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28/11/2023

Assignment 2

Exercise 1: Ice flow

Question 1.1 [8 pts]: What sliding speed (in ma^{-1}) do you expect, and what assumptions do you need to make for being able to answer the question?

We have the following information:

The glacier ice thickness is known to be 536 m: $H = 536\,m$

The surface mass balance is virtually "zero", i.e. the point is very close to the glacier's equilibrium line altitude, so we assume a Temperature of 0°C and with that an ice density of $\rho_{ice} = 916.7\ \mathrm{kg\ m^{-3}}$ (Cogley et al., 2011, p. 103), a factor of $n = 3$ and $A = 2{,}4 * 10^{-24}\ \mathrm{s^{-1}Pa^{-3}}$(Cuffey & Paterson, 2010, p. 75).

We furthermore assume that it is a homogeneous glacier with a block size (and not glacier geometry), the velocity and slopes (no ablation and accumulation) stay the same for the whole year as well as there were no other processes than basal sliding and deformation and a crystal glacier bed.

As we have two positions of the pole (P1 and P2) for two following years, we can calculate the vector of the speed and the absolute horizontal:

$$Vector(v) = P2 - P1 = \begin{matrix} 89.34 \\ 201.83 \\ 9.51 \end{matrix}$$

$$v = |Vector(v)| = \sqrt{89.34^2 + 201.83^2} = 220.72\,m/a$$

The point is in an area where the glacier surface topography looks smooth; we can therefore assume that the slope is the change of the position of the pipe in the two years: $tan(\alpha) = \Delta z/\Delta xy = 0.0431$

We want to use the following formula:

$$v = v_{deformation} + v_{sliding}$$

$$(1)\ v_{sliding} = v - v_{deformation}$$

To get the depth-averaged deformation velocity, we employ the equation:

$$(2)\ v_{deformation} = \frac{2A}{n+1} {\tau_d}^n H^{n+1}$$

Furthermore, the driving stress can be calculated by:

$$(3)\ \tau_d = \rho_{ice} * g * H * \tan\alpha$$

Applying the values of the beginning in equation (3) we get the following value: $\tau_d = 207.68\ kPa$.

For (2) our result is therefore: $v_{deformation} = 181.7\ m/a$ and now we can apply equation (1) and get:

$$v_{sliding} = v - v_{deformation} = 39.02\ m/a$$

Result:

In conclusion, we determine a sliding speed of 39.02 m/a.

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Question 1.2 [5 pts]: Discuss the various hints given below. Qualitatively: what changes do you expect if the individual assumptions were not fulfilled?

Hint 1: From the climatic setting of the region, you can assume that the glacier is temperate.

If this hint is not fulfilled, we would have to assume that it would be a cold-based glacier, and therefore the contribution of sliding to the surface ice flow velocity would not have a huge impact i.e., $v_{\text{sliding}} \approx$ few % of v_{surface}. Temperate glaciers normally have a higher sliding velocity due to a thin layer of water on the bed, which reduces the friction. The calculation of τ_d only holds for temperate glaciers, therefore our calculation for a cold glacier would be more difficult to obtain.

Hint 2: From the geological setting, you can assume that the glacier bed is crystalline and has no till.

If the bed had a deformable material and with that roughness, we would have to assume a higher bed deformation velocity. The presence of a till could also increase the friction and would result in a smaller sliding velocity. Boulton has plotted the effect of different geological settings like the following:

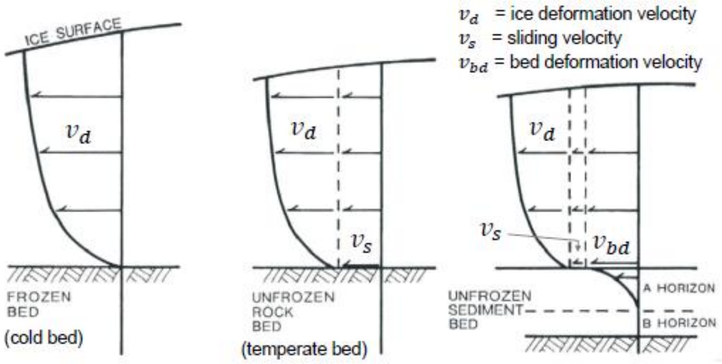

Figure 1: Schematic diagrams showing the horizontal velocity distribution in sections through a glacier on different surfaces. The vertical scale in the sediment is exaggerated. (Boulton, 1996)

Hints 1 and 2 are closely related because the presence of a till only has a major impact if it is a temperate glacier.

Hint 3: For simplicity, you can assume that the entire driving stress is resisted by basal drag (i.e. no effects from the glacier´s margins).

If this assumption is not fulfilled and other factors such as the glacier margin are involved, it would change the speed of the glacier. The deformation velocity would be smaller and the sliding velocity higher if we considered the friction at the margins (and assume the observed surface velocity).

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Exercise 2: Glacier instabilities

Question 2.1 [6 pts]: Can you tell whether the data are indeed cause of concern, and if so, under what conditions?

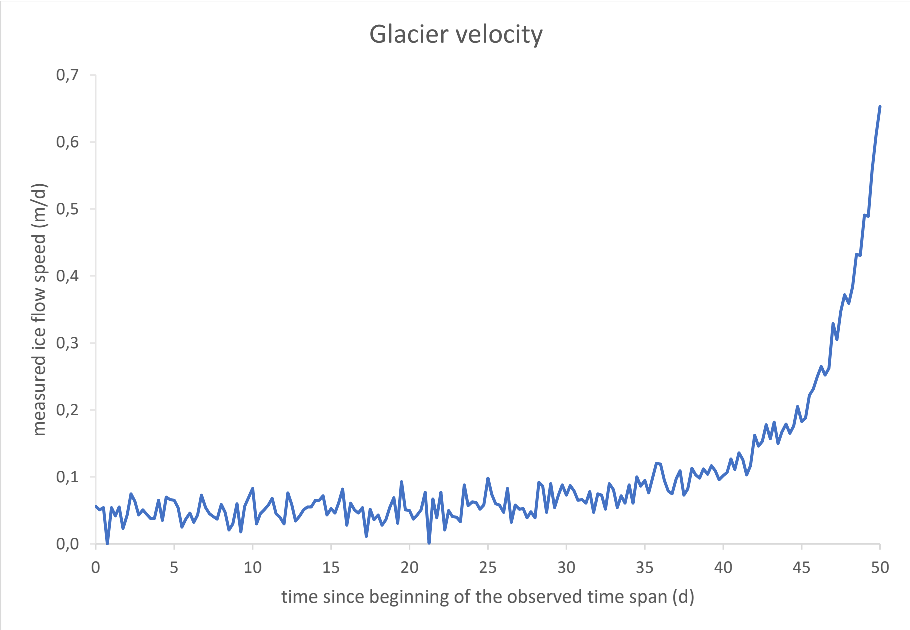

Figure 2 - Measured ice flow speed of the glacier in the observed period of day 5586 (≙ day 0) until day 5636 (≙ day 50) since the beginning of the measurements. Measurements were made every six hours, summing up to 201 data points.

When we plot the ice flow speed of the glacier over the observed period (see Table 1), we can see an exponential increase in glacier velocity within the last couple of days. Due to a lack of information about the type of glacier and the surrounding circumstances, it is impossible to conclude with a general prediction. However, under several assumptions, the data can be interpreted as follows:

<u>A cold-based glacier</u>

- Generally, for cold-based glaciers, the surface velocity increases before rupture, following an exponential law
- As the rupture occurs within the ice, and is based on an accumulation of damages, it is possible to predict the break-off time by observing the glacier velocity

➔ Under the assumption that we are talking about a cold-based glacier, a rupture is likely to occur soon and the data would be indeed concerning

A temperate-based glacier

- This means the glacier tongue is temperate and the glacier is sliding at the base
- Ruptures of temperate-based glaciers are caused by massively increased sliding
- The higher velocity is an indicator, but not a sufficient condition for a break-off so it is impossible to predict the time of the break-off solely based on the development of the glacier velocity.
- However, there are several indices for a rupture to occur:
    - A critical geometry of the glacier tongue
    - When the glacier is in an “active phase” (= ongoing speed-up)
    - A distributed drainage network
    - Period of decreasing water input (causing glacier-bed re-coupling and inefficient drainage)
    - Water pulse triggering the final rupture

➔ For a temperate-based glacier, the increased speed is an indication that a fracture could occur soon, but without further information on the criteria listed, it is not possible to make a clear prediction.

Polythermal-based glacier

- This means the glacier is partially cold-, partially temperate-based
- Instabilities usually occur in two phases: A long-lasting phase during which parts of the ice become temperate, and a fast (few days long) rupture phase
- There are no visible changes until a few days before the collapse

➔ If the glacier is polythermal-based it is not possible to predict the rupture.

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Question 2.2 [7 pts]: If you think that the glacier (or parts of it) is likely to break off, when (timing) has this to be expected?

Assuming that the data is obtained from a cold-based glacier, the following equation can be used to calculate the velocity of the glacier:

$$(1)\ v(t) = v_0 + a \cdot (t_c - t)^{-m}$$

where $v(t)$ = the velocity at time t, v_0 = the constant velocity, t_c = time of failure / break off and a and m = fitting parameters (>0).

To calculate the timing when the glacier (or parts of it) will break off, we convert the formula to:

$$(2)\ t_c = t + \sqrt[-m]{\frac{(v(t) - v_0)}{a}}$$

To calculate the timing of the glacier break-off, we first needed to find values for the parameters v_0, *a* and *m* before we could solve the second equation after t_c.

Before using the data, we adjusted the time column to start from 0 by subtracting the starting time (day 5586). This way we only focused on the period that we obtained the data from. A start estimation for v_0 could be calculated as the mean velocity within the stable phase of the glacier (until about day 30). After that, we performed a nonlinear least squares regression to generate a model for the glacier velocity. This method fits a curve to the given dataset and by comparing the relation between the different parameters one can get estimations for the optimized parameters v_0, *a* and *m*. Figure 3 shows our regression line over the given glacier velocity data points. We started with initial values of v_0 = 0,05 (the calculated mean velocity), t_c = 5636 (the last day of the observed period), *m* = 1, and *a* = 1. By doing this regression we get better and better estimations of the parameters ending up at *a* = 27,36, *m* = 2,14, and v_0= 0,04, which leads to the best fit to the data. Together with a random pair of v and t values from the dataset, we could use those parameters at the end in equation 2 to calculate t_c.

According to our results, the glacier will break down in t_c = 5,95 days after the last day of the observed period.

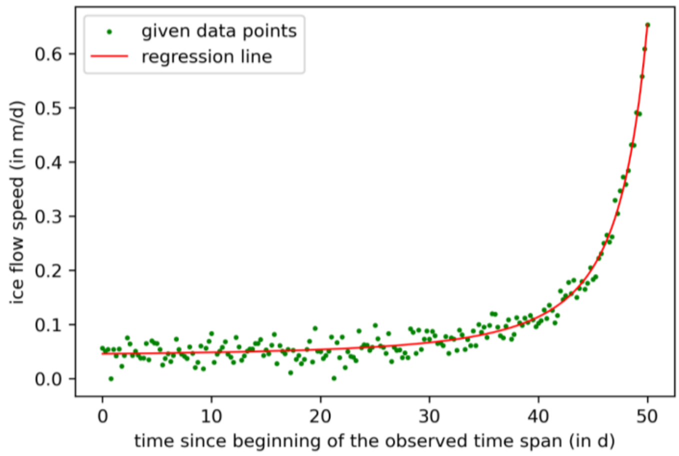

Figure 3 - The glacier velocity data over the observed period together with our calculated regression line.

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Exercise 3: Glacier lake outburst floods

Question 3.1 [6 pts]: How did the lake look like and what was the lake volume in each year? Plot each DEM as a contour-line plot or as a heatmap (i.e. as a map with color-coded elevations), mark the lake extents (referred to below as lake basin) and approximate glacier extents, and express the lake volumes in 10^6 m^3.

Using the Python function plt.contour, we get the following contour line plots for each year:

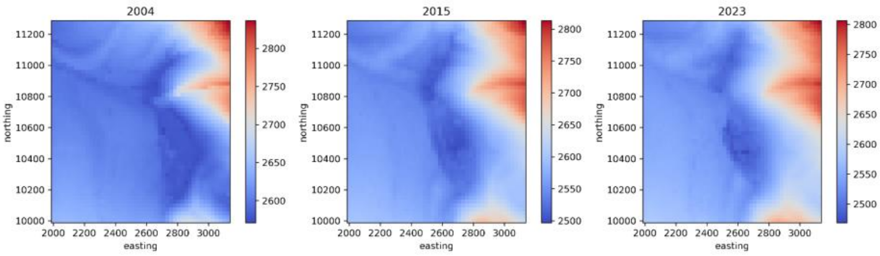

Figure 4 - Contour line plots of the glacier in the years 2004, 2015 and 2023

We can find the areas where the difference in elevation between years is above a significant threshold (in our case, between 2015 and 2023 and >10m) which we approximate to represent the glacier extent. We can then use the areas below the altitude of the overflow point to map out the lake basin for each year. The approximative glacier extent is plotted below in white/blue to mark where the glacier ends and the lake basin for each year is plotted in black:

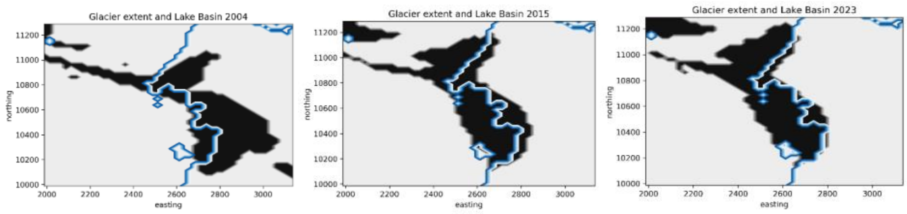

Figure 5 - The approximative extent of the glacier (white/blue line) and the lake basin (black) for the years 2004, 2015 and 2023

We find the altitude of the overflow point and mark it as the surface level of the lake during that year. We then take every grid below that altitude and consider it as the lake bathymetry. Subtracting the lake surface level with its bathymetry gives us the depth at every lake grid, which when multiplied by the grid area gives us its volume. Summing all the lake grid volumes together gives us the lake volumes for 2004, 2015, and 2023, respectively $3.2*10^6$ m^3, $4.4*10^6$ m^3, and $5.0*10^6$ m^3.

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Question 3.2 [6 pts]: What is the value of the parameter k for the data you have been given? Make sure to plot your data and the fitted relation on a log-log plot.

The Clague-Mathews equation is as follows:

$$Q_{max} = kV^{\alpha}$$

With alpha fixed to 2/3.

To find the k, we first have to linearize the equation by applying a logarithm on both sides which gives:

$$\log Q_{max} = \frac{2}{3}\log \mathrm{V} + \log \mathrm{k}$$

With the slope being fixed, we use the method of linear regression, in which we iteratively find the logk that minimizes the square of the error between the predicted and the observed values. This yields a value of k being $2.2*10^{-3}$m/s.

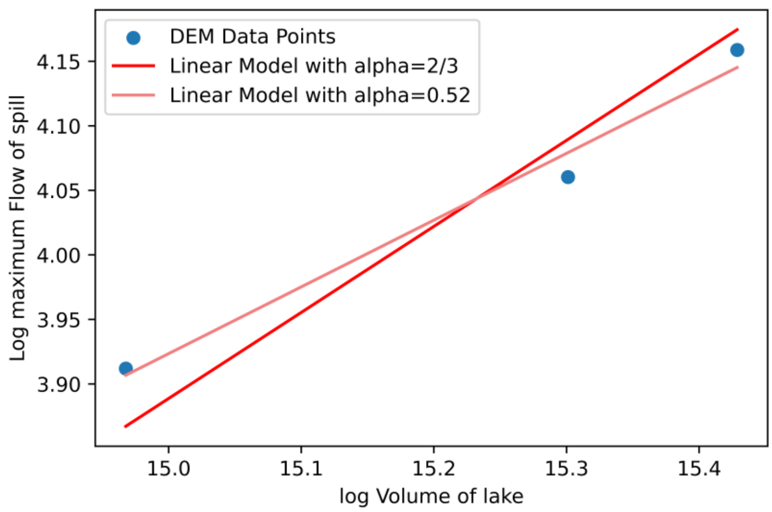

Figure 6 – Linear regression of the log Flows and log volumes for the three years (2004, 2015, 2023) followed the Clague Mathews equation.

We see however that if we apply a linear regression without fixing alpha = 2/3, it fits a slope of 0,51 which seems to fit the value better. The k factor thus also becomes around 10x higher at around $2.2*10^{-2}$m/s.

Question 3.3a [1 pts]: What does the elevation change rate look like? Plot the spatial distribution of the elevation change. Note that outside the glacier, the elevation change is negligible.

We subtract the 2023 DEM from the 2015 DEM, approximatively put a cutoff filter at 10m elevation change between these two years, and divide every value by 8 to transform it to the elevation change rate per year. There are still some cutoff imprecisions likely due to instrument imprecisions or exceptional perturbations during the creation of the two DEMs in 2023 and 2015. More DEMs over the years could smooth out the assessment of the glacier contours. Despite that, we get the following spatial distribution of elevation change, from dark blue meaning little change to bright red meaning large change.

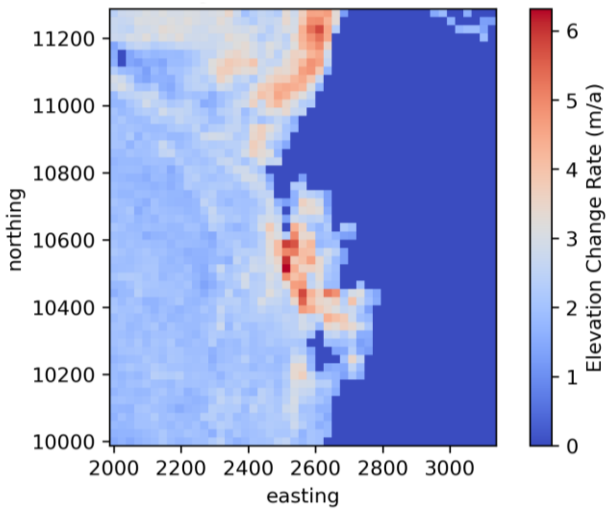

Figure 7 - Elevation change rate of the glacier between 2015 and 2023

We can note that as is common with other glaciers, the glacier tongue is where the most change is happening. The elevation change rate can rise to 6,1m/a (red coloured contours).

Question 3.3b [1 pts]: State the two lowering rates in m/a.

We use a mask to find where the glacier extent and the lake basin in 2023 are to extrapolate their lowering rates. The lake basin has ice-covered and ice-free areas which both have very different lowering rates, which was considered in calculating the overall lake basin lowering rate. The glacier lowering did not take into consideration the parts covered by the lake. With this in mind, the average lowering of the glacier between 2015 and 2023 was 2.15m/a and the average lowering of the lake basin was 2.39 m/a. It can be seen that there is no big change in the lowering rate even if the biggest elevation change rates were in the lake basin. This is because the ice-free part of the lake basin has virtually no elevation change rate, which balances the rates out.

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Question 3.3c [1 pts]: Forecast the lake volume for the years 2024, 2027, and 2032 (if you did not reply to Question 3.3b, assume mean lowering rates on the glacier (sans lake basin) at −3 m/a and in the lake basin at −4 m/a).

We use the glacier lowering rate to forecast where the lake surface elevation will be, and use the lake lowering rate to forecast where the lake bathymetry will be. Since only averages were used to calculate both values, we just simply assume that the lake depth will increase by the lowering rate of the lake basin subtracted by the lowering rate of the glacier. Multiplied by the lake area, this yields a volume increase of $74*10^3$ m^3/a. We can just multiply this value by the number of years between 2023 and the prediction dates to find the volume in 2024, 2027, and 2032. Their volume is predicted to be $5.1*10^6$ m^3, $5.3*10^6$ m^3 and $5.6*10^6$ m^3 respectively.

Kommentiert [MC1]: A really huge step from 2024 to 2027, is a , missing at the other numbers???

Kommentiert [YL2R1]: Oups yeah it was missing a comma

Kommentiert [MC3]: Useful to add the equation here?

Table 1 - Predicted volumes of the glacier lake in the years 2024, 2027 and 2032

2024	2027	2032
$5,1*10^6$ m^3	$5,3*10^6$ m^3	$5,6*10^6$ m^3

Question 3.3d [1 pts]: What are the peak discharges that have to be expected in 2024, 2027, and 2032? Express the results in $m^3 s^{-1}$.

We can simply apply the Clague Mathews equation (see above) and find that the peak discharges as follows:

Table 2 - Predicted peak discharge of the glacier lake in the years 2024, 2027 and 2032

2024	2027	2032
66 m^3/s	68 m^3/s	71 m^3/s

Question 3.3e [2 pts]: What are the uncertainties in the projected lake volumes and what is your qualitative assessment of how big are they? Please state with six sentences or less how those uncertainties arise. How big are thus the uncertainties in the peak discharges you predict and what is the worst-case discharge?

We assume a constant rate of change in glacier height, a fixed spillway location over time, and a direct correlation between lake volume and a simple topography, disregarding variables like temperature and precipitation. Consequently, uncertainties arise regarding the actual glacier topology and lake surface elevation, critical factors in our volume estimations. While we anticipate rather small uncertainty from glacier topology changes over the given timeframe, the primary source of uncertainty stems from accelerated glacier mass and elevation loss due to climate change, which influences spillway elevation, and location along with different temperatures, precipitation, and extreme weather patterns from climate change that could significantly impact lake volume. Our k-parameter from the Clague-Mathews equation might overestimate the predicted peak discharges but ultimately, our assessment indicates substantial uncertainties in peak discharge, particularly since the magnitude of these uncertainties was not precisely quantified. This would mean that the worst-case discharge could be larger than what we predict.

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