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Skid Steering Mobile Robot Modeling and Control

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***Abstract*—In this paper, a reduced order model of dynamic and drive models augmentation of a skid steering mobile robot is presented. Moreover, a Linear Quadratic Regulator (LQR) controller augmented with a feed-forward part is designed for controlling this reduced order model. The controller is simple in terms of design and implementation in comparison with complex nonlinear control schemes that are usually designed for this system. Moreover, it provides good performance for the plant comparable with a nonlinear controller based on the inverse dynamics which depends on the availability of an accurate model describing the system. Simulation results are included.**

***Index Terms*—Skid Steering Mobile Robots, Reduced order model, Linear Quadratic Regulator, Feed-Forward Compensation, Inverse Dynamics.**

I. INTRODUCTION

Skid steering mobile robots are widely used as outdoor mobile robots. They are suitable for terrain traversal such as loaders, farm machinery, mining and military applications, due to the simple and robust mechanical structure, faster response, high maneuverability, strong traction, and high mobility[1], [2]. Due to complex kinematic constraints and wheel/ground interactions, considering the dynamical model and designing a proper controller for skid-steering mobile robots (SSMR) are challenging tasks.

A number of research papers have been published on the topic of modelling and control of skid steering mobile robots. Wheeled skid steering mobile robots stability has been studied by some authors using model based nonlinear control techniques by explicitly considering dynamics and drive models [3], [4], [5]. Furthermore, in some works the kinematics have been addressed as the relation of linear and angular velocities with the position of the vehicle [6], [7]. But, major skid effects have not been considered, which arise at a lower level, in the relation between drive velocities and vehicle velocities. An online adaptive control for wheeled skid steering mobile robot has been considered for estimating tire/ground friction of a simplified dynamic model [8]. Control methods of wheeled skid-steering mobile robot trajectory tracking on a rough terrain were presented in [9] including practical fuzzy lateral control, longitudinal control and sensor pan-tilt control; the authors used ADAMS and MATLAB co-simulation platform to assess these control laws. In [10],[11], a thorough dynamic analysis of a skid-steered vehicle has been introduced; this analysis considers steady-state (i.e., constant linear and angular velocities) dynamic models for circular motion of tracked vehicles.

Most of the previous works stated above do a design of a control system for dynamic and drive independently without considering the combination between dynamic and drive model. Therefore, our main contribution in this research is to design one controller for an augmented dynamic-drive model in a reduced order form. The consideration of the drive model with the dynamics of the SSMR is essential to enable a direct control for the motors.

In this paper, a reduced order state space model of the dynamic-drive parts of the SSMR based on [3] is developed. Then, an LQR controller augmented with a feed-forward part to compensate nonlinearities of a part of the dynamic-drive model, which highly affects the system, is developed. For comparison, an inverse dynamics controller is designed. The main advantage of the proposed LQR controller is simplicity of design and experimental implementation in comparison with nonlinear controllers which are highly complex for implementation.

The rest of the paper is organized as follows: In section II an SSMR dynamic-drive models are presented in a systematic way. Section III focuses on the development of the controller using LQR with feed-forward compensation for the system. An inverse dynamics controller is presented to ensure robustness to the nonlinearities of dynamical model. Section IV is dedicated for extensive simulation results considering trajectory tracking problem. Section V is devoted to conclusions and ideas for future work. Finally, acknowledgments and references complete the paper.

II. SSMR MODEL

A mathematical description of the dynamics of an SSMR moving on a planar surface is reviewed in this section. The mathematical model of the vehicle [3] can be divided into three parts: kinematics, dynamics and drive subsystems, see Fig. 1. In this paper, we focus on the first two blocks, i.e., the drive and dynamics subsystems, and we use them for reference tracking control of both the linear and angular velocities.

A. Dynamic Model

The main equation that describes the dynamic subsystem of the SSMR moving on a planar surface as shown in Fig. 2 is

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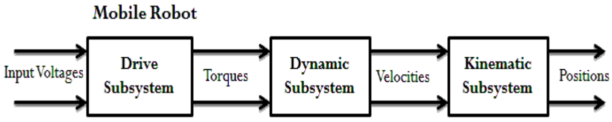

Fig. 1. An electrically driven mobile robot decomposition

given by [3], [4]:

$$\overline{M}(q)\dot{\eta} + \overline{C}(\dot{q})\eta + \overline{R}(\dot{q}) = \overline{B}(q)\tau \tag{1}$$

where $\eta = [v_x \, w]^T$, v_x is the longitudinal velocity, w is the angular velocity of the robot, τ is the torque control input, and $q = [X \, Y \, \theta]^T$ represents the generalized coordinates of the center of mass(COM) of the robot, i.e., the COM position, with X and Y; and θ is the orientation of the local coordinate frame with respect to the inertial frame. The matrices $\overline{M}$, $\overline{C}$, $\overline{R}$, and $\overline{B}$ are given, respectively, by:

$$\overline{M} = \begin{bmatrix} m & 0 \\ 0 & m x_{ICR}^2 + I \end{bmatrix}, \tag{2}$$

$$\overline{C} = \begin{bmatrix} 0 & m x_{ICR}\dot{\theta} \\ -m x_{ICR}\dot{\theta} & m x_{ICR}\dot{x}_{ICR} \end{bmatrix}, \tag{3}$$

$$\overline{R} = \begin{bmatrix} F_{rx}(\dot{q}) \\ x_{ICR}F_{ry}(\dot{q}) + M_r \end{bmatrix}, \tag{4}$$

$$\overline{B} = \begin{bmatrix} \frac{1}{r} & \frac{1}{r} \\ \frac{-c}{r} & \frac{c}{r} \end{bmatrix}. \tag{5}$$

Equation (1) can be written as:

$$\dot{\eta} = \overline{M}^{-1}(q)\overline{B}(q)\tau - \overline{M}^{-1}(q)\overline{C}\eta - \overline{M}^{-1}(q)\overline{R}(\dot{q}) \tag{6}$$

where $\overline{M}$ is nonsingular for all q, $2c$ is the vehicle width, m represents the mass of the robot, I is the moment of inertia of the robot about the COM, r denotes the wheel radius, the coordinate of the instantaneous center of rotation (ICR) is defined as (x_{ICR}, y_{ICR}), $F_{rx}(\dot{q})$ and $F_{ry}(\dot{q})$ are the resultant forces expressed in the inertial frame, and $M_r(\dot{q})$ is the resistant moment around the center of mass. Equations that describe $F_{rx}(\dot{q})$, $F_{ry}(\dot{q})$ and $M_r(\dot{q})$ can be found in [3].

B. Drive Model

It is assumed that the robot is driven by two DC motors, one at each side, with mechanical gears. In Fig. 3, a simplified scheme of the drive on the right side of the robot is depicted. Considering only one drive and assuming that the two motors and gears have the same parameters, the relation between the torque τ and voltage u_{va} can be written as follows:

$$\tau = nK_i i_a, \tag{7}$$

$$u_{va} = L_a \frac{d}{dt} i_a + R_a i_a + nK_e w_i, \tag{8}$$

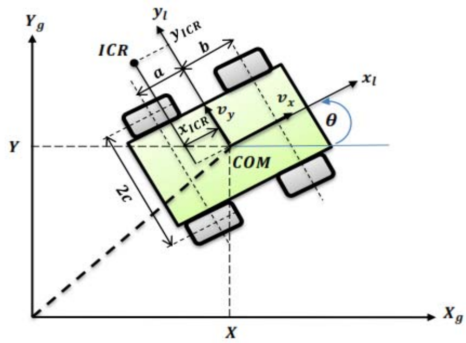

Fig. 2. Schematic diagram of SSMR

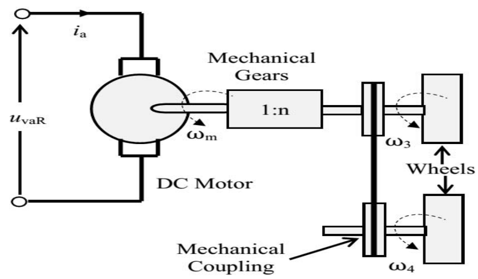

Fig. 3. Drive system on the right side of the vehicle

where i_a is the armature current, K_i is the motor torque constant, n is the gear ratio($n > 1$), L_a and R_a denote the series inductance and resistance of the rotors, respectively, K_e is the electromotive force coefficient, and $w_i = [w_L \;\; w_R]^T$. The left w_L and right w_R sides angular velocities can be obtained from the following formulas:

$$w_L = \frac{v_x - cw}{r}, \tag{9}$$

$$w_R = \frac{v_x + cw}{r}. \tag{10}$$

In order to obtain the overall model of the two motors of the drive system, (8) can be written as follows:

$$\begin{bmatrix} \dot{i}_{a1} \\ \dot{i}_{a2} \end{bmatrix} = \frac{1}{L_a}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} - \frac{R_a}{L_a}\begin{bmatrix} i_{a1} \\ i_{a2} \end{bmatrix} - \frac{nK_e}{rL_a}\begin{bmatrix} v_x - cw \\ v_x + cw \end{bmatrix}, \tag{11}$$

which can be reformulated as:

$$\begin{bmatrix} \dot{i}_{a1} \\ \dot{i}_{a2} \end{bmatrix} = \begin{bmatrix} \frac{-nK_e}{rL_a} & \frac{ncK_e}{rL_a} \\ \frac{-nK_e}{rL_a} & \frac{-ncK_e}{rL_a} \end{bmatrix}\begin{bmatrix} v_x \\ w \end{bmatrix} - \frac{R_a}{L_a}\begin{bmatrix} i_{a1} \\ i_{a2} \end{bmatrix} + \frac{1}{L_a}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}, \tag{12}$$

where V_1 and V_2 are the left and right side motor voltage signals, respectively.

C. Overall System State Space Model

Next, the drive and the dynamic subsystems are combined in one state space representation. Substitution from (7) into

(6), gives:

$$\begin{bmatrix} \dot{v_x} \\ \dot{w} \end{bmatrix} = nK_i\overline{M}^{-1}\overline{B}\begin{bmatrix} i_{a1} \\ i_{a2} \end{bmatrix} - \overline{M}^{-1}\overline{C}\begin{bmatrix} v_x \\ w \end{bmatrix} - \overline{M}^{-1}\overline{R}, \tag{13}$$

which can be mathematically manipulated to obtain,

$$\begin{bmatrix} \dot{v_x} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} \frac{nK_i}{mr}i_{a1} & \frac{nK_i}{mr}i_{a2} \\ \frac{-ncK_i}{r(mx_{ICR}^2+I)}i_{a1} & \frac{ncK_i}{r(mx_{ICR}^2+I)}i_{a2} \end{bmatrix} - \begin{bmatrix} X_{ICR}ww \\ \frac{-mx_{ICR}w}{mx_{ICR}^2+I}v_x \end{bmatrix} - \begin{bmatrix} \frac{F_{rx}}{m} \\ \frac{x_{ICR}F_{ry}+M_r}{mx_{ICR}^2+I} \end{bmatrix}. \tag{14}$$

The dynamic-drive model of the SSMR vehicle described by (12) and (14) can be represented by the following state space representation:

$$\begin{bmatrix} \dot{v_x} \\ \dot{w} \\ \dot{i}_{a1} \\ \dot{i}_{a2} \end{bmatrix} = A_1\begin{bmatrix} v_x \\ w \\ i_{a1} \\ i_{a2} \end{bmatrix} + B_1\begin{bmatrix} V_1 \\ V_1 \end{bmatrix} + D_1, \tag{15}$$

where

$$A_1 = \begin{bmatrix} 0 & -x_{ICR}w & \frac{nK_i}{mr} & \frac{nK_i}{mr} \\ \frac{mx_{ICR}w}{mx_{ICR}^2+I} & 0 & \frac{-ncK_i}{r(mx_{ICR}^2+I)} & \frac{ncK_i}{r(mx_{ICR}^2+I)} \\ \frac{-nK_e}{rL_a} & \frac{ncK_e}{rL_a} & \frac{-R_a}{L_a} & 0 \\ \frac{-nK_e}{rL_a} & \frac{-ncK_e}{rL_a} & 0 & \frac{-R_a}{L_a} \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{1}{L_a} & 0 \\ 0 & \frac{1}{L_a} \end{bmatrix} \quad D_1 = \begin{bmatrix} \frac{-F_{rx}}{m} \\ \frac{-x_{ICR}F_{ry}-M_r}{mx_{ICR}^2+I} \\ 0 \\ 0 \end{bmatrix} \tag{16}$$

The order of the model (15) can be reduced to 2 by neglecting the motor inductance, i.e., $L_a = 0$. Therefore, (8) can be written as :

$$u_{va} = R_a i_a + nK_e w_i. \tag{17}$$

From this equation, we can obtain the current of the motors as:

$$\begin{bmatrix} i_{a1} \\ i_{a2} \end{bmatrix} = \begin{bmatrix} \frac{1}{R_a} & 0 \\ 0 & \frac{1}{R_a} \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} + \begin{bmatrix} \frac{-nK_e}{rR_a} & \frac{ncK_e}{rR_a} \\ \frac{-nK_e}{rR_a} & \frac{-ncK_e}{rR_a} \end{bmatrix}\begin{bmatrix} v_x \\ w \end{bmatrix}. \tag{18}$$

The torque can be calculated as:

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} \frac{nK_i}{R_a} & 0 \\ 0 & \frac{nK_i}{R_a} \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} + \begin{bmatrix} \frac{-n^2K_iK_e}{rR_a} & \frac{n^2cK_iK_e}{rR_a} \\ \frac{-n^2K_iK_e}{rR_a} & \frac{-n^2cK_iK_e}{rR_a} \end{bmatrix}\begin{bmatrix} v_x \\ w \end{bmatrix}. \tag{19}$$

By using (19), the state space representation of the reduced order overall system can be given by:

$$\begin{bmatrix} \dot{v_x} \\ \dot{w} \end{bmatrix} = A_2\begin{bmatrix} v_x \\ w \end{bmatrix} + B_2\begin{bmatrix} V_1 \\ V_1 \end{bmatrix} + D_2. \tag{20}$$

where

$$A_2 = \begin{bmatrix} \frac{-2n^2K_iK_e}{mr^2R_a} & -x_{ICR}w \\ \frac{mx_{ICR}w}{mx_{ICR}^2+I} & \frac{-2n^2c^2K_iK_e}{r^2R_a(mx_{ICR}^2+I)} \end{bmatrix}$$

$$B_2 = \begin{bmatrix} \frac{nK_i}{mrR_a} & \frac{nK_i}{mrR_a} \\ \frac{-ncK_i}{rR_a(mx_{ICR}^2+I)} & \frac{ncK_i}{rR_a(mx_{ICR}^2+I)} \end{bmatrix}$$

$$D_2 = \begin{bmatrix} \frac{-F_{rx}}{m} \\ \frac{-x_{ICR}F_{ry}-M_r}{mx_{ICR}^2+I} \end{bmatrix}. \tag{21}$$

Equation (20) can be written in the general form of the state space representation as follows:

$$\dot{X} = A(X)X + BU + D(X) \tag{22}$$

where the system states are represented by:

$$X = [v_x \;\; w]^T. \tag{23}$$

The control input vector U is represented by:

$$U = [V_1 \;\; V_2]^T \tag{24}$$

Finally, we consider $D(X)$, the last term of (22) as a disturbance; this will be neglected in the design of the LQR controller; however, it will be considered in the design of the feed-forward controller to overcome its effect on the system as shown below.

III. Controller Design

In this section, we introduce two control laws, LQR with feed-forward compensation and inverse dynamics controller to be used for a reference tracking.

A. LQR State Feedback Design

LQR control plays a crucial role in optimal control systems and it has many applications, e.g. airplane flight control, chemical process control, and motor control. The main purpose of LQR control is to obtain an optimal control law in order to minimize a cost function along the trajectory of a linear system. Consider the state space representation of a system [12]:

$$\dot{x} = Ax + Bu, \tag{25}$$

$$y = Cx. \tag{26}$$

with $x(t) \in R^n$, $u(t) \in R^m$ and the initial condition is $x(0)$. We assume here that all the states are measurable and seek to find a state-variable feedback control law as:

$$u = -Kx \tag{27}$$

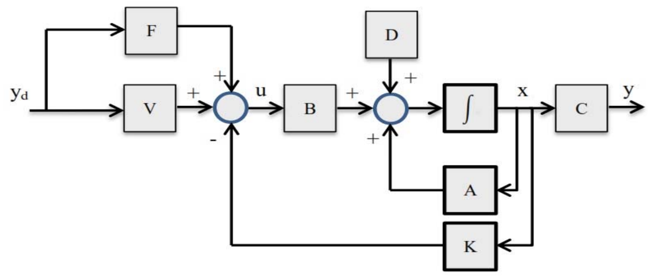

Fig. 4. Linear quadratic control with feed-forward compensation.

that gives desirable closed-loop properties such that K is the feedback gain vector. The optimal feedback state regulation minimizes the quadratic cost function defined by:

$$J = \frac{1}{2}\int_0^{\infty}(x^TQx + u^TRu)dt, \tag{28}$$

where Q is a symmetric positive semi-definite matrix and R is a symmetric positive definite matrix. The optimal feedback gain vector can be calculated by:

$$K = R^{-1}B^TP \tag{29}$$

where P is the solution of the Algebraic Riccati Equation (ARE) defined by :

$$A^TP + PA + Q - PBR^{-1}B^TP = 0 \tag{30}$$

Our objective is to design an optimal control law to provide a reference tracking controller of a linearized model of the skid steering mobile robot vehicle using the voltages of the two motors as the inputs to the system. The closed loop reference tracking control system for the SSMR is described by (25) and (26). The control law for reference tracking LQR controller can be defined as [13]:

$$u = -Kx + Vy_d \tag{31}$$

where $V = -(C^T(A - BK)^{-1}B)^{-1}$ to insure zero steady-state error and y_d is the desired output.

B. LQR Control with feed-forward compensation

Next, we design an LQR controller for reference tracking of both linear and angular velocities of a linearized model. In this model, we consider the system defined by (20) to be represented by:

$$\dot{x} = A(x)x + Bu + D(x) \tag{32}$$

In the simulation, we consider the term $D(x)$ of (32) as a disturbance; therefore, we test the system response with and without this term to check its effect. We propose a feed-forward compensation to overcome the effect of this term. The overall closed-loop block diagram of the system with the LQR controller and the feed-forward compensation is depicted in Fig. 4, where F is a vector which can be calculated by:

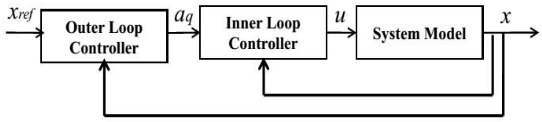

Fig. 5. Inner-outer loop control for inverse dynamics

$$F = \begin{bmatrix} \frac{R_a r}{2nK_i}\left[\left(\frac{-x_{ICR}F_{ry}(\dot{q})}{c} - \frac{M_r}{c}\right) + F_{rx}(\dot{q})\right] \\ \\ \frac{R_a r}{2nK_i}\left[\left(\frac{x_{ICR}F_{ry}(\dot{q})}{c} + \frac{M_r}{c}\right) + F_{rx}(\dot{q})\right] \end{bmatrix} \tag{33}$$

This vector represents the compensation of the two values of $D(x)$.

C. Inverse Dynamics Control

Next, we design an inverse dynamics controller for the system based on the idea presented in [14]. Consider again the SSMR dynamical model described by (32). The idea of inverse dynamics is to seek a nonlinear feedback control law described by:

$$u = f(x, t) \tag{34}$$

The block diagram of the scheme of the inverse dynamics control is shown in Fig. 5. Consider a new input to the system a_q such that:

$$a_q = A(x)x + Bu + D(x) \tag{35}$$

By mathematical manipulation of (35), we can obtain the control input u as follows:

$$a_q - A(x)x - D(x) = Bu, \tag{36}$$

$$u = B^{-1}(a_q - A(x)x - D(x)). \tag{37}$$

The new control input a_q can be given by:

$$a_q = k(x - x_{ref}) \tag{38}$$

where k is the gain to be designed for the controller and x_{ref} is the reference system states.

IV. Simulation Results

In this simulation, the dynamic and drive models described by the reduced order model (20) is considered, so the inputs to the system are the two motor voltages V_1 and V_2 and the outputs of the system are the linear velocity v_x and angular velocity w. The system parameters applied for simulation are shown in Table I. In practice, it is difficult to measure x_{ICR} value, so it is assumed here to be [3], [4]:

$$x_{ICR} = constant = x_0 \quad x_0 \in (-a, b) \tag{39}$$

where a and b are positive kinematic parameters of the robot depicted in Fig. 2,

TABLE I
PARAMETERS OF THE MODEL

Variable	Value	Unit
$a = b$	39	mm
c	34	mm
r	26.5	mm
m	1	Kg
I	0.0036	$Kg.m^2$
x_0	-15	mm
Ra	3.9	Ω
K_i	8.55	$mN.m/A$
K_e	8.55	$mV.s/rad$
n	12	$-$

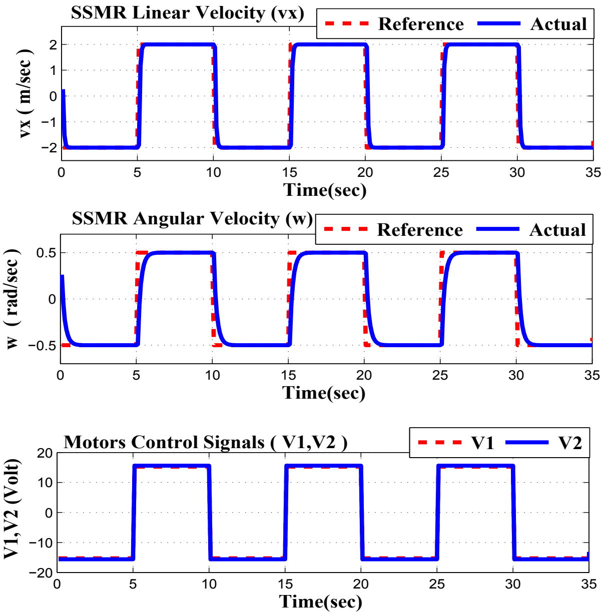

Fig. 6. System response (without the nolinear term $D(x)$)

A. LQR Controller Results

For the LQR controller, we choose the matrices Q and R to be:

$$Q = \begin{bmatrix} 0.1 & 0 \\ 0 & 0 \end{bmatrix} \qquad R = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \tag{40}$$

In simulation, we test three cases for the system. First, we assume that the system is described by (32) without the $D(x)$ term which is considered as disturbance. Second, we add this term to the system to check its effect on the response and asses the controller performance. Finally, we add the feed forward compensation part to overcome the effect of the disturbance part. Fig. 6 shows the system response to a square reference for both the linear and angular velocities. The system can track the desired inputs quickly and without any overshoot. Also, Fig. 6 provides the control signals of the system motors and it is obvious that the control signals are in the limits of the maximum motor voltages ($\pm 24VDC$).

In order to demonstrate the effectiveness of the controller, the term $D(x)$ is added to the system and the system response

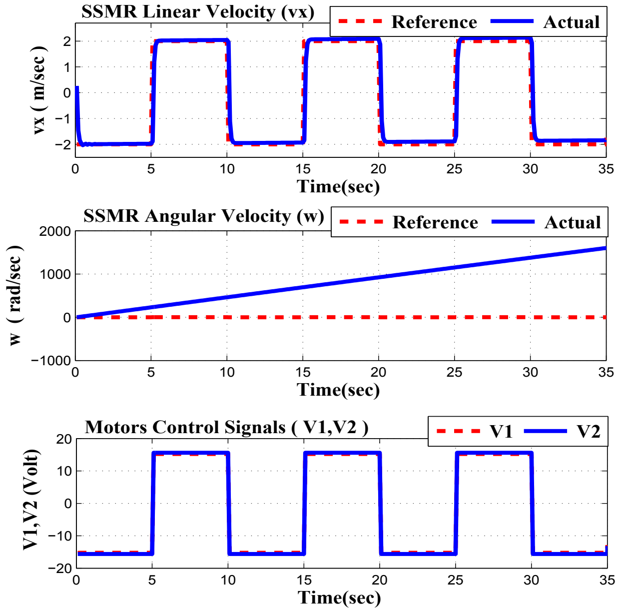

Fig. 7. System response (including $D(x)$ as part of the system)

and control signals are shown in Fig. 7. It can be shown that the system is affected by this part which destabilizes the angular velocity output. In order to overcome the effect of $D(x)$, The feed-forward compensation is added to the controller. The system response and control signals are shown in Fig. 8. It is shown that the system can again track the desired references with small steady state errors which can be removed by the proper design of the LQR controller by suitable selection of weight matrices Q and R. Moreover, we see that the control signals are different from those that are depicted in Fig. 6 and Fig. 7 as the control law is changed, see in Fig. 4.
From the above analysis of the results, the simulation results illustrate that the LQR performance is enhanced for the linearized model; however, the stability and tracking of the reference input may not be guaranteed if this controller is applied to the nonlinear system.

B. Inverse Dynamics Controller Results

Next, we apply the inverse dynamics controller to the nonlinear system described by (20) directly. The gain k found in (38) was selected to be 10. The system response and control signals are shown in Fig. 9. These results clearly illustrate the performance enhancement of the inverse dynamics controller that can deal with nonlinear systems. Also, the two control signals are within the limits of the motor voltages. But, it is clear from the inverse dynamics controller design that it is required to have an accurate model of the nonlinear system for proper design of this controller as it depends on the system matrices. Finally, it should be noted that the inverse dynamics controller is applied to the nonlinear model of the SSMR

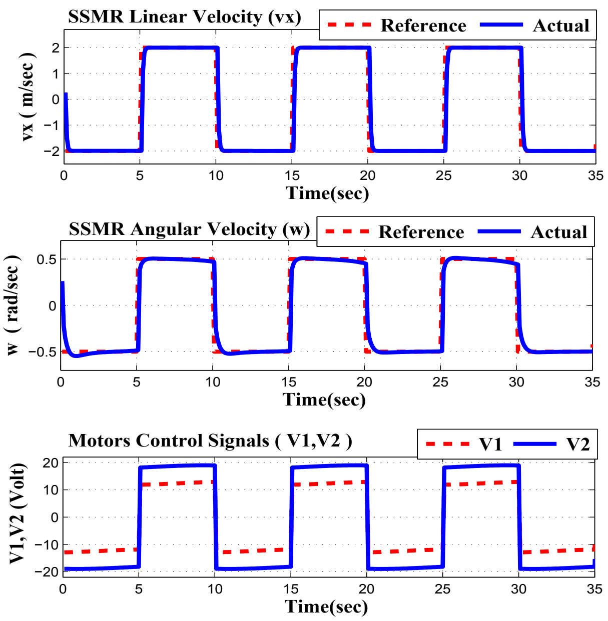

Fig. 8. System response with feed-forward compensation

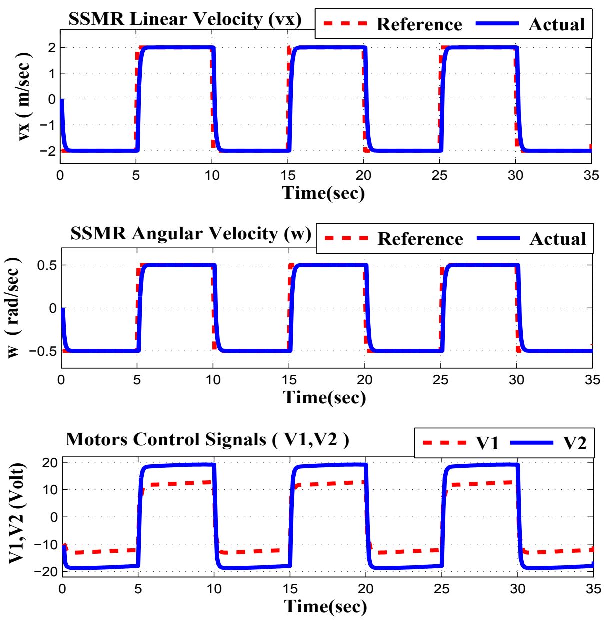

Fig. 9. System response with an inverse dynamic controller

directly while the LQR with the feed forward compensation is applied to a linearized model of the system.

V. Conclusion

An LQR with feed-forward compensation algorithm is presented in this paper for controlling a reduced order model of augmented dynamic and drive models of an SSMR. The controller is considered by merging a linear quadratic regulator controller with a feed-forward compensation to improve tracking accuracy and overcome effects of nonlinearities. For comparison, an inverse dynamics controller is designed. The LQR controller with the feed-forward compensation shows satisfactory results.

In the future work, a development of a time-varying LQR controller to deal with the nonlinearities of the system will be investigated. Experimental implementation will be considered as well.

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References

[1] W. Yu, O. Y. Chuy, E. G. Collins, and P. Hollis, "Analysis and experimental verification for dynamic modeling of a skid-steered wheeled vehicle," *IEEE transactions on robotics*, vol. 26, no. 2, pp. 341–353, 2010.

[2] A. Mandow, J. L. Martinez, J. Morales, J. L. Blanco, A. G. Cerezo, and J. Gonzalez, "Experimental kinematics for wheeled skid-steer mobile robots," in *In Proc. of the IEEE/RSJ International Conference on Intelligent Robots and Systems*, USA, 2007, pp. 1222–1227.

[3] K. Kozlowski and D. Pazderski, "Modeling and control of a 4-wheel skid-steering mobile robot," *Int. J. Appl. Math. Comput. Sci.*, vol. 14, no. 4, pp. 477–496, 2004.

[4] Z. Jian, W. S. Shuang, L. Hua, and L. Bin, "The sliding mode control based on extended state observer for skid steering of 4-wheel-drive electric vehicle," in *In Proc. Of the 2nd International Conference on Consumer Electronics, Communications and Networks (CECNet)*, China, 2012, pp. 2195–2200.

[5] L. Caracciolo, A. D. Luca, and S. Iannitti, "Trajectory tracking control of a four-wheel differentially driven mobile robot," in *In Proc. of IEEE International Conference on Robotics and Automation*, USA, 1999, pp. 2632–2638.

[6] E. Maalouf, M. Saad, and H. Saliah, "A higher level path tracking controller for a four-wheel differentially steered mobile robot," *Journal of Robotics and Autonomous Systems*, vol. 54, pp. 23–33, 2006.

[7] K. Kozlowski and D. Pazderski, "Practical stabilization of a skid steering mobile robot - a kinematic-based approach," in *In Proc. of IEEE 3rd International Conference on Mechatronics*, Hungary, 2006, pp. 519–524.

[8] J. Yi, D. Song, J. Zhang, and Z. Goodwin, "Adaptive trajectory tracking control of skid-steered mobile robots," in *In Proc. of the IEEE International conference on robotics and automation*, Italy, 2007, pp. 2605–2610.

[9] Y. Yi, F. Mengyin, Z. Hao, X. Guangming, and S. Changsheng, "Control methods of mobile robot rough-terrain trajectory tracking," in *In Proc. of the 8th IEEE International Conference on Control and Automation*, China, 2010, pp. 731–738.

[10] J. Y. Wong, *Theory of Ground Vehicles*, 3rd ed. New York: John Wiley & Sons, 2001.

[11] J. Y. Wong and C. F. Chiang, "A general theory for skid steering of tracked vehicles on firm ground," *In Proc. of Institution of Mechanical Engineering, Part D, J. Automobile Eng.*, vol. 215, pp. 343–355, 2001.

[12] K. Ogata, *Modern Control Engineering*, 5th ed. Pearson Prentice-Hall, 2010.

[13] M. Ruderman, J. Krettek, F. Hoffmann, and T. Bertram, "Optimal state space control of dc motor," in *In Proc. of the 17th World Congress The International Federation of Automatic Control*, Korea, 2008, pp. 5796–5801.

[14] M. W. Spong, S. Hutchinson, and M. Vidyasagar, *Robot Modeling and Control*, 5th ed. New York: John Wiley & Sons, 2006.