Introduction to Formal Concept Analysis

Yannick Toussaint[1]
ENSMN R407, LORIA B160
Yannick.Toussaint@loria.fr

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Summary of the lecture

Formal Concepts and Concept Lattices

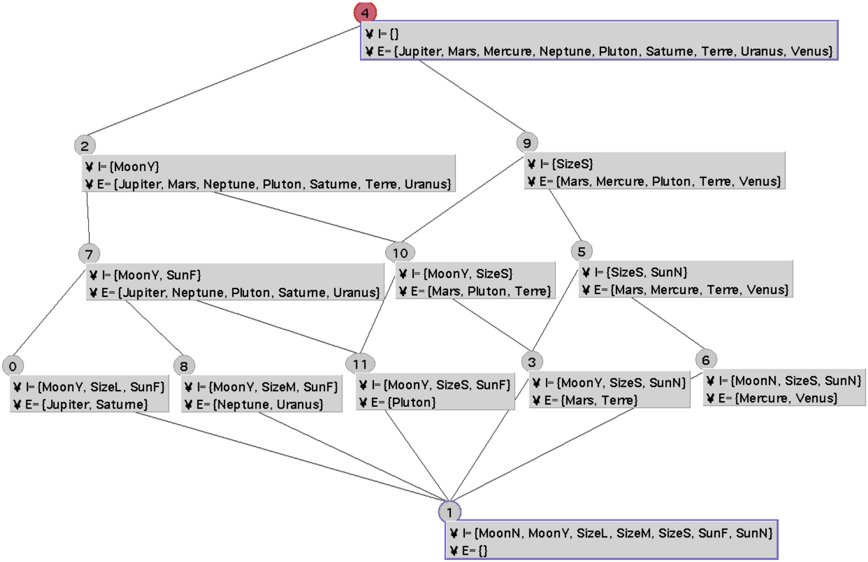
The approach of FCA

- The basic procedure of Formal Concept Analysis (FCA) is based on a simple representation of data, i.e. a binary table called a formal context.
- Each formal context is transformed into a mathematical structure called concept lattice.
- The information contained in the formal context is preserved.
- The concept lattice is the basis for data analysis.
  It is represented graphically to support communication, analysis, and interpretation.

The context of planets

Planet	Size			Distance to Sun		Moon(s)	
	small	medium	large	near	far	yes	no
Jupiter			x		x	x	
Mars	x			x		x	
Mercure	x			x			x
Neptune		x			x	x	
Pluton	x				x	x	
Saturne			x		x	x	
Terre	x			x		x	
Uranus		x			x	x	
Venus	x			x			x

The concept lattice of planets



The notion of a formal context

Objects / Attributes	m1	m2	m3	m4
g1	x	x	x	x
g2	x	x		
g3		x	x	x
g4		x		
g5		x	x	

- $(\mathtt{G}, \mathtt{M}, \mathtt{I})$ is called a formal context where $\mathtt{G}$ and $\mathtt{M}$ are sets, and $\mathtt{I} \subseteq \mathtt{G} \times \mathtt{M}$ is a binary relation between $\mathtt{G}$ and $\mathtt{M}$.
- The elements of $\mathtt{G}$ are the objects, while the elements of $\mathtt{M}$ are the attributes, $\mathtt{I}$ is the incidence relation of the context $(\mathtt{G}, \mathtt{M}, \mathtt{I})$.

Basic notations

- For $\mathtt{A} \subseteq \mathtt{G}$ we define:
  $\mathtt{A}' = \{\mathtt{m} \in \mathtt{M}/(\mathtt{g}, \mathtt{m}) \in \mathtt{I} \texttt{ for all } \mathtt{g} \in \mathtt{A}\}$
- Dually, for $\mathtt{B} \subseteq \mathtt{M}$ we define:
  $\mathtt{B}' = \{\mathtt{g} \in \mathtt{G}/(\mathtt{g}, \mathtt{m}) \in \mathtt{I} \texttt{ for all } \mathtt{m} \in \mathtt{B}\}$

The derivation operators and the Galois connection

The derivation operators establish a Galois connection between the power sets $\wp(\mathtt{G})$ and $\wp(\mathtt{M})$ (and thereby a dual isomorphism between two closure systems).

A Galois connection is defined as follows:

- Let $\mathtt{P}$ and $\mathtt{Q}$ be ordered sets.
  A pair of maps $\phi : \mathtt{P} \longrightarrow \mathtt{Q}$ and $\psi : \mathtt{Q} \longrightarrow \mathtt{P}$ is called a Galois connection between $\mathtt{P}$ and $\mathtt{Q}$ if:
  - (i) $\mathtt{p}_1 \leq \mathtt{p}_2 \Longrightarrow \phi(\mathtt{p}_1) \geq \phi(\mathtt{p}_2)$
  - (ii) $\mathtt{q}_1 \leq \mathtt{q}_2 \Longrightarrow \psi(\mathtt{q}_1) \geq \psi(\mathtt{q}_2)$
  - (iii) $\mathtt{p} \leq \psi \circ \phi(\mathtt{p})$ and $\mathtt{q} \leq \phi \circ \psi(\mathtt{q})$

The Galois connection and the closure operators

- $' : \wp(\mathtt{G}) \longrightarrow \wp(\mathtt{M})$ with $\mathtt{A} \longrightarrow \mathtt{A}'$
- $' : \wp(\mathtt{M}) \longrightarrow \wp(\mathtt{G})$ with $\mathtt{B} \longrightarrow \mathtt{B}'$
- These two applications induce a Galois connection between $\wp(\mathtt{G})$ and $\wp(\mathtt{M})$ when sets are ordered by set inclusion relation.
- The composition operators $''$ are closure operators.

A closure operator on a set $\mathtt{H}$ is a map κ such that:

- $\kappa : \wp(\mathtt{H}) \longrightarrow \wp(\mathtt{H})$
- For all $\mathtt{A}_1, \mathtt{A}_2 \subseteq \mathtt{H}$:
  - (i) $\mathtt{A}_1 \subseteq \kappa(\mathtt{A}_1)$ (idempotency)
  - (ii) $\mathtt{A}_1 \subseteq \mathtt{A}_2$ then $\kappa(\mathtt{A}_1) \subseteq \kappa(\mathtt{A}_2)$ (monotonicity)
  - (iii) $\kappa(\kappa(\mathtt{A}_1)) = \kappa(\mathtt{A}_1)$ (extensitivity)

The notion of formal concept

Given a formal context $(\mathtt{G}, \mathtt{M}, \mathtt{I})$:

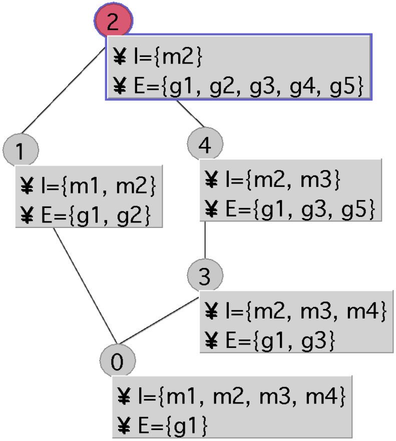
- $\mathtt{A}' = \{\mathtt{m} \in \mathtt{M}/(\mathtt{g}, \mathtt{m}) \in \mathtt{I} \text{ for all } \mathtt{g} \in \mathtt{A}\}$
- $\mathtt{B}' = \{\mathtt{g} \in \mathtt{G}/(\mathtt{g}, \mathtt{m}) \in \mathtt{I} \text{ for all } \mathtt{m} \in \mathtt{B}\}$
- $(\mathtt{A}, \mathtt{B})$ is a formal concept of $(\mathtt{G}, \mathtt{M}, \mathtt{I})$ iff:
  $\mathtt{A} \subseteq \mathtt{G}$, $\mathtt{B} \subseteq \mathtt{M}$, $\mathtt{A}' = \mathtt{B}$, and $\mathtt{A} = \mathtt{B}'$.
- $\mathtt{A}$ is the extent and $\mathtt{B}$ is the intent of $(\mathtt{A}, \mathtt{B})$.
- The mappings $\mathtt{A} \longrightarrow \mathtt{A}''$ and $\mathtt{B} \longrightarrow \mathtt{B}''$ are closure operators.

The notion of concept lattice

- Formal concepts can be ordered by:
  $(\mathtt{A}_1, \mathtt{B}_1) \leq (\mathtt{A}_2, \mathtt{B}_2) \iff \mathtt{A}_1 \subseteq \mathtt{A}_2$ (dually $\mathtt{B}_2 \subseteq \mathtt{B}_1$).
- The set $\underline{\mathfrak{B}}(\mathtt{G}, \mathtt{M}, \mathtt{I})$ of all formal concepts of $(\mathtt{G}, \mathtt{M}, \mathtt{I})$ with this order is a complete lattice called the concept lattice of $(\mathtt{G}, \mathtt{M}, \mathtt{I})$.
- Recall that: a set $(\mathtt{P}, \leq)$ is a complete lattice if the supremum $\bigvee \mathtt{S}$ and the infimum $\bigwedge \mathtt{S}$ exist for any subset $\mathtt{S}$ of $\mathtt{P}$.
- Every complete lattice has a top or unit element denoted by $\top$, and a bottom or zero element denoted by $\perp$.

An example

Objects / Attributes	m1	m2	m3	m4
g1	×	×	×	×
g2	×	×		
g3		×	×	×
g4		×		
g5		×	×	



The basic theorem of FCA

Given two concepts (A_1, B_1) and (A_2, B_2) of the lattice $\underline{\mathfrak{B}}(\mathtt{G}, \mathtt{M}, \mathtt{I})$

- the infimum, the greatest commun subconcept,

$$(A_1, B_1) \wedge (A_2, B_2) = (A_1 \cap A_2, (B_1 \cup B_2)'')$$

- the supremum, the least general superconcept,

$$(A_1, B_1) \vee (A_2, B_2) = ((A_1 \cup A_2)'', B_1 \cap B_2)$$

- Note: an intersection of closed sets is a closed set but a union of closed sets is not necessarily a closed set.

What if the data set is large?

With growing size of the data set, it quickly becomes unreasonable to display the full data in a single lattice diagram.
The theory of Formal Concept Analysis offers methods to:

- split large diagrams into smaller ones, so that the information content is preserved,
- browse through lattices and thereby build conceptual landscapes of information,

Examples are given hereafter (conceptual scaling).

The need for scaling

- There are many series of formal contexts that have an suggestive interpretation. Such formal contexts will be called scales.
- So formally, a scale is the same as a formal context. But it is meant to have a special interpretation.
- Examples of scales are nominal, ordinal, and dichotomic scales.

Conceptual scaling

- The basic data type of Formal Concept Analysis is that of a formal context.
  But data is often given in form of a many-valued context.
- Many-valued contexts are translated to one-valued context via conceptual scaling.
  This is not automatic; it is an act of interpretation.

The example of the context of planets

Planet	Size			Distance to Sun		Moon(s)	
Jupiter			large		far	yes	
Mars	small			near		yes	
Mercure	small			near			no
Neptune		medium			far	yes	
Pluton	small				far	yes	
Saturne			large		far	yes	
Terre	small			near		yes	
Uranus		medium			far	yes	
Venus	small			near			no

The context of planets after nominal scaling

Planet	Size			Distance to Sun		Moon(s)	
	small	medium	large	near	far	yes	no
Jupiter			x		x	x	
Mars	x			x		x	
Mercure	x			x			x
Neptune		x			x	x	
Pluton	x				x	x	
Saturne			x		x	x	
Terre	x			x		x	
Uranus		x			x	x	
Venus	x			x			x

Examples of scaling

- Nominal: $\mathtt{K} = (\mathtt{N}, \mathtt{N}, =)$
- Ordinal: $\mathtt{K} = (\mathtt{N}, \mathtt{N}, \leq)$
- Interordinal: $\mathtt{K} = (\mathtt{N}, \mathtt{N}, \leq \cup \geq)$

About the derivation operators

About the derivation operators

The derivation operator $'$ satisfy the following rules:

- $\mathtt{A}' = \{\mathtt{m} \in \mathtt{M}/(\mathtt{g}, \mathtt{m}) \in \mathtt{I} \texttt{ for all } \mathtt{g} \in \mathtt{A}\}$
- $\mathtt{B}' = \{\mathtt{g} \in \mathtt{G}/(\mathtt{g}, \mathtt{m}) \in \mathtt{I} \texttt{ for all } \mathtt{m} \in \mathtt{B}\}$

Iterating derivation

- The derivation operators can be combined: Starting with a set $\mathtt{A} \subseteq \mathtt{G}$, we obtain that $\mathtt{A}'$ is a subset of $\mathtt{M}$.
- Applying the second operator on this set, we get $(\mathtt{A}')'$, or $\mathtt{A}''$ for short, which is a set of objects.
- Continuing, we obtain $\mathtt{A}'''$, $\mathtt{A}''''$, and so on.
- It can be noticed that all these sets are not different!

Properties of the derivation operators

The derivation operator $'$ satisfy the following rules:

- $\mathtt{A}' = \{\mathtt{m} \in \mathtt{M}/(\mathtt{g},\mathtt{m}) \in \mathtt{I} \text{ for all } \mathtt{g} \in \mathtt{A}\}$
- $\mathtt{B}' = \{\mathtt{g} \in \mathtt{G}/(\mathtt{g},\mathtt{m}) \in \mathtt{I} \text{ for all } \mathtt{m} \in \mathtt{B}\}$

- $\mathtt{A}_1 \subseteq \mathtt{A}_2 \Longrightarrow \mathtt{A}_2' \subseteq \mathtt{A}_1'$
- $\mathtt{B}_1 \subseteq \mathtt{B}_2 \Longrightarrow \mathtt{B}_2' \subseteq \mathtt{B}_1'$

- $\mathtt{A} \subseteq \mathtt{A}''$ and $\mathtt{A}' = \mathtt{A}'''$
- $\mathtt{B} \subseteq \mathtt{B}''$ and $\mathtt{B}' = \mathtt{B}'''$

Examples

Objects / Attributes	m1	m2	m3	m4
g1	x	x	x	x
g2	x	x		
g3		x	x	x
g4		x		
g5		x	x	

- $\mathtt{A}_1 \subseteq \mathtt{A}_2 \Longrightarrow \mathtt{A}_2' \subseteq \mathtt{A}_1'$
  $\mathtt{B}_1 \subseteq \mathtt{B}_2 \Longrightarrow \mathtt{B}_2' \subseteq \mathtt{B}_1'$
- $\mathtt{A} \subseteq \mathtt{A}''$ and $\mathtt{A}' = \mathtt{A}'''$
  $\mathtt{B} \subseteq \mathtt{B}''$ and $\mathtt{B}' = \mathtt{B}'''$

Extent closure, intent closure

The derivation operator $'$ satisfy the following rules:

- For $\mathtt{A} \subseteq \mathtt{G}$ we have that $\mathtt{A}'' \subseteq \mathtt{G}$.
  The set $\mathtt{A}''$ is called the extent closure of $\mathtt{A}$.
- Dually, when $\mathtt{B} \subseteq \mathtt{M}$ we have also that $\mathtt{B}'' \subseteq \mathtt{M}$.
  The set $\mathtt{B}''$ is called the intent closure of $\mathtt{A}$.

Extent closure, intent closure

From $\mathtt{A} \subseteq \mathtt{A}''$ and $\mathtt{B} \subseteq \mathtt{B}''$ it comes:

- (i) whenever all objects from a set $\mathtt{A} \subseteq \mathtt{G}$ have a common attribute $\mathtt{m}$, then also all objects from $\mathtt{A}''$ have that attribute.
- (ii) whenever an object $\mathtt{g} \in \mathtt{G}$ has all attributes from $\mathtt{B} \subseteq \mathtt{M}$, then this object also has all attributes from $\mathtt{B}''$.

Other properties of the derivation operators

For $\mathtt{A}_1, \mathtt{A}_2 \subseteq \mathtt{G}$, and dually for $\mathtt{B}_1, \mathtt{B}_2 \subseteq \mathtt{M}$, we have:

- $\mathtt{A}_1 \subseteq \mathtt{A}_2 \Longrightarrow \mathtt{A}_1'' \subseteq \mathtt{A}_2''$
- $\mathtt{B}_1 \subseteq \mathtt{B}_2 \Longrightarrow \mathtt{B}_1'' \subseteq \mathtt{B}_2''$
- $(\mathtt{A}'')'' = \mathtt{A}''$
- $(\mathtt{B}'')'' = \mathtt{B}''$

Closure operators

- As already mentioned, the operators $'$ and $''$ satisfying the above properties are closure operators.
- The sets which are images of a closure operator are the closed sets.
- Thus, in the case of a closure operator $\mathtt{X} \longrightarrow \mathtt{X}''$ the closed sets are the sets of the form $\mathtt{X}''$.

Closed sets are intents and extents

- If $(\mathtt{G}, \mathtt{M}, \mathtt{I})$ is a formal context and $\mathtt{A} \subseteq \mathtt{G}$, then $\mathtt{A}''$ is an extent.
- Conversely, if $\mathtt{A}$ is an extent of $(\mathtt{G}, \mathtt{M}, \mathtt{I})$, then $\mathtt{A} = \mathtt{A}''$.
- Dually if $\mathtt{B}$ is an intent of $(\mathtt{G}, \mathtt{M}, \mathtt{I})$, then $\mathtt{B} = \mathtt{B}''$, and every intent $\mathtt{B}$ satisfies $\mathtt{B} = \mathtt{B}''$.
- This follows from the fact that for each subset $\mathtt{A} \subseteq \mathtt{G}$, the pair $(\mathtt{A}'', \mathtt{A}')$ is a formal concept, and that similarly, for each subset $\mathtt{B} \subseteq \mathtt{M}$, $(\mathtt{B}', \mathtt{B}'')$ is a formal concept.
- Therefore, the closed sets of the closure operator $\mathtt{A} \longrightarrow \mathtt{A}'', \mathtt{A} \subseteq \mathtt{G}$ are precisely the extents of $(\mathtt{G}, \mathtt{M}, \mathtt{I})$,
  and the closed sets of the operator $\mathtt{B} \longrightarrow \mathtt{B}'', \mathtt{B} \subseteq \mathtt{M}$, are precisely the intents.

The structure of the concept lattice

The reduced labeling

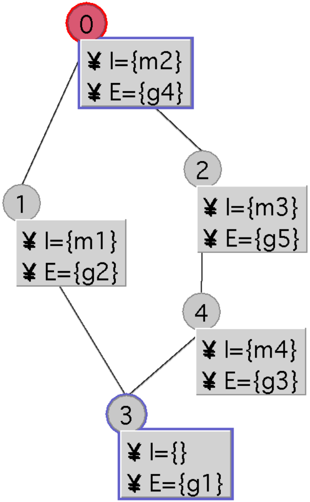
- A reduced labeling may be used allowing that each object and each attribute is entered only once in a diagram.
- The name of the object g is attached to the "lower half" of the corresponding object concept $\gamma(g) = (\{g\}'', \{g\}')$.
- The object concept of an object $g \in \mathtt{G}$ is the concept $(\{g\}'', \{g\}')$ where $\{g\}'$ is the object intent $\{\mathtt{m} \in \mathtt{M}/\mathtt{gIm}\}$ of g.
- The object concept of g, denoted by $\gamma(g)$, is the smallest concept (for the lattice order) with g in its extent.

The reduced labeling

- The name of the attribute $\mathtt{m}$ is located to the "upper half" of the corresponding attribute concept $\mu(\mathtt{m}) = (\{\mathtt{m}\}', \{\mathtt{m}\}'')$.
- Correspondingly, the attribute concept of an attribute $\mathtt{m} \in \mathtt{M}$ is the concept $(\{\mathtt{m}\}', \{\mathtt{m}\}'')$ where $\{\mathtt{m}\}'$ is the attribute extent $\{\mathtt{g} \in \mathtt{G}/\mathtt{gIm}\}$ of $\mathtt{m}$.
- The attribute concept of $\mathtt{m}$, denoted by $\mu(\mathtt{m})$ is the largest concept (for the lattice order) with $\mathtt{m}$ in its intent.

An example

Objects / Attributes	m1	m2	m3	m4
g1	×	×	×	×
g2	×	×		
g3		×	×	×
g4		×		
g5		×	×	



The reduced labeling (3)

- For any concept $(\mathtt{A}, \mathtt{B})$ we have:
- $\mathtt{g} \in \mathtt{A} \iff \gamma(\mathtt{g}) \leq (\mathtt{A}, \mathtt{B})$
- $\mathtt{m} \in \mathtt{B} \iff (\mathtt{A}, \mathtt{B}) \leq \mu(\mathtt{m})$

Largest and smallest concepts

- Given a concept lattice, the context associated to the lattice can be read using the following general rule:
  $(\mathtt{g}, \mathtt{m}) \in \mathtt{I} \iff \gamma(\mathtt{g}) \leq \mu(\mathtt{m})$
- Just as a set of concepts can be uniquely determined from a given context, so the context can be reconstructed from its concepts.
- The set $\mathtt{G}$ is the extent of the largest concept $(\mathtt{G}, \mathtt{G}')$.
- The set $\mathtt{M}$ is the intent of the smallest concept $(\mathtt{M}', \mathtt{M})$.

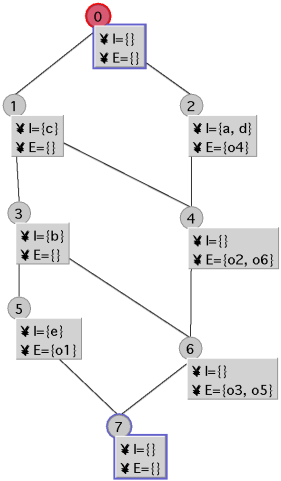
The implications in a formal context

Implications in a formal context (exact rules)

- Let $(\mathtt{G}, \mathtt{M}, \mathtt{I})$ be a formal context and let $\mathtt{A}, \mathtt{B} \subseteq \mathtt{M}$.
- We say that the implication $\mathtt{A} \longrightarrow \mathtt{B}$ holds in $(\mathtt{G}, \mathtt{M}, \mathtt{I})$ iff every object that has all the attributes from $\mathtt{A}$ also has all the attributes from $\mathtt{B}$.
- There are simple rules for implication inference in formal contexts (Armstrong rules).
- The theory of contextual implications can be applied to functional dependencies, association rules.

An example

Extracting rules from the lattice: mutual implications between local attributes



- Equivalence between local attributes: $\mathtt{a} \longleftrightarrow \mathtt{d}$ for $(23456, \mathtt{ad})$.
- Local attributes imply the inherited attributes: $\mathtt{b} \longrightarrow \mathtt{c}$ for $(135, \mathtt{bc})$ and $\mathtt{e} \longrightarrow \mathtt{bc}$ for $(1, \mathtt{bce})$.

Example

Contrast set mining

Constrat set mining

- A well established data mining area
- Aims at discovering conjunctions of attributes and values that differ meaningfully in their distributions across groups
- Many techniques such as subgroup discovery and emerging patterns
- Use of discriminative power
- Contrast sets are highly useful in supervised tasks to solve real world problems in many domains

Emerging Patterns and Jumping Emerging Patterns

- Consider a dataset of objects partitioned into several classes, each object being described by binary attributes.
- Emerging patterns (EPs) are patterns whose frequency strongly varies between two datasets.
- A Jumping Emerging Pattern (JEP) is an EP which has the notable property to occur only in a single class.

Emerging Patterns and Jumping Emerging Patterns

- JEPs are greatly valuable to obtain highly accurate rule-based classifiers
- A minimal JEP designates a JEP where none of its proper subsets is a JEP.
- Minimal JEPs also called Hypothesis
- Using more attributes may not help and even add noise in a classification purpose.

An experiment on word desambiguisation

Candidate	Frequency	Positive Occurrences	Terminological Degree	Category
Adjective	216	207	95.83%	highly terminological
Reception	47	41	87.23%	highly terminological
Collocation	109	90	82.56%	highly terminological
Sentence	311	238	76.52%	enough terminological
Speaker	233	178	76.39%	enough terminological
Corpus	688	510	74.12%	enough terminological
Language	926	549	59.28%	ambiguous
Statement	289	164	56.74%	ambiguous
Context	302	147	48.67%	ambiguous
Text	568	266	46.83%	ambiguous
Speech	534	248	46.44%	ambiguous
Form	462	122	26.40%	slightly terminological
Relation	676	171	25.29%	slightly terminological
Expression	197	48	24.36%	slightly terminological
Semantic	413	80	19.37%	very slightly terminological
Lexical	477	84	17.61%	very slightly terminological
Model	250	13	5.20%	very slightly terminological

An experiment on word desambiguisation

Word	Frequency	Positive Examples	Terminolog. Degree	Words used only in T_+	Hypotheses Generated from T_+	Proportion of Positive Hypotheses	Shared Words	Negative Examples	Words used only in T_-	Hypotheses Generated from T_-
Adjective	216	207	95.83%	966	301	97,41%	64	9	59	8
Corpus	688	510	74.12%	1035	1347	81,93%	713	178	535	297
Text	568	266	46.83%	735	913	52,32%	772	302	792	832
Relation	676	171	25.29%	159	183	11,48%	629	505	1427	1410
Semantic	413	80	19.37%	272	108	8,88%	560	333	1258	1107

An experiment on word desambiguisation

Candidate	Category	Freq.	Ex2tag	Hypotheses Generated (+)	Hypotheses Projected (+)	Hypotheses Generated (-)	Hypotheses Projected (-)	Unclassified Positive Examples	Unclassified Negative Examples
Adjective	highly term.	216	27	263	46	7	0	1.375	0.125
Reception	highly term.	47	5.87	28	3	5	0	1.571	0.714
Collocation	highly term.	109	13.62	132	10	16	0	3.5	0.875
Sentence	enough term.	311	38.87	394	94	77	14	4	1.875
Speaker	enough term.	233	29.12	397	96	64	10	2.875	0.25
Corpus	enough term.	688	86	1126	268	249	61	25.5	5.25
Language	ambig.	926	115,75	1201	418	617	231	21.5	13.125
Statement	ambig.	289	36.12	309	51	146	18	5.5	2.125
Context	ambig.	302	37.75	261	76	326	74	4.5	4.5
Text	ambig.	568	71	757	194	706	145	7.625	5.75
Speech	ambig.	534	66.75	396	80	535	83	5.75	6.125
Form	slightly term.	462	57.75	134	33	955	333	1.125	4.875
Relation	slightly term.	676	142,25	160	16	1172	244	1.5	9.5
Expression	slightly term.	197	24.62	44	5	275	67	1.25	3.0
Semantic	very slightly term.	413	51.62	92	20	915	288	0.5	5.5
Lexical	very slightly term.	477	59.62	77	12	950	312	0.375	2.625
Model	very slightly term.	250	31.25	8	0	384	80	0	1

Positive description of occurrences

Support	Stability	Hypotheses in T_+	Hypotheses in T_+ *-english-*
7	0.7968	[sdrt, être, argument]	[sdrt, be, argument]
9	0.7792	[argument, plus]	[argument, more]
6	0.73437	[être, argument, aussi]	[be, argument, also]
6	0.7187	[argument, verbal]	[argument, verbal]
...	...	...	...
5	0.6562	[être, argument, indique]	[be, argument, denote]
4	0.5	[argument, syntaxique]	[argument, syntactic]
...	...	...	...
6	0.3281	[être, argument, rst]	[be, argument, rst]
8	0.25	[argument, nucleus]	[argument, nucleus]

An experiment on word desambiguisation

Negative description of occurrences

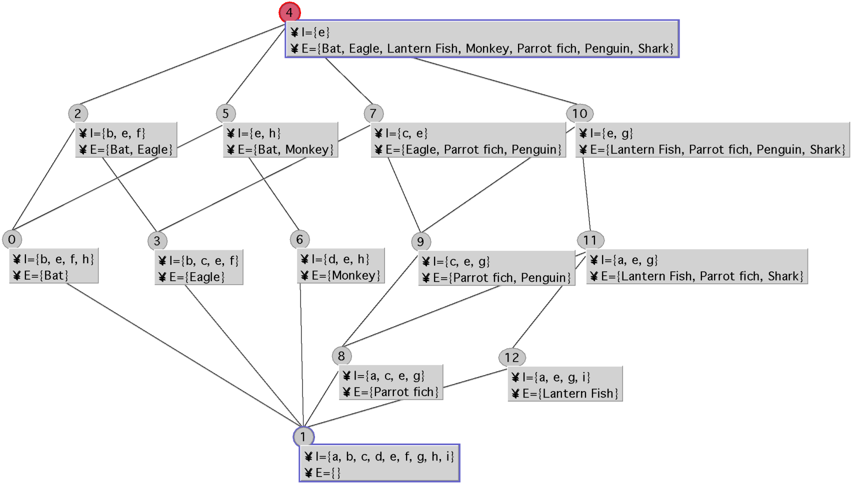
Support	Stability	Hypotheses in T_+	Hypotheses in T_+ *-english-*
7	0.7968	[sdrt, être, argument]	[sdrt, be, argument]
9	0.7792	[argument, plus]	[argument, more]
6	0.73437	[être, argument, aussi]	[be, argument, also]
6	0.7187	[argument, verbal]	[argument, verbal]
...	...	...	...
5	0.6562	[être, argument, indique]	[be, argument, denote]
4	0.5	[argument, syntaxique]	[argument, syntactic]
...	...	...	...
6	0.3281	[être, argument, rst]	[be, argument, rst]
8	0.25	[argument, nucleus]	[argument, nucleus]

Classifying animals

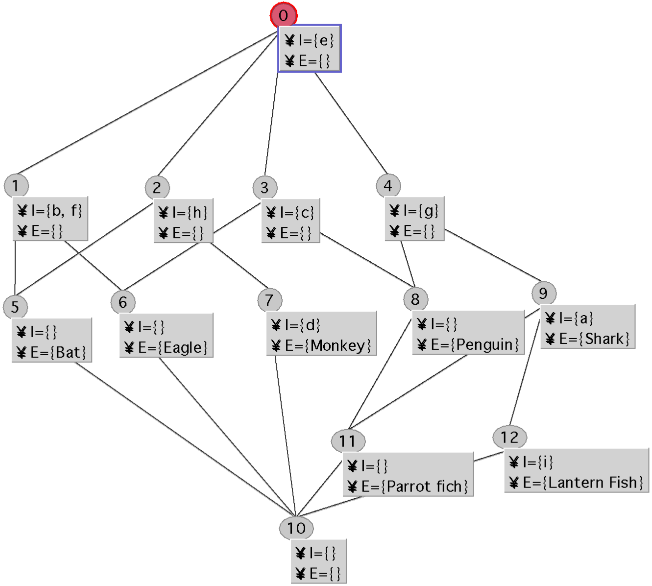
Animal/Features	a	b	c	d	e	f	g	h	i
Bat		x			x	x		x	
Eagle		x	x		x	x			
Monkey				x	x			x	
Parrot Fish	x		x		x		x		
Penguin			x		x	x	x		
Shark	x				x		x		
Lantern Fish	x				x		x		x

with (a) breath in the water, (b) can fly, (c) has beak, (d) has hands, (e) has skeleton, (f) has wings, (g) lives in water, (h) is viviparous, (i) produces light.

Classifying animals – the lattice



Classifying animals – the reduced lattice



Classyfying animals

Implications

Elements of bibliography on concept lattices

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- B. Ganter and R. Wille, Formal Concept Analysis, Springer, 1999.
- R. Godin and R. Missaoui, An incremental concept formation approach for learning from databases, Theoretical Computer Science, 133(2):387–419, 1994.

Tools for building and visualizing concept lattices

- The Conexp program: `http://sourceforge.net/projects/conexp`
- The Galicia Platform: `http://www.iro.umontreal.ca/~galicia/`
- The Toscana platform: `http://tockit.sourceforge.net/toscanaj/index.html`
- The Formal Concept Analysis Homepage: `http://www.upriss.org.uk/fca/fca.html`