Lecture: Kinematics

Lecture: Content

- Forward kinematics differential drive

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- Holonomic robots

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- Car with trailer
- Nonholonomicity
- Holonomic robots
- ICC - Instantaneous Center of Curvature (COR)

Lecture: Sand buggy at high speed

Dynamics becomes important - slip, skidding

Lecture: Bulldozer - in contact

Tracked vehicles has more advanced kinematics due to friction

Lecture: Hägglunds BAE BV206

Hägglunds BV206 in water - river rafting

Lecture: Wheel Kinematic Constraints

- Moves on a horizontal plane

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- Steering axes are orthogonal to surface

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- Moves on a horizontal plane
- Point contact on wheel
- Wheel will not deform
- No slip, skid, or sliding
- No friction around contact point
- Steering axes are orthogonal to surface
- Wheels are mounted on a frame

Lecture: Kinematics differential driven robot

Differential driven robot - a power wheelchair

Lecture: Kinematics differential driven robot

$$v = \frac{v_R + v_L}{2}$$
$$\omega = \frac{v_R - v_L}{B}$$
$$R = B \cdot \frac{(v_R + v_L)}{(v_R - v_L)}$$
$$v = \omega \cdot R$$

Kinematic equations for a differential driven robot

- Baseline between wheels, B

Lecture: Kinematics differential driven robot

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Kinematic equations for a differential driven robot

- Baseline between wheels, B
- Velocity left wheel, v_L
- Velocity right wheel, v_R
- Vehicle angluar velocity, ω
- Vehicle forward velocity, v
- Vehicle turn radius, R (can be negative)

Lecture: Time Continous Model - wheelchair

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix}_G = \begin{bmatrix} \cos(\theta) & 0 \\ \sin(\theta) & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} v \\ \omega \end{bmatrix}$$

Time continous kinematic model

- Differential driven robot (wheelchair)

Lecture: Time Continous Model - wheelchair

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Time continous kinematic model

- Differential driven robot (wheelchair)
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Time continous kinematic model

- Differential driven robot (wheelchair)
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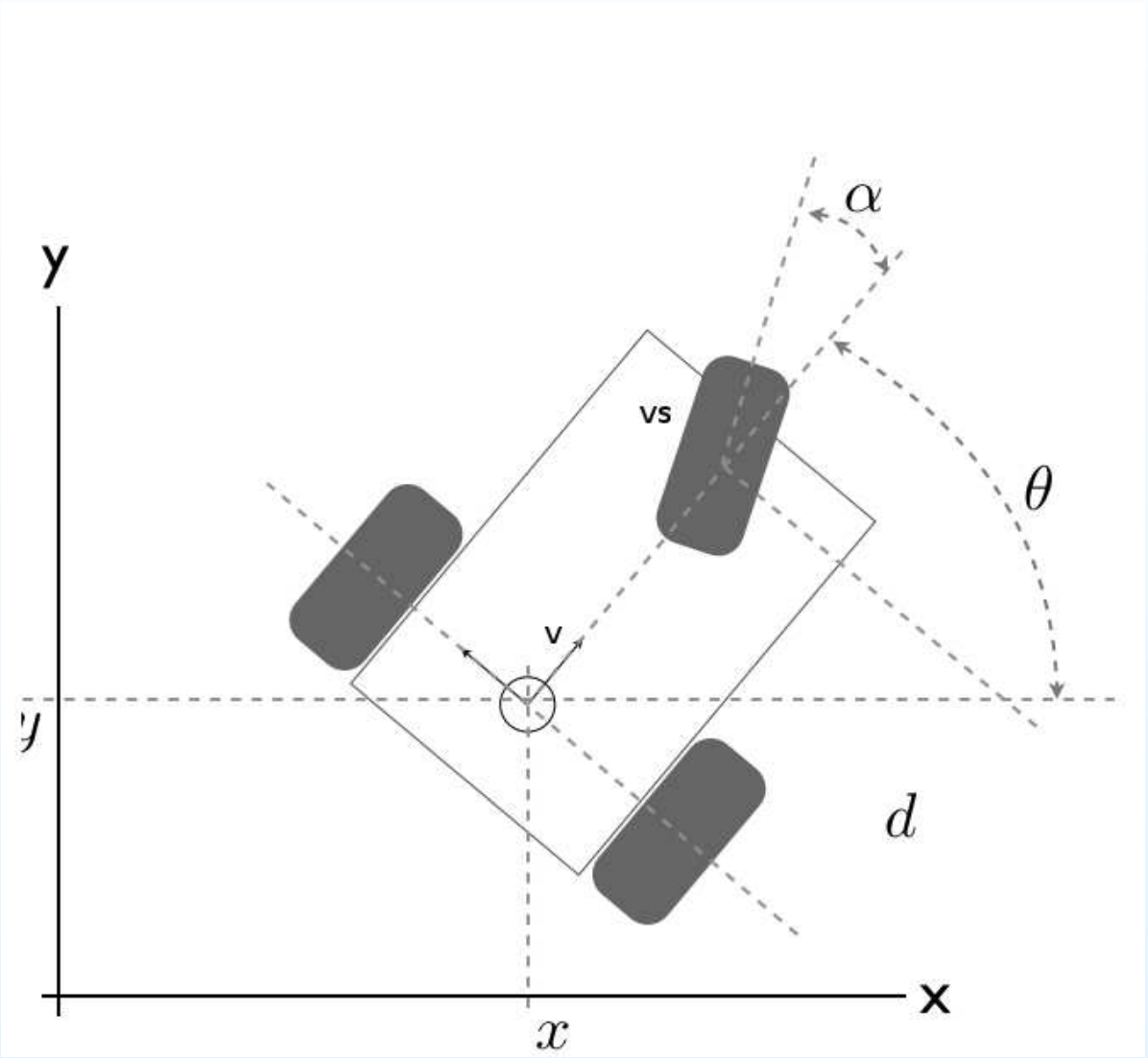
Time continous kinematic model

- Differential driven robot (wheelchair)
- Vehicle orientation, θ
- Vehicle position, (x, y)
- Vehicle change in position, $(\dot{x}, \dot{y})$
- Forward velocity, v
- Turning rate, ω

Lecture: Tricycle model

A typical tricycle

Lecture: Tricycle model



Tricycle with parameters marked

Lecture: Time continous tricycle model

$$\dot{x} = v\cos(\theta)$$
$$\dot{y} = v\sin(\theta)$$
$$\dot{\theta} = \omega$$
$$v = v_s\cos(\alpha)$$
$$\omega = \frac{v_s\sin(\alpha)}{L}$$

Time continous vehicle model

- v_s - velocity on steering wheel

Lecture: Time continous tricycle model

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Time continous vehicle model

- v_s - velocity on steering wheel
- v - forward velocity
- (x, y) - position of vechicle in global coordinate
- α - steering angle

Lecture: Car model

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{\alpha} \end{bmatrix} = \begin{bmatrix} \cos(\theta) \\ \sin(\theta) \\ \frac{\tan(\alpha)}{L} \\ 0 \end{bmatrix} v + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \omega_s$$

Kinematic model for a car

- Vehicle model with steering wheel, steering angle α

Lecture: Car model

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- (x, y, θ) is the pose of the vehicle (orientation)

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- ω_s - angular velocity direction of front wheel
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- COR, $[x_c, y_c]^T = R[-\sin\theta, \cos\theta]^T + [x, y]^T$

Lecture: Car model

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- COR, $[x_c, y_c]^T = R[-\sin\theta, \cos\theta]^T + [x, y]^T$
- Turn radius, $R = L/\sin(\alpha)$

Lecture: Time discrete movement

$$\begin{bmatrix} \Delta x_k \\ \Delta y_k \\ \Delta \theta_k \end{bmatrix} = \left(\frac{2v}{\omega}\right) \sin\left(\frac{\omega T}{2}\right) \begin{bmatrix} \cos\left(\theta_k + \frac{\omega T}{2}\right) \\ \sin\left(\theta_k + \frac{\omega T}{2}\right) \\ \dots \end{bmatrix} \approx T \begin{bmatrix} v \cos\left(\theta_k + \frac{\omega}{2}\right) \\ v \sin\left(\theta_k + \frac{\omega}{2}\right) \\ \omega_k \end{bmatrix}$$

Time discrete movement

- State Vector

Lecture: Time discrete movement

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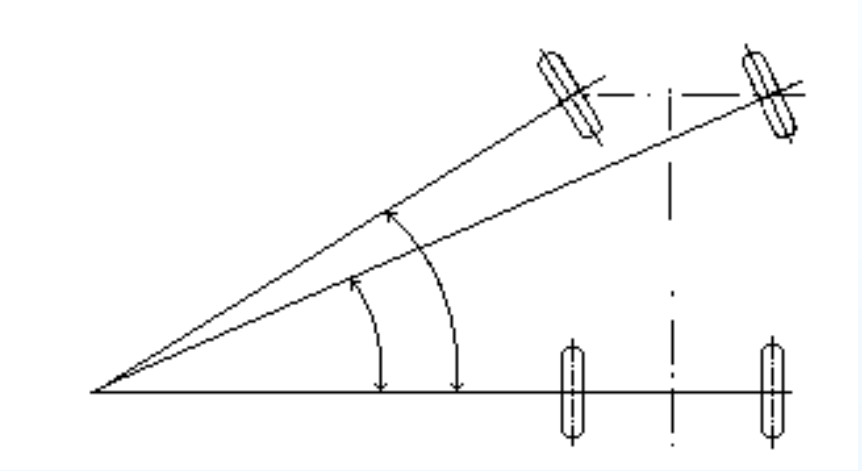
Time discrete movement

- State Vector

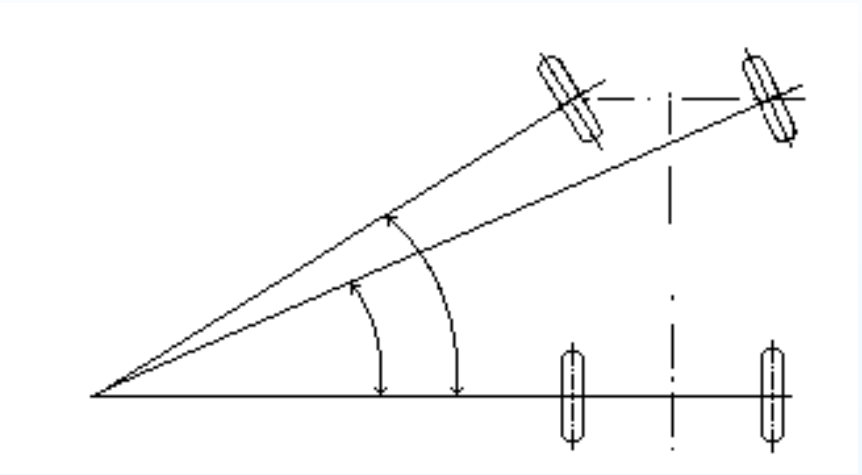
Lecture: Ackermann steering



Ackermann steering

- Four wheeled vehicle

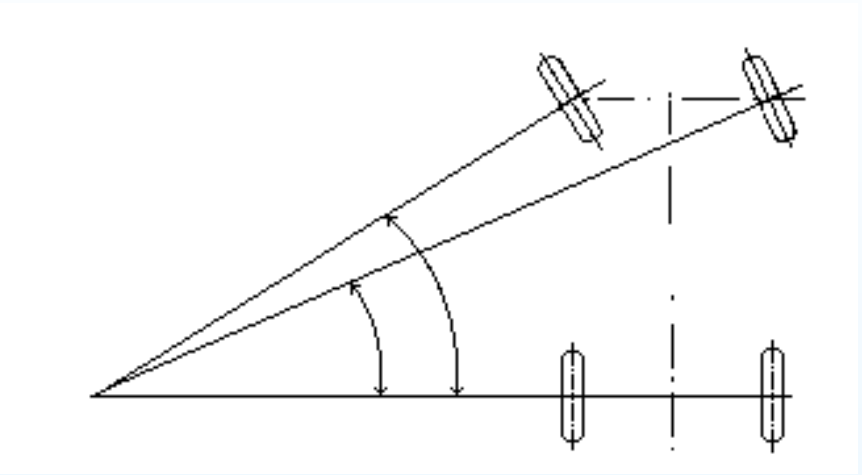
Lecture: Ackermann steering



Ackermann steering

- Four wheeled vehicle
- Steering wheels have different angles

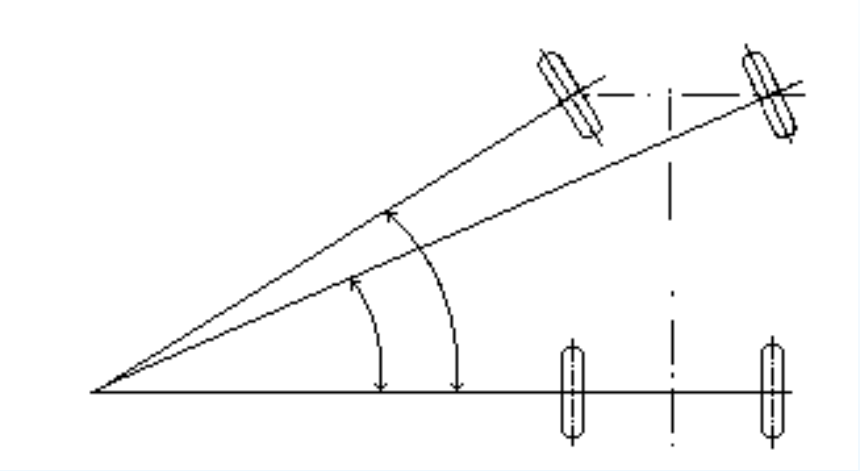
Lecture: Ackermann steering



Ackermann steering

- Four wheeled vehicle
- Steering wheels have different angles
- Four different turning radii

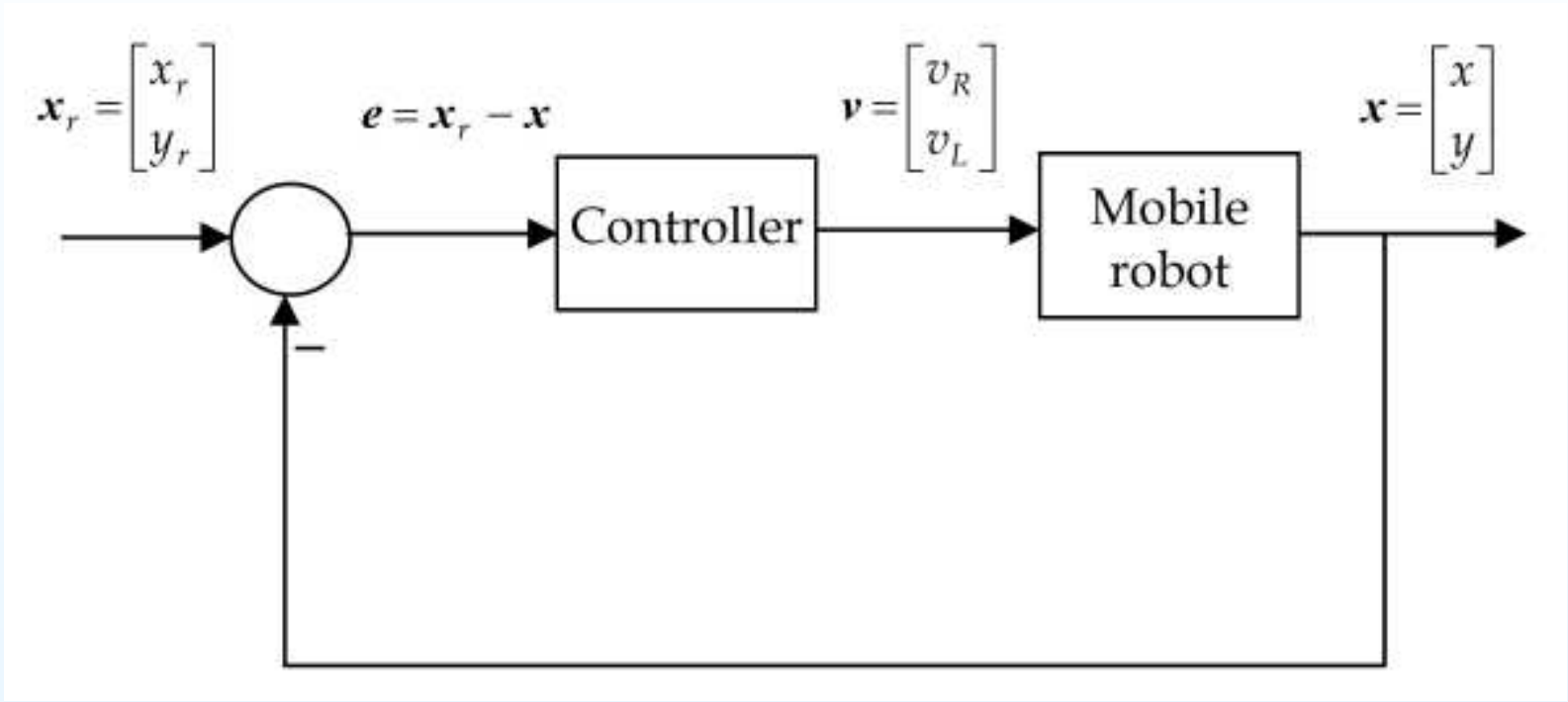
Lecture: Ackermann steering



Ackermann steering

- Four wheeled vehicle
- Steering wheels have different angles
- Four different turning radii
- Center of rotation is the same (intersection point)

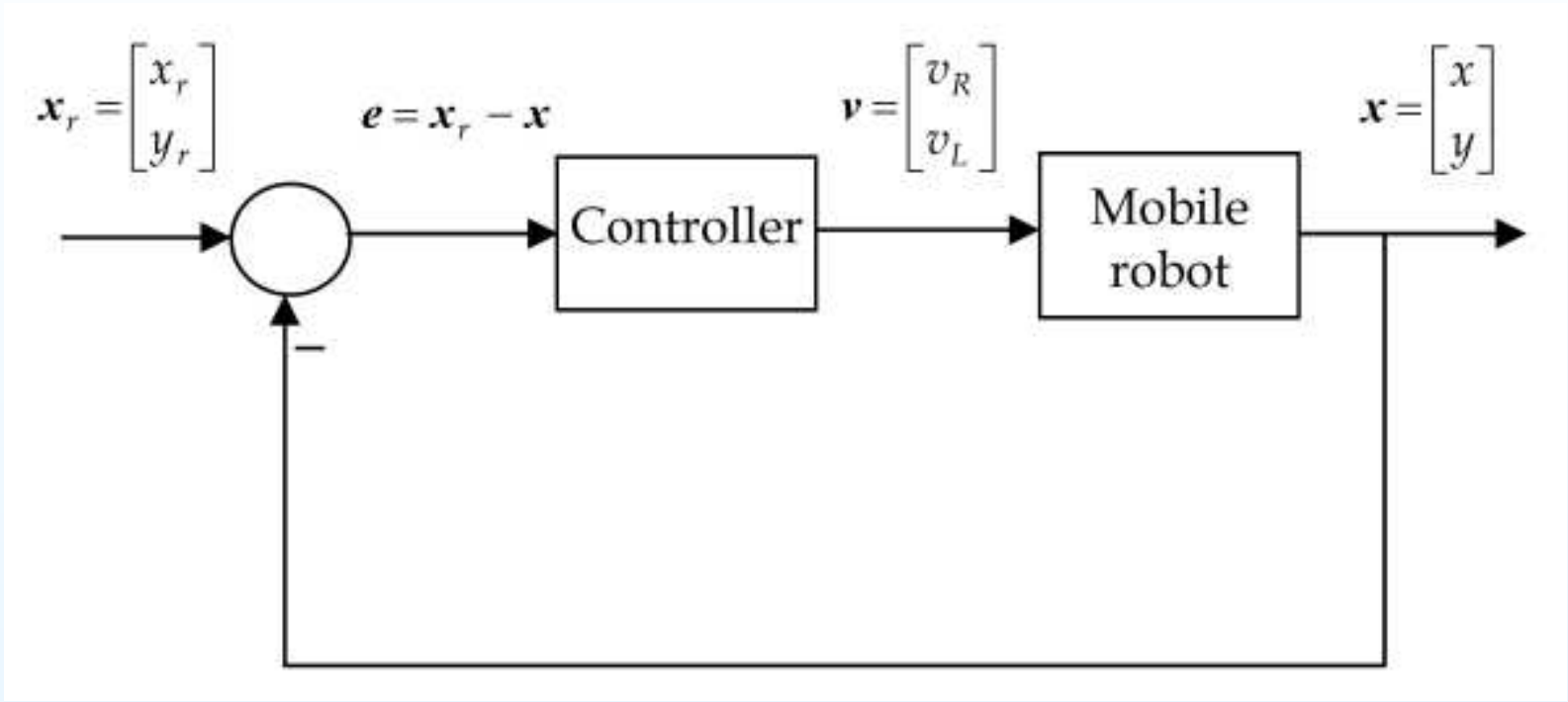
Lecture: Controller



Controller for a mobile robot

- Objective to follow a path or a velocity

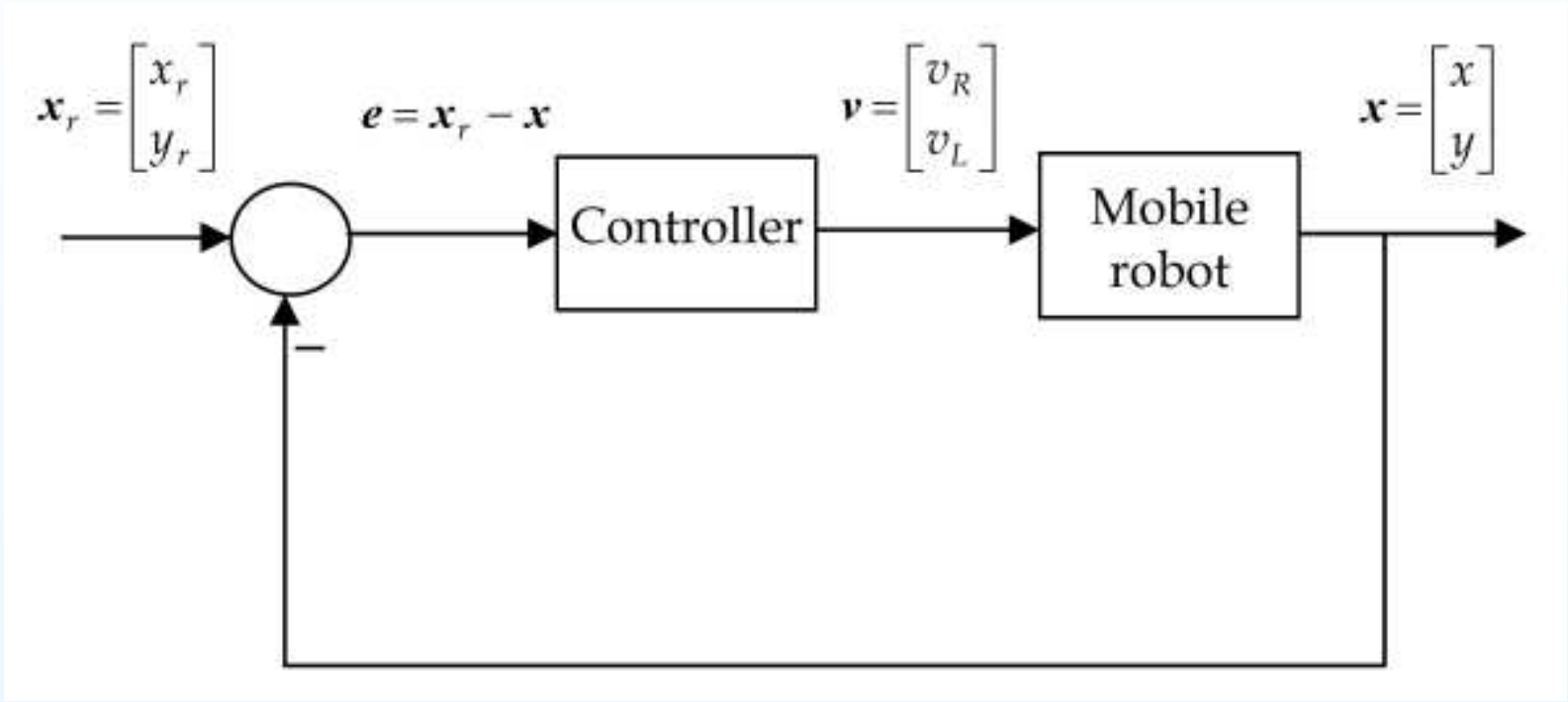
Lecture: Controller



Controller for a mobile robot

- Objective to follow a path or a velocity
- Not straighforward non-holonomic system

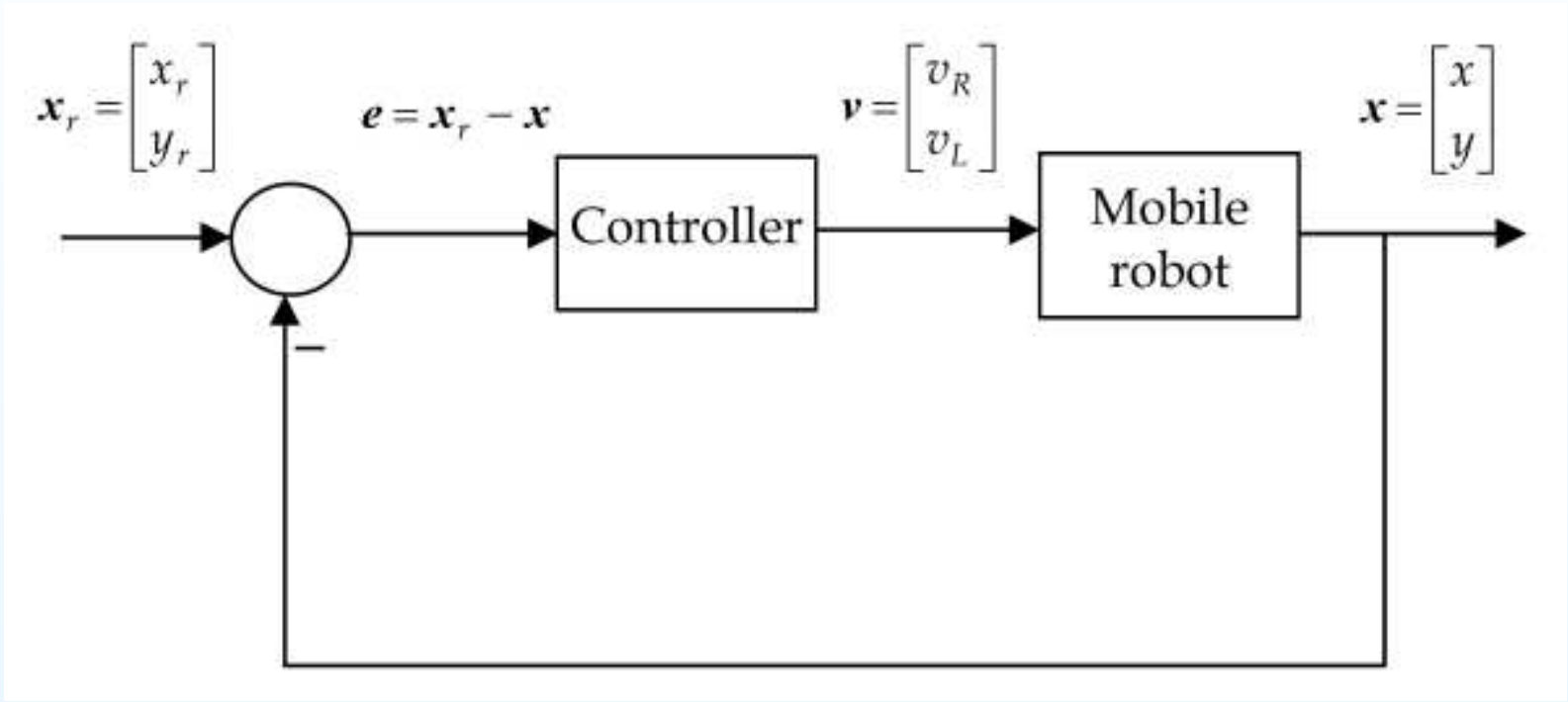
Lecture: Controller



Controller for a mobile robot

- Objective to follow a path or a velocity
- Not straighforward non-holonomic system
- Kinematics controller does not include dynamics

Lecture: Controller



Controller for a mobile robot

- Objective to follow a path or a velocity
- Not straighforward non-holonomic system
- Kinematics controller does not include dynamics
- Arrive at specific position or pose

Lecture: Car with trailer model

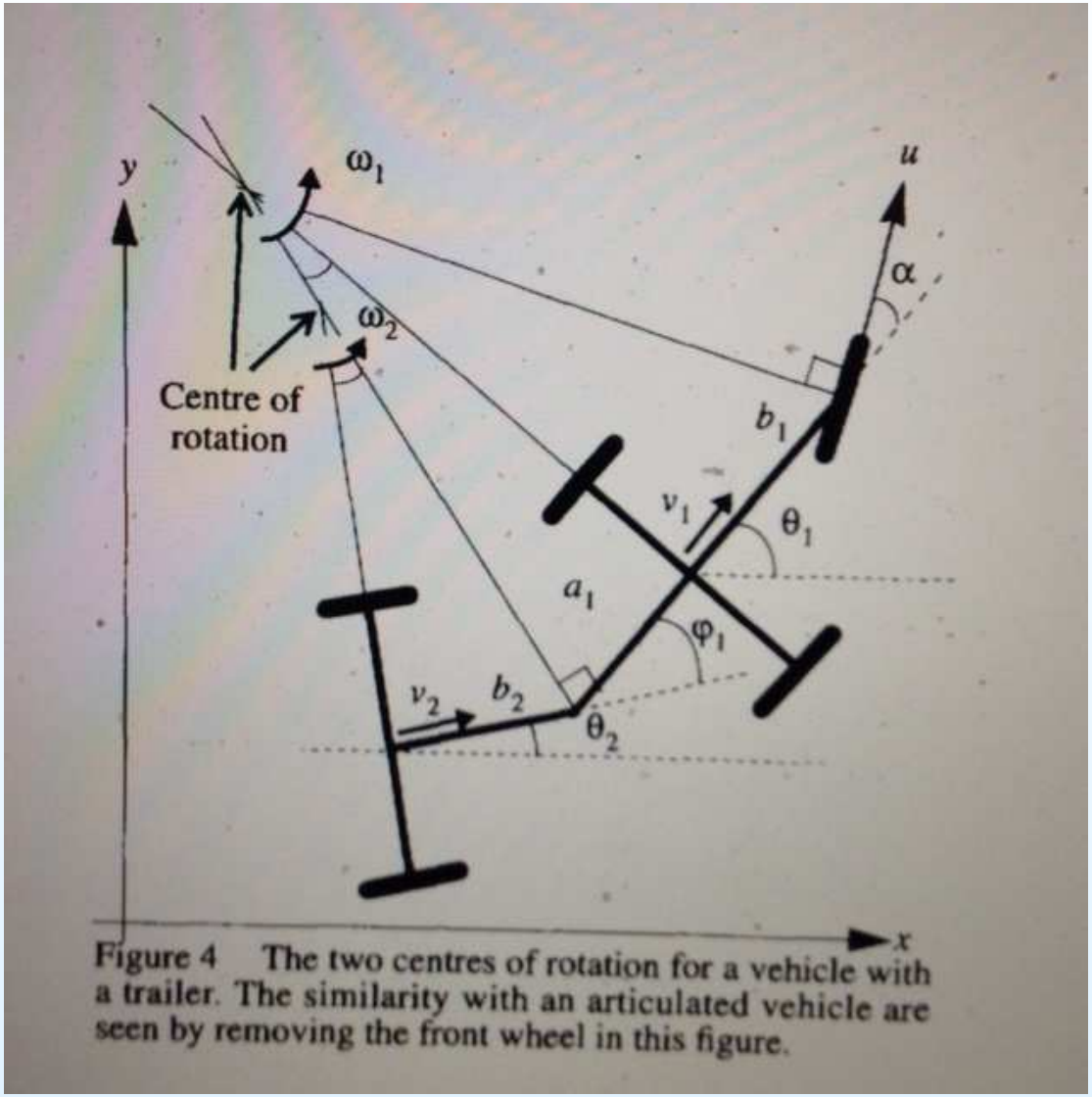


Figure 4 The two centres of rotation for a vehicle with a trailer. The similarity with an articulated vehicle are seen by removing the front wheel in this figure.

Car with trailer - Ake We.

Lecture: Car with trailer

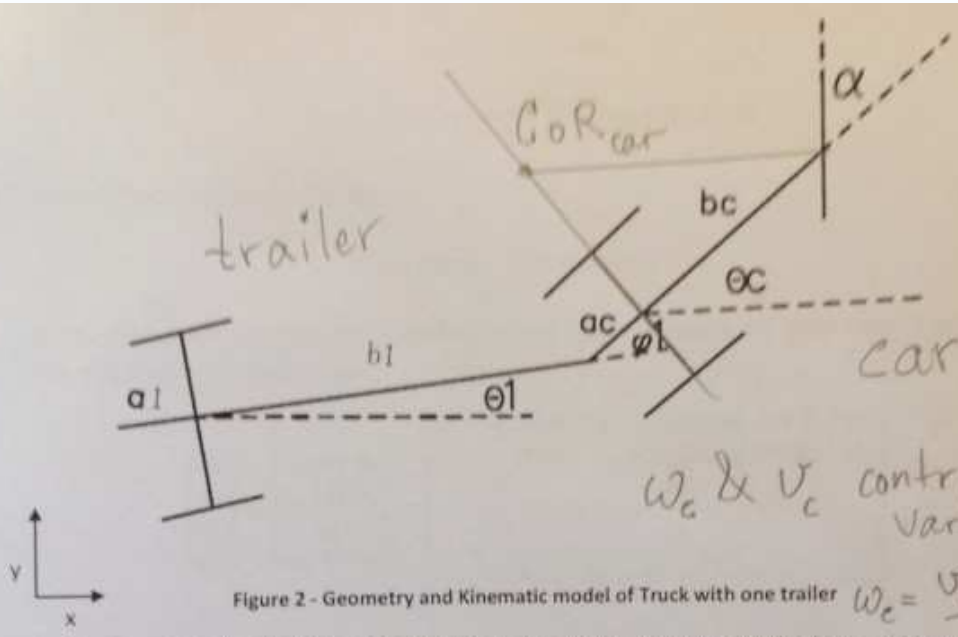


Figure 2 - Geometry and Kinematic model of Truck with one trailer

$\omega_c = \frac{v_c}{b_c} \cdot \tan\alpha$

In this theoretical part of the project, the first step was to interpret the kinematic equations dominant on the system. Considering a single trailer is sufficient to start with and yields to the following nonlinear set of differential equations in matrix form as mentioned before.

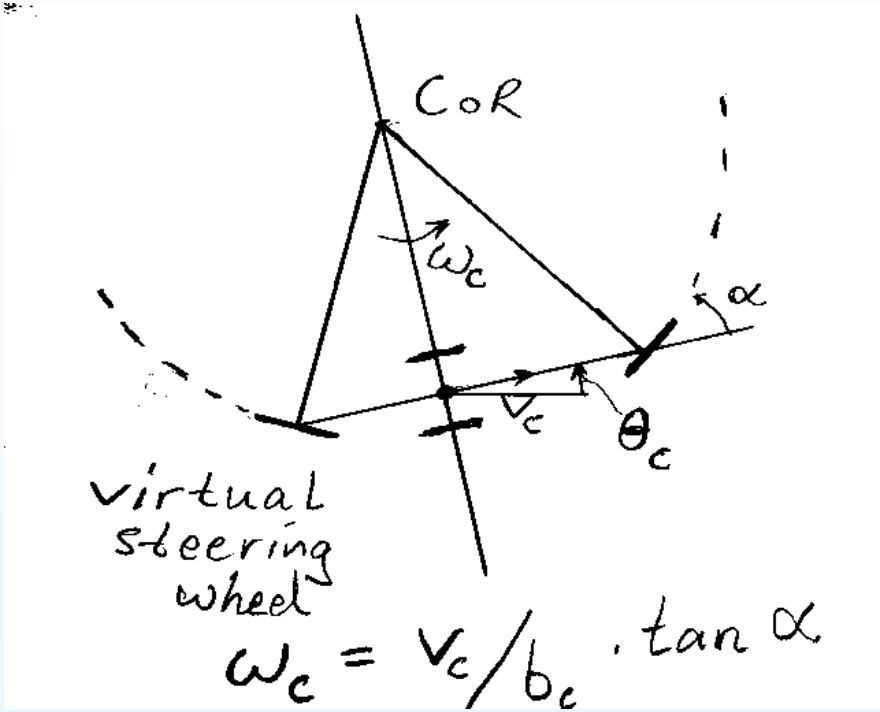
state $= (x_1, y_1, \theta_1, \varphi_1)$ in trailer coordinates

$$\begin{bmatrix} \dot{x}_1 \\ \dot{y}_1 \\ \dot{\theta}_1 \\ \dot{\varphi}_1 \end{bmatrix} = \begin{bmatrix} v_1 \cos\theta_1 \\ v_1 \sin\theta_1 \\ \omega_1 \\ \omega_c - \omega_1 \end{bmatrix} = \begin{bmatrix} v_1 \cos\theta_1 \\ v_1 \sin\theta_1 \\ \frac{v_c}{b_1}\sin\varphi_1 - \omega_c \cdot \frac{a_c}{b_1}\cos\varphi_1 \\ -\frac{v_c}{b_1}\sin\varphi_1 + \omega_c\left(1 + \frac{a_c}{b_1}\cos\varphi_1\right) \end{bmatrix}$$

a_c Distance between connection point of tractor and tractor head of trailer
a_1 Distance between head of second trailer and first trailer center of back wheels
b_c Distance between steering wheel and center of rear wheels of tractor

Car with trailer - Ake We.

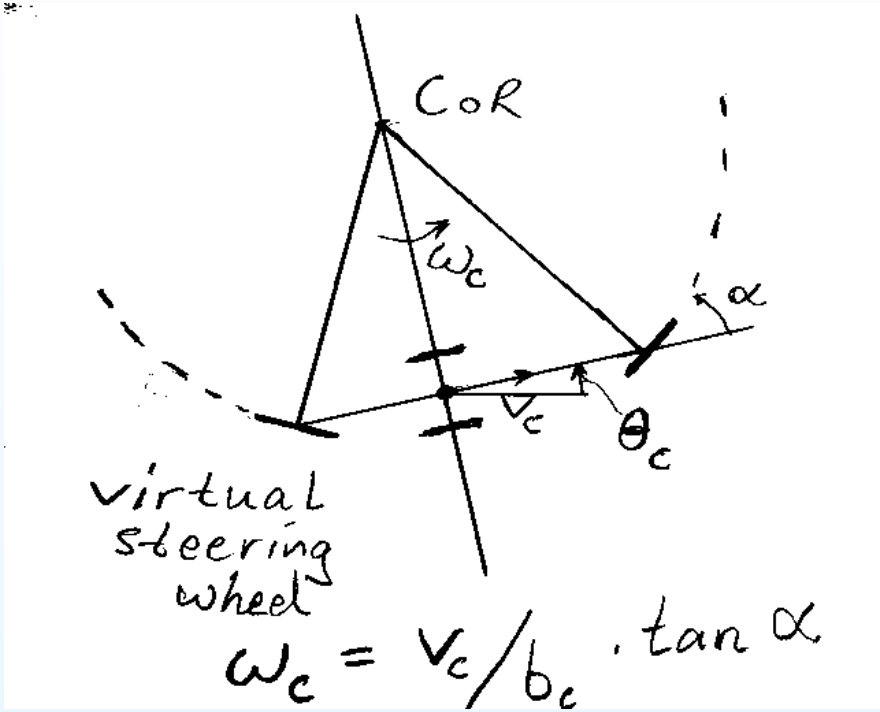
Lecture: Virtual Steering Wheel



Virtual steering wheel - Ake We.

- Useful to put out virtual steering wheel, reversing

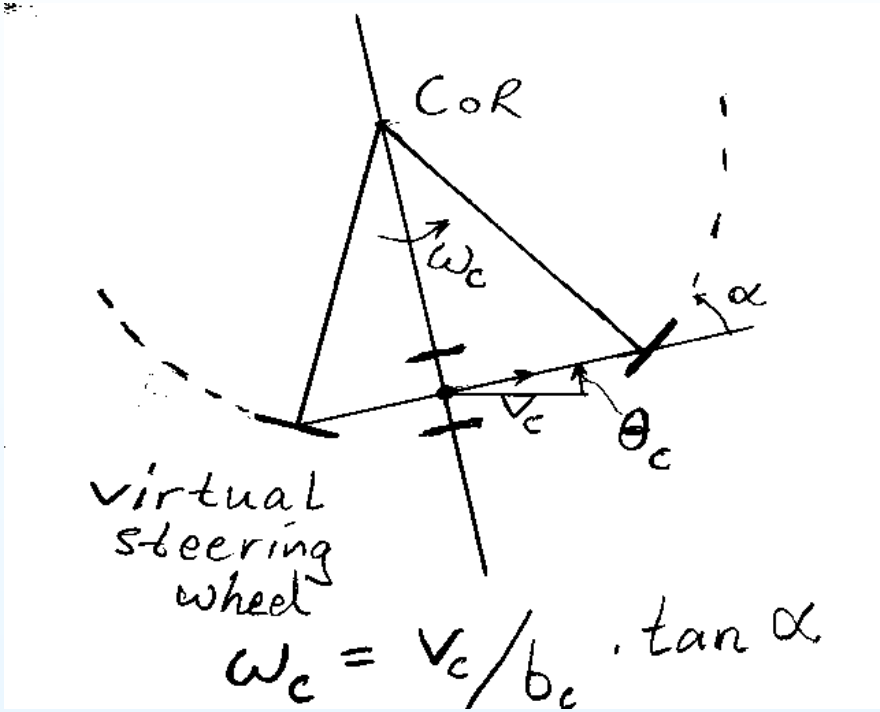
Lecture: Virtual Steering Wheel



Virtual steering wheel - Ake We.

- Useful to put out virtual steering wheel, reversing
- New kinematic model needs to be created

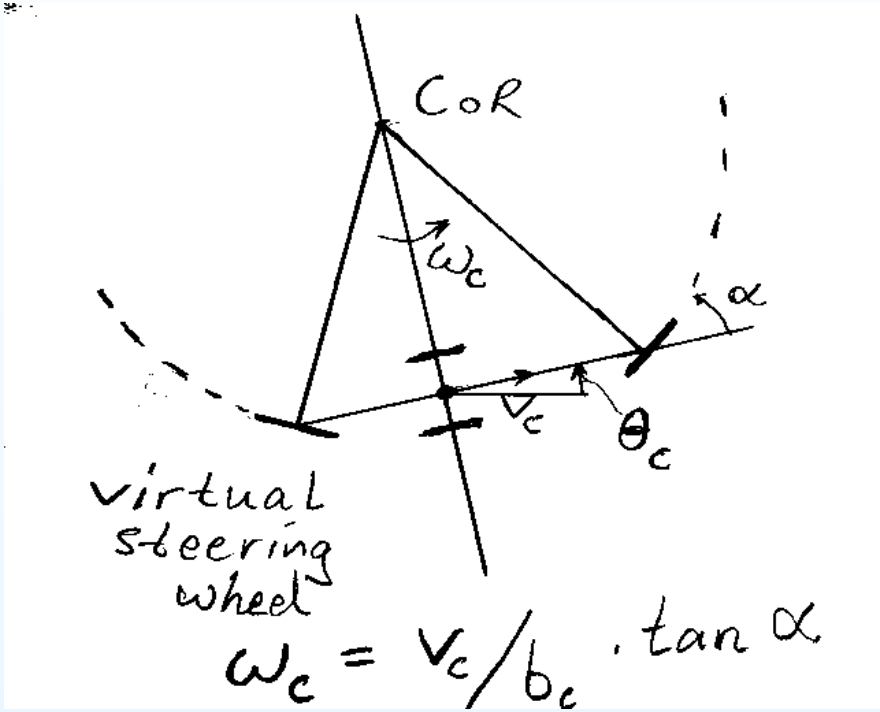
Lecture: Virtual Steering Wheel



Virtual steering wheel - Ake We.

- Useful to put out virtual steering wheel, reversing
- New kinematic model needs to be created
- Virtual wheel can be placed arbitrary

Lecture: Virtual Steering Wheel



Virtual steering wheel - Ake We.

- Useful to put out virtual steering wheel, reversing
- New kinematic model needs to be created
- Virtual wheel can be placed arbitrary
- Useful for dog rabbit control