Modeling and Control of Mobile Robot with Differential Chassis

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Modeling and Control of Mobile Robot with Differential Chassis

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***Abstract* – The main focus of this article is modeling, simulation and control of the mobile robot with differential chassis. The article presents a mathematical model of the robot with a differential chassis, which consists of kinematic model, dynamic model and the internal feedback control loop to suppress the influences of dynamics. The dynamic model of the robot includes the mass and moment of inertia for each part of the robot. Model is validated in posture control structure. The simulation model of the robot with differential chassis and control algorithm are implemented in simulation environment Simulink.**

***Keywords* – mathematical modeling, simulation, mobile robot with differential wheeled chassis, control algorithm to posture, Simulink.**

I. Introduction

In recent years, the robotics and control of robotic systems is still an actual theme. In past, the static robots were used mostly in industrial tasks as manipulators, but the mobile robots were almost exclusively applied in research. But today, the mobile robots are gaining popularity common users, as can be seen in our households. The automatic vacuum cleaner or lawn movers, that use differential chassis concept are quite common now. Based on the assumptions that the robot has a non-zero mass and moment of inertia [1] [2], we can obtain the mathematical model of the mobile robot. Usually, the implemented control algorithm uses this model to complete its defined task. In presented case, the task for the mobile robot is get to a point with the desired orientation angle - to posture [3]. This article is output of bachelor thesis, which title is "Modeling, simulation and control of the two-wheel mobile robot" [4].

II. Mathematical model of the robot with differential wheeled chassis

The mathematical model of the robot with differential wheeled chassis describes the behaviour of the mobile robot in the plane [5].

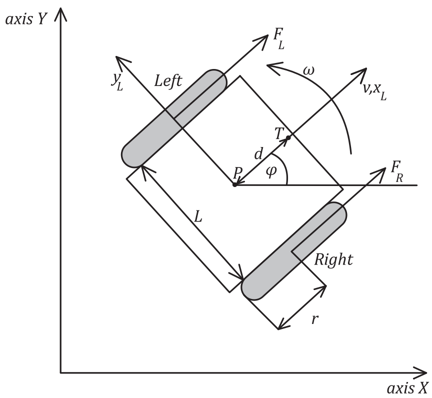

Fig. 1 Schematic illustration of the mobile robot

The model inputs are angular velocities of the right and left wheel ω_R, ω_L. Based on the angular velocities, the posture of the robot is calculated as x and y position coordinates together with an orientation angle the mobile robot φ. To create a simulation model of mobile robot that can be represented by scheme depicted on fig. 2, we require both kinematic and dynamic models of the mobile robot along with the internal feedback control loop which can suppress the influence of dynamics.

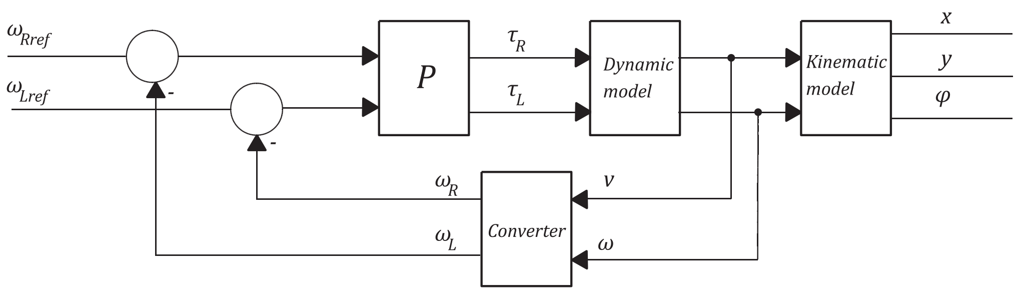

Fig. 2 Simulation scheme of mathematical model of mobile robot

A. Kinematic model of the mobile robot

Based on the assumption that the robot's wheels cannot move to side ensuring that the velocity in Y axis of robot' local coordinate system is zero $v_y = 0$, the kinematic model of the robot can be obtained from geometric properties of the reference point movement in plane. The movement of the robot in the direction of it's X axis is the robot's overall linear velocity v. The situation is described on fig. 3. Robot's posture in the plane is described x and y coordinate together with an orientation angle φ. [6]

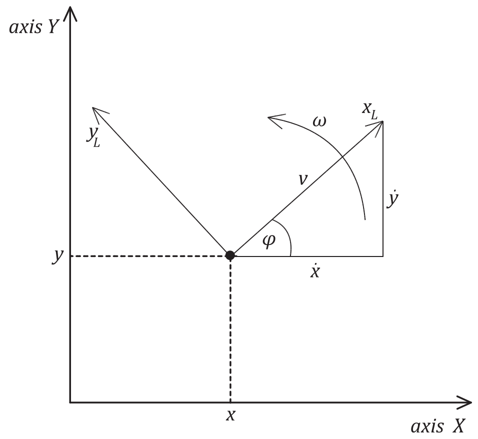

Fig. 3 Schematic illustration of the robot in the plane

The kinematic model of the robot with differential chassis can be written in matrix form [5]:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\varphi} \end{bmatrix} = \begin{bmatrix} \cos\varphi & 0 \\ \sin\varphi & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix}, \tag{1}$$

where v is overall linear velocity of robot, ω is overall angular velocity of robot, x, y are position coordinates in global coordinate system and φ is orientation angle the mobile robot.

For implementation and control reasons, it is often required to control the robot by wheel's linear velocities v_R, v_L instead of the linear and angular velocity of the robot v, ω. The transformation between velocities is

$$\begin{bmatrix} v_R \\ v_L \end{bmatrix} = \begin{bmatrix} 1 & \frac{L}{2} \\ 1 & -\frac{L}{2} \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix}, \tag{2}$$

where L is distance between wheels. Also, usually is better to use wheel angular velocities ω_R, ω_L, the transformation have simple form

$$\begin{bmatrix} \omega_R \\ \omega_L \end{bmatrix} = \begin{bmatrix} \frac{1}{r} & 0 \\ 0 & \frac{1}{r} \end{bmatrix} \begin{bmatrix} v_R \\ v_L \end{bmatrix}, \tag{3}$$

where r is radius of the wheel. Contra, it is possible to convert the angular velocities of right and left wheel ω_R, ω_L to overall linear and angular velocity of mobile robot v, ω as

$$\begin{bmatrix} v \\ \omega \end{bmatrix} = \begin{bmatrix} \frac{r}{2} & \frac{r}{2} \\ \frac{r}{L} & -\frac{r}{L} \end{bmatrix} \begin{bmatrix} \omega_R \\ \omega_L \end{bmatrix}. \tag{4}$$

B. Dynamic model of mobile robot for partial moments of inertia

The dynamic model describes the mass m and moment of inertia J for individual parts of the robot as engines, wheels and chassis. The equation of the dynamic model in general can be written as [1]

$$M\dot{\eta} + V\eta = B\tau, \tag{5}$$

where M is matrix of the moments of inertia, V is a matrix of coriolis forces, B is matrix of input, τ is matrix of motors torques, which are calculated as $\tau = rF$, m_c is mass of chassis and J_w is the moment of inertia of each driving wheel with a motor about the wheel axis. These matrices can be expressed as

$$M = \begin{bmatrix} J_w + \frac{r^2}{L^2}(m\frac{L^2}{4} + J) & \frac{r^2}{L^2}(m\frac{L^2}{4} - J) \\ \frac{r^2}{L^2}(m\frac{L^2}{4} - J) & J_w + \frac{r^2}{L^2}(m\frac{L^2}{4} + J) \end{bmatrix}, \tag{6}$$

$$V = \begin{bmatrix} 0 & \frac{r^2}{L}m_c d\omega \\ -\frac{r^2}{L}m_c d\omega & 0 \end{bmatrix}, \tag{7}$$

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \tag{8}$$

$$\dot{\eta} = \begin{bmatrix} \dot{\theta}_R \\ \dot{\theta}_L \end{bmatrix} = \begin{bmatrix} \omega_R \\ \omega_L \end{bmatrix}, \tau = \begin{bmatrix} \tau_R \\ \tau_L \end{bmatrix}. \tag{9}$$

The robot's overall mass m and moment of inertia J are calculated as

$$m = m_c + 2m_w, \tag{10}$$

$$J = m_c d^2 + m_w \frac{L^2}{2} + J_c + 2J_m, \tag{11}$$

where m_w is the mass of each driving wheel with engine, J_c the moment of inertia of the chassis about the vertical axis through the centre of mass and J_m is the moment of inertia of each driving wheel with a motor about the wheel diameter. The distance between the origin of mobile robot's coordinate system P and centre of robot's mass T is marked as d.

The mobile robot's dynamic model equation (5) needs to be expressed in form where the outputs of the model are the robot's overall linear and angular velocity. By using the transformation (4) and substitution of equations (4) it is possible to rewrite (5) as [2]

$$J_w\omega_R + \frac{rmv}{2} + \frac{rJ}{L}\omega + \frac{r^2}{L}m_c d\omega\theta_L = \tau_R, \tag{12}$$

$$J_w\omega_L + \frac{rmv}{2} - \frac{rJ}{L}\omega - \frac{r^2}{L}m_c d\omega\theta_R = \tau_L. \tag{13}$$

The equations (12), (13) can be rewritten more convenient form as

$$(m + \frac{2J_w}{r^2})\dot{v} - m_c d\omega^2 = \frac{1}{r}(\tau_R + \tau_L), \tag{14}$$

$$(J + \frac{L^2}{2r^2}J_w)\dot{\omega} + m_c d\omega v = \frac{L}{2r}(\tau_R - \tau_L). \tag{15}$$

The dynamic model of mobile robot represented by (14), (15) is used to create the simulation model of mobile robot with differential chassis. To have inputs of mobile robot defined as wheel angular velocities, it is required to incorporate the internal control loop that will also suppress the influences of dynamics.

C. Internal feedback control loop to suppress the influence of dynamics

Internal feedback control loop have to suppress the influences of dynamics and the control objective is to minimize difference between the desired $\omega_{Rref/Lref}$ and actual $\omega_{R/L}$ angular velocities of the right/left wheel. The actuators, wheel motors generate traction forces based and the forces acting on the chassis of the robot can be expressed as [5].

$$F_{R/L} = K_{in} \cdot (\omega_{Rref/Lref} - \omega_{R/L}) \; for \; |K_{in} \cdot (\omega_{Rref/Lref} - \omega_{R/L})| < F_{max},$$
$$F_{R/L} = F_{max} \cdot sign(\omega_{Rref/Lref} - \omega_{R/L}) \; for \; |K_{in} \cdot (\omega_{Rref/Lref} - \omega_{R/L})| \geq F_{max}, \tag{16}$$

where $\omega_{Rref/Lref}$ is desired angular velocity right or left wheel, $\omega_{R/L}$ is actual angular velocity right or left wheel, K_{in} is a gain and F_{max} is maximal traction force of engines.

Putting it all together, the simulation model proposed in 2 can be implemented in Simulink environment as shown on fig. 4. This simulation model is verified in control experiments.

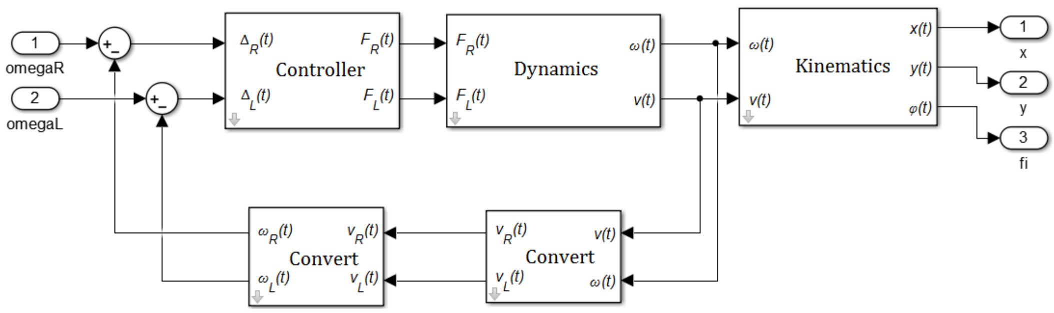

Fig. 4 Simulation scheme of mathematical model of mobile robot

III. Posture control algorithms for mobile robot

The posture control algorithms of mobile robot depends on position and the final orientation of the robot. In presented case, the requester final orientation angle φ has to be zero. Input to the control algorithm is the actual posture of robot x, y, orientation angle φ and position of a reference point x_{ref}, y_{ref}. Based on these coordinates, the algorithm calculates the distance between the reference point and robot current position as ρ together with the orientation angle error α and the heading angle error δ [3]. The situation and errors are depicted no fig. 5.

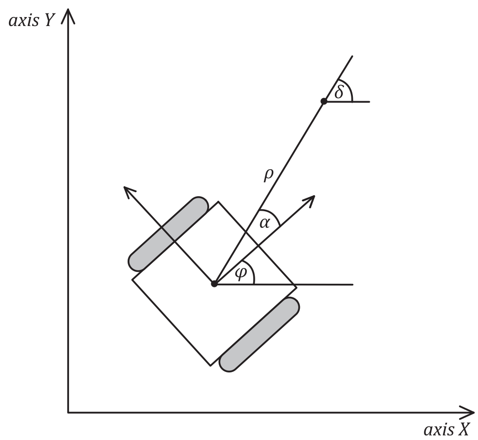

Fig. 5 Schematic illustration ρ, δ, α

The distance ρ is calculated as Euclidean distance between the reference and current position

$$\rho = \sqrt{(x_{ref} - x)^2 + (y_{ref} - y)^2}, \tag{17}$$

while the heading angle error δ is calculated using trigonometric function

$$\beta = \tan^{-1} \frac{(y_{ref} - y)}{(x_{ref} - x)}, \tag{18}$$

together with the set of rules for angles

$$\begin{aligned} \text{If } x_{ref} < x \text{ and } y_{ref} \geq 0 \text{ then } \delta &= \beta + \pi, &(19)\\ If\ x_{ref} < x \text{ and } y_{ref} < 0 \text{ then } \delta &= \beta - \pi, &(20)\\ If\ x_{ref} \geq x \text{ then } \delta &= \beta. &(21) \end{aligned}$$

and the orientation angle error α is calculated as

$$\alpha = \delta - \varphi. \tag{22}$$

The chosen control algorithm produces outputs as desired linear velocity v and angular velocity ω of the robot, calculated for control law [3]

$$\begin{aligned} v &= K_1 \rho \cos\alpha, &(23)\\ \omega &= K_2\alpha + K_1 \frac{\cos\alpha \sin\alpha}{\alpha}(\alpha + K_3\delta), &(24) \end{aligned}$$

where K_1, K_2, K_3 are gain coefficients.

A. Simulation experiments

The control experiment includes four points A, B, C, D around the robot, their distance from on X axis from Y axis $0.1\ m$. The initial position of the robot x, y is always the origin of the global coordinate system $[0\ 0]$ and starting orientation angle is $\varphi = 0\ rad$. The experiment depicted on fig. 6 proved that mobile robot in presented control structure was able to successfully reach all of the points with final robot orientation angle $\varphi_{final} = 0\ \ rad$.

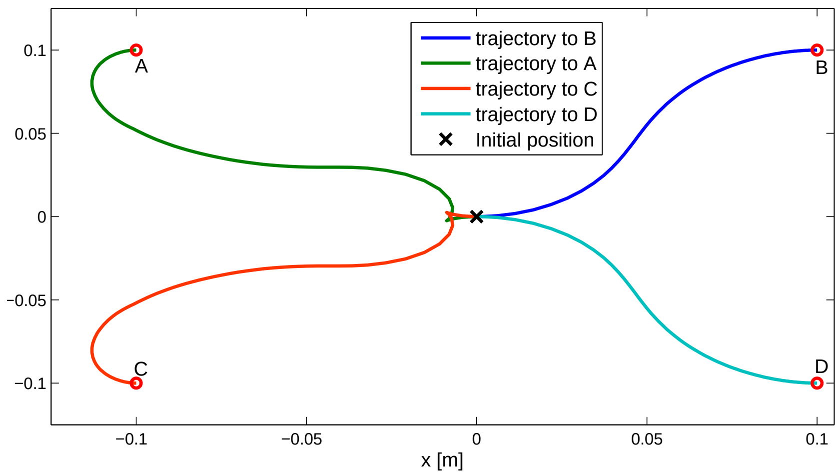

Fig. 6 Behaviour of the mobile robot in control to the four different points

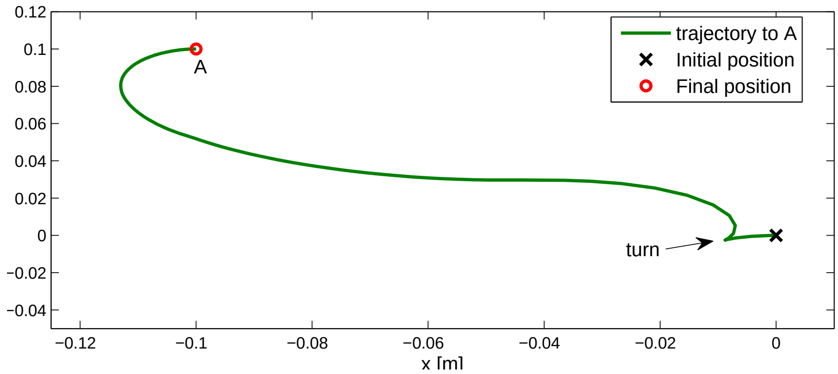

Fig. 7 Experiment in detail - control of mobile robot to the point $A = [-0.1\ 0.1]$

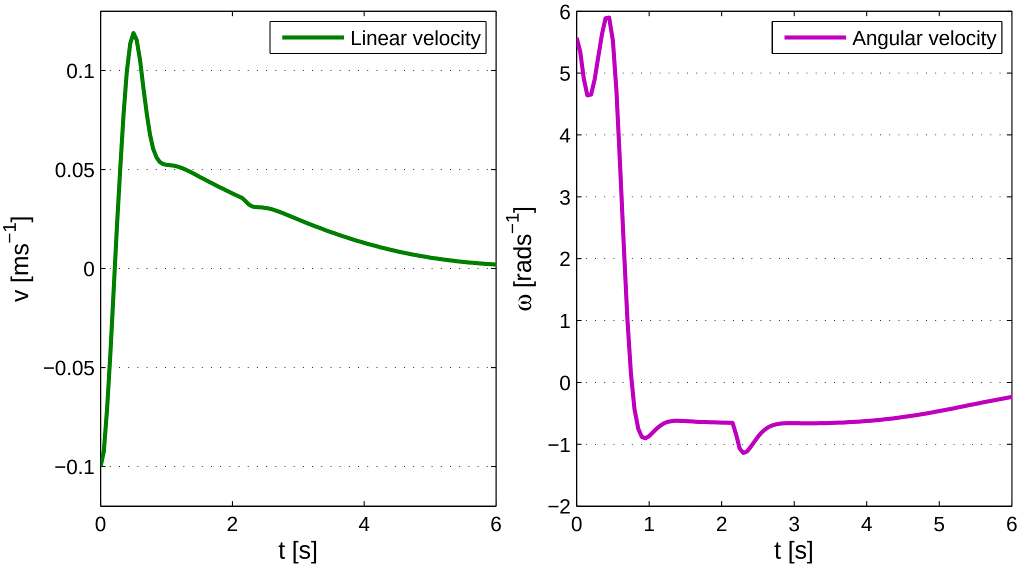

Fig. 8 Experiment in detail - mobile robot's linear and angular velocities

It is obvious from the experiment results shown on fig. 7 and 8 that, if the point, in this case $A = [-0.1\ 0.1]$, then linear velocity v of the robot is initially negative, which means that the robot reverse. After a while the robot starts to turn and get to the point with desired orientation angle $\varphi = 0\ rad$.

IV. Conclusion

The article presents a mathematical model of the robot with a differential wheeled chassis, which consists of the kinematic model, the dynamic model for partial moments of inertia together with internal feedback control loop to suppress the influence of dynamics. The model was validated in closed loop experiment. The posture control algorithm used in experiments was able to achieve all reference points around the robot with desired orientation.

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References

[1] T. Fukao, H. Nakagawa, and N. Adachi, "Adaptive," *Robotics and Automation, IEEE Transactions on*, vol. 16, no. 5, pp. 609–615, 2000.
[2] R. Dhaouadi and A. Hatab, "Dynamic modelling of differential-drive mobile robots using lagrange and newton-euler methodologies: A unified framework," *Adv Robot Autom*, vol. 2, no. 107, p. 2, 2013.
[3] M. Aicardi, G. Casalino, A. Bicchi, and A. Balestrino, "Closed loop steering of unicycle like vehicles via lyapunov techniques," *Robotics & Automation Magazine, IEEE*, vol. 2, no. 1, pp. 27–35, 1995.
[4] T. Klein, "Modelovanie, simulácia a riadenie dvojkolesového mobilného robota," *Bachelor Thesis*, 2015.
[5] J. Šembera and F. Šolc, "Modelování a řízení mobilního robotu s diferenciálním podvozkem," *ATP journal*, 2007.
[6] J. Mireles Jr, "Kinematic models of mobile robots," *Automation and Robotics Research Institute, University of Texas at Austin*, 2004.