Chapter 3. One-Dimensional Kinematics

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- 3.2 Displacement and Velocity
- 3.3 Instantaneous Velocity
- 3.4 Acceleration
- 3.5 The Use of Areas
- 3.6 The Equations of Kinematics for Constant Acceleration
- 3.7 Vertical Free-Fall
- 3.8 Terminal Speed

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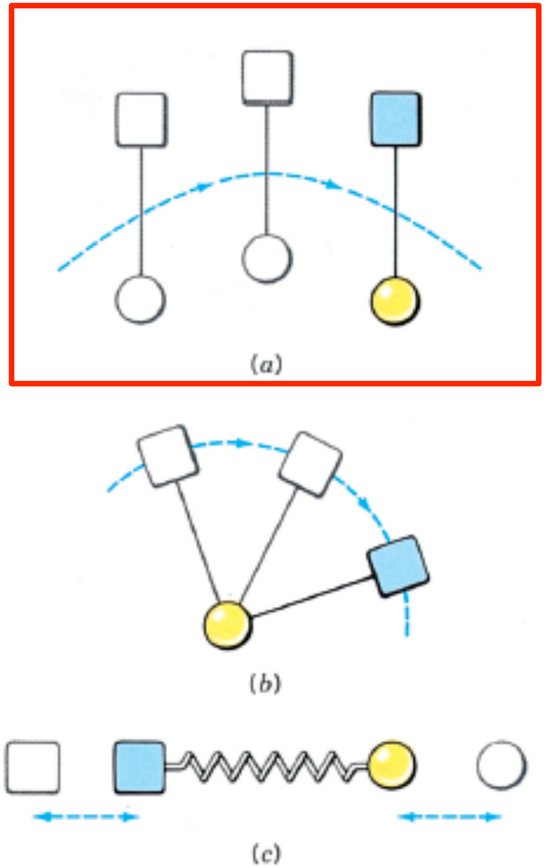
3.1 Particle Kinematics

- Kinematics (運動學、動力學) is concerned with the description of how a body moves in space and time.
  - **Translational motion:** all parts of a body undergo the same change in position
  - Rotational motion
  - Vibrational motion
  - Combination



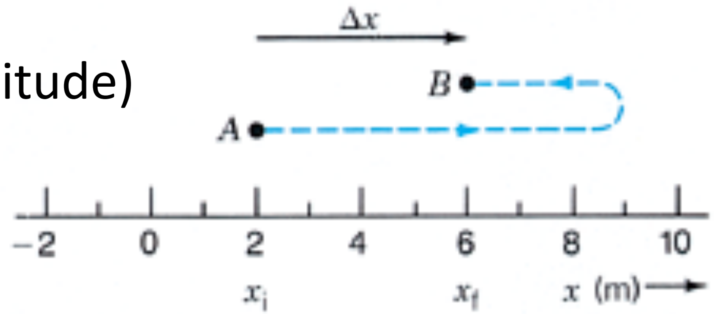
- The translational motion of an object can be completely described by the movement of any single point on it.

  ➔ Object may be treated as a particle.

3.2 Displacement and Velocity

Displacement (位移):

- **Vector**, defined as: $\Delta x = x_f - x_i$
- The sign of Δx indicates its direction (not magnitude)
- Displacement gives no details of the journey



Distance traveled (距離):

- **Scalar** (always positive), defined as *the length of the journey*

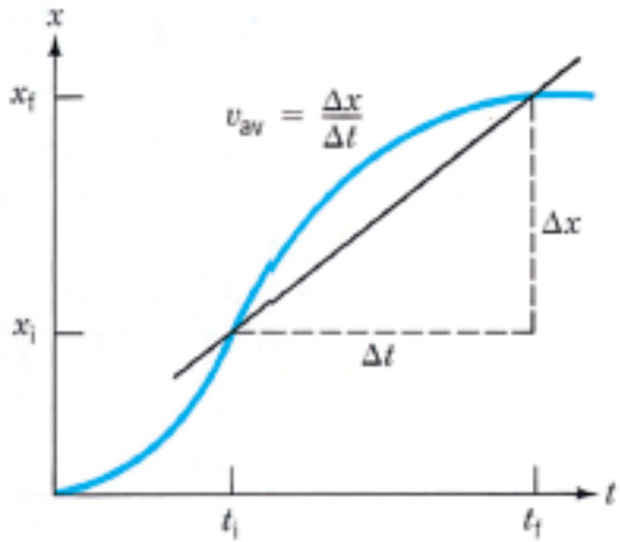
Average speed (平均速度):

- **Scalar**, defined as $\dfrac{\text{Distance traveled}}{\text{Time interval}}$

Average velocity (平均速率)

- **Vector**, defined as $\dfrac{\text{Displacement}}{\text{Time interval}}$

$$v_{av} = \frac{\Delta x}{\Delta t}$$



3.2 Displacement and Velocity

- Example 3.2

A jogger runs her first 100 m at 5 m/s and the second 100 m at 4 m/s in the same direction. What is her average velocity?

Solution:

Use $v = \frac{\Delta x}{\Delta t} \rightarrow \Delta t = \frac{\Delta x}{v}$

$$\Delta t_1 = 100/5 = 20s$$

$$\Delta t_2 = 100/4 = 25s$$

$$\begin{aligned} v_{tot.} &= \frac{\Delta x_{tot.}}{\Delta t_{tot}} \\ &= \frac{200\ m}{20 + 25\ s} \\ &= 4.44\ m/s \end{aligned}$$

*Note: it is not (5+4)/2 = 4.5 m/s

3.3 Instantaneous Velocity

- The velocity at any instant of time, or point in space, is called the **instantaneous velocity**

  "The instantaneous velocity at any instant is given by the slope of the tangent to the position-time graph at that time"

- Mathematical way to describe:

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

- In the notation of calculus:

$$v = \frac{dx}{dt}$$

  The instantaneous velocity is equal to the *derivative* of the position with respect to time.

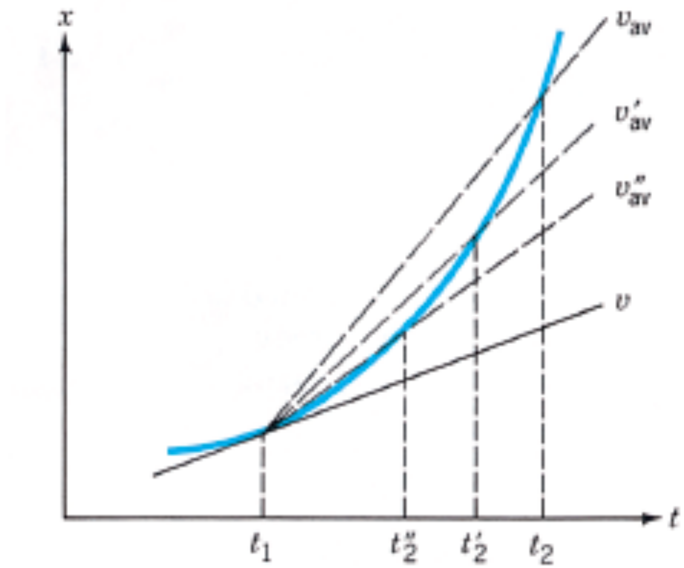


FIGURE 3.7 The instantaneous velocity v at time t_1 is the slope of the tangent to the curve at t_1.

3.3 Instantaneous Velocity

- Example 3.3

  The position of a particle is given by the equation x = 3t^2 m.

  Find the instantaneous velocity at 2 s by using (a) a limiting process, and (b) the derivative of the function.

  Solution:

  (a) by definition:

$$v = \frac{\Delta x}{\Delta t}$$

$$\Delta x = 3(2 + \Delta t)^2 - 3(2)^2$$

$$v = \frac{3(4 + 4\Delta t + \Delta t^2) - 12}{\Delta t}$$

$$= \frac{12\Delta t + 3\Delta t^2)}{\Delta t}$$

$$= 12 + 3\Delta t$$

v = 15 m/s, Δt = 1s
v = 13.5 m/s, Δt = 0.5 s
v = 12.003 m/s, Δt = 0.001 s

  (b) by definition:

$$v = \frac{dx}{dt}$$

$$= 3(2t) = 6t = 12\ m/s$$

$$where\ t = 2\ s$$

*Note: the smaller Δt, the better result

3.4 Acceleration

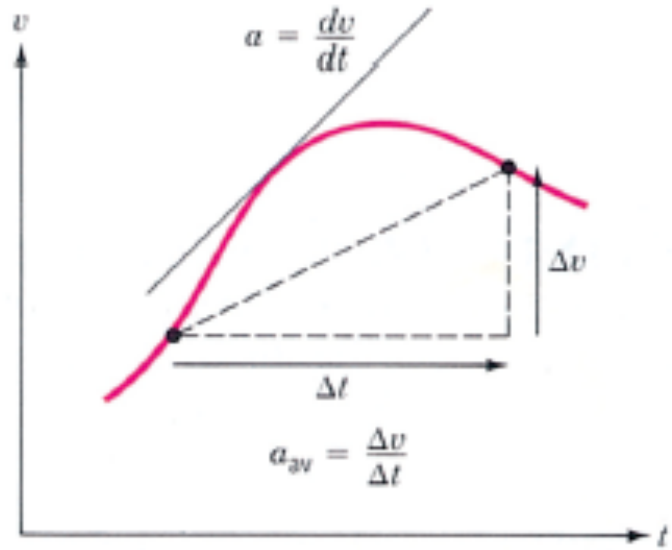
- The average acceleration for a finite time interval is defined as

$$\text{Average acceleration} = \frac{\text{Change in velocity}}{\text{Time interval}}$$

$$a_{av} = \frac{\Delta v}{\Delta t}$$

- We can defined the **instantaneous acceleration** by following the same logic as the instantaneous velocity

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$
$$= \frac{dv}{dt}$$



Accelerated motion in the vertical direction.

- Example 3.4

  At t = 0 a car is moving east at 10 m/s. Find its average acceleration between t = 0 and each of the following times at which it has the given velocities: (a) t = 2 s, 15 m/s east; (b) t = 5 s, 5 m/s east; (c) t = 10 s, 10 m/s west; (d) t = 20, 20 m/s west.

  Solution:

  Use $a_{av} = \dfrac{\Delta v}{\Delta t} = \dfrac{v_f - v_i}{t_f - t_i}$

  (a) a_{av} = (15 – 10) / 2 = 2.25 m/s^2

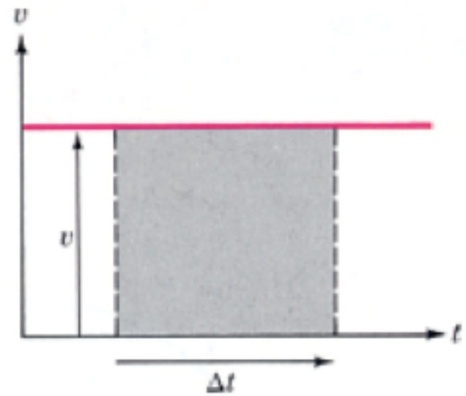
  (b) a_{av} = (5 – 10) / 5 = -1 m/s^2

  (c) and (d) : try it by yourself!

  Remember a_{av} is a **vector**

3.5 The Use of Areas

- Now, lets learn how to determine x from a graph of v versus t and v from a graph of a versus t.



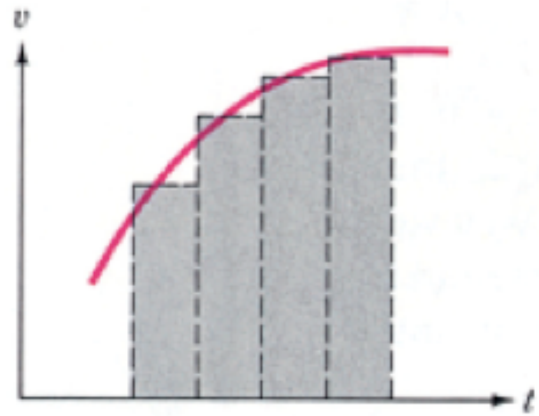
$$v = \frac{\Delta x}{\Delta t}$$

$$\Delta x = v\Delta t \quad (\text{unit is } (\text{m/s})(\text{s}) = \text{m})$$

➔ *The displacement Δx in some time interval is given by the area under the v versus t graph for that interval*

- The approximation improves as the number of rectangles is increased

  ➔ Calculus: Integration



- Similarly, $\Delta v = a\Delta t$, for a given time interval, the area under the a versus t graph gives the change in velocity Δv during that interval.

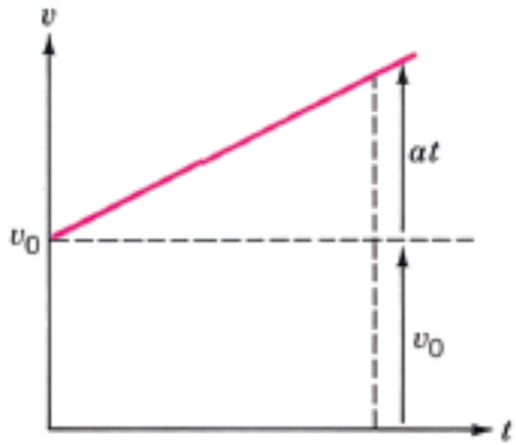
3.6 The Eqs. of Kin. for Const. Accel.

- Under constant acceleration condition, we can quantitatively write down the **equations of kinematics**

$$a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i} = \frac{v - v_0}{t}$$

$$(\text{where } v_f = v,\ v_i = v_0,\ t_f = t\ and\ t_i = 0)$$

$$\boxed{v = v_0 + at}$$

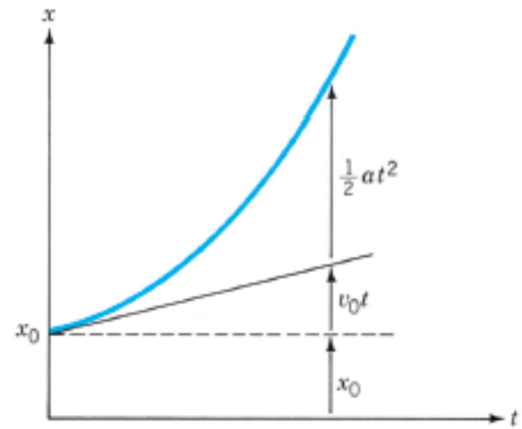


$$v_{av} = \frac{\Delta x}{\Delta t}$$

$$\Delta x = v_{av}\Delta t = \frac{1}{2}(v_i + v_f)\Delta t$$

$$x_f - x_i = \frac{1}{2}(v_i + v_f)(t_f - t_i)$$

$$\boxed{x = x_0 + \frac{1}{2}(v_0 + v)t}$$



$$(where\ x_f = x,\ x_i = x_0,\ v_i = v_0,\ v_f = v,\ t_f = t\ and\ t_i = 0)$$

$$\boxed{x = x_0 + v_0 t + \frac{1}{2}at^2}$$ exercise → $$\boxed{v^2 = v_0^2 + 2a(x - x_0)}$$

3.6 The Eqs. of Kin. for Const. Accel.

- Example 3.6

  A car accelerates with constant acceleration from rest to 30 m/s in 10 s. It then continues at constant velocity. Find: (a) its acceleration; (b) how far it travels while speeding up; (c) the distance it covers while its velocity changes from 10 m/s to 20 m/s.

  Solution:

  (a) use $v = v_0 + at$

  $$30 = 0 + a(10)$$

  $$a = \frac{30}{10}\ m/s^2$$

  $$a = 3\ m/s^2$$

  (b) use $x = x_0 + v_0 t + \frac{1}{2}at^2$

  $$= 0 + 0 + \frac{1}{2}3(10)^2$$

  $$= 150\ m$$

  (c) Find the total traveled time first,

  use $v = v_0 + at$

  $$20 = 10 + 3t$$

  $$t = 10/3\ s$$

  then, use $x = x_0 + v_0 t + \frac{1}{2}at^2$

  $$= 0 + 10(\frac{10}{3}) + \frac{1}{2}3(\frac{10}{3})^2$$

  $$= \frac{100}{3} + (\frac{1}{2})(\frac{100}{3})$$

  $$= 50\ m$$

3.6 The Eqs. of Kin. for Const. Accel.

- Example 3.7

A particle is at x = 5 m at t = 2 s and has a velocity v = 10 m/s. Its acceleration is constant at -4 m/s^2. Find the initial position at t = 0.

Solution:

The key equation: $x = x_0 + v_0 t + \frac{1}{2}at^2$

Find v_0 first by using $v = v_0 + at$

$$10 = v_0 + (-4)(2)$$

$$v_0 = 18$$

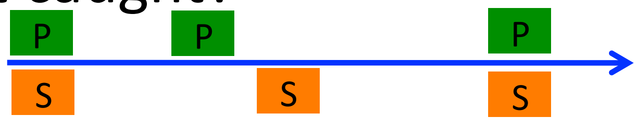
Then use $x = x_0 + v_0 t + \frac{1}{2}at^2$ to find x_0

$$5 = x_0 + 18(2) + \frac{1}{2}(-4)(2^2)$$

$$x_0 = 36 - 8 - 5 = 23\ m$$

- Example 3.8

A speeder moves at a constant 15 m/s in a school zone. A police car starts from rest just as the speeder passes it. The police car accelerates at 2 m/s^2 until it reaches its maximum velocity of 20 m/s. Where and when does the speeder get caught?



Solution:

Key equation: $x = x_0 + v_0 t + \frac{1}{2} a t^2$

First check how far Police traveled when he reaches its max velocity:

$$v_{pM} = v_{p0} + a_p t_{pM}$$
$$20 = 0 + 2t_{pM}$$
$$t_{pM} = 10\ s$$
$$x_{pM} = 0 + 0 + \frac{1}{2}(2)(10^2)$$
$$x_{pM} = 100\ m$$

During this time interval, speeder traveled: $x_s = v_{s0} t = 150\ m$

for police
$$x_p = x_{p0} + v_{p0} t_p + 0$$
$$x_p = -50 + 20 t_p$$

for speeder
$$x_s = x_{s0} + v_{s0} t_s + 0$$
$$x_s = 0 + 15 t_s$$

$$\because x_p = x_s \ and \ t_p = t_s$$
$$\therefore 15t = -50 + 20t$$
$$t = 10\ s$$

$$t_{tot.} = t_{pM} + t$$
$$= 20\ s$$
$$x_{tot.} = v_{s0} t_{tot.} = 15(20) = 300\ m$$
$$= x_{p0} + v_{p0} t_{pM} = 100 + 20(10) = 300\ m$$

3.6 The Eqs. of Kin. for Const. Accel.

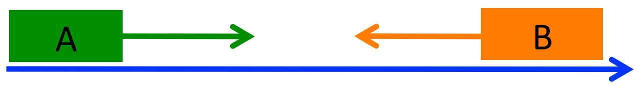
- Example 3.9

Two cars approach each other on a straight road. Car A moves at 16 m/s and car B moves at 8 m/s. When they are 45 m apart, both drivers apply their brakes. Car A slows down at 2 m/s^2, while car B slows down at 4 m/s^2. Where and when do they collide?

Solution:

Key equation: $x = x_0 + v_0 t + \frac{1}{2}at^2$

Remember velocity and acceleration are **vectors**!



$$x_A = x_{A0} + v_{A0}t_A + \frac{1}{2}a_A t_A^2$$

$$x_A = 0 + 16t_A + \frac{1}{2}(-2)t_A^2$$

$$x_A = 16t_A - t_A^2$$

$$x_B = x_{B0} + v_{B0}t_A + \frac{1}{2}a_B t_B^2$$

$$x_B = 45 + (-8)t_B + \frac{1}{2}(4)t_A^2$$

$$x_B = 45 - 8t_B + 2t_B^2$$

when they collide, $x_A = x_B$

$$16t - t^2 = 45 - 8t + 2t^2$$

$$3t^2 - 24t + 45 = 0$$

$$3(t^2 - 8t + 15) = 0$$

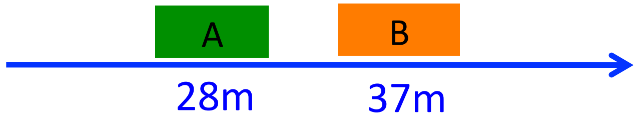
$$3(t-3)(t-5) = 0$$

$$t = 3 \ or \ 5 \ s$$

BUT... something strange here!!!

3.6 The Eqs. of Kin. for Const. Accel.

- Example 3.9



Solution (continue):

If you look at the final velocity of A and B:

$$v_A = v_{A0} + a_A t = 16 - 2(3) = 10\ m/s$$

$$v_B = v_{B0} + a_B t = -8 + 4(3) = 4\ m/s$$ direction is wrong!! This answer is unphysical!

Correct way, first check when one of them stops, does they collide?

$$v_A = v_{A0} + a_A t \Rightarrow 0 = 16 - 2(t_A) \Rightarrow t_A = 8\ s$$

$$v_B = v_{B0} + a_B t \Rightarrow 0 = -8 + 4(t_B) \Rightarrow t_B = 2\ s$$

At t = 2 s, $x_A = x_{A0} + v_{A0}t + \frac{1}{2}a_A t^2$

$$= 0 + 16(2) - \frac{1}{2}(2)(2^2)$$

$$= 28\ m$$

$$x_B = x_{B0} + v_{B0}t + \frac{1}{2}a_B t^2$$

$$= 45 - 8(2) + \frac{1}{2}(4)(2^2)$$

$= 37\ m$ now, B stops

Since B stops,
they will collide at 37m:

$$x_A = x_{A0} + v_{A0}t + \frac{1}{2}a_A t^2$$

$$37 = 0 + 16t - \frac{1}{2}(2)t^2$$

$$t^2 - 16t + 37 = 0$$

$$t = \frac{16 \pm \sqrt{16^2 - 4(37)}}{2}$$

$$= \frac{16 \pm 10.39}{2}$$

$$= \underline{2.8}\ or\ 13.2\ s$$

3.7 Vertical Free-Fall

- Motion that occurs solely under the influence of gravity is called **free-fall (自由落體)**

  From Galileo: In the absence of air resistance, all falling bodies have the same acceleration due to gravity, regardless of their sizes or shapes.

- The acceleration of gravity: **a** = - g**j**, g ≈ 9.8 m/s^2

  ➔ The equations of kinematics becomes:

$$v = v_0 - gt$$

$$y = y_0 + \frac{1}{2}(v_0 + v)t$$

$$y = y_0 + v_0 t - \frac{1}{2} g t^2$$

$$v^2 = v_0^2 - 2g(y - y_0)$$

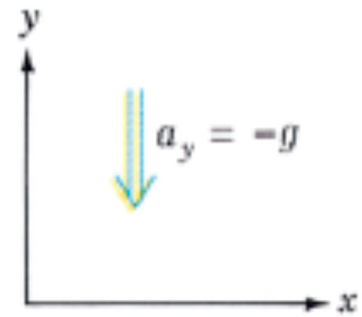


FIGURE 3.24 If the y axis is chosen to point upward, the acceleration of a particle in free-fall is $a_y = -g$, where $g = 9.8\ \mathrm{m/s^2}$ is the magnitude of the acceleration due to gravity.

- Example 3.10

A ball thrown up from the ground reaches a maximum height of 20m. Find: (a) its initial velocity; (b) the time taken to reach the highest point; (c) its velocity just before hitting the ground; (d) its displacement between 0.5 and 2.5 ; (e) the time at which it is 15 m above the ground.

Solution:

(a) When it reaches its maximum height

➔ v = 0 m/s

Use $v^2 = v_0^2 - 2g(y - y_0)$

$$0 = v_0^2 - 2g(20 - 0)$$

$$v_0^2 = 40(9.8)$$

$$v_0 = 19.8\ m/s$$

(b) Use $v = v_0 - gt$

$$0 = 19.8 - 9.8t$$

$$t = 2.02\ s$$

(c) Use $v^2 = v_0^2 - 2g(y - y_0)$

$$v^2 = 0 - 2(9.8)(0 - 20)$$

$$v = -19.8\ m/s$$

Note: v_0 = v for free-fall

3.7 Vertical Free-Fall

- Example 3.10

A ball thrown up from the ground reaches a maximum height of 20m. Find: (a) its initial velocity; (b) the time taken to reach the highest point; (c) its velocity just before hitting the ground; (d) its displacement between 0.5 and 2.5 s; (e) the time at which it is 15 m above the ground.

Solution:

(d) Key equation: $y = y_0 + v_0 t - \frac{1}{2}gt^2$

$$\Delta y = y_f - y_i$$

$$y_f = 19.8(2.5) - \frac{1}{2}(9.8)(2.5)^2$$

$$= 18.87\ m$$

$$y_i = 19.8(0.5) - \frac{1}{2}(9.8)(0.5)^2$$

$$= 8.675\ m$$

$$\Delta y = 18.87 - 8.675 = 10.195\ m$$

(e) Use $y = y_0 + v_0 t - \frac{1}{2}gt^2$

$$15 = 0 + 19.8t - 4.9t^2$$

$$t = \frac{19.8 \pm \sqrt{19.8^2 - 4(4.9)(15)}}{2(4.9)}$$

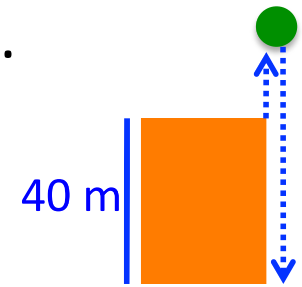
$$t = 1.01\ or\ 3.03\ s$$

on the way up and down

3.7 Vertical Free-Fall

- Example 3.11

A ball is thrown upward with an initial velocity of 12 m/s from a rooftop 40 m high. Find: (a) its velocity on hitting the ground; (b) the time of flight; (c) the maximum height; (d) the time to return to roof level; (e) the time it is 15 m below the rooftop.



Solution:

(a, b, c) Find the time when it reaches the maximum height ➔ v = 0

$$v = v_0 - gt_{up}$$

$$0 = 12 - 9.8t_{up}$$

$$t_{up} = 1.22\ s$$

$$y = y_0 + v_0 t - \frac{1}{2}gt^2$$

$$y = 40 + 12(1.22) - \frac{1}{2}(9.8)(1.22^2)$$

$$\underline{y = 47.35\ m}$$

$$y = y_0 + v_0 t - \frac{1}{2}gt^2$$

$$0 = 47.35 + 0 - 4.9t_{down}^2$$

$$t_{down} = 3.11\ s$$

$$t_{tot.} = t_{up} + t_{down} = \underline{4.33\ s}$$

$$v = v_0 - gt_{down}$$

$$v = 0 - 9.8(3.11) = \underline{-30.47\ m/s}$$

- Example 3.11

(d) the time to return to roof level; (e) the time it is 15 m below the rooftop.

Solution:

(d) We already know the time to reach the maximum height, now we only need to know the time from there to rooftop

$$y = y_0 + v_0 t_{downR} - \frac{1}{2} g t_{downR}^2$$

$$40 = 47.35 + 0 - \frac{1}{2}(9.8) t_{downR}^2$$

$$t_{downR} = 1.22\ s$$

$$t_{tot.} = t_{up} + t_{downR} = 2.44\ s$$

OR

$$y = y_0 + v_0 t - \frac{1}{2} g t^2$$

$$40 = 40 + 12t - \frac{1}{2}(9.8) t^2$$

$$t(4.9t - 12) = 0$$

$$t = 0\ or\ \underline{2.45\ s}$$

(e) 15 m below means 25 m above ground:

$$y = y_0 + v_0 t - \frac{1}{2} g t^2$$

$$25 = 40 + 12t - \frac{1}{2}(9.8) t^2$$

$$4.9t^2 - 12t - 15 = 0$$

$$t = \frac{12 \pm \sqrt{12^2 + 4(4.9)(15)}}{2(4.9)}$$

$$t = -0.91\ or\ \underline{3.36\ s}$$

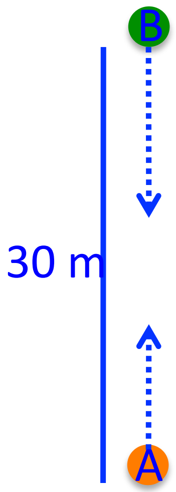
Note: learn how to choose the physical answer!

3.7 Vertical Free-Fall

- Example 3.12

Two balls are thrown toward each other: ball A at 16.0 m/s upward from the ground, ball B at 9.00 m/s downward from a roof 30.0 m high, one second later. (a) Where and when do they meet? (b) What are their velocities on impact?

Solution:



(a) $$y_A = y_{A0} + v_{A0}t_A - \frac{1}{2}gt_A^2$$
$$= 0 + 16t_A - 4.9t_A^2$$
$$= 16t_A - 4.9t_A^2$$

$$y_B = y_{B0} + v_{B0}t_B - \frac{1}{2}gt_B^2$$
$$= 30 - 9t_B - 4.9t_B^2$$

and $t_B = t_A - 1$,

when they meet, $y_A = y_B$

$$16t - 4.9t^2 = 30 - 9(t-1) - 4.9(t-1)^2$$
$$16t - 4.9t^2 = 30 - 9t + 9 - 4.9(t^2 - 2t + 1)$$
$$15.2t = 34.1$$
$$t = \underline{2.24\ s}$$
$$y_A = y_B$$
$$= 16t_A - 4.9t_A^2$$
$$= 16(2.24) - 4.9(2.24)^2$$
$$= \underline{11.25\ m}$$

3.7 Vertical Free-Fall

- Example 3.12

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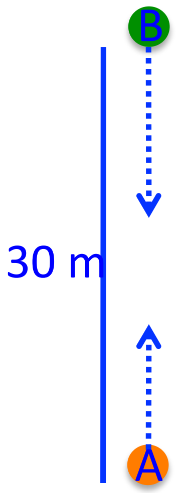
Solution:



(b) $t = 2.24\ s$

$$\begin{aligned} v_A &= v_{A0} - gt \\ &= 16 - 9.8(2.24) \\ &= \underline{-5.95\ m/s} \end{aligned}$$

$$\begin{aligned} v_B &= v_{B0} - gt_B \\ &= v_{B0} - g(t-1) \\ &= -9 - 9.8(1.24) \\ &= \underline{-21.15\ m/s} \end{aligned}$$

3.8 Terminal Speed

- An object dropped from a great enough height does not accelerate indefinitely. The object ultimately reaches a terminal speed, v_T, and then continues to fall at this constant rate.
- The value of v_T depends on the weight and shape of the falling object and the density of the air.