

## Mutable Functions

## Functions with behavior that changes over time

```
def square(x):
    return x * x

def f(x):
    ...
```

>>> square(5)  
25  
>>> square(5)  
25  
>>> square(5)  
25

Returns the same value when called with the same input

Return value is different when called with the same input

>>> f(5)  
25  
>>> f(5)  
26  
>>> f(5)  
27

## Example - Withdraw

Let's model a bank account that has a balance of \$100

```
>>> withdraw(25)
75
>>> withdraw(25)
50
>>> withdraw(60)
'Insufficient funds'
>>> withdraw(15)
35
>>> withdraw = make_withdraw(100)
```

Return value: remaining balance

Different return value!

Argument: amount to withdraw

Second withdrawal of the same amount

Where's this balance stored?

Within the parent frame of the function!

A function has a body and a parent environment

## Persistent Local State Using Environments

Global frame

make\_withdraw

withdraw

f1: make\_withdraw [parent=Global]

f2: withdraw [parent=f1]

f3: withdraw [parent=f1]

amount

balance

withdraw

Return value

The parent frame contains the balance, the local state of the withdraw function

Every call decreases the same balance by (a possibly different) amount

All calls to the same function have the same parent

## Reminder: Local Assignment

```
def percent_difference(x, y):
    difference = abs(x - y)
    return 100 * difference / x
diff = percent_difference(40, 50)
```

Assignment binds name(s) to value(s) in the first frame of the current environment

Global frame

percent\_difference

f1: percent\_difference [parent=Global]

x

y

difference

Execution rule for assignment statements:

1. Evaluate all expressions right of =, from left to right
2. Bind the names on the left to the resulting values in the **current frame**

## Non-Local Assignment & Persistent Local State

```
def make_withdraw(balance):
    """Return a withdraw function with a starting balance."""
    def withdraw(amount):
        nonlocal balance
        if amount > balance:
            return 'Insufficient funds'
        balance = balance - amount
        return balance
    return withdraw
```

Declare the name "balance" nonlocal at the top of the body of the function in which it is re-assigned

Re-bind balance in the first non-local frame in which it was bound previously

(Demo)

## Non-Local Assignment

## The Effect of Nonlocal Statements

nonlocal <name>, <name>, ...

**Effect:** Future assignments to that name change its pre-existing binding in the **first non-local frame** of the current environment in which that name is bound.

Python Docs: an "enclosing scope"

From the Python 3 language reference:

Names listed in a nonlocal statement must refer to pre-existing bindings in an enclosing scope.

Names listed in a nonlocal statement must not collide with pre-existing bindings in the **local scope**.

[http://docs.python.org/release/3.1.3/reference/simple\\_stmts.html#the-nonlocal-statement](http://docs.python.org/release/3.1.3/reference/simple_stmts.html#the-nonlocal-statement)  
<http://www.python.org/dev/peps/pep-3104/>

## The Many Meanings of Assignment Statements

Status	Effect
<ul style="list-style-type: none"> <li>• No nonlocal statement</li> <li>• "x" is <b>not</b> bound locally</li> </ul>	Create a new binding from name "x" to object 2 in the first frame of the current environment
<ul style="list-style-type: none"> <li>• No nonlocal statement</li> <li>• "x" is bound locally</li> </ul>	Re-bind name "x" to object 2 in the first frame of the current environment
<ul style="list-style-type: none"> <li>• nonlocal x</li> <li>• "x" is bound in a non-local frame</li> </ul>	Re-bind "x" to 2 in the first non-local frame of the current environment in which "x" is bound
<ul style="list-style-type: none"> <li>• nonlocal x</li> <li>• "x" is <b>not</b> bound in a non-local frame</li> </ul>	SyntaxError: no binding for nonlocal 'x' found
<ul style="list-style-type: none"> <li>• nonlocal x</li> <li>• "x" is bound in a non-local frame</li> <li>• "x" also bound locally</li> </ul>	SyntaxError: name 'x' is parameter and nonlocal

## Python Particulars

Python pre-computes which frame contains each name before executing the body of a function.

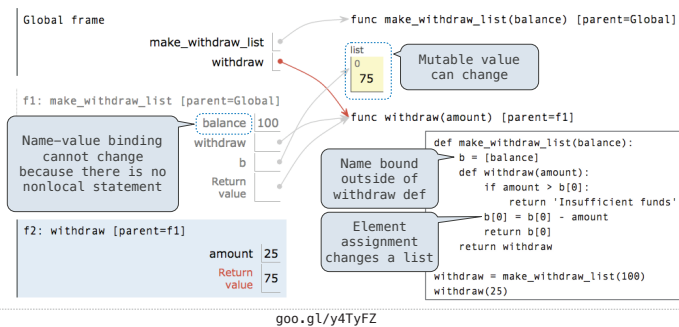
Within the body of a function, all instances of a name must refer to the same frame.

```
def make_withdraw(balance):  
    def withdraw(amount):  
        if amount > balance:  
            return 'Insufficient funds'  
        balance = balance - amount  
        return balance  
    return withdraw  
  
wd = make_withdraw(20)  
wd(5)
```

UnboundLocalError: local variable 'balance' referenced before assignment

## Mutable Values & Persistent Local State

Mutable values can be changed *without* a nonlocal statement.

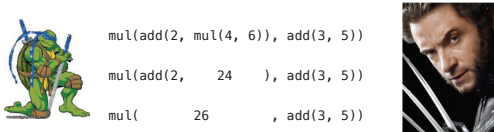


## Multiple Mutable Functions

( Demo )

## Referential Transparency, Lost

- Expressions are **referentially transparent** if substituting an expression with its value does not change the meaning of a program.



- Mutation operations violate the condition of referential transparency because they do more than just return a value; **they change the environment.**

## Referential Transparency in Environment Diagrams

```
def f(x):
    x = 4
    def g(y):
        def h(z):
            nonlocal x
            x = x + 1
            return x + y + z
        return h
    return g
```

Go Bears!

Diagram illustrating the execution of a lambda expression in a multi-frame environment.

**Code:**

```
def oski(berk):
    def cal(berk):
        nonlocal berk
        if berk(berk) == 0:
            return [berk+1, berk-1]
        berk = lambda ley: berk-ley
        return [berk, cal(berk)]
    return cal(2)

oski(abs)
```

**Execution Flow:**

- Global frame oski:** Contains `func oski(berk) [parent=G]`.
- f1: oski [parent=G]:** Contains `berk` (value 2), `cal`, and `Return Value`. It calls `func cal(berk) [parent=f1]`.
- f2: cal [parent=f1]:** Contains `berk` (value 2), `Return Value`, and a `list` containing 0 and 1. It calls `func abs(...) [parent=G]`.
- f3: cal [parent=f1]:** Contains `berk` (value 2), `Return Value`, and a `list` containing 0, 3, and 1. It calls `func oski(berk) [parent=G]`.
- f4: λ [parent=f2]:** Contains `ley` (value 2), `Return Value`, and a `list` containing 0 and 1. It calls `func λ(ley) [parent=f2]`.

The final result is the list `[2, 3, 1]`.

## Summary

- Nonlocal allows for functions whose behavior changes over time
- When declaring a variable nonlocal, we move part of the function's local state to its parent
- There are various rules for which variables may be declared nonlocal
- Nonlocal gives us a new type of assignment, where we change the binding in a parent instead
- Next time, we'll see more examples of functions which change state outside their local frame!

