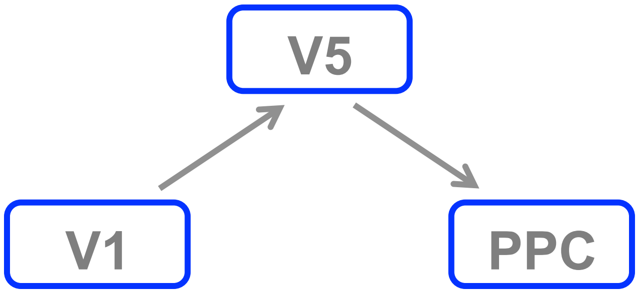
Module 25:
Effective Connectivity

Brain Connectivity

- Effective Connectivity
  - Directed influence of one brain region on the physiological activity recorded in other brain regions.
  - Claims to make statements about causal effects among tasks and regions.
  - Usually makes anatomically motivated assumptions and restricts inference to networks comprising of a number of pre-selected regions of interest.

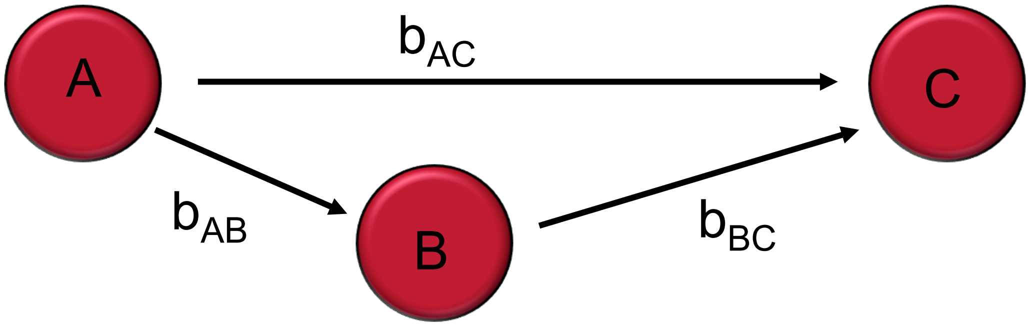


Effective Connectivity

- Methods include:
  - Structural Equation Modeling
  - Granger Causality
  - Dynamic Causal Modeling
  - Bayes Net

SEM

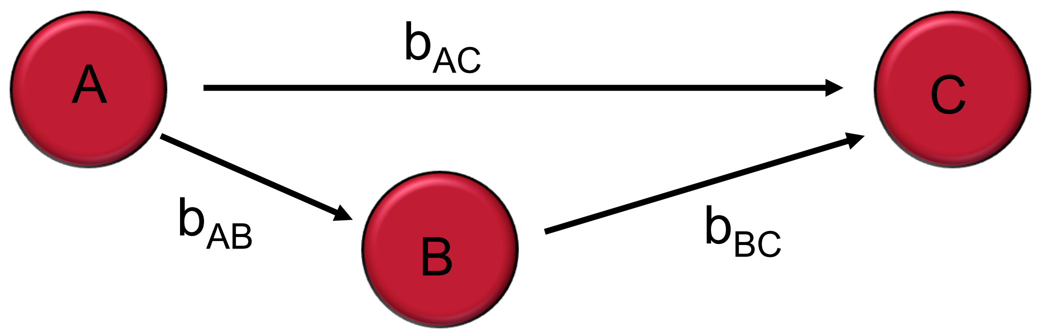
- Structural Equation Models comprise a set of regions and a set of directed connections.



- Path coefficients defined between pairs of nodes.

- Directional relationships are assumed *a priori*.
  - Often given a causal interpretation.

Example



$$\begin{bmatrix} y_A \\ y_B \\ y_C \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ b_{AB} & 0 & 0 \\ b_{AC} & b_{BC} & 0 \end{bmatrix} \begin{bmatrix} y_A \\ y_B \\ y_C \end{bmatrix} + \begin{bmatrix} e(1) \\ e(2) \\ e(3) \end{bmatrix}$$

$$y_t = My_t + e_t \qquad e_t \sim N(0,R) \qquad t = 1,\ldots T$$

Set-Up

- We can rewrite:

$$y_t = My_t + e_t$$

as

$$y_t = (I - M)^{-1} e_t$$

- Hence, we can write the covariance matrix of y_t as

$$\Sigma(\theta) = (I - M)^{-1} R((I - M)^{-1})^T$$

- The parameters θ are the unknown elements of the matrices M and R.

Estimation

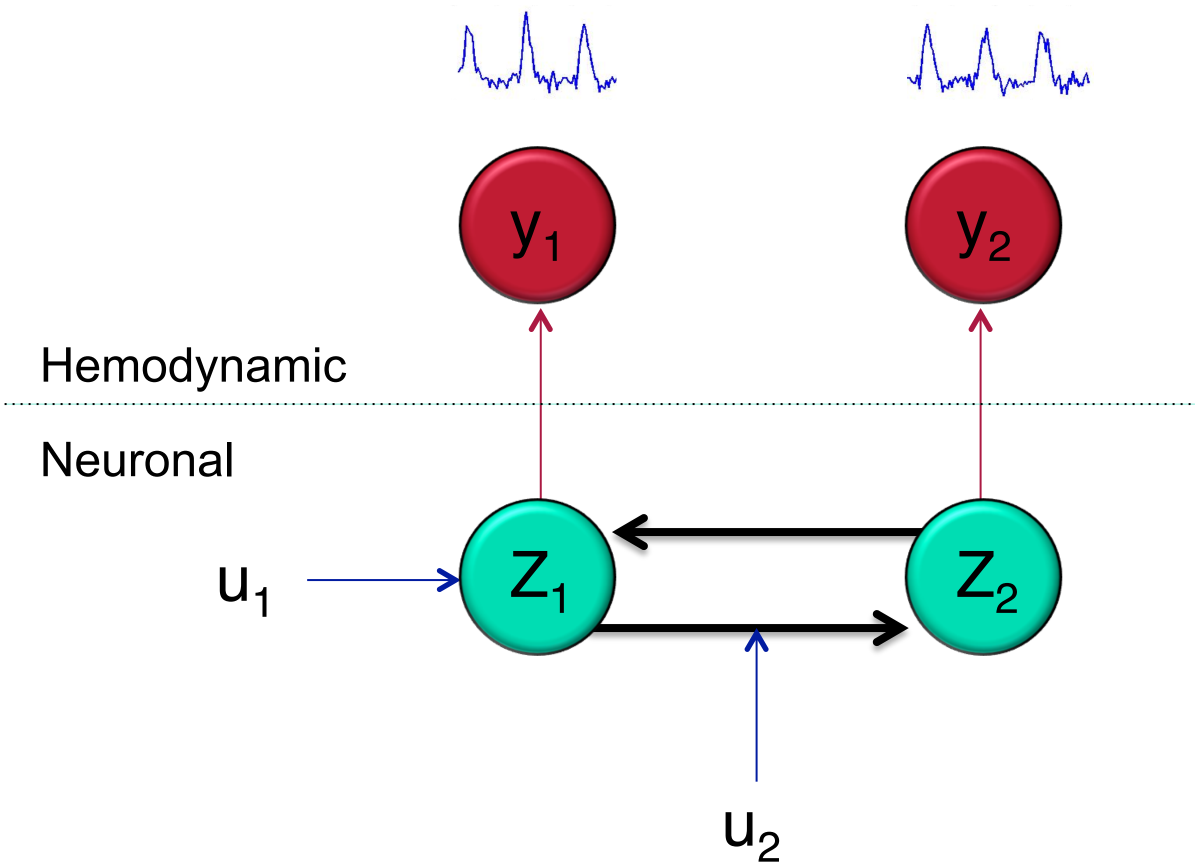
- The covariance of the data represents how activities in two or more regions are related.

- In SEM we seek to minimize the difference between the observed covariance matrix and the one implied by the structure of the model.
  - The parameters of the model are adjusted to minimize this difference.
  - Typically maximum likelihood estimation is used to estimate the parameters.

Dynamic Casual Modeling

- DCM attempts to model latent neuronal interactions using hemodynamic time series.
  - Based on a neuronal model of interacting regions, supplemented with a forward model of how neuronal activity is transformed into the observed response.
- Effective connectivity is parameterized in terms of the coupling among unobserved neuronal activity in different regions.
  - We can estimate these parameters by perturbing the system and measuring the response.

Illustration



Neuronal Model

- Define the neuronal states as:

$$z = (z_1, \ldots z_N)^T$$

- The effective connectivity model is described by:

$$\dot{z}_t = \left(A + \sum_{j=1}^{J} u_t(j) B^j \right) z_t + C u_t$$

where z_t is the neuronal activity at time t (latent) and $u_t(j)$ is the j^{th} of J inputs at time t (known).

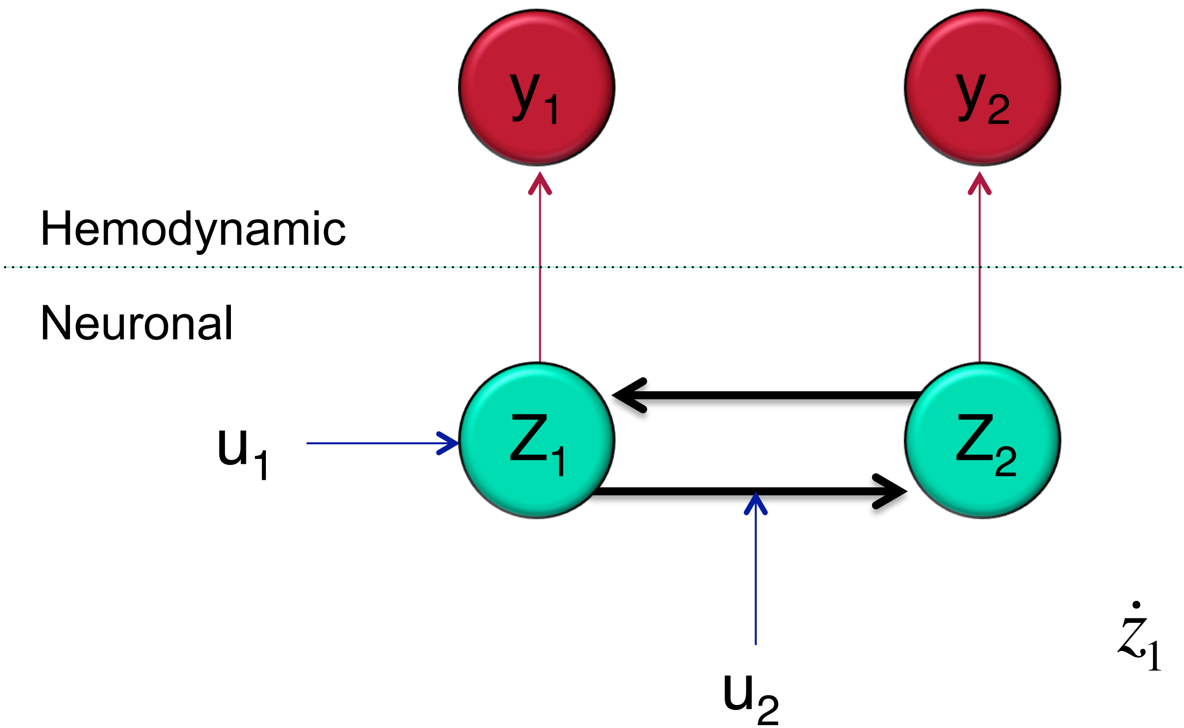
Interpretation

- The matrix A represents the first order connectivity among regions in the absence of input.
  - Specifies how regions are connected and whether these connections are uni- or bidirectional.

- The matrix C represents the extrinsic influence of inputs on neuronal activity.
  - Specifies how inputs are connected to regions.

- The matrices B_j represent the change in coupling induced by the jth input.
  - Specifies how connections are changed by inputs.

$$\dot{z}_t = \left(A + \sum_{j=1}^{J} u_t(j) B^j \right) z_t + C u_t$$



$$\dot{z}_1 = a_{11} z_1 + a_{12} z_2 + c_{11} u_1$$

$$\dot{z}_2 = a_{21} z_1 + a_{22} z_2 + b_{21}^2 u_2 z_1$$

Hemodynamic Model

- Neuronal activity causes changes in blood volume and deoxyhmoglobin that cause changes in the observed BOLD response.

- The hemodynamics are described using an extended Balloon model, which involves a set of hemodynamic state variables, state equations and hemodynamic parameters θ^h.

Extended Balloon Model

Activity-dependent signal: $\dot{s} = z - \kappa s - \gamma(f - 1)$

Flow induction: $\dot{f} = s$

Changes in volume: $\tau\dot{v} = f - v^{1/\alpha}$

Changes in dHb: $\tau\dot{q} = fE(f,\rho)/\rho - v^{1/\alpha}\, q/v$

Hemodynamic response $y = \lambda(v,q)$

State Equations

Neuronal state:

Neuronal activity - z_t with parameters θ^c.

Hemodynamic states:

Vasodilatory signal - s_t

Inflow - f_t

Blood volume - v_t

Deoxygenation content - q_t

The observed data: $y_t=\lambda(q_t,v_t)$ with parameters θ^h.

Bayesian Analysis

- Combining the neuronal and hemodynamic states $x=\{z, s, f, v, q\}$ gives us the following state-space model:

$$\dot{x} = f(x, u, \theta)$$

$$y = \lambda(x, \theta)$$

- Analysis performed using Bayesian methods
  - Normal priors are placed on θ.
  - The posterior density is used to make inferences about the connections.

End of Module

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