Case 1

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The key idea of Treasury bond arbitrage is to use two coupon bonds of the same maturity to build a replicating portfolio of the zero-coupon bond and exploit the potential price discrepancy.

Suppose we have 2 coupon bonds whose coupon rates are C_1 and C_2, and our replicating portfolio has weights of x, 1-x in the two bonds respectively.

To replicating a zero coupon bond, at each coupon payment date, we have to make sure that we have zero net cash flow.

$$C_1 * x + C_2 * (1 - x) = 0$$

solve for x:

$$x = \frac{C_2}{C2 - C1}$$

where the price of the synthetic zero coupon bond is:

$$P_{synthetic} = x * P_1 + (1 - x) * P_2$$

and the profit of each trade is:

$$Profit = |P_{synthetic} - P_{ZCB}|$$

The outputs of bonds mature at 11/15/2022

```
ZCB_11_15_22=86.15
small_11_15_22=98+(18.5/32)+(1.625)/2*55/181 #price of the 1.625 coupon rate bond
large_11_15_22=143+2.5/32+(7.625)/2*55/181 # price of the 7.625 coupon rate bond

# use the two coupon bonds to sythesize the ZCB and check the price discrapency
coup_price=c(small_11_15_22,large_11_15_22)
weights=c(7.625/(7.625-1.625),1-7.625/(7.625-1.625))
sythetic_val=coup_price%*%weights
profit_each_trade=sythetic_val-ZCB_11_15_22

output=list("ZCB_price"=ZCB_11_15_22,"small_coupon_bond"=small_11_15_22,"large_coupon_bond"=large_11_15_
            "weights"=weights,"sythetic_ZCB"=sythetic_val,"profit per trade"=profit_each_trade)
output
```

```
## $ZCB_price
## [1] 86.15
##
## $small_coupon_bond
## [1] 98.82502
##
## $large_coupon_bond
## [1] 144.2366
##
## $weights
## [1]  1.2708333 -0.2708333
```

```
##
## $sythetic_ZCB
##          [,1]
## [1,] 86.52604
##
## $`profit per trade`
##           [,1]
## [1,] 0.3760417
```

The outputs of bonds mature at 02/15/2023:

```
ZCB_02_15_23=85.43
small_02_15_23=101+(7/32)+(2.000)/2*144/184 #price of the 1.625 coupon rate bond
large_02_15_23=140+2.5/32+(7.125)/2*144/184 # price of the 7.625 coupon rate bond

# use the two coupon bonds to sythesize the ZCB and check the price discrapency
coup_price=c(small_02_15_23,large_02_15_23)
weights=c(7.125/(7.125-2.00),1-7.125/(7.125-2.00))
sythetic_val=coup_price%*%weights
profit_each_trade=sythetic_val-ZCB_02_15_23
output=list("ZCB_price"=ZCB_02_15_23,"small_coupon_bond"=small_02_15_23,"large_coupon_bond"=large_02_15_
            "weights"=weights,"sythetic_ZCB"=sythetic_val,"profit per trade"=profit_each_trade)
output
```

```
## $ZCB_price
## [1] 85.43
##
## $small_coupon_bond
## [1] 102.0014
##
## $large_coupon_bond
## [1] 142.8662
##
## $weights
## [1]  1.3902439 -0.3902439
##
## $sythetic_ZCB
##          [,1]
## [1,] 86.05412
##
## $`profit per trade`
##           [,1]
## [1,] 0.6241159
```

The outputs of bonds mature at 08/15/2023 :

```
ZCB_08_15_23=84.29
small_08_15_23=104+(27.5/32)+(2.50)/2*147/184 #price of the 1.625 coupon rate bond
large_08_15_23=135+2/32+(6.25)/2*147/184 # price of the 7.625 coupon rate bond

# use the two coupon bonds to sythesize the ZCB and check the price discrapency
coup_price=c(small_08_15_23,large_08_15_23)
weights=c(6.25/(6.25-2.50),1-6.25/(6.25-2.50))
sythetic_val=coup_price%*%weights
profit_each_trade=sythetic_val-ZCB_08_15_23
```

```
output=list("ZCB_price"=ZCB_08_15_23,"small_coupon_bond"=small_08_15_23,"large_coupon_bond"=large_08_15_
            "weights"=weights,"sythetic_ZCB"=sythetic_val,"profit per trade"=profit_each_trade)
output
```

```
## $ZCB_price
## [1] 84.29
##
## $small_coupon_bond
## [1] 105.858
##
## $large_coupon_bond
## [1] 137.5591
##
## $weights
## [1]  1.6666667 -0.6666667
##
## $sythetic_ZCB
##          [,1]
## [1,] 84.72396
##
## $`profit per trade`
##           [,1]
## [1,] 0.4339583
```

The outputs of bonds mature at 11/15/2024 are as followed:

```
#compute the dirty prices for the three bonds mature at 02/15/2022

ZCB_11_15_24=81.23
small_11_15_24=102+(17.5/32)+(2.25)/2*55/181 #price of the 1.625 coupon rate bond
large_11_15_24=150+2/32+(7.50)/2*55/181 # price of the 7.625 coupon rate bond

# use the two coupon bonds to sythesize the ZCB and check the price discrapency
coup_price=c(small_11_15_24,large_11_15_24)
weights=c(7.50/(7.50-2.25),1-7.50/(7.50-2.25))
sythetic_val=coup_price%*%weights
profit_each_trade=sythetic_val-ZCB_11_15_24

output=list("ZCB_price"=ZCB_11_15_24,"small_coupon_bond"=small_11_15_24,"large_coupon_bond"=large_11_15_
            "weights"=weights,"sythetic_ZCB"=sythetic_val,"profit per trade"=profit_each_trade)
output
```

```
## $ZCB_price
## [1] 81.23
##
## $small_coupon_bond
## [1] 102.8887
##
## $large_coupon_bond
## [1] 151.202
##
## $weights
## [1]  1.4285714 -0.4285714
##
## $sythetic_ZCB
```

```
##          [,1]
## [1,] 82.18304
##
## $`profit per trade`
##           [,1]
## [1,] 0.9530357
```

From above, we can conclude our strategy: because the price of the synthetic zero-coupon bond is higher than the price of traded zero-coupon bond, we exploit this price discrepancy by longing the traded zero-coupon bond and shorting the synthetic zero-coupon bond.

Our expected profit per unit trade for these four trades are 0.380013,0.6241159,0.4339583 and 0.9530357.