# Module 12:
## Model Building I

# General Linear Model

A standard GLM can be written:

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon} \qquad \boldsymbol{\varepsilon} \sim N(\mathbf{0}, \mathbf{V})$$

where

$$\begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{bmatrix} = \begin{bmatrix} 1 & X_{11} & \cdots & X_{1p} \\ 1 & X_{21} & \cdots & X_{2p} \\ \vdots & \vdots & & \vdots \\ 1 & X_{np} & \cdots & X_{np} \end{bmatrix} \times \begin{bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_p \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{bmatrix}$$

fMRI Data

Design matrix

Model parameters

Noise

V is the covariance matrix whose format depends on the noise model.

The quality of the model depends on our choice of X and V.

# Model Building

- Proper construction of the design matrix is critical for effective use of the GLM.

- This process can be complicated by the following properties of the BOLD response:
  - It includes low-frequency noise and artifacts related to head movement and cardiopulmonary-induced brain movement.
  - The neural response shape may not be known.
  - The hemodynamic response varies in shape across the brain.

# BOLD Response

- Predict the shape of the BOLD response to a given stimulus pattern. Assume the shape is known and the amplitude is unknown.

- The relationship between stimuli and the BOLD response is typically modeled using a linear time invariant (LTI) system.

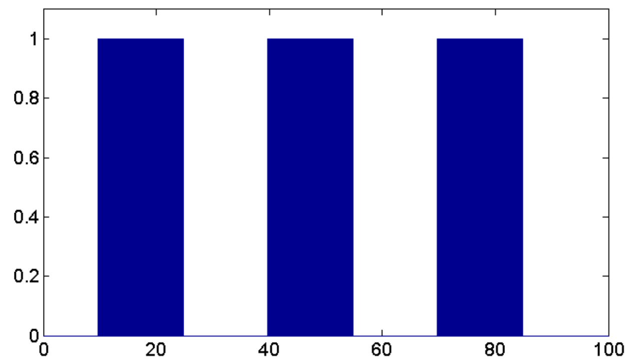
- In an LTI system an impulse (i.e., neuronal activity) is convolved with an impulse response function (i.e., HRF).

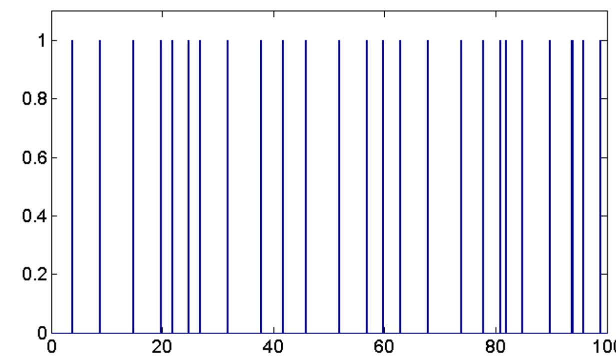
# Convolution Examples

Block Design

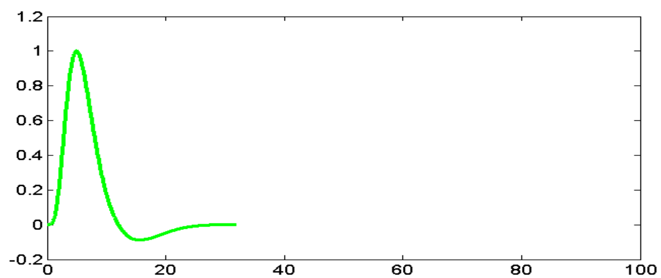
Event-Related

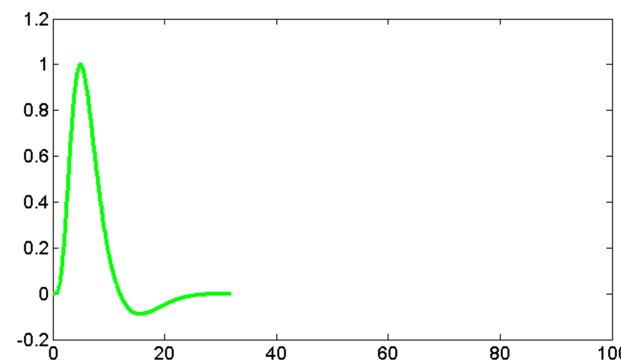
Experimental Stimulus Function



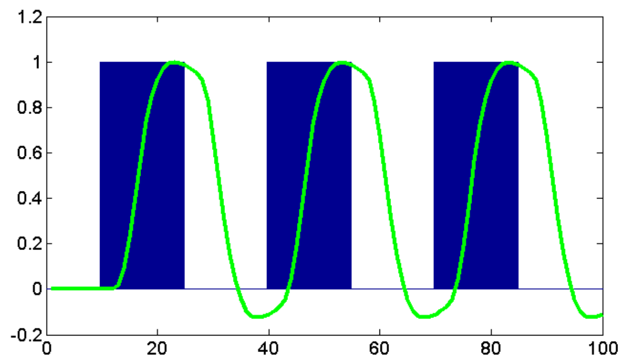



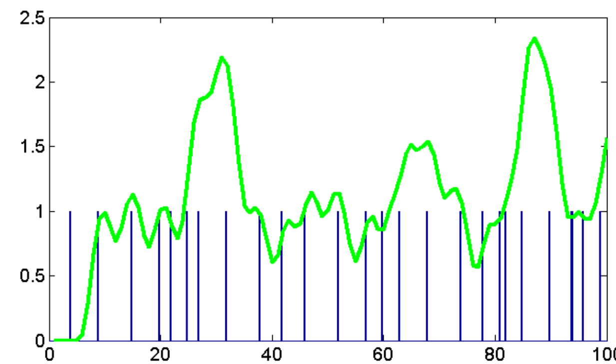

Hemodynamic Response Function



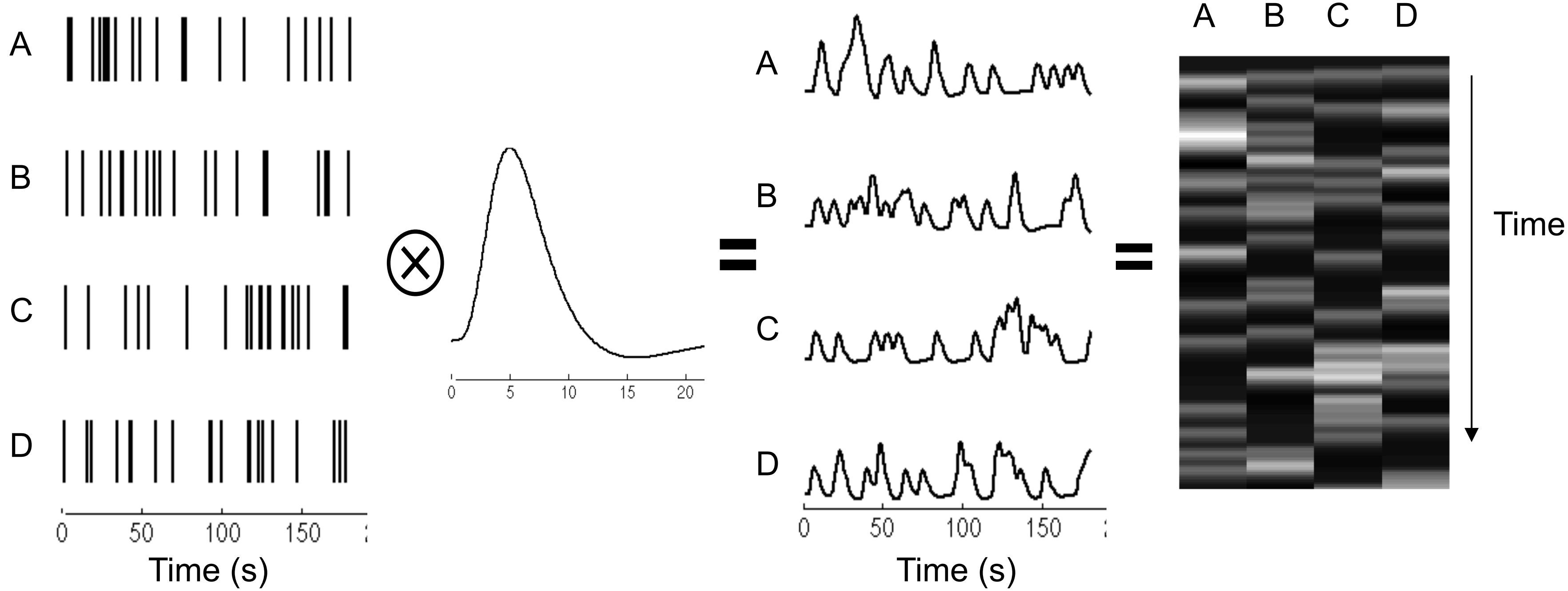

Predicted Response



# Multiple Conditions

Indicator functions (Onsets) | Assumed HRF (Basis function) | Design Matrix ($X^T$) | Design Matrix ($X$)




## Assumptions:

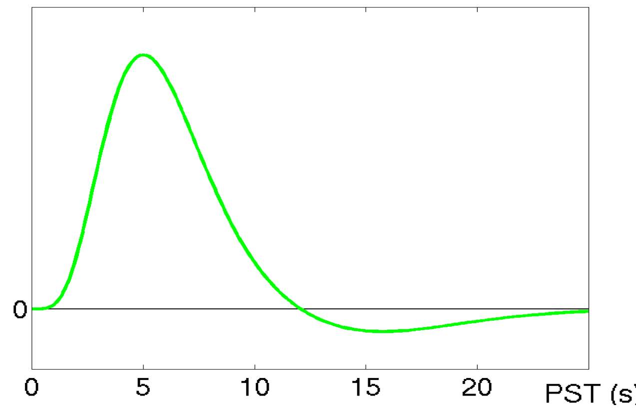
Assume neural activity function is correct

Assume HRF is correct

Assume LTI system

# HRF Models

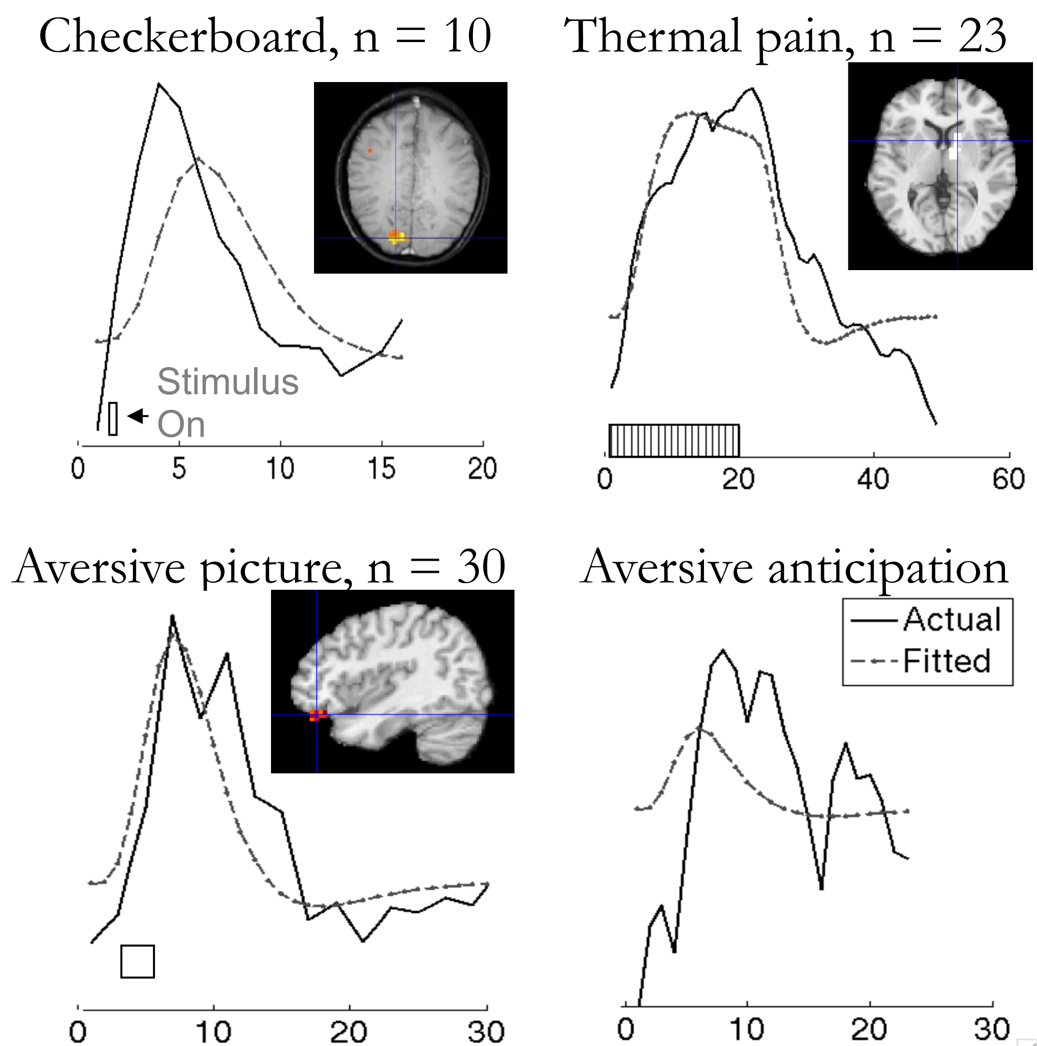
- Often a fixed canonical HRF is used to model the response to neuronal activity
  - Linear combination of 2 gamma functions.
  - Optimal if correct.
  - If wrong, leads to bias and power loss.
    - Unlikely that the same HRF is valid for all voxels.
    - True response may be faster/slower
    - True response may have smaller/bigger undershoot



# Problems

The HRF shape depends both on the vasculature and the time course of neural activity.

Assuming a fixed HRF is usually not appropriate.



# Temporal Basis Functions

- To allow for different types of HRFs in different brain regions, it is typically better to use temporal basis functions.
- A linear combination of functions can be used to account for delays and dispersions in the HRF.
  - The stimulus function is convolved with each of the basis functions to give a set of regressors.
  - The parameter estimates give the coefficients that determine the combination of basis functions that best models the HRF for the trial type and voxel in question.

# Temporal Basis Functions

- In an LTI system the BOLD response is modeled

$$x(t) = (s * h)(t)$$

  where $s(t)$ is a stimulus function and $h(t)$ the HRF.

- Model the HRF as a linear combination of temporal basis functions, $f_i(t)$, such that

$$h(t) = \sum \beta_i f_i(t)$$

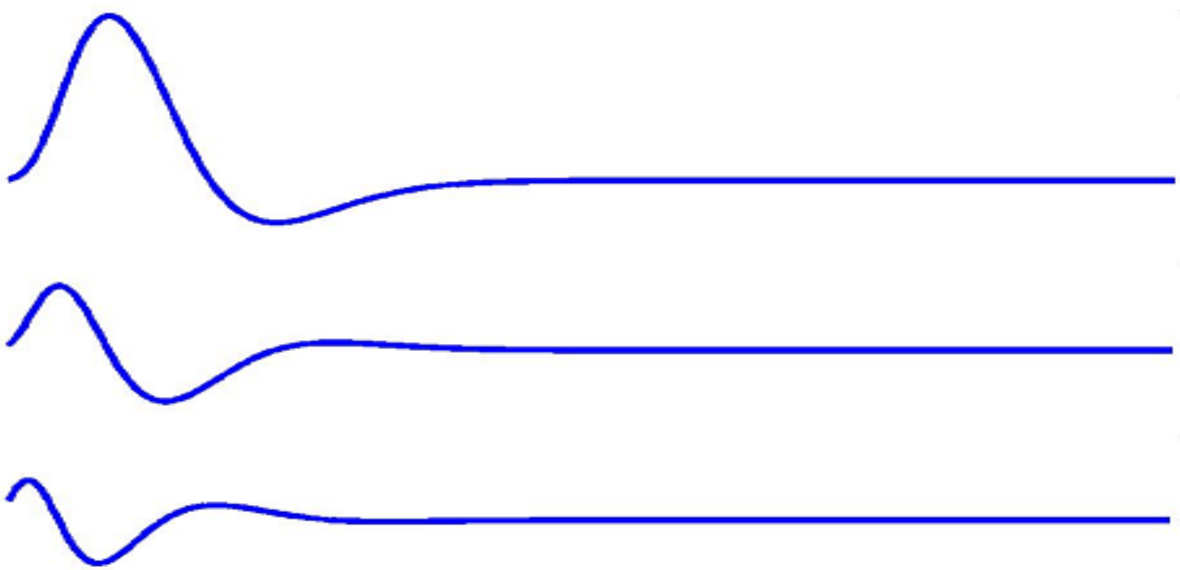
# Temporal Basis Functions

$$h(t) = \sum \beta_i f_i(t)$$

$$h(t) = \beta_1$$

$$+\beta_2$$

$$+\beta_3$$



# Temporal Basis Functions

- The BOLD response can be rewritten:

$$x(t) = \sum \beta_i (s * f_i)(t)$$

- In the GLM framework the convolution of the stimulus function with each basis function makes up a separate column of the design matrix.

- Each corresponding $\beta_i$ describes the weight of that component.

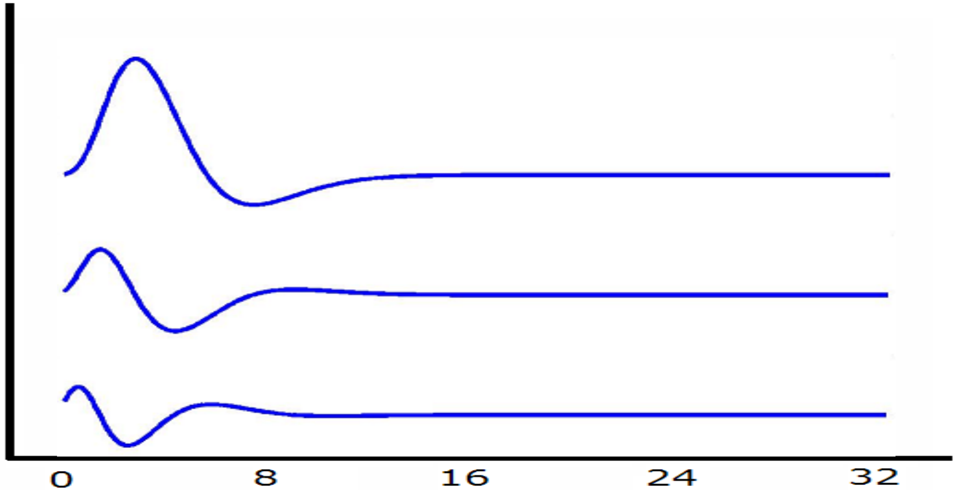
# Temporal Basis Functions

- Typically-used models vary in the degree they make *a priori* assumptions about the shape of the response.

- In the most extreme case, the shape of the HRF is fixed and only the amplitude is allowed to vary.

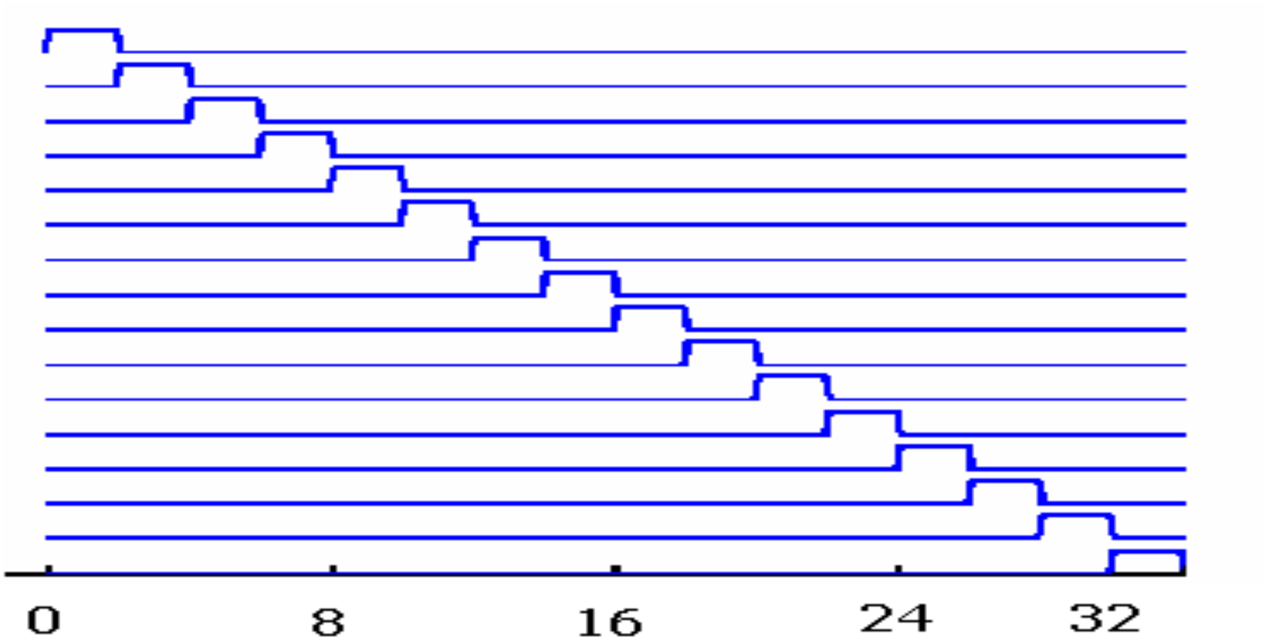
- By contrast, a finite impulse response (FIR) basis set, contains one free parameter for every time-point following stimulation for every cognitive event type.

# Canonical HRF + Derivatives




Including the derivatives allows for a shift in delay and dispersion.

# Finite Impulse Response




The model estimates an HRF of arbitrary shape for each event type in each voxel of the brain

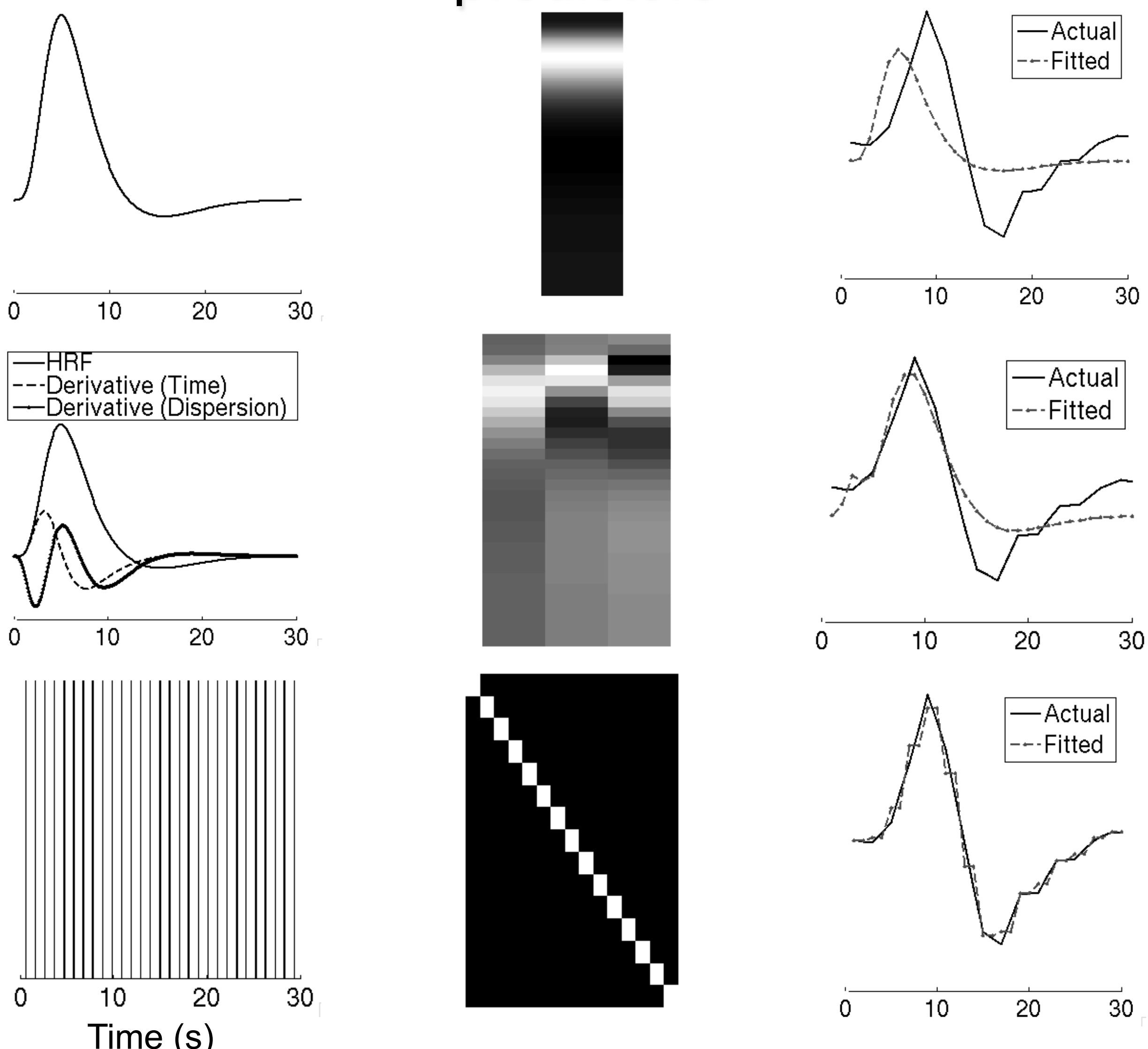
# Basis sets

Model | Image of predictors | Data & Fitted

Single HRF

HRF + derivatives

Finite Impulse Response (FIR)



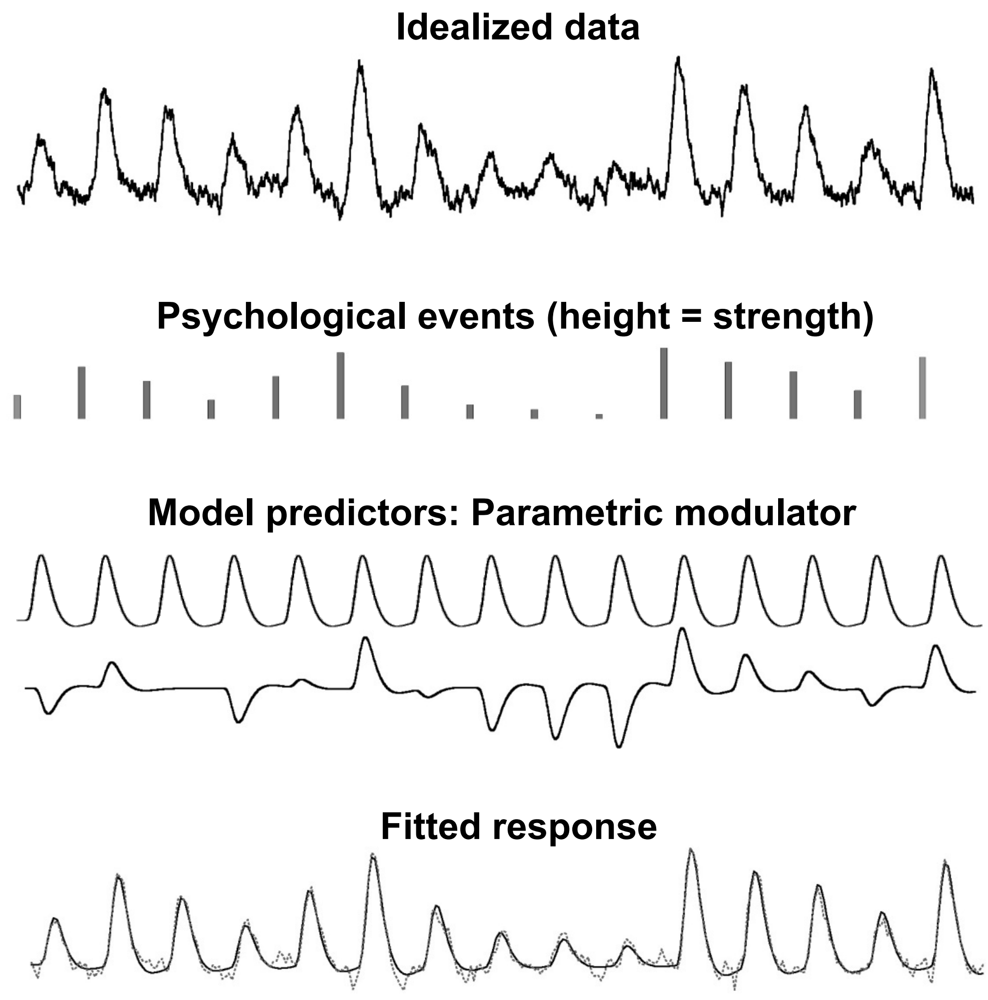
# Parametric Modulation

- Often a stimulus can be parametrically varied across repetitions, and it is thought that this may be reflected in the strength of the neuronal response.

- In these types of situations the parametric modulation can be modeled by including an additional regressor in the design matrix that accounts for the possible variation in neural response.

# Parametric Modulators

Parametric modulators are often used to model trial-to-trial variation in a psychological process.



# End of Module