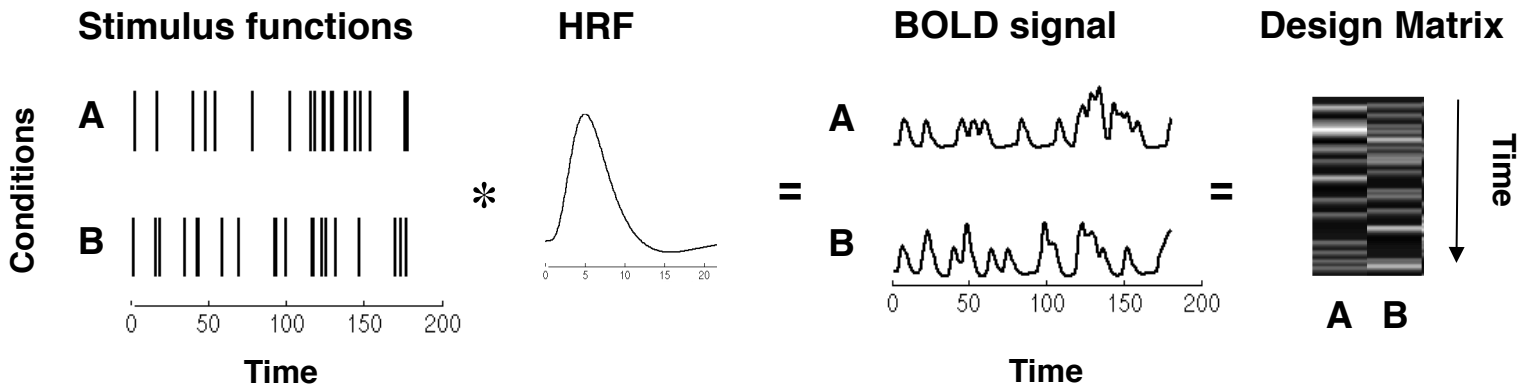
Module 18:
Multiple Comparisons

Localizing Activation

1. Construct a model for each voxel of the brain.
   - "Massive univariate approach"
   - Regression models (GLM) commonly used.

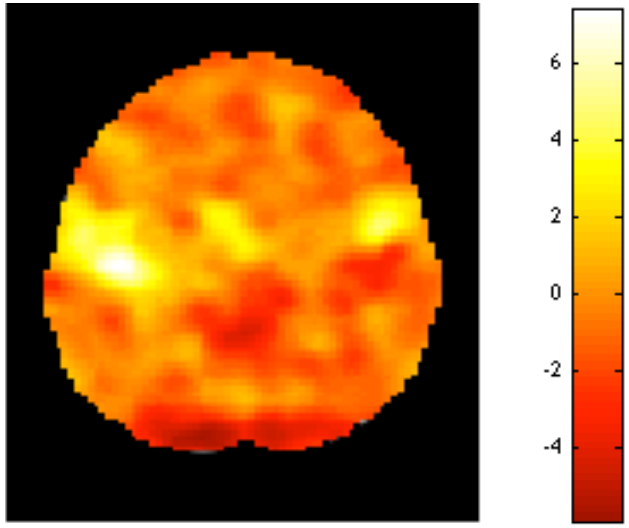


$$\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon} \qquad \boldsymbol{\varepsilon} \sim N(\mathbf{0}, \mathbf{V})$$

Localizing Activation

2. Perform a statistical test to determine whether task related activation is present in the voxel.

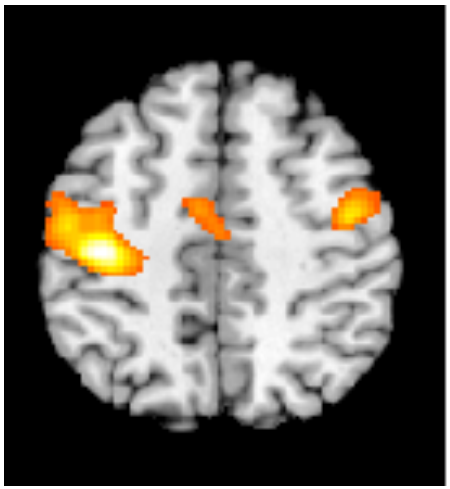
$$H_0 : \mathbf{c}^T \boldsymbol{\beta} = 0$$



Statistical image:
Map of t-tests across all voxels (a.k.a t-map).

Localizing Activation

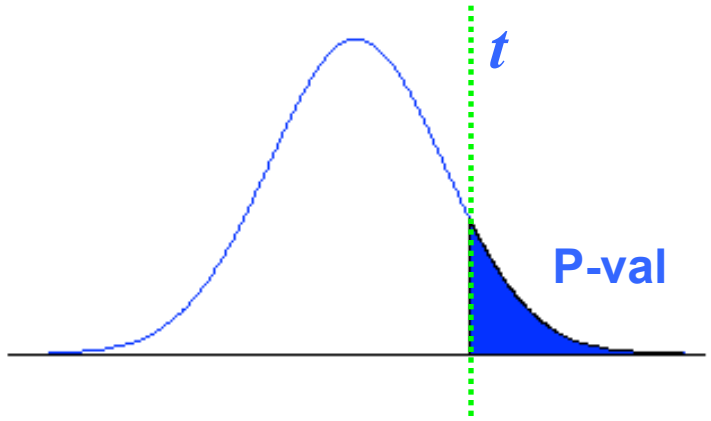
3. Choose an appropriate threshold for determining statistical significance.



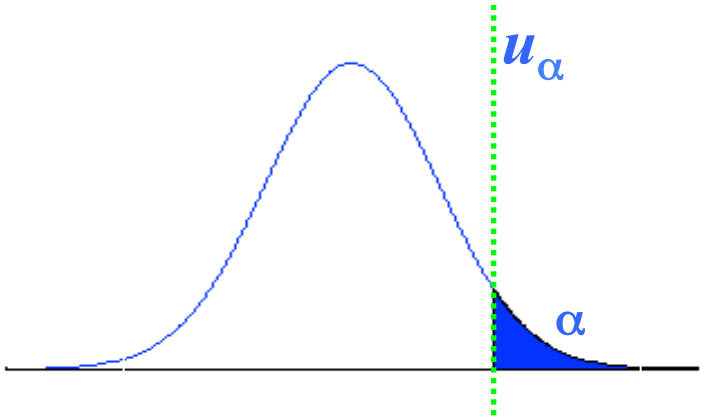
Statistical parametric map: Each significant voxel is color-coded according to the size of its p-value.

Hypothesis Testing

- Null Hypothesis H_0
  - Statement of no effect (e.g., β_1-β_2=0).
- Test statistic T
  - Measures compatibility between the null hypothesis and the data.
- P-value
  - Probability that the test statistic would take a value as or more extreme than that actually observed if H_0 is true, i.e $P(T > t \mid H_0)$.
- Significance level
  - Threshold u_α controls false positive rate at level $\alpha = P(T>u_\alpha \mid H_0)$



Null Distribution of T



Null Distribution of T

Making Errors

- There are two types of errors one can make when performing significance tests:
  - Type I error
    - H_0 is true, but we mistakenly reject it (False positive).
    - Controlled by significance level α.
  - Type II error
    - H_0 is false, but we fail to reject it (False negative)
- The probability that a hypothesis test will correctly reject a false null hypothesis is the power of the test.

Multiple Comparisons

- Choosing an appropriate threshold is complicated by the fact we are dealing with a family of tests.

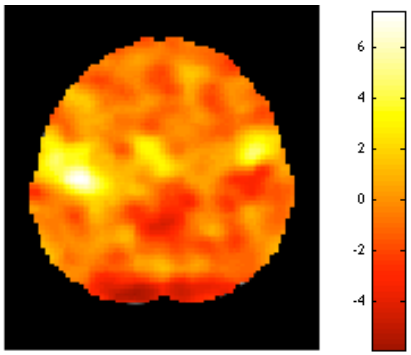
- If more than one hypothesis test is performed, the risk of making at least one Type I error is greater than the α value for a single test.

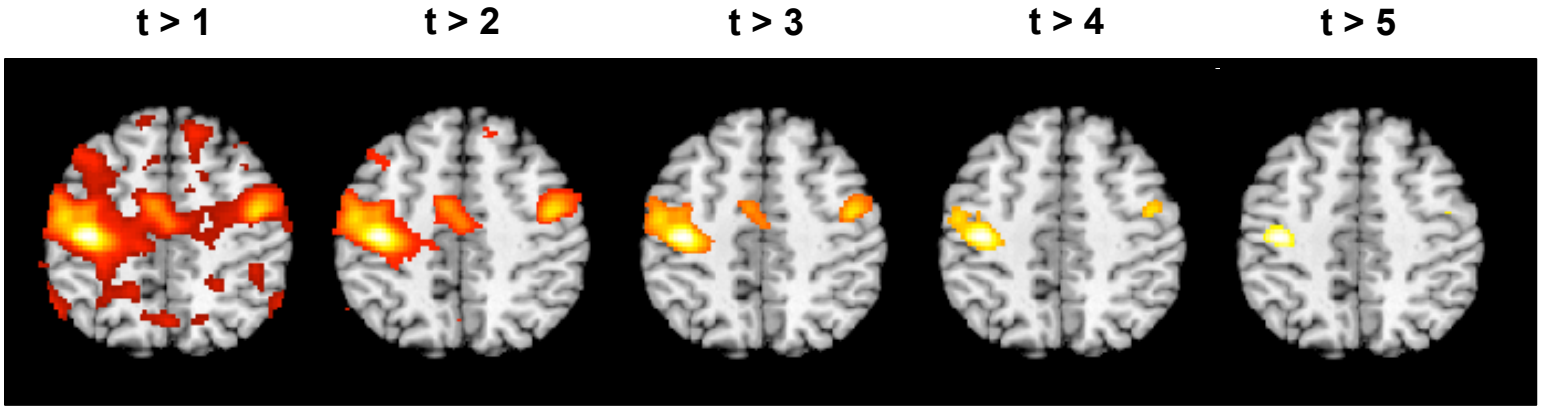
- The more tests one performs, the greater the likelihood of getting at least one false positive.

Multiple Comparisons

- Which of 100,000 voxels are significant?
  - α=0.05 $\Rightarrow$ 5,000 false positive voxels

- Choosing a threshold is a balance between sensitivity (true positive rate) and specificity (true negative rate).





Measures of False Positives

- There exist several ways of quantifying the likelihood of obtaining false positives.

- Family-Wise Error Rate (FWER)
  - Probability of any false positives

- False Discovery Rate (FDR)
  - Proportion of false positives among rejected tests

End of Module

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