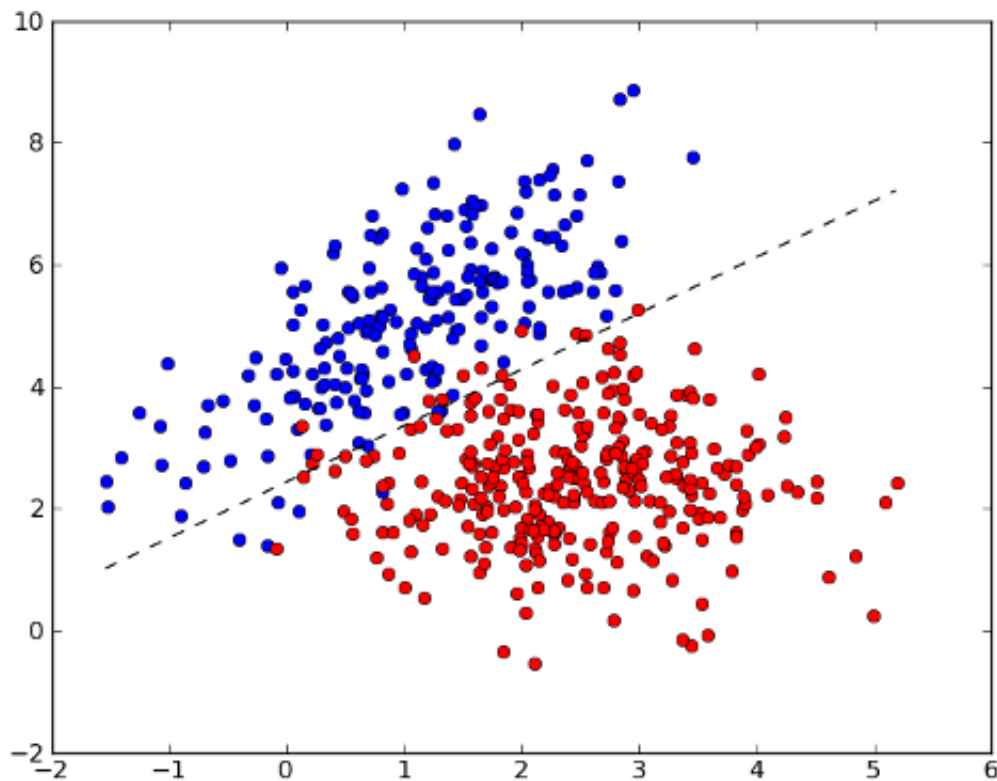


In []:

Logistic Regression



In logistic regression, our goal is to find a hyperplane, or a decision boundary, that separates the data. The hyperplane is always 1 dimension less than the dimension of the data. For example, if you have a 2 dimensional data, the hyperplane is 1 dimensional, or a line. And, if you have a 3 dimensional data, the hyperplane is a 2 dimensional, or a plane. Anyways, we'll focus on the 2 dimensional data case.

The decision boundary determines whether or not a sample is in one class or the other. In the image above, the decision boundary determines whether a sample is blue, $P(blue|sample)$, or red $P(red|sample)$.

Like in linear classification, the decision boundary in logistic regression is a line, which as we saw earlier, can be represented by $X\theta$. When a sample is on the decision line $X\theta$, we say that

$P(blue|sample) = P(red|sample) = .5$. Normally, however, since the range of $X\theta$ is $[-\infty, \infty]$, we need to convert this to a probability and the way to do that is by using a sigmoid function

$$P(blue|sample) = \frac{1}{1+e^{-\theta^T x}}.$$

Logistic Regression Model

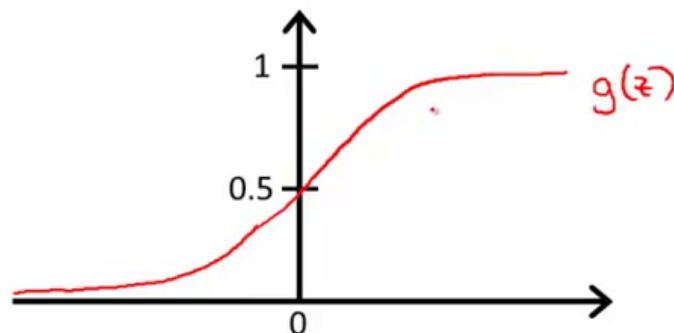
Want $0 \leq h_{\theta}(x) \leq 1$

$$h_{\theta}(x) = g(\theta^T x)$$

$$\rightarrow g(z) = \frac{1}{1 + e^{-z}}$$

$\theta^T x$

$$h_{\theta}(x) = \frac{1}{1 + e^{-\theta^T x}}$$



→ Sigmoid function
→ Logistic function

Andrew Ng

Let's rewrite this another way:

$$\begin{aligned} P(\text{blue}|\text{sample}) &= \frac{1}{1 + e^{-\theta^T x}} \\ 1 + e^{-\theta^T x} &= \frac{1}{P(\text{blue}|\text{sample})} \\ e^{-\theta^T x} &= \frac{1}{P(\text{blue}|\text{sample})} - 1 \\ e^{-\theta^T x} &= \frac{1}{\frac{P(\text{blue}|\text{sample})}{1 - P(\text{blue}|\text{sample})}} - \frac{P(\text{blue}|\text{sample})}{P(\text{blue}|\text{sample})} \\ e^{-\theta^T x} &= \frac{1 - P(\text{blue}|\text{sample})}{P(\text{blue}|\text{sample})} \\ \log(e^{-\theta^T x}) &= \log\left(\frac{1 - P(\text{blue}|\text{sample})}{P(\text{blue}|\text{sample})}\right) \\ -\theta^T x &= \log\left(\frac{1 - P(\text{blue}|\text{sample})}{P(\text{blue}|\text{sample})}\right) \\ \theta^T x &= \log\left(\frac{P(\text{blue}|\text{sample})}{1 - P(\text{blue}|\text{sample})}\right) \end{aligned}$$

In []: