Yahoo_Case

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Assume we hold one unit of short position on the junk bond (because it is a liability), and buy x unit of T-bill and y unit of T-note to hedge interest rate risk. In addition, assume we want to do a duration and convexity hedge. In essense, we want:

$$\Delta P_{port} = x\Delta P_{bill} + y\Delta P_{note} - \Delta P_{corp} = 0$$

Recall the 2nd order approximate of price change:

$$\Delta P = -D^* P\Delta r + \frac{1}{2}C_T P(\Delta r)^2$$

Where D^* is the Modified duration of the bond, and $C_T$is the Convexity of the bond.

Therefore, we want:

$$\Delta P_{port} = xP_{bill}(-D^*_{bill}\Delta r+\frac{1}{2}C_{Tbill}(\Delta r)^2)+yP_{note}(-D^*_{note}\Delta r+\frac{1}{2}C_{Tnote}(\Delta r)^2)-P_{corp}(-D^*_{corp}\Delta r+\frac{1}{2}C_{Tcorp}(\Delta r)^2) = 0$$

To make it strictly 0, we must have its first and second order co-efficient equal 0. So we have:

$$D^*_{Port} = -xP_{bill}D^*_{bill} - yP_{note}D^*_{note} + P_{corp}D^*_{corp} = 0$$

and

$$C_{TPort} = xP_{bill}C_{Tbill} + yP_{note}C_{Tnote} - P_{corp}C_{Tcorp} = 0$$

solve this two system of equation in matrix form

$$\begin{pmatrix} -D^*_{bill} & -D^*_{note} \\ C_{Tbill} & C_{Tnote} \end{pmatrix} \begin{pmatrix} \frac{xP_{bill}}{P_{corp}} \\ \frac{yP_{note}}{P_{corp}} \end{pmatrix} = \begin{pmatrix} -D^*_{corp} \\ C_{Tcorp} \end{pmatrix}$$

where the vector $\begin{pmatrix} \frac{xP_{bill}}{P_{corp}} \\ \frac{yP_{note}}{P_{corp}} \end{pmatrix}$ is the weights we want solve for

```r
# import library and input info
library(derivmkts)
```

```
## Warning: package 'derivmkts' was built under R version 3.5.2
```

```r
#initalization
bond_data=data.frame(matrix(data = 0,nrow = 3,ncol = 3))
rownames(bond_data)=c('10-year','1-year','Corp-Bond')
colnames(bond_data)=c('Coupon','Maturity','Yield')
#note the corp bond paid coupon semi-annually but the 10-yr bond paid coupon annully
bond_data$Coupon=c(5,0,11)
bond_data$Maturity=c(10,1,7)
bond_data$Yield=c(0.05,0.0075,0.08)

get_price_ModiD_Conv=function(df)
  {for (i in 1:2)
    {
```

```r
    #compute bond price
    df$Price[i]=bondpv(coupon = df$Coupon[i],mat = df$Maturity[i],yield = df$Yield[i],principal = 100,f
    #compute bond modified duration
    df$ModiD[i]=duration(price = df$Price[i],coupon = df$Coupon[i],mat = df$Maturity[i],principal = 100
    #compute bond convexity
    df$Conv[i]=convexity(price = df$Price[i],coupon = df$Coupon[i],mat = df$Maturity[i],principal = 100
  }
  # compute price, duration and convexity for corp bond (semi-annul)
    #compute bond price
    df$Price[3]=bondpv(coupon = df$Coupon[3],mat = df$Maturity[3],yield = df$Yield[3],principal = 100,f
    #compute bond modified duration
    df$ModiD[3]=duration(price = df$Price[3],coupon = df$Coupon[3],mat = df$Maturity[3],principal = 100
    #compute bond convexity
    df$Conv[3]=convexity(price = df$Price[3],coupon = df$Coupon[3],mat = df$Maturity[3],principal = 100
    return(df)
}

bond_data=get_price_ModiD_Conv(df = bond_data)

get_hedge_weights=function(df){
    # construct A matrix and b vector
  A_mtx=as.matrix(df[1:2,5:6])
  A_mtx=t(A_mtx)
  A_mtx[1,]=-A_mtx[1,]
  b=c(-df$ModiD[3],df$Conv[3])
  weights=solve(a = A_mtx,b = b)
  weights=as.vector(append(x = weights,values = -1.0,after = 3))
  return(weights)
}

Port_weights=get_hedge_weights(df=bond_data)
```

Therefore, the weights we should invest in the T-notes, T-bill and corp bond are:

```r
Port_weights
```

```
## [1]  0.374823  2.132244 -1.000000
```

To compute the capital gain or loss, we evalutae our initial hedged portfolio, the portfolio after rates change and take the difference.

To compute the new hedge ratio, we recompute the duration and convexity of the three bonds and recompute the perfect hedge ratio.

```r
# compute the initial value of the portfolio
ini_Port_value=t(Port_weights)%*%bond_data$Price
#compute the new bond info when rates go up 50bp
up_bond=bond_data[,1:3]
up_bond$Yield=bond_data$Yield+0.005
# recompute all the prices, modified duration and convexity
up_bond=get_price_ModiD_Conv(df=up_bond)
#compute the value difference in portfolio
up_Port_diff=t(Port_weights)%*%up_bond$Price-ini_Port_value
#compute the new hedge weights
up_hedge_weights=get_hedge_weights(df = up_bond)
#compute change in positions
```

```r
up_change_positions=up_hedge_weights-Port_weights
```

When the rates go up by 50 basis point, the capital gain or loss as a percentage of market value of high yield bond would be:

```r
up_Port_diff/bond_data$Price[3]
```

```
##              [,1]
## [1,] 0.003438226
```

The position change for 10-year bond, 1-year bond and Corp bond would be

```r
up_change_positions
```

```
## [1]  0.002225315 -0.013984016  0.000000000
```

To compute the gain/loss when rates go down by 50 bs, and the new hedge ratio

```r
# compute the initial value of the portfolio
ini_Port_value=t(Port_weights)%*%bond_data$Price
#compute the new bond info when rates go up 50bp
down_bond=bond_data[,1:3]
down_bond$Yield=bond_data$Yield-0.005
# recompute all the prices, modified duration and convexity
down_bond=get_price_ModiD_Conv(df=down_bond)
#compute the value difference in portfolio
down_Port_diff=t(Port_weights)%*%down_bond$Price-ini_Port_value
#compute the new hedge weights
down_hedge_weights=get_hedge_weights(df = down_bond)
#compute change in positions
down_change_positions=down_hedge_weights-Port_weights
```

When the rates go down by 50 basis point, the capital gain or loss as a percentage of market value of high yield bond would be:

```r
down_Port_diff/bond_data$Price[3]
```

```
##               [,1]
## [1,] -0.003549314
```

The position change for 10-year bond, 1-year bond and Corp bond would be

```r
down_change_positions
```

```
## [1] -0.002209589  0.013894952  0.000000000
```