Fixed Income

HW4

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02/07/2020

```
#import the data
Rate_paths=data.frame(matrix(data = c(0.04,0.04,0.04,0.04,0.04,0.05,0.03,0.04,0.07,0.02,
0.07,0.02,0.05,0.06,0.08,0.06,0.04,0.07,0.04,0.09,0.03,0.07,0.08,0.04,0.12,0.06,0.06,
0.10,0.05,0.09),nrow = 5,ncol = 6))
colnames(Rate_paths)=seq(0,5,1)
Rate_paths
```

```
##      0    1    2    3    4    5
## 1 0.04 0.05 0.07 0.06 0.03 0.06
## 2 0.04 0.03 0.02 0.04 0.07 0.06
## 3 0.04 0.04 0.05 0.07 0.08 0.10
## 4 0.04 0.07 0.06 0.04 0.04 0.05
## 5 0.04 0.02 0.08 0.09 0.12 0.09
```

1

Recall from the slides, under risk neutral valuation, the PV formula:

$$PV = E_0^Q[e^{-AT}\Phi(X_T)].$$

where A is the average rate from 0 to T and $\Phi(X_T)$ is the payoff function at T.

```
valuate_risk_neutral=function(paths,FUN=100,...){
   #initialization
    DerivPrice=rep(0,ncol(paths)-1)
    for (mar in 2:ncol(paths)){
    trajectories=paths[,1:mar]
    A=rowMeans(trajectories)
    DerivPrice[mar-1]=mean(exp(-A*(mar-1))*FUN(trajectories,...))
  }
  DerivPrice=data.frame(t(DerivPrice))
  colnames(DerivPrice)=seq(1,(ncol(paths)-1),1)
  return(DerivPrice)
}
```

Formulated in code, the prices of zero coupon bonds would be

```
#wrirte the payoff function of ZCBs
ZCP_Phi=function(df)
{
  return(100)
}
valuate_risk_neutral(paths = Rate_paths,FUN = ZCP_Phi)
```

```
##          1        2        3        4        5
## 1 95.98646 91.22631 86.23078 80.89408 75.55902
```

2

The payoff function of an interest rate caplet is:

$$Max(0, R_T - 0.05).$$

Write the payoff function of the interest rate caplet:

```
Phi_cap=function(df,K){
  Payoff=pmax(0,df[,ncol(df)]-K)
  return(Payoff)
}
```

Compute the present value of each interest rate caplet:

```
valuate_risk_neutral(paths = Rate_paths,FUN = Phi_cap,K=0.05)
```

```
##             1          2          3          4          5
## 1 0.003785941 0.01084635 0.01187109 0.01878544 0.01598513
```

The cap is the sum of these values:

```
sum(valuate_risk_neutral(paths = Rate_paths,FUN = Phi_cap,K=0.05))
```

```
## [1] 0.06127394
```

3

The payoff function of an interest rate floorlet is:

$$Max(0, K - R_T).$$

Write the payoff function of the floorlet:

```
Phi_floor=function(df,K){
  Payoff=pmax(0,K-df[,ncol(df)])
  return(Payoff)
}
```

Compute the interest rate floorlet present values:

```
valuate_risk_neutral(paths = Rate_paths,FUN = Phi_floor,K=0.07)
```

```
##            1          2          3          4           5
## 1 0.02701803 0.01487128 0.01226406 0.01146223 0.006270275
```

The floor is the sum of these values:

```
sum(valuate_risk_neutral(paths = Rate_paths,FUN = Phi_floor,K=0.07))
```

```
## [1] 0.07188588
```

4

Compute the price of a five year call on short interest rates:

```
valuate_risk_neutral(paths = Rate_paths,FUN = Phi_cap,K=0.065)[ncol(Rate_paths)-1]
```

```
##             5
## 1 0.008565218
```

Compute the price of a five year put on short interest rates:

```
valuate_risk_neutral(paths = Rate_paths,FUN = Phi_floor,K=0.065)[ncol(Rate_paths)-1]
```

```
##             5
## 1 0.003913938
```

Therefore a five year call on short term rates is more valuable.

5

The payoff of the Asian option should be:

$$Max(0, \bar{R} - K).$$

Write the payoff fuction

```
Phi_Asian_cap=function(df,K){
  Payoff=pmax(0,rowMeans(df)-K)
  return(Payoff)
}
```

Compute the price of the Asian call:

```
valuate_risk_neutral(paths = Rate_paths,FUN = Phi_Asian_cap,K=0.06)[ncol(Rate_paths)-1]
```

```
##             5
## 1 0.002333824
```

Compute the price of the five-year call:

```
valuate_risk_neutral(paths = Rate_paths,FUN = Phi_cap,K=0.06)[ncol(Rate_paths)-1]
```

```
##             5
## 1 0.009986832
```

The European call is more valuable in this case.

6

The standard deviation of the short-term rate at year 5:

```
sd(Rate_paths[,6])
```

```
## [1] 0.02167948
```

The standard deviation of the average short-term rate year 5

```
sd(rowMeans(Rate_paths))
```

```
## [1] 0.0119257
```

Obviously, the volatility of average short-term rates is much smaller than that of the terminal short-term rate. Because option prices are positively correlated the volatility of the underlying asset, the European call is more valuable than the Asian Call.

7

In the lecture, we proved that, by the condition of no abitrage, the future prices at time 0 must equal to the martingale expectation of the underlying asset at maturity T:

$$K = E_0^Q(S_T).$$

Therefore, the futures prices for short-term rate (Eurodollar futures) should be:

```r
colMeans(Rate_paths)
```

```
##     0     1     2     3     4     5
## 0.040 0.042 0.056 0.060 0.068 0.072
```