Module 24:
Multivariate Decomposition Methods

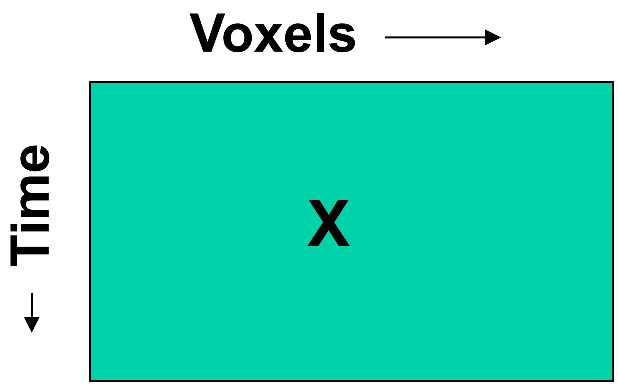
Decomposition Methods

- We often use multivariate decomposition methods to study functional connectivity.
  - Provides a decomposition of the data into separate components.
  - Can be used to find coherent brain networks.
  - Provides information on how different brain regions interact with one another.

- The most common decomposition methods are principal components analysis and independent components analysis.

Data Structure

- Throughout we organize the fMRI data in an $M \times N$ matrix **X**.
  - The row dimension is the number of time points and the column dimension the number of voxels.



Principal Components Analysis

- Principal Components Analysis (PCA) is a multivariate procedure concerned with explaining the variance-covariance structure of a high dimensional random vector.

- In PCA, a set of correlated variables are transformed into a set of uncorrelated variables, ordered by the amount of variability in the data that they explain.

Principal Components Analysis

- In fMRI principal components analysis involves finding spatial modes, or eigenimages, in the data.
  - These are the patterns that account for most of the variance-covariance structure in the data.
  - They are ranked in order of the amount of variation they explain.

- The eigenimages can be obtained using singular value decomposition (SVD), which decomposes the data into two sets of orthogonal vectors that correspond to patterns in space and time.

Singular Value Decomposition

- Singular value decomposition (SVD) is an operation that decomposes a matrix $\mathbf{X}$ as:

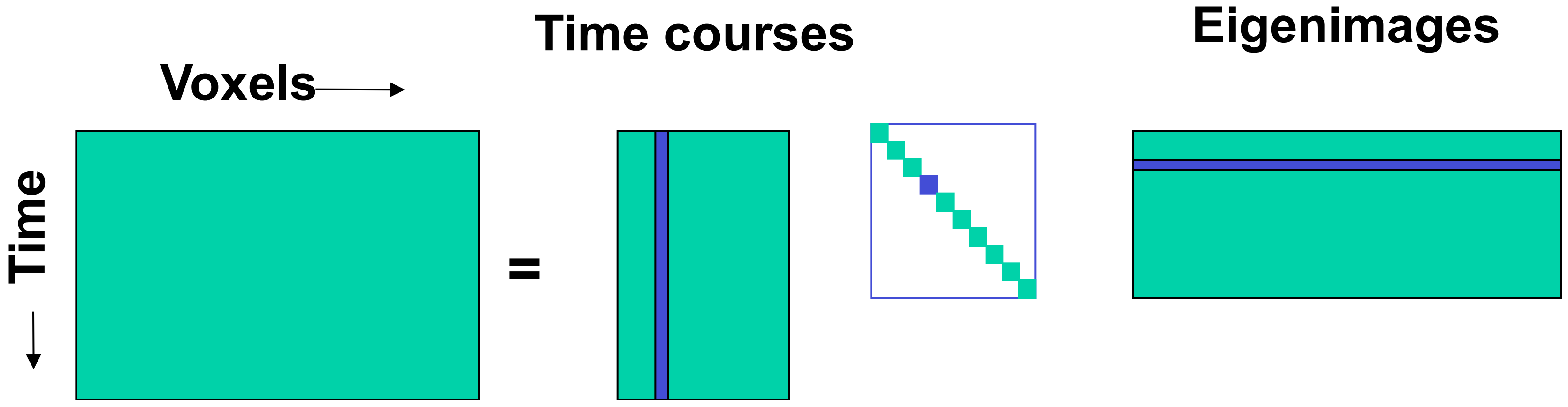
$$\mathbf{X} = \mathbf{U}\mathbf{S}\mathbf{V}^T$$

where

$$\mathbf{V}^T\mathbf{V} = \mathbf{I}$$

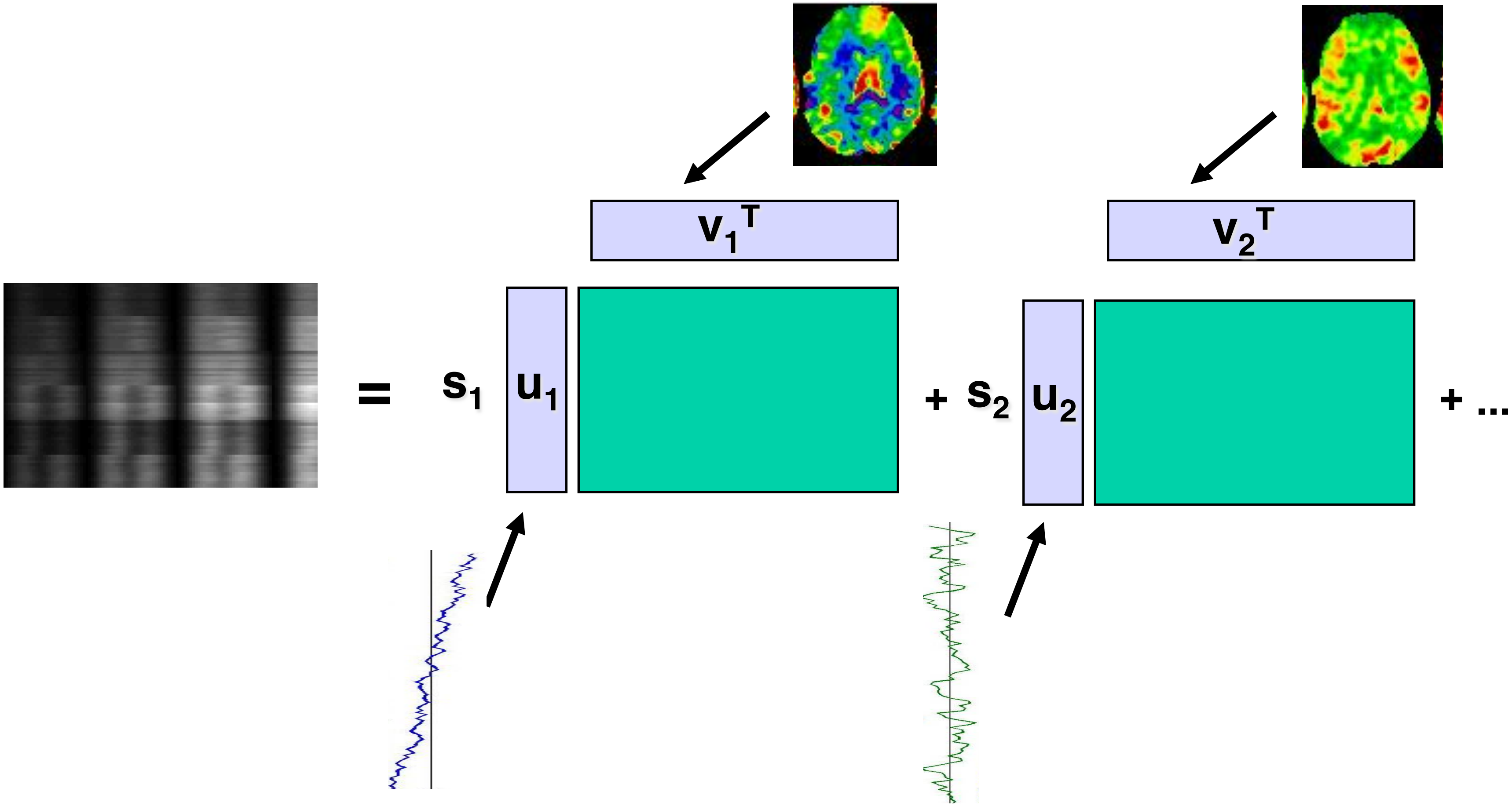
$$\mathbf{U}^T\mathbf{U} = \mathbf{I}$$

and $\mathbf{S}$ is a diagonal matrix whose elements are called singular values.

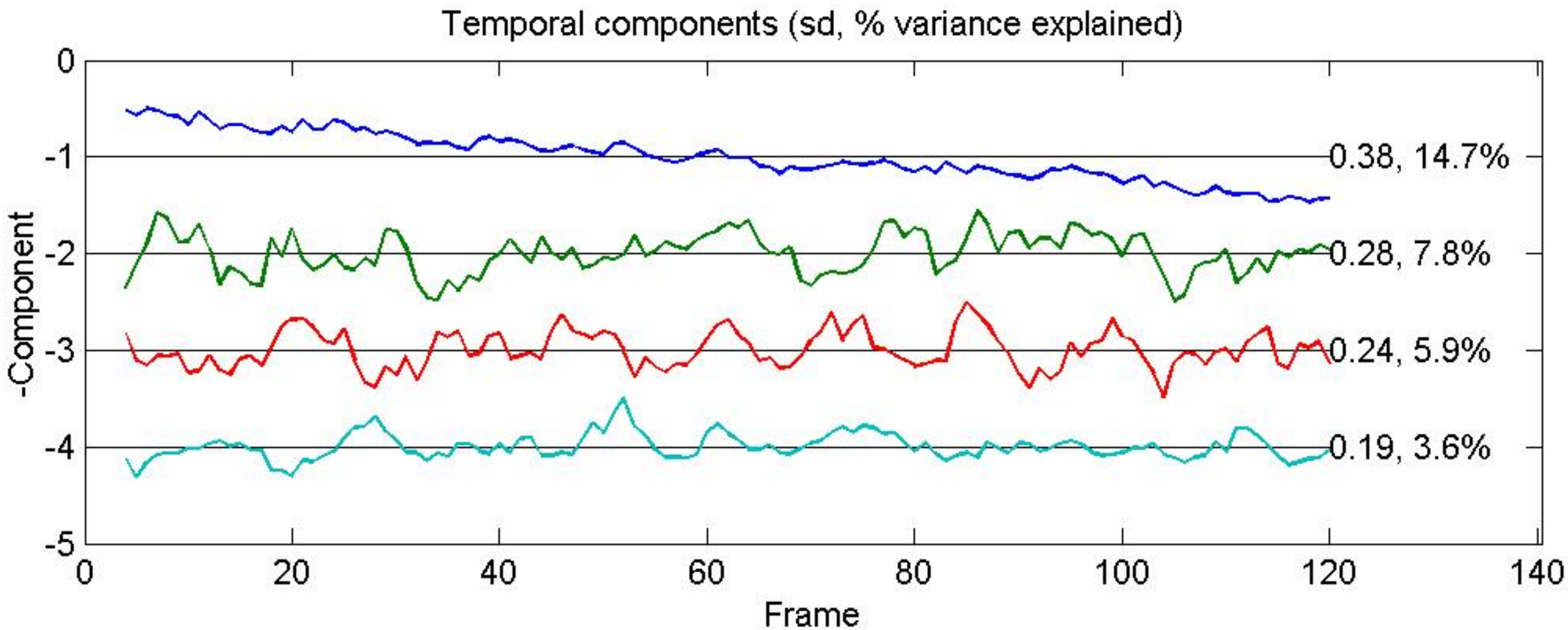


$$\mathbf{X} = \mathbf{USV}^{T}$$

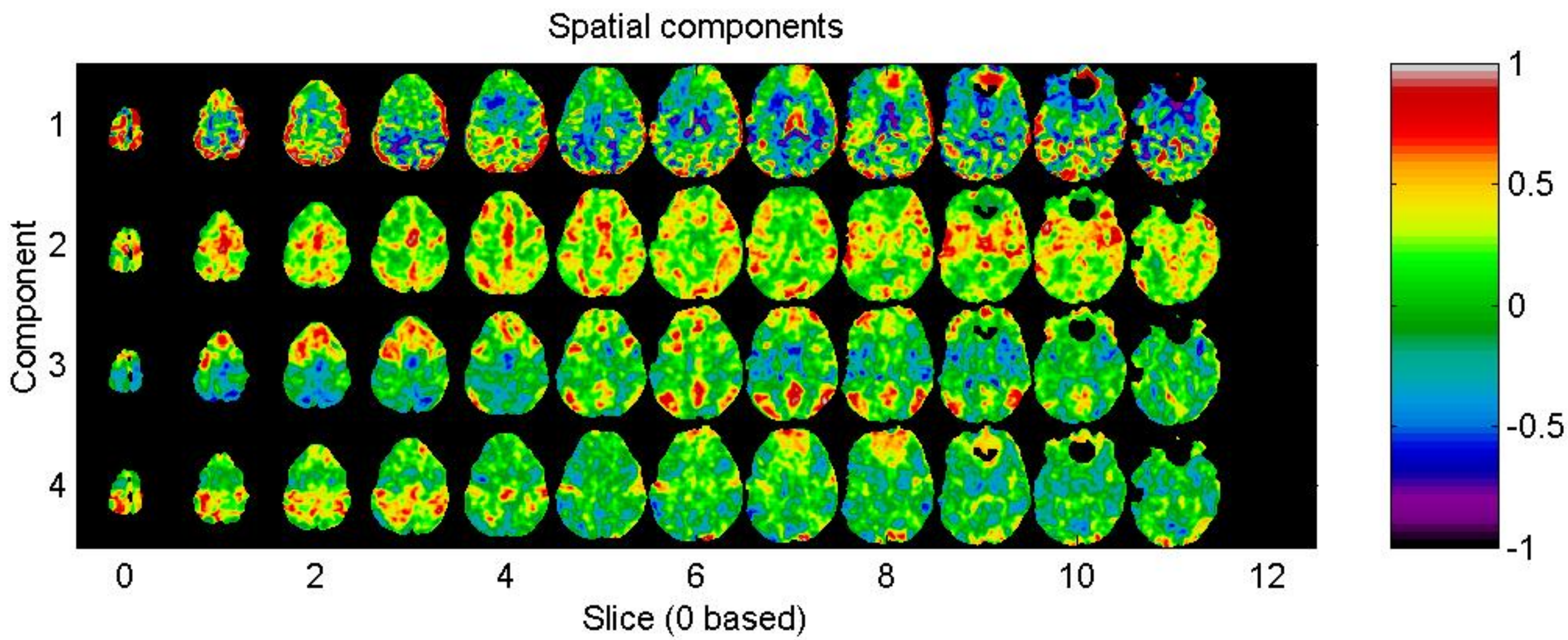
$$\mathbf{X} = s_1 \mathbf{u}_1 \mathbf{v}_1^T + s_2 \mathbf{u}_2 \mathbf{v}_2^T + \ldots + s_N \mathbf{u}_N \mathbf{v}_N^T$$



=
s_1
u_1
v_1^T
+ s_2
u_2
v_2^T
+ ...



Temporal components (sd, % variance explained)
0
-1
-2
-3
-4
-5
-Component
0.38, 14.7%
0.28, 7.8%
0.24, 5.9%
0.19, 3.6%
0
20
40
60
80
100
120
140
Frame



Spatial components
1
2
3
4
Component
0
2
4
6
8
10
12
Slice (0 based)
1
0.5
0
-0.5
-1

Worsley

Independent Components Analysis

- Independent Components Analysis (ICA) is a family of techniques used to extract independent signals from some source signal.

- ICA provides a method to blindly separate the data into spatially independent components.

- The key assumption is that the data set consists of *p* spatially independent components, which are linearly mixed and spatially fixed.

Independent Components Analysis

- The ICA Model:

$$\mathbf{X} = \mathbf{AS}$$

- Here A is referred to as the mixing matrix and S as the source matrix.

- Our goal is to find an un-mixing matrix **W** such that

$$\mathbf{Y} = \mathbf{WX}$$

provides a good approximation to **S**.

Assumptions

- If the mixing matrix is known, the problem is straight forward.

- However, ICA solves this problem without knowing the mixing parameters.

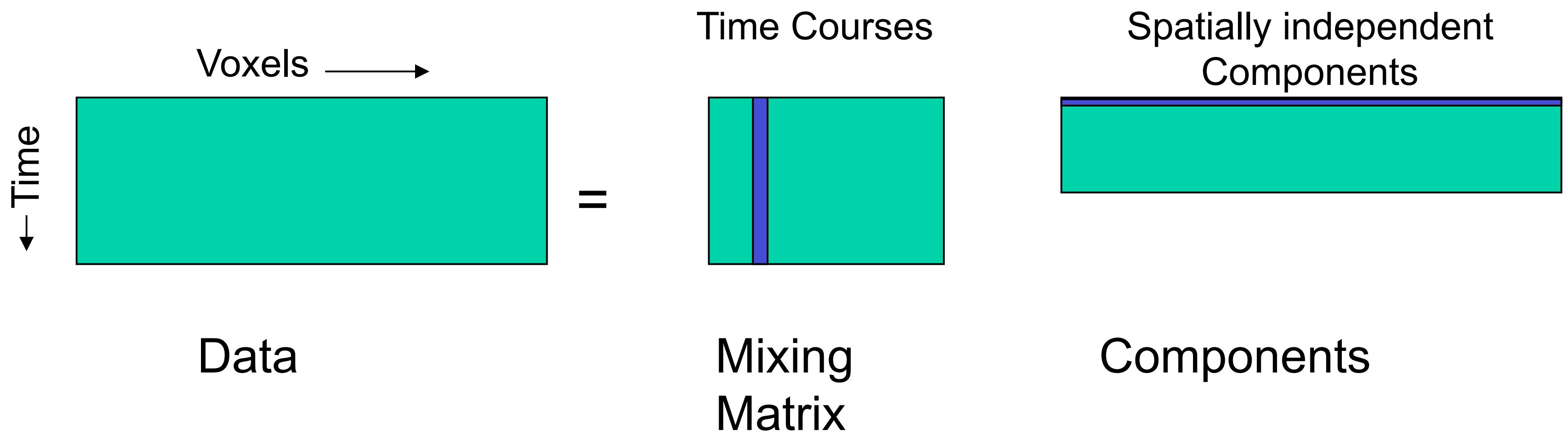
- Instead it exploits some key assumptions:
  - Linear mixing of sources.
  - The components s_i are statistically independent.
  - The components s_i are non-Gaussian.

ICA for fMRI

- It is assumed that the fMRI data can be modeled by identifying sets of voxels whose activity both vary together over time and are different from the activity in other sets.

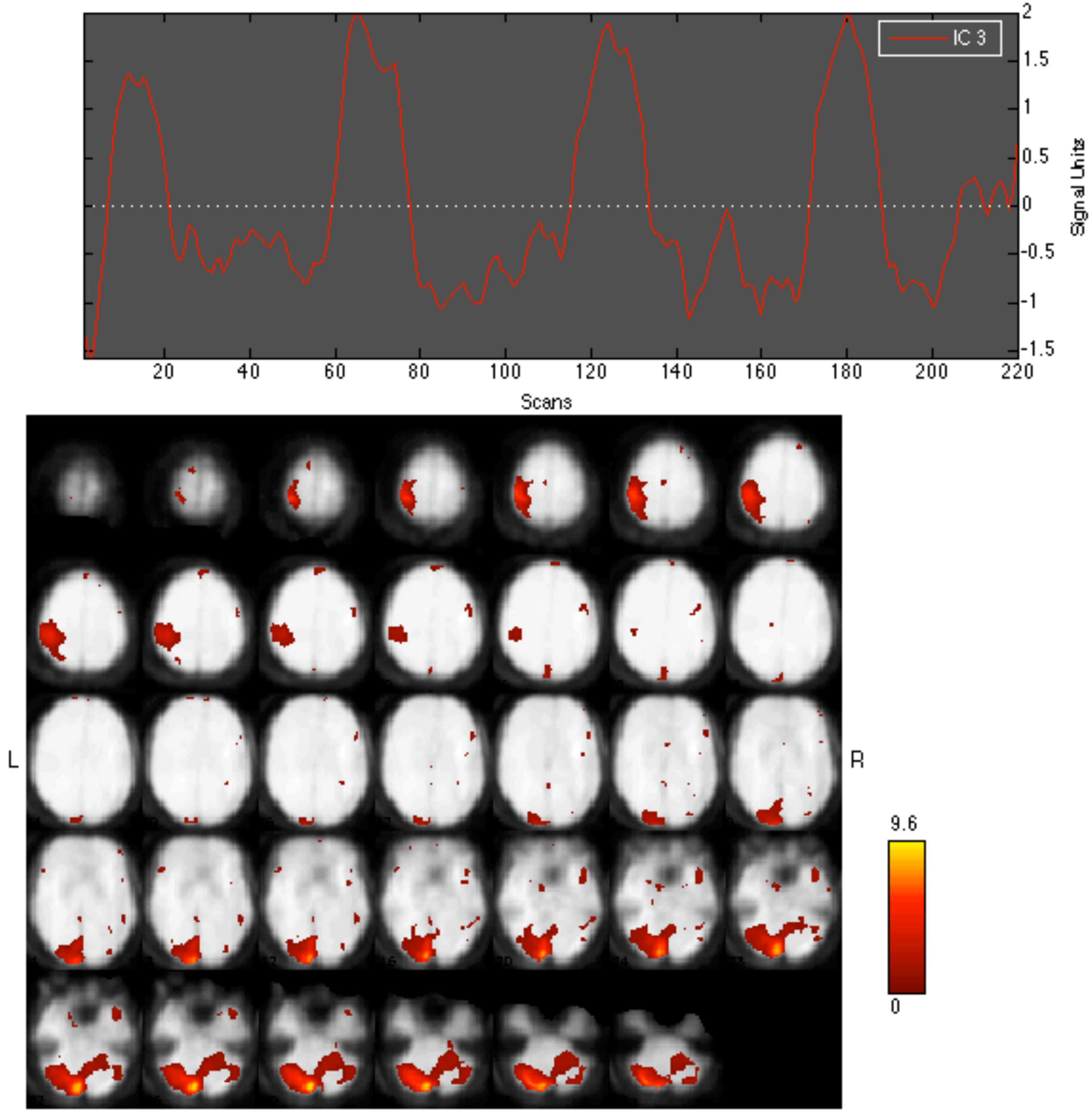
- Decompose the data set into a set of spatially independent component maps with a set of corresponding time-courses.

Overview

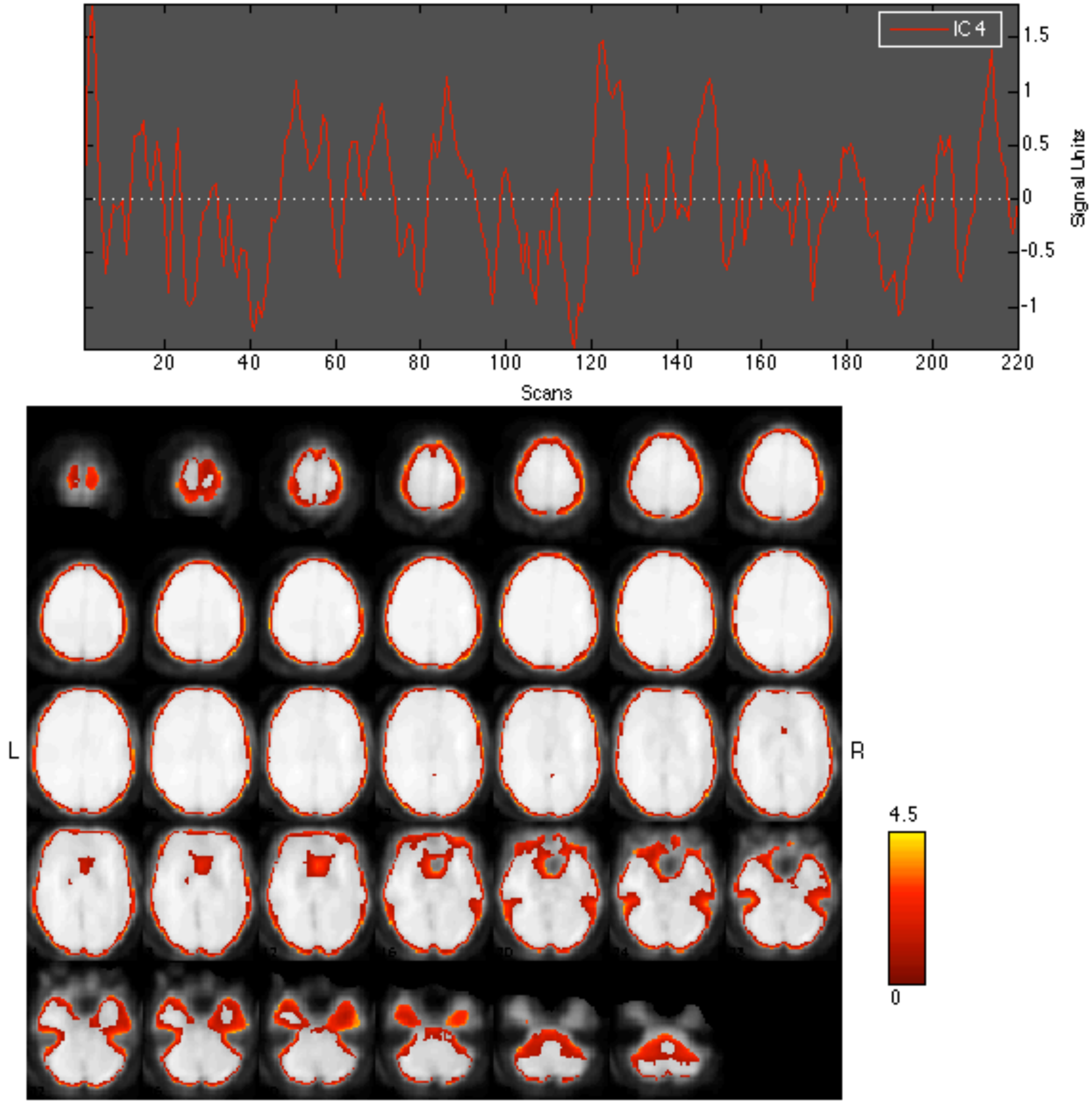


$$\mathbf{X} = \mathbf{AS}$$

Use an ICA algorithm to find A and S.



IC 3
Signal Units
Scans
L
R
9.6
0


IC 4
1.5
1
0.5
0
-0.5
-1
Signal Units
20
40
60
80
100
120
140
160
180
200
220
Scans
L
R
4.5
0

Comments

- Unlike PCA which assumes an orthonormality constraint, ICA assumes statistical independence among a collection of spatial patterns.
  - Independence is a stronger requirement than orthonormality.
- However, in ICA the spatially independent components are not ranked in order of importance as they are when performing PCA.

End of Module

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