sliding surfaces, we choose the derivatives of the sliding surfaces as follows:

$$\begin{aligned} \dot{s_1} &= -k_1 s_1 - q_1 sat(s_1) \\ \dot{s_2} &= -k_2 s_2 - q_2 sat(s_2) \end{aligned} \tag{8}$$

Where sat is the saturation function and:

$$\begin{aligned} \dot{s_1} &= r e_2 - u_d \cos(e_3) + u \\ \dot{s_2} &= r - r_d + \tfrac{e_2}{1+(u_d e_2)^2} \dot{u}_d + \tfrac{u_d}{1+(u_d e_2)^2} \dot{e}_2 \end{aligned} \tag{9}$$

r_d is the desired yaw velocity. u_d and r_d can be calculated using the following relations [6]:

$$u_d = \sqrt{\dot{x}_d^2 + \dot{y}_d^2} \tag{10}$$

$$r_d = \frac{\ddot{y}_d \dot{x}_d - \ddot{x}_d \dot{y}_d}{\dot{x}_d^2 + \dot{y}_d^2} \tag{11}$$

Where (x_d, y_d) represents the desired instantaneous posture.

Now, we can derive the following control law:

$$\begin{aligned} u &= -r e_2 + u_d \cos(e_3) - k_1 s_1 - q_1 sat(s_1) \\ r &= r_d - \tfrac{e_2}{1+(u_d e_2)^2} \dot{u}_d - \tfrac{u_d^2}{1-(u_d e_2)^2} \sin(e_3) - k_2 s_2 \\ &\quad - q_2 sat(s_2) \end{aligned} \tag{12}$$

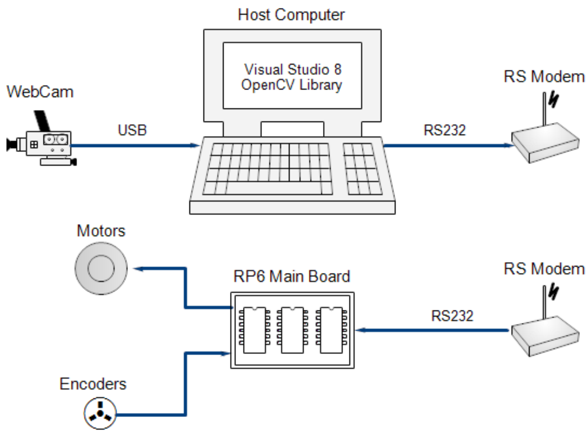

Fig. 3. the schematic hardware/software structure of the testbed

Fig. 4. RP6 robot

B. Second Strategy

This strategy consider the model skid-steering robot proposed in [4]. Therefore, we choose three sliding surfaces as follows:

$$\begin{aligned} s_1 &= e_1 \\ s_2 &= e_2 \\ s_3 &= e_3 \end{aligned} \tag{13}$$

We choose the following derivatives of sliding surfaces to force the error to slide on the sliding surfaces:

$$\begin{aligned} \dot{s_1} &= -k_1 s_1 - q_1 sat(s_1) \\ \dot{s_2} &= -k_2 s_2 - q_2 sat(s_2) \\ \dot{s_3} &= -k_3 s_3 - q_3 sat(s_3) \end{aligned} \tag{14}$$

Where:

$$\begin{aligned} \dot{s_1} &= re_2 - u_d \cos(e_3) + u - v_d \sin(e_3) \\ \dot{s_2} &= -re_1 - v_d \cos(e_3) + v + u_d \sin(e_3) \\ \dot{s_3} &= r - r_d \end{aligned} \tag{15}$$

$u_d = \dot{x_d}, v_d = \dot{y_d}, r_d = \dot{\theta_d}$; u, v, r are the control inputs.

As a result, we get the following control laws:

$$\begin{aligned} r &= r_d - k_3 s_3 - q_3 sat(s_3) \\ v &= re_1 - u_d \sin(e_3) + v_d \cos(e_3) - k_2 s_2 \\ &\quad -q_2 sat(s_2) \\ u &= -re_2 + v_d \sin(e_3) + u_d \cos(e_3) - k_1 s_1 \\ &\quad -q_1 sat(s_1) \end{aligned} \tag{16}$$

IV. SIMULATIONS

We present here the study of the proposed methods based on a simulation built in MatLab environment. The model of the robot is defined according to the equation (3) and (u, v, r) are calculated using Equation (4)in which we consider the following values of the $ICRs$:

$$\begin{aligned} y_{ICRr} &= -\tfrac{d}{2} \\ y_{ICRl} &= \tfrac{d}{2} \\ x_{ICRv} &= rand[-\tfrac{d}{2}, \tfrac{d}{2}] \end{aligned} \tag{17}$$

d is the distance between the driving wheels.
We supposed that there is an angle between the overall velocity and surge velocity β (See figure (1)). Our overall kinematic model has two inputs (left and right wheels angular velocities(W_L, W_R)) and three outputs (x, y, θ). The model inputs should be related to the control outputs (u, v, r) by the relations:

$$U = \sqrt{u^2 + v^2} \tag{18}$$

$$\begin{aligned} W_R &= \tfrac{2U + d.r}{2R} \\ W_L &= \tfrac{2U - d.r}{2R} \end{aligned} \tag{19}$$

Where U is the overall Linear velocity, R is driving wheels Radius.
However, (W_R, W_L) are passed to the model through first order transfer function in order to express the closed loop system that control the angular velocities of the wheels. U needs to be scaled with respect to the saturation values of the wheels angular velocities.
In the first strategy, U is supposed to equal u and $\beta = 0$.

Figures $(5, 6)$ show the simulation results of tracking a circular trajectory and a straight line based on the first strategy of modeling. The maximum error rate was $5cm$ distance in all directions. Figures $(7, 8)$ show the simulation results using the model of the second strategy. However, the maximum error rate was $10cm$ distance in all directions. We can see that the second strategy shows a better performance than the first one especially in tracking the circle. The denoted errors in the first strategy are due to the fact that the controller does not take into account the errors in detecting the instantaneous position of the robot. These errors are treated with the assumption of a sway velocity that must be taken into account when designing the control law according to the second strategy.

V. EXPERIMENTAL RESULTS

In order to validate our results, real experiments were carried out in one of the laboratories at HIAST (Higher Institute for Applied Science and Technology).

A. Experimental Setup

The area of the testbed is a $(2.4m \times 2.7m)$. We chose to test the proposed methods on the RP6 which is an example of a skid steering mobile robot with $(18 \times 11 \times 5.5)cm^3$ volume and $(0.5Kg)$weight (Figure (4)). The vehicle is equipped with two rear driving wheels mounted on the same axis.

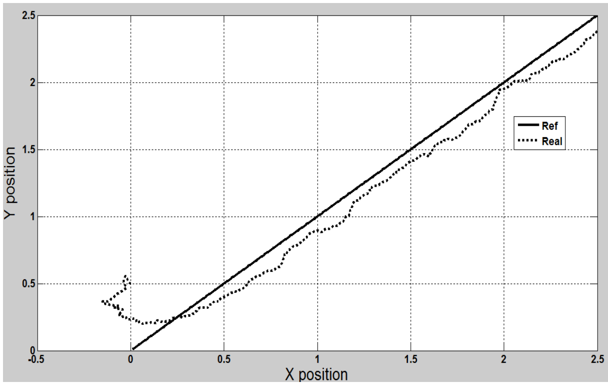

Fig. 5. Simulated Line Tracking (First strategy)

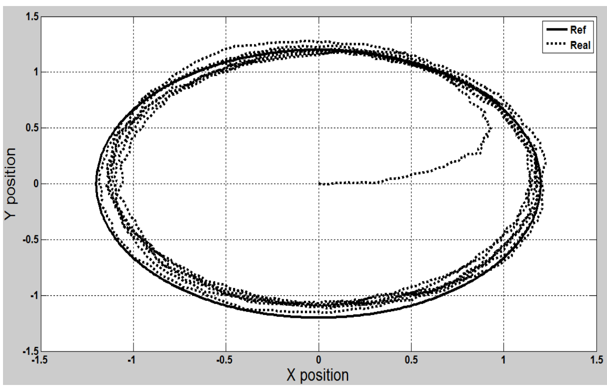

Fig. 6. Simulated Circle Tracking (First strategy)

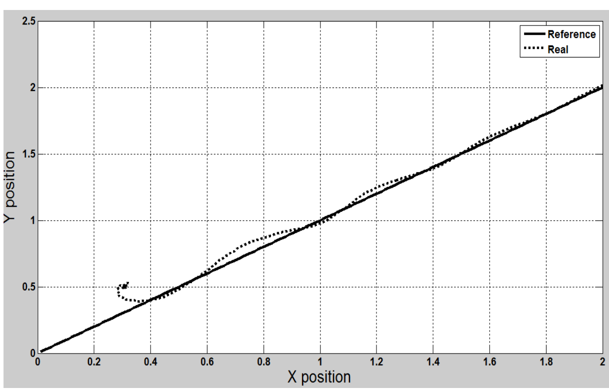

Fig. 7. Simulated Line Tracking (Second strategy)

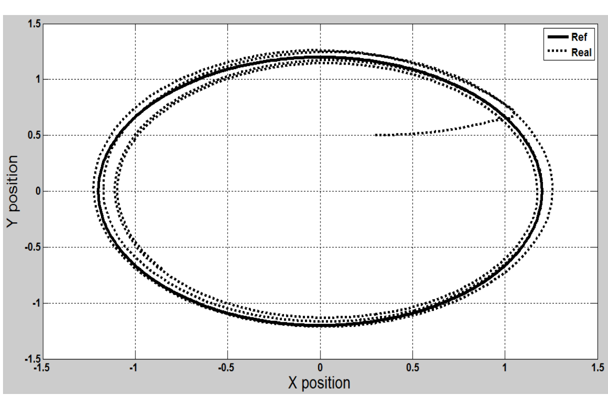

Fig. 8. Simulated Circle Tracking (Second strategy)

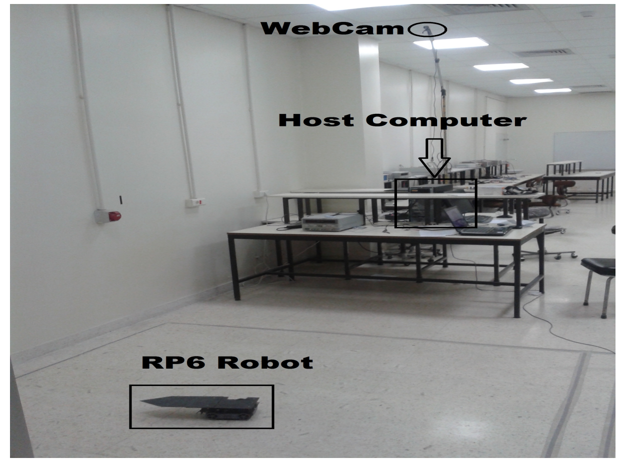

Fig. 9. Experiment Workspace

Every wheel is connected to a frontal one via a chain. The vehicle can be controlled by driving the angular velocities of the wheels. In our experiments, all high-level control algorithms including the trajectory tracker are performed on a host computer $(2.4GHz, 2GRAM)$ running $Windows7$. We used an $RSModem(JZ863)$ for the wireless communications between the host computer and the RP6 robot. The algorithms were programmed under $VisualStudio2008$ including $OpenCV$ library. An overhead WebCam capturing image frames at 15 frames per second is mounted at a pose where it can capture all the area of the testbed. Image processing was employed to determine the current position and the orientation of the robot (Figure (3)). Actually, we added a rocket shaped cartoon-made plate to the top of the RP6 to facilitate the heading detection. Therefore, the position of the center of the detected object in the image is calculated and the farthest point on its contour to the center is determined in order to calculate the orientation. However, the resulted heading is in the range $[-\pi, \pi]$ and that does not guarantee a smooth behavior of the vehicle because of the sudden changes that may occur in orientation. To avoid this problem, we used a sub-routine to extend this range to $[-\infty, \infty]$. Finally, we added some small stones on the testbed and covered them with adhesive tape to simulate disturbances. Figure (9) shows the experiment workspace.

B. Experiments

The present study addresses the trajectory tracking problem for a mobile robots to prove the feasibility of the control using Sliding Mode controller and to compare between two strategies of choosing the kinematic models.

We chose two reference trajectories; a circular and a straight line. The comparison is based on the docility of the tracking behavior to the trajectory being followed.

Figure (10) presents the tracking of a straight line using the first strategy where the robot starts from a relatively far position from the origin and with a big difference in direction. In figure (11), the robot is tracking a circle with a big initial error. The controller presents a good performance of tracking against the multiple error sources. The error in position is due to the camera calibration errors because of the big distortion of the Webcam. Besides, we have a random error in detecting the

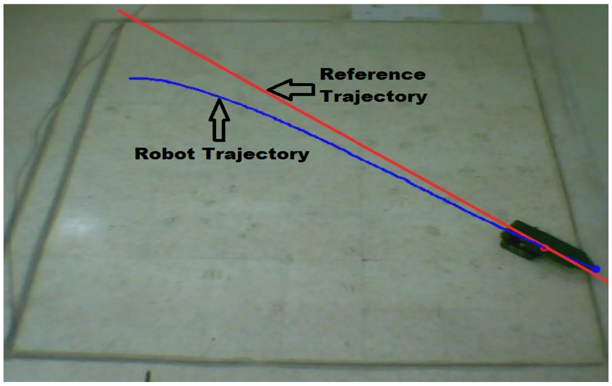

Fig. 10. Line Tracking (First strategy)

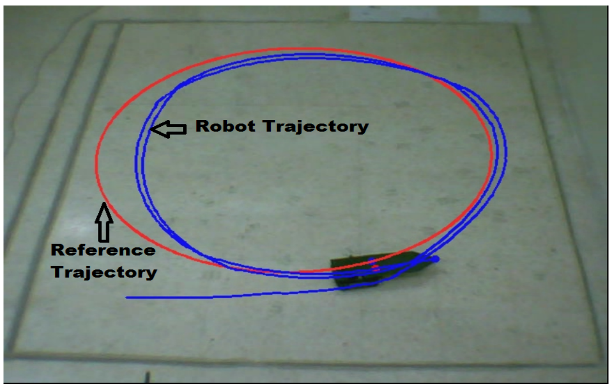

Fig. 11. Circle Tracking (First strategy)

position of the robot in the image and it can be considered as a noise in position and direction. Moreover, the testbed contains some small stones playing the role of disturbances. However, the results show that the controller of the first strategy rejected these disturbances.
In figures (12, 13), the controller based on the second strategy shows a better performance than the first strategy as the robot trajectory is smoother.
Figures(14, 15) show the tracking errors in position and direction for the two trajectories (Line and Circle).

As a result, we can say that this work demonstrate the effectiveness of the sliding mode control in trajectory tracking

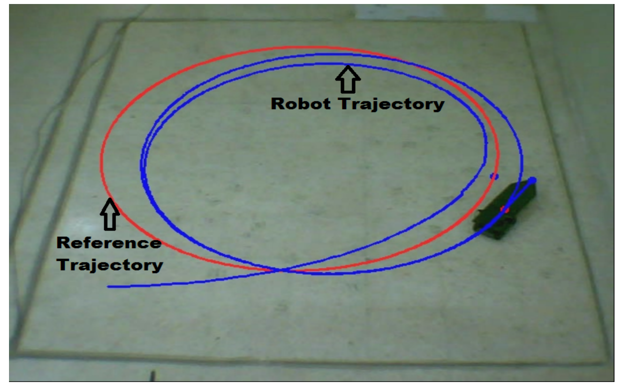

Fig. 13. Circle Tracking (Second strategy)

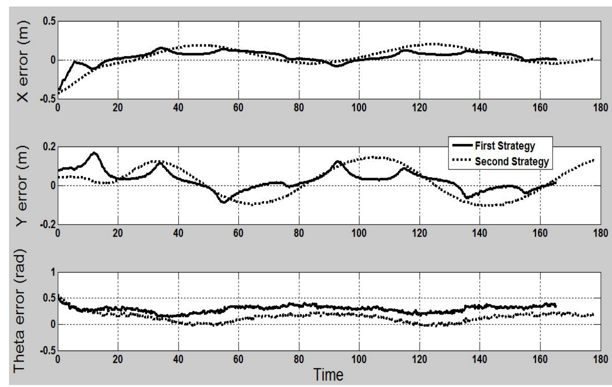

Fig. 14. Errors for circle tracking of the two strategies

for skid steering mobile robots. Moreover, despite the smoothness of trajectory tracking of the second strategy, the first strategy yielded also good results in an indoor environment, which proves that treating the skid-steering mobile robot as a nonholonomic system is a valid strategy for control purposes. However, the main drawback of sliding mode control is that its performance depends on the chosen kinematics which influences the choosing of the control outputs and consequently, of the sliding surfaces.

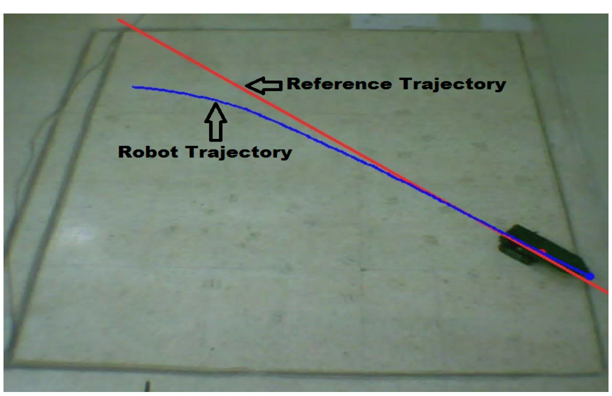

Fig. 12. Line Tracking (Second strategy)

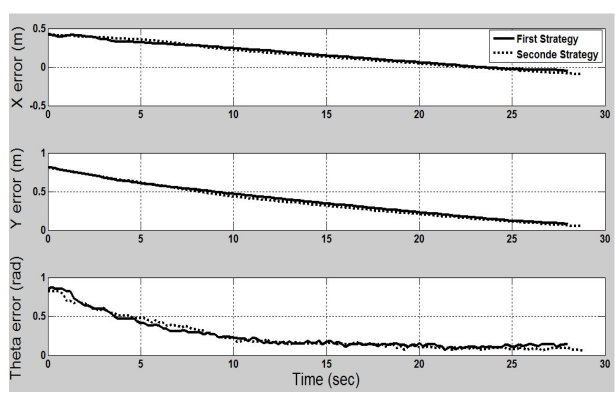

Fig. 15. Errors for line tracking of the two strategies

VI. CONCLUSIONS

Trajectory tracking for Skid-Steering robots was studied in this paper based on sliding mode controllers. The major contribution of this paper lie in the design of two tracking control schemes with different number of inputs to asymptotically stabilize the tracking of a desired trajectory. Two reference trajectories were chosen; a line and a circle. The results presented in this paper prove that, by applying the sliding mode control, the behavior of a mobile robot is robust against initial tracking errors and external disturbances. However, the comparison between the two proposed schemes shows that the performance is related to the control scheme depending on the chosen kinematics model. The results of the first scheme prove that by using the proposed control methodology, the skid-steering mobile robot can be considered as a nonholonomic system with only two inputs.

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