Mika Chadzelek
Maike Friedel
Yitao Li

12.12.2023

Assignment 3

Exercise 1: Lake ice

Question 1.1 [7 pts]: How do you comment on the likelihood of this accident to happen?

The area and mass of the PistenBully 100 can be found on the brochure and data profiles found on the PistenBully website [1]. The area where the mass is exerting force on the lake has been assumed only to be on the chain tracks. As the type of chain used is unclear, the smaller version has been considered as it exerts more pressure on the lake than the larger one. The dimensions are given below:

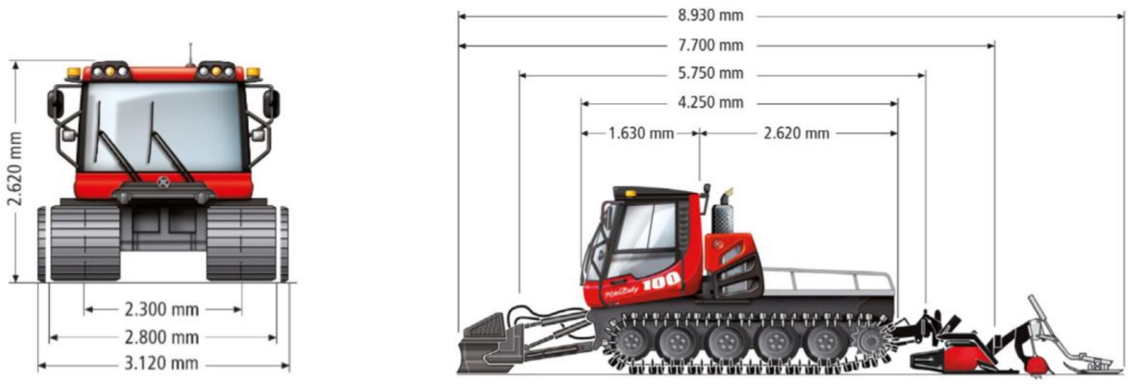

Figure 1 - Piston Bully 100 dimensions

With these information, the area where the chain track is exerting pressure on the ice lake is calculated to be 2*500mm*4250mm = 4,25m^2.

Furthermore, the weight of the snow grooming machine can be found below:

Weights		
Dead weight	with combination chain	from 5,000 kg
	with X-Track chain	from 4,760 kg
Permissible total weight		8,500 kg
Payload loading area		1,500 kg

Table 1: Table of details of the pisten bully

We have decided to take the largest weight estimation as to ensure no underestimation. We therefore assumed the machine to be weighing 8,500kg.

From there, we can use three different considerations to estimate the maximum load, which give three different appreciations of the likelihood of and accident:

Consideration 1: Everything should stay afloat.

To do so, the simple Archimedes principle is applied to the ice on the ice lake:

$$M_{potential} = (\rho_w - \rho_{ice}) \cdot h_{ice} \cdot \Delta x \cdot \Delta y$$

With ρ_w = density of water (1000 $\frac{kg}{m^3}$), ρ_{ice} = density of the ice (917 $\frac{kg}{m^3}$), Δx and Δy = the side lengths of the area, h_{ice} = thickness of the ice (here 0,28 m).

We assume that $\Delta x \cdot \Delta y$ can be assumed to be the area of the chain tracks, and using the equation above with the known values, we get:

$$M_{potential} = (1000 - 917) \cdot 0.28 \cdot 4.25 = 98.77\ [kg]$$

which is lower than our assumed weight of the Pistenbully.

Therefore, to stay afloat the piston bully (8500 kg) would require an ice height of 24 meters or an area of 366m². This result shows that under this first very simplistic consideration, the likelihood of accident is very high.

Consideration 2: Ice can bend

With this consideration, the maximum permissible (cylindrically) distributed load for short time scales (<100 [s]) is given by the following formula:

$$P_w = \frac{n\, h_{ice} \rho_w g}{1 + aC_0} \pi r^2$$

with n, C_0, a and lambda are given as

$$n = 1 - \frac{\rho_{ice}}{\rho_{water}}$$

$$C_0 = -\frac{1}{a} + \frac{1}{8}\pi a - \frac{1}{16}(1.3659 - \ln a)a^3$$

$$a = \frac{r}{\lambda}$$

$$\lambda = \left(\frac{E \cdot h_{ice}^3}{12 \cdot \rho_w \cdot g \cdot (1 - \mu^2)}\right)^{\frac{1}{4}}$$

and with E = Young's (or elastic) modulus (≈ 8.7 GPa), g = gravitational acceleration (≈ 9.81 m/s²) and μ = Poisson's ratio for ice (≈ 0.33).

r in our equations symbolizes the radius of our cylindrically distributed load. We can assume that the 4.25m² area covered can be modelled as a circular area of radius r = 1.16m in order to make sense in the limit load calculation.

We can input previously known values and get the following results:

$$\lambda = \left(\frac{8.7 * 10^9 \cdot (0.28)^3}{12 \cdot 1000 \cdot 9.81 \cdot (1 - 0.33^2)}\right)^{\frac{1}{4}} = 6.53\ [m]$$

$$a = \frac{1.16}{6.53} = 0.177$$

$$C_0 = -\frac{1}{0.177} + \frac{1}{8}\pi * 0.177 - \frac{1}{16}(1.3659 - \ln(0.177)) * 0.177^3 = -5.56$$

$$n = 1 - \frac{917}{1000} = 0.083$$

This gives us the maximum distributed load of:

$$P_w = \frac{0.083 * \ 0.28 * 1000 * 9.81}{1 + 0.177 * \ -5.56} \pi 1.16^2 = 78999[N] \cong 8053[kg]$$

which is lower than our assumed weight of the Pistenbully.

In conclusion, this consideration makes it likely that if the piston bully stayed longer than 100 seconds in a static position, the likelihood of the accident is very high. However, the faster the piston bully is, the shorter duration the ice needs to stretch, which makes the likelihood of the accident lower. In summary, the ice height is not sufficient to provide safe practice if we only consider that the ice can bend.

Consideration 3: Ice can break.

P_{cr} is the load at which the first cracks occur, but empirical tests show that the load at which failure occurs P_f is higher and approximated as follows:

$$P_f^U \approx 2 \cdot P_{cr}$$

$$P_{cr} = \frac{\sigma_f \cdot \ h_{ice}^2}{3 \cdot (1 + \mu) \cdot C_1}$$

With σ_f = Flexural strength of ice (≈ 1.5 MPa) and μ = Poisson's ratio for ice (≈ 0.33) . Other variables include:

$$C_1 = \frac{(0{,}619 - \ln{(a)} \cdot \frac{a}{2} + \frac{\pi \cdot a^3}{64}}{\pi \cdot a} + \cdots$$

$$a = \frac{r}{\lambda}$$

With r = characteristic size of the loaded area (again 1,16 m) and λ = the characteristic (or elastic) length calculated above. The variable a therefore has the same value as above. This then leads to:

$$C_1 = \frac{(0{,}619 - \ln(0.177)) \cdot \frac{0.177}{2} + \frac{\pi \cdot (0.177)^3}{64}}{\pi \cdot 0.177} = 0.374$$

This finally gives us:

$$P_{cr} = \frac{1.5 * 10^6 \cdot \ 0.28^2}{3 \cdot (1 + 0.33) \cdot 0.374} = 78'806[N]$$

And gives us a final limit weight of:

$$P_f^U \approx 2 \cdot P_{cr} = 157'613[N] \cong 16'065\ [kg]$$

which is higher than our assumed weight of the Pistenbully.

Conclusion

Taking into account these three considerations, it can be seen that 2/3 show high likelihood of accident. However, considering the ice thickness (0.28m), consideration 3 is the most important one and consideration 1 and 2 are of minor importance. The last consideration however shows that using the piston bully is safe and has a very small likelihood of accident. We have assumed that factors such as the temperature increase was not relevant and that the type of ice and quality of ice is good and uniform over the entire lake surface.

With these assumptions, we further assume that the first two considerations are not enough to reflect the reality of the load capacity of the lake ice. We find the last consideration to be the most realistic under perfect conditions, and therefore believe that the likelihood of accident is low.

We have to remind that many assumptions were made, such as the type of ice, the weight of the Piston Bully 100, the area of the chain track, the influence of the chain track profile, and the values of the structural strength of ice and its elastic modulus, which both show great variability. These informations are lacking and need to be included to do a proper analysis of the likelihood of the accident.

Mika Chadzelek
Maike Friedel
Yitao Li

12.12.2023

Question 1.2 [4 pts]: Is the temperature change likely to have played a role? Why?

Yes, it is likely to have contributed to the instability:

Due to high increase of temperature, the ice close to the air expanded, however the bottom part of the ice that touches the lake stayed at a similar temperature which might have produced extension cracks that might not be that good visible from the top. The expansion can be calculated by multiplying the distance and temperature with the thermal coefficient:

$$a \approx 5{,}0 \times 10^{-5}\ °C^{-1}\ [2]$$

Knowing the extent of the lake, we can calculate the total displacement in the shortest and longest direction:

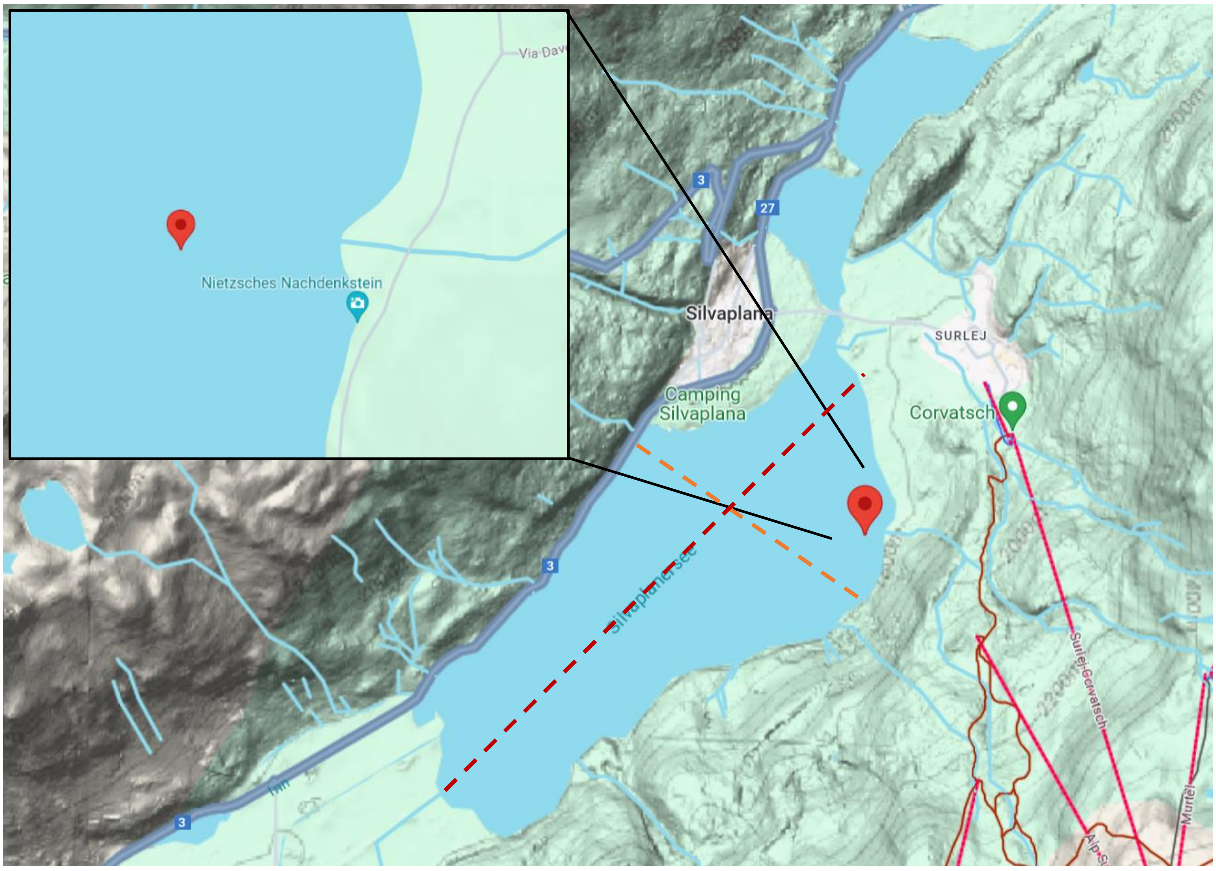

Figure 2: Location of the accident on the Lake Silvaplaner (source: google maps); red dashed line is air distance a (3 km), orange dashed line is air distance b (1.42 km)

In the long direction of the lake (a, red dashed line, 3 km), we expect an expansion of 1.05 m. In the short direction (b, orange dashed line, 1.42 km) an expansion of 0.50 m is expected. These expansions will occur in the form of extension cracks caused by the rise in temperature. As we do not know where exactly the cracks have formed, this could affect the ability of the ice to keep the Pistenbully afloat if they are all close to each other and in the vicinity of the Pistenbully (see Consideration 1 in Task 1.1).

In addition, the temperature rise might have also led to an increased temperature of inflowing water from e.g. streams, which as a result have been warmer than the lake water. This could have caused local instabilities near the inflowing points as the “warmer” water entered until it mixed with the colder lake water. The point where the accident happened, was closely located to such an incoming stream (see figure 2).

From the later investigations, we know that the local temperature rose from -11 to -4° C; however, it could have reached even higher values as these are just averaged temperatures for the whole day. Consequently, during midday, temperatures could have surpassed 0°C, the critical temperature of ice melting, leading to a reduction in the ice layer of the top ice levels (which was black ice and more stable).

Question 1.3 [4 pts]: Are there other factors that may have played a role?

A stream is nearby the place where the accident happened (see figure 2). This might have influenced the ice's stability and might potentially have caused cracks due to an increasing water temperature by water movement producing mixing of the colder surface water and the warmer lower water. Furthermore, the stream water itself might have been warmer than the lake water due to the reported increase in air temperature. Although the ice has a specific thickness (in our case 28cm), the density and stability are not the same everywhere. The equations in task one are assumed for so called "black ice" (primary ice), however, there can be a top of snow ice (superimposed ice) and spring ice (secondary (congelation) ice) at the bottom, which are not as stable as the primary ice.

It would also be important to consider the meteorological conditions on the day. Factors like high solar radiation from clear sky or strong winds could also have led to increased sublimation and produce cracking of the lake ice, however we expect this impact to be rather small. If there were pre-existing small cracks that went unnoticed, they could have also contributed to instability of the ice. In addition to that, any activity like ice fishing with making a hole into the ice would be a reason for the development of cracks.

The duration of time spent at a specific spot, such as for a smoking break, could have gradually reduced the ice's bearing capacity, especially if constant loading persisted for an extended period, with a noticeable decrease after 2 hours and 20 minutes. At the same time, the frequency and speed at which a tractor passed through the area can also be a factor, as repeated passages over time could weaken the ice.

The profile of the chain tracks could also exert an additional high localised pressure to the ice. Piston Bully 100 has two different chains that could have been used for the maintenance of the runway, with different profiles and material types. One is made of rubber and the other one of steel, so the pressure exerted on the ice could be different depending on the material. The vibration of the machine should also be considered as it could have weakened the structural integrity and stability of the ice.

Sources

[1] PistenBully 100. (o. J.). PistenBully | Pistenraupen & Pistenfahrzeuge. Abgerufen 9. Dezember 2023, von https://www.pistenbully.com/fahrzeuge/pistenbully-100

[2] Butkovich, T. R. (1959). Thermal expansion of ice. *Journal of Applied Physics*, *30*(3), 350–353. https://doi.org/10.1063/1.1735166