Business Analytics

Lecture 4

Normal Distribution

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UCL SCHOOL OF MANAGEMENT

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- Normal Distribution
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Reminder: Continuous Random Variable

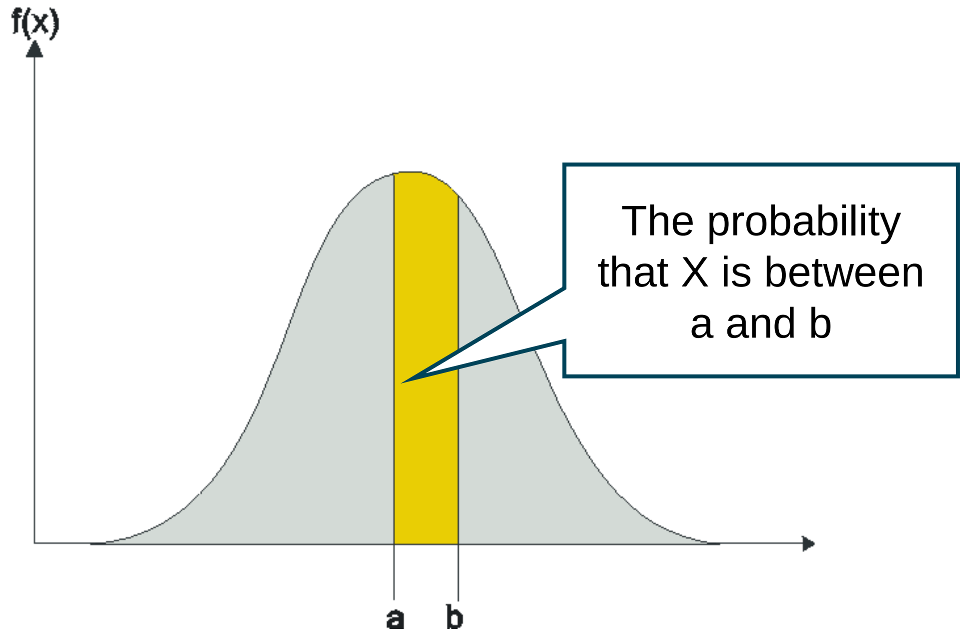
Definition. A continuous random variable is a random variable that can take on any value in an interval of numbers.

Examples:

- Time (e.g. time required to complete a stage in a project)
- Investment risk
- Profit (though one may argue that money has a finite resolution)

Reminder: Probability Density Function

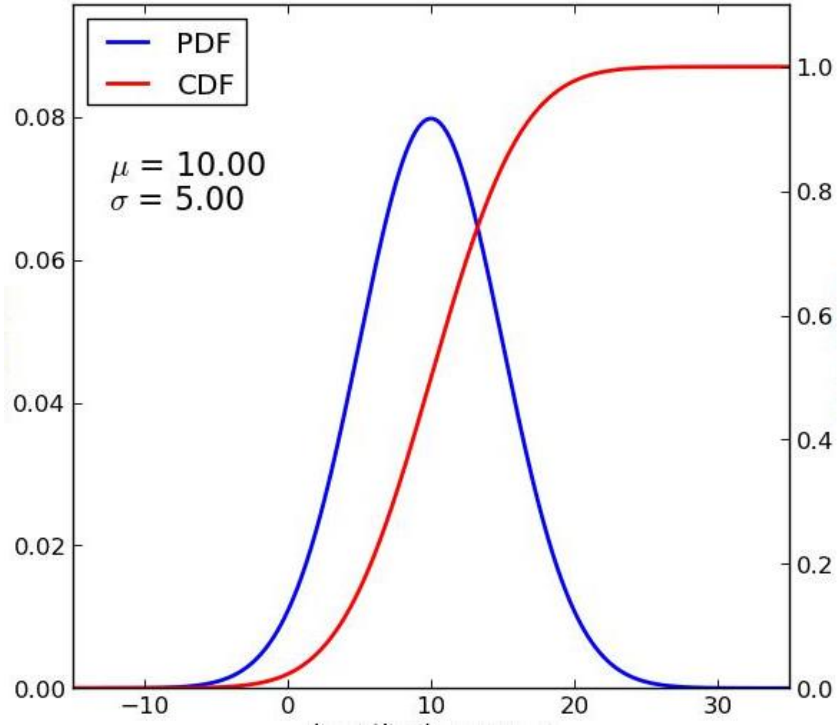
- Properties of the probability density function f(x):
  1. $f(x) \geq 0$ for all x
  2. The total area under f(x) is 1
  3. The probability that X is between a and b is the area under f(x) between a and b.



Reminder: Cumulative Distribution Function

- Definition. The Cumulative Distribution Function (CDF) of a continuous random variable is $F(x)=P(X \leq x)$.

- The meaning of the cumulative distribution function:
  - The area under $f(x)$ between the smallest possible value of X and x
  - The probability that $X \leq x$.



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Normal Distribution: PDF

- The probability density function of a normal distribution with mean μ and standard deviation σ is

$$f(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

- Example: A normal distribution with mean 3 and standard deviation 5:

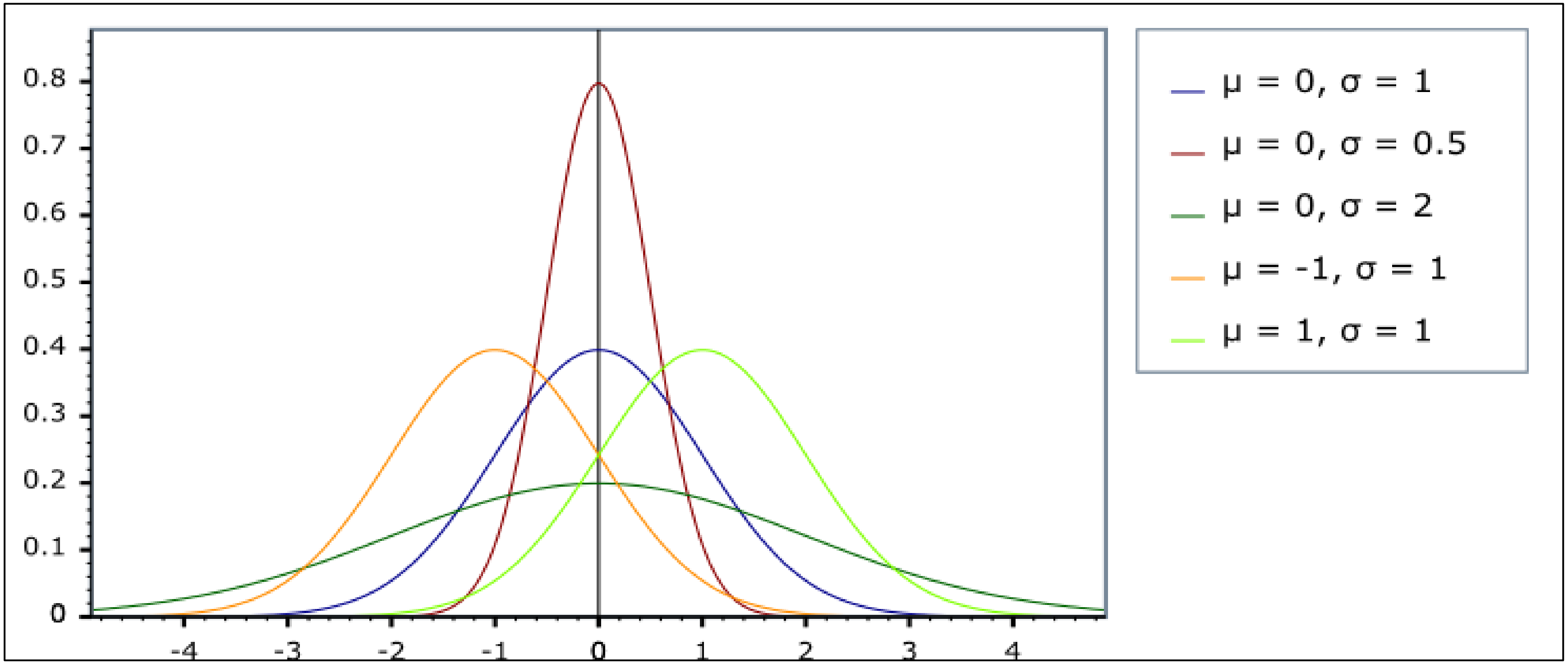
$$f(x) = \frac{1}{\sqrt{2\pi 5}} e^{-\frac{1}{2}\left(\frac{x-3}{5}\right)^2}$$

- Example: A normal distribution with mean 4 and standard deviation 6

$$f(x) = \frac{1}{\sqrt{2\pi 6}} e^{-\frac{1}{2}\left(\frac{x-4}{6}\right)^2}$$

Examples of Normal Distribution

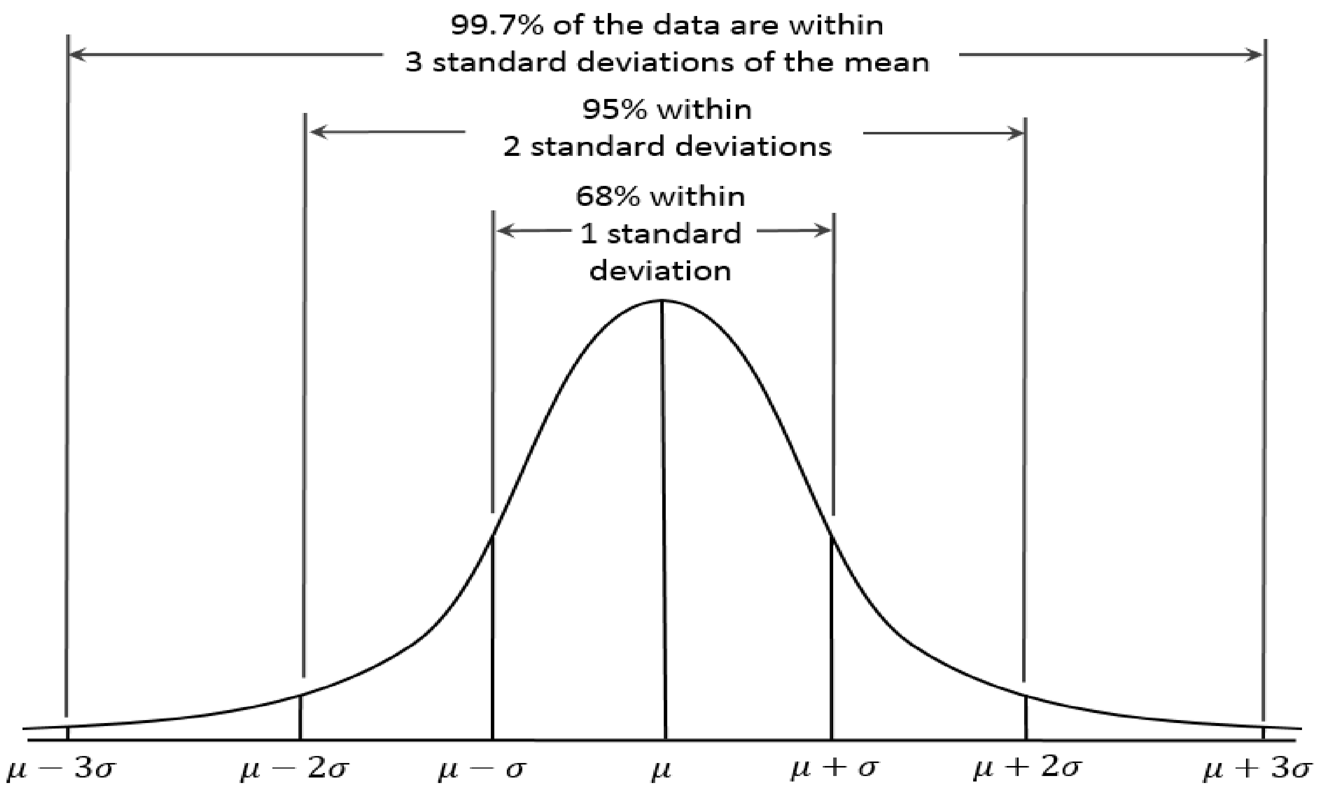
$$f(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$



- μ determines the location of the maximum of the function
- ɓ determines how wide it is.

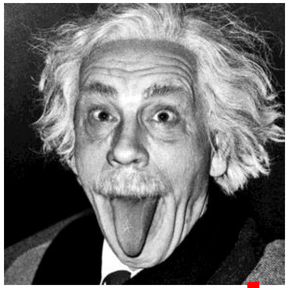
Properties of Normal Distribution

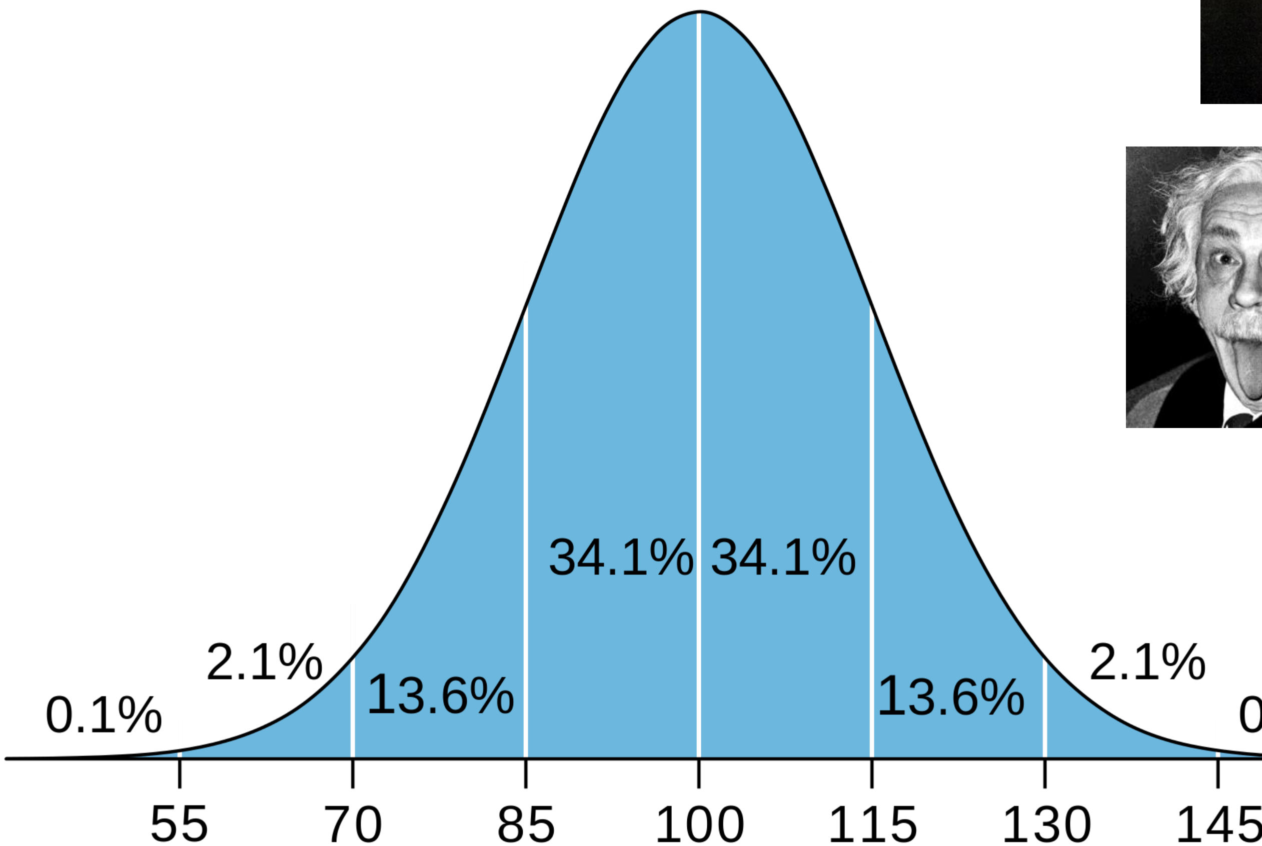
- The total area under the density curve = 1
- 68.26% of the area under the curve is between μ-σ, μ+σ,
- 95.44% of the area under the curve is between μ-2σ, μ+2σ,
- 99.72% of the area under the curve is between μ-3σ, μ+3σ.



Example: IQ Distribution

- It is a Normal distribution: $$f(x) = \frac{1}{\sqrt{2\pi 15}} e^{-\frac{1}{2}\left(\frac{x-100}{15}\right)^2}$$





Example

- Watch

https://www.youtube.com/watch?v=dr1DynUzjq0

The Sum of Normal Distributions

- Theorem. Let $X_1, X_2, \ldots, X_n$ be normally distributed independent random variables. Then the sum $S=X_1+X_2+\ldots+X_n$ is also normally distributed with
  $E(S)=E(X_1)+E(X_2)+\ldots+E(X_n)$
  $V(S)= V(X_1)+V(X_2)+\ldots+V(X_n)$.

- Example. Let X_1, X_2, X_3, X_4 be normally distributed independent random variables, suppose
  $E(X_1)=7$, $E(X_2)=5$, $E(X_3)=3$, $E(X_4)=1$, and $V(X_1)=2$, $V(X_2)=4$, $V(X_3)=6$, $V(X_4)=8$.
  Find the distribution of the sum $S=X_1+X_2+X_3+X_4$. Report the mean and variance of S.

- Solution. As the variables are independent and normally distributed, S is also normally distributed with
  $E(S)=7+5+3+1=16$ and
  $V(S)=2+4+6+8=20$.

Linear Combination of Normal Distributions

- Theorem. Let X_1, X_2,…, X_n be normally distributed random variables, which are independent. Then their linear combination $L=a_1X_1+a_2X_2+\ldots+a_nX_n$ is also normally distributed with

  $E(L)=a_1E(X_1)+a_2E(X_2)+\ldots+a_nE(X_n)$ and

  $V(L)=a_1^2V(X_1)+a_2^2V(X_2)+\ldots+a_n^2V(X_n)$.

- Example. Let X_1, X_2, X_3 , X_4 be normally distributed random variables, which are independent. Assume that

  $E(X_1)=7$, $E(X_2)=5$, $E(X_3)=3$, $E(X_4)=1$, and $V(X_1)=2$, $V(X_2)=4$, $V(X_3)=6$, $V(X_4)=8$.

  Find the distribution of the linear combination $L=2X_1+X_2+3X_3+X_4$. Report the mean, variance, and standard deviation of L.

- Solution. As the variables are independent and normally distributed, L is also normally distributed with

  $E(L)=2\cdot7+5+3\cdot3+1=29$ and

  $V(L)=2^2\cdot2+4+3^2\cdot6+8=74$. $SD(L)=74^{0.5}=8.6023$.

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Generalisation

Theorem. Let X_1, X_2,…, X_n be normally distributed random variables, which are independent. Then $Q=a_1X_1+a_2X_2+\ldots+ a_nX_n+b$ is also normally distributed with:

$$E(Q)=a_1E(X_1)+a_2E(X_2)+\ldots+a_nE(X_n)+b$$

$$V(Q)= a_1^2V(X_1)+a_2^2V(X_2)+\ldots+a_n^2V(X_n).$$

Example (Aczel, 4.4)

A cost accountant needs to forecast the unit cost of a product for next year.

- Each unit of the product requires 12 hours of labour and 5.8 pounds of raw material.
- Each unit of the product is assigned an overhead cost of $184.50.
- The cost of an hour of labour next year will be normally distributed with an expected value of $45.75 and a standard deviation of $1.80.
- The cost of the raw material per pound will be normally distributed with an expected value of $62.35 and a standard deviation of $2.52.

Find the distribution of the unit cost of the product. Report its expected value, variance, and standard deviation.

Example (Aczel, 4.4)

Solution. Denote:
L: the unit cost of labour
M: the unit cost of the raw material
Q: unit cost of the product

Then: $Q = 12L + 5.8M + 184.50$.

If L does not influence the M, we can assume that the two are independent. Therefore Q is a normal random variable and:

$E(Q) = 12 * 45.75 + 5.8 * 62.35 + 184.5 = \$1095.13$
$V(Q) = 12^2 * 1.80^2 + 5.8^2 * 2.52^2 = 680.19$
$SD(Q) = 680.19^{0.5} = \$26.08$.

Standard Normal Distribution

- Definition. The standard normal random variable Z is the normal random variable with mean 0 and standard deviation1. That is:

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$$

- Notation. Z~N(0,1).

Calculation of Probabilities

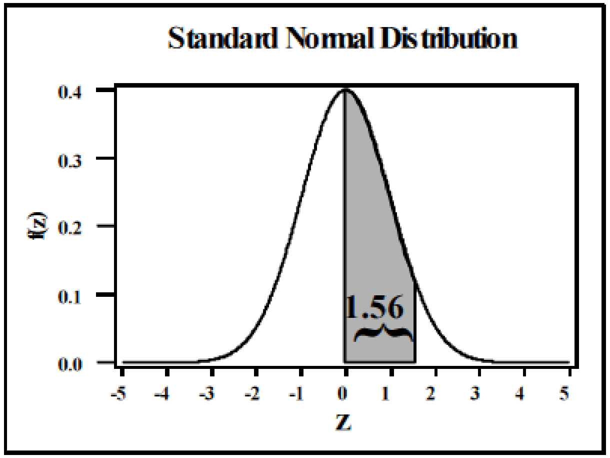
- Example. Assume that the standardized daily price change of an asset is normally distributed with mean 0 and sd 1. What is the probability that the daily price change is between 0 and 1.56?

- Solution. we are interested in $P(0<Z<1.56)$. That is, the area marked in grey.

- In Aczel, page 152 there is a table which gives the area between 0 and each value of z

  (also available in Moodle)

Using Standard Normal Distribution Table



Look in row labeled 1.5 and column labeled .06 to find $P(0 \leq Z \leq 1.56) =$ 0.4406

Standard Normal Probabilities

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

Exercises: using the table

- Exercise 1. What is P(0<Z<0.67)?
  P(0<Z<0.67)=0.2486.

- Exercise 2. What is P(0.56<Z<0.93)?
  P(0.56<Z<0.93)
  =P(0<Z<0.93)-P(0<Z<0.56)
  =0.3238-0.2123=0.1115.

- Exercise 3. Calculate P(0.24<Z<0.33).
  P(0.24<Z<0.33)
  =0.1293-0.0948=0.0345.

FIGURE 4–4 The Table Area TA for a Point *z* of the Standard Normal Distribution

The area given in the standard normal probability table is the area under the curve between 0 and a given point z

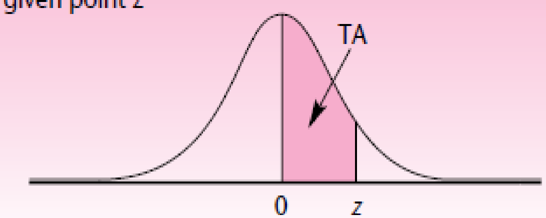


TABLE 4–1 Standard Normal Probabilities

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
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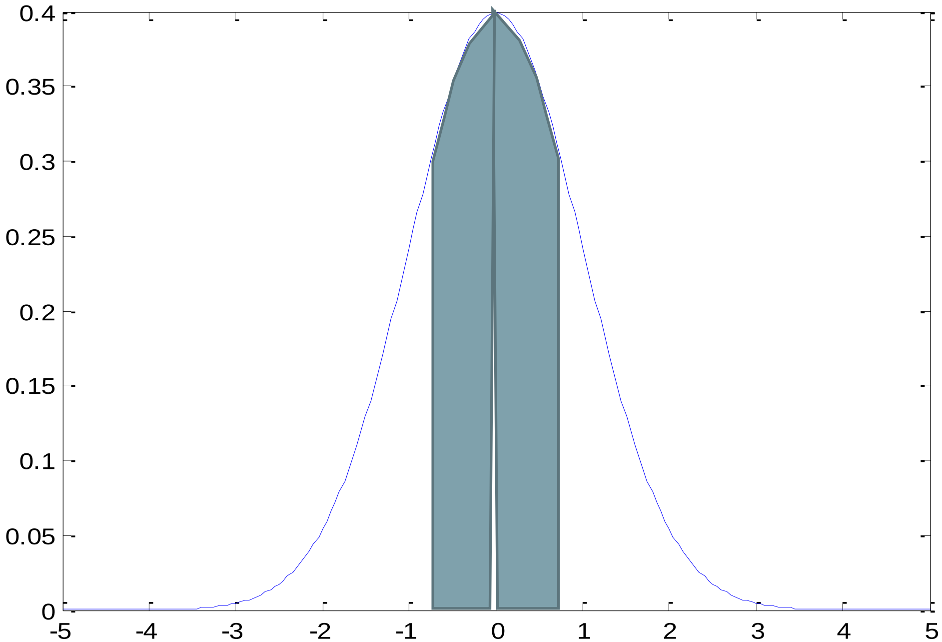
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

Symmetry of Normal Distribution:

- Example:

  $P(-0.82<Z<0.82)=2*P(0<Z<0.82)= 2*0.2939=0.5878$



Tricks

- If $a>0$, $P(0<Z<a)$ is the value written in the table.
- If $a>0$, $P(-a<Z<0)=P(0<Z<a)$ (symmetry)
- $P(Z<0)=P(-\infty<Z<0)= P(0<Z< \infty)=P(Z>0)=0.5$
- If $a>0$, $P(Z<a)=P(-\infty<Z<0)+P(0<Z<a)=0.5+P(0<Z<a)$.
- $P(Z>a)=1-P(Z<a)$

Exercises

1. P(Z<0.66)

=0.2454+0.5=0.7454

2. P(Z<-0.66)

=P(Z>0.66) (symmetry)

=1-P(Z<0.66)

=1-0.7454=0.2546

3. P(-0.23<Z<0.34)

=P(-0.23<Z<0)+P(0<Z<0.34)

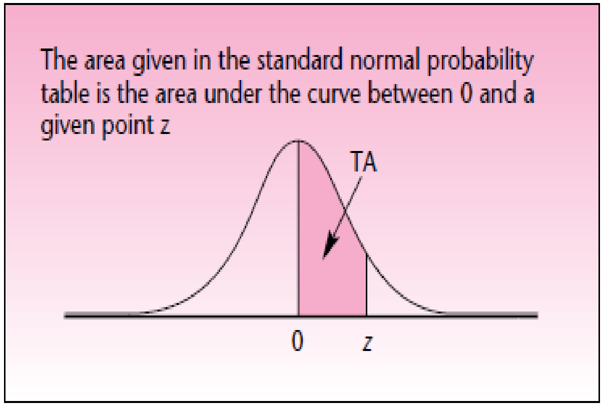
=P(0<Z<0.23)+ P(0<Z<0.34)

=0.0910+0.1331=0.2241.

4. Find z such that
P(0<Z<z) is 0.27.

The nearest value is z=0.74.

FIGURE 4–4 The Table Area TA for a Point *z* of the Standard Normal Distribution



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[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

Caution: Different Tables

Normal Deviate z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-4.0	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-3.9	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-3.8	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-3.7	.0001	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-3.6	.0002	.0002	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001
-3.5	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483

FIGURE 4–4 The Table Area TA for a Point *z* of the Standard Normal Distribution

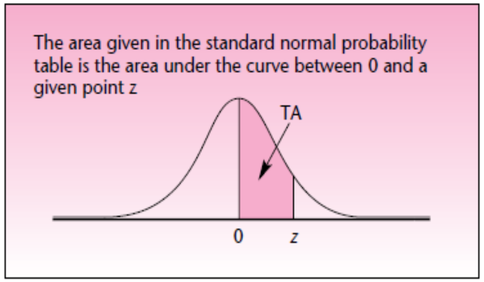


TABLE 4–1 Standard Normal Probabilities

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389

Why Standard Normal Distribution?

- In order to calculate probabilities of the type $P(X<x)$ or $P(a \leq X \leq b)$, we need to calculate the area under part of a normal distribution of X

- In general, it is difficult for a "normal" Normal distribution

- However, for the standard normal distribution Z, there are tables that help us calculate the probabilities.

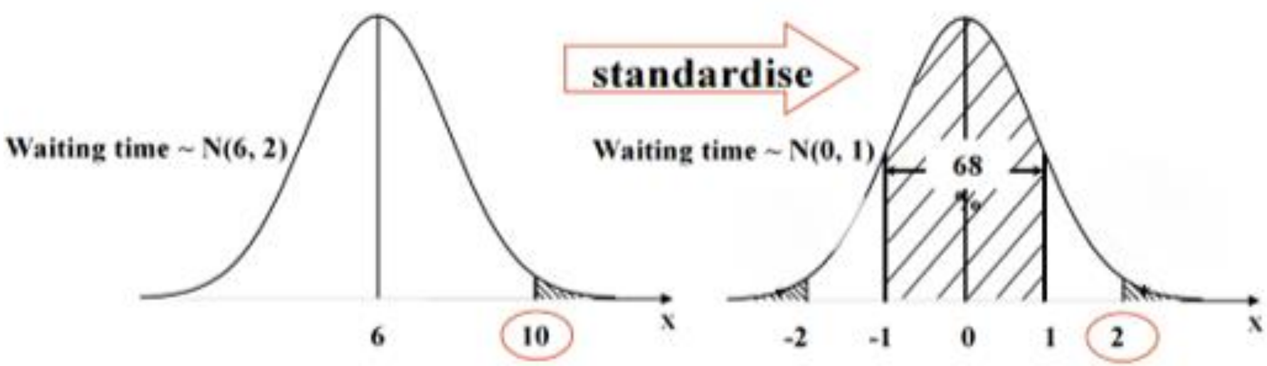
- To calculate the area under the relevant normal distribution X, we will transform the question from X to Z and use the result of the table for Z.

How to transform the question from X to Z? Namely, how to standardize normal distributions?

Standardization of Normal Distribution

$$X = \mu + \sigma Z, \quad Z = (X - \mu)/\sigma$$

Example. Suppose the waiting time for customer service is normally distributed with a mean of 6 min. and standard deviation of 2 min. What is the probability that a customer will wait more than 10 minutes?



P(Time>10)=P(Z>(10-6/2)=P(Z>2)=2.3%

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Example

- Let X be the amount of money spent by customers in a shop. Assume that $X \sim N(50, 25^2)$. What is the probability that a customer spends more than £100 in the shop?

- Solution.

$$P(X > 100) = P\left(Z > \frac{100-\mu}{\sigma}\right)$$

$$= P\left(Z > \frac{100-50}{25}\right) = P(Z > 2)$$

$$= 1 - P(Z < 2)$$

$$= 1 - [P(-\infty < Z < 0) + P(0 < Z < 2)] = [1 - (0.5 + 0.4772)] = 0.0228.$$

Exercise

The number of shares traded daily on the New York Stock Exchange (NYSE) is referred to as the *volume of* trading. Assume that the number of shares traded on the NYSE is a normally distributed random variable, with a mean of 1.8 billion and a standard deviation of 0.15 billion.

For a randomly selected day, what is the probability that the volume is below 1.35 billion?

Solution.

$X \sim N(1.8, 0.15^2)$.

$$P(X < 1.35) = P\left(Z < \frac{1.35 - \mu}{\sigma}\right) = P\left(Z < \frac{1.35 - 1.8}{0.15}\right)$$

$$P(Z < -3) = 0.5 - P(0 < Z < 3) = 0.5 - 0.4987 = 0.0013.$$

The Inverse Approach: example

- The length of phone calls in a company is normally distributed, with the mean 50 seconds and the standard deviation 12 seconds. What is the duration of a call which 80% of all calls are longer?

Solution. We have $X \sim N(50,12^2)$. First, we find z such that $P(Z>z)=0.8$, or, $P(Z<z)=0.2$

z is negative

$\Rightarrow P(Z<z)=0.5-P(z<Z<0)$

$=0.5-P(0<Z<-z)=0.2$

$\Rightarrow P(0<Z<-z)=0.5-0.2=0.3$

$\Rightarrow -z=0.84,\ z=-0.84$

$\Rightarrow X=\mu+\sigma z=50-12\cdot 0.84=39.92$[sec]

Additional Exercise 1

An investor puts some money into each of two stocks, A and B: £400 in A to stock and £600 in to stock B.

The mean rates of return for the stocks are £0.13 and £0.1 (So, if one puts a dollar into stock A, the expected profit at the end of the year is £0.13).

The standard deviations of the rates of returns are σA = £0.02 and σB = £0.03. Assume that the rates of return are independent random variables.

a. Find the mean of the total amount the investor earns in one year.

b. Find the standard deviation of the total amount the investor earns in one year.

Solution. a. Let RA and RB be the rates of return on the two stocks A and B. Let X be the total profit on the investment in one year. Then

X=400 RA + 600 RB.

a. The mean of the total profit is:

E(X) = 400×E(RA) + 600×E(RB) = 400×0.13 + 600×0.1 = £112.

b. V(X) = V(400 RA) + V(600 RB) = $(400)^2 \times (0.02)^2 + (600)^2 \times (0.03)^2$ = 388 and the standard deviation of X is $388^{0.5}$=£19.6977.

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Additional Exercise 2

- The price of one of the parts of the main component of SeaStar250 is normally distributed with mean £20 and variance 4.

  The price of the second part of the same component is normally distributed with mean £30 and variance 5.

  Assume that the prices of the two parts are independent. What is the probability that the price of this component is between £52 and £55?

Solution. Denote by P1 the price of the first part and by P2 the price of the second part. Let TP=P1+P2.

As the prices of both parts are normally distributed, and they are independent, TP is also normally distributed.

E(TP)=E(P1)+E(P2)=20+30=£50. V(TP)=V(P1)+V(P2)=4+5=9. SD(TP)=V(TP)$^{0.5}$=£3

TP~N(50,3^2)

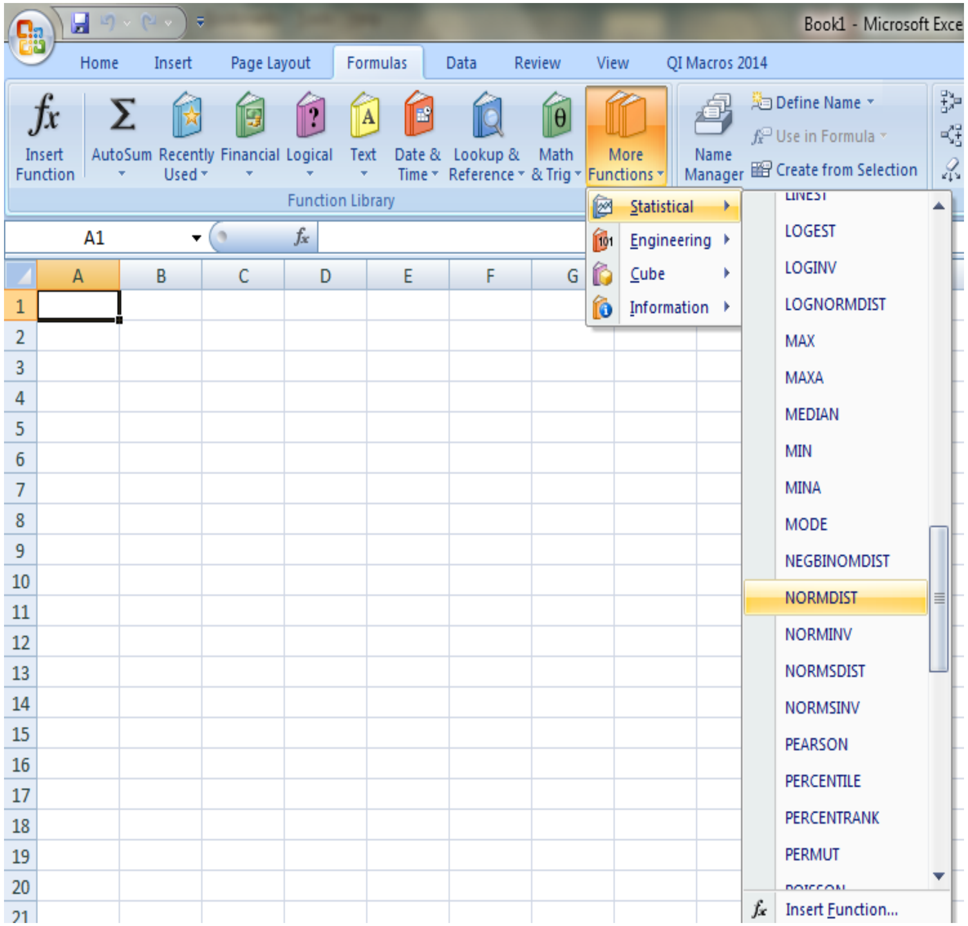
$$P(52 < X < 55) = P\left(\frac{52-\mu}{\sigma} < Z < \frac{55-\mu}{\sigma}\right) = P\left(\frac{52-50}{3} < Z < \frac{55-50}{3}\right) = P(0.67 < Z < 1.67)$$
$$= P(0 < Z < 1.67) - P(0 < Z < 0.67) = 0.4525 - 0.2486 = 0.2039.$$

Excel: Calculation of Normal Distribution

Excel: Normal Distribution

- Choose: Formulas
  ->More functions
  ->Statistical

- Then choose NORM.DIST
  NORM.DIST(x,mean,std,true) calculates
  $P(X<x)=P(-\infty<X<x)$
  for X~N(mean,std).

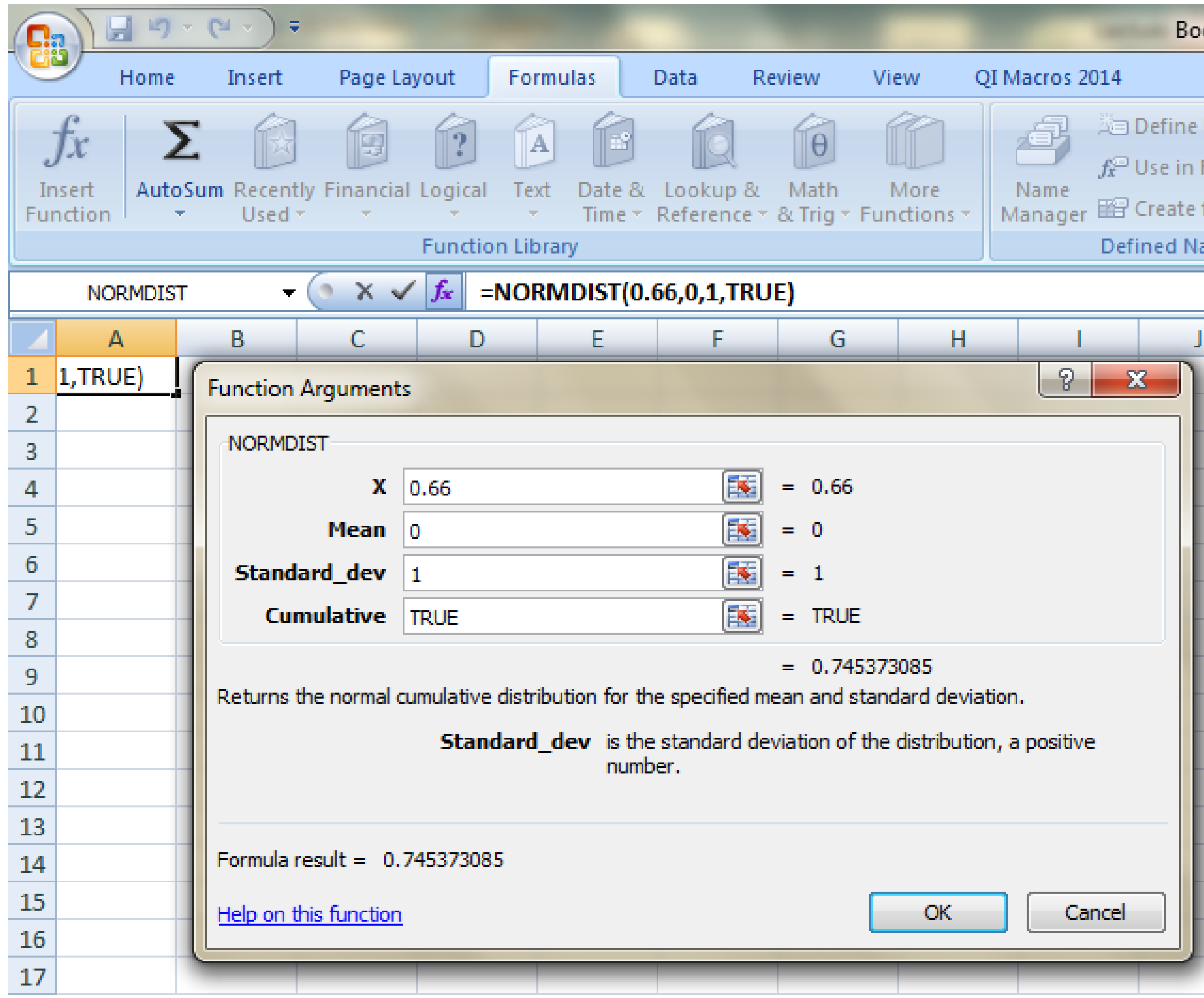


Example

- Let Z be a random variable which have a standard normal distribution. Calculate P(Z<0.66).

- Solution.

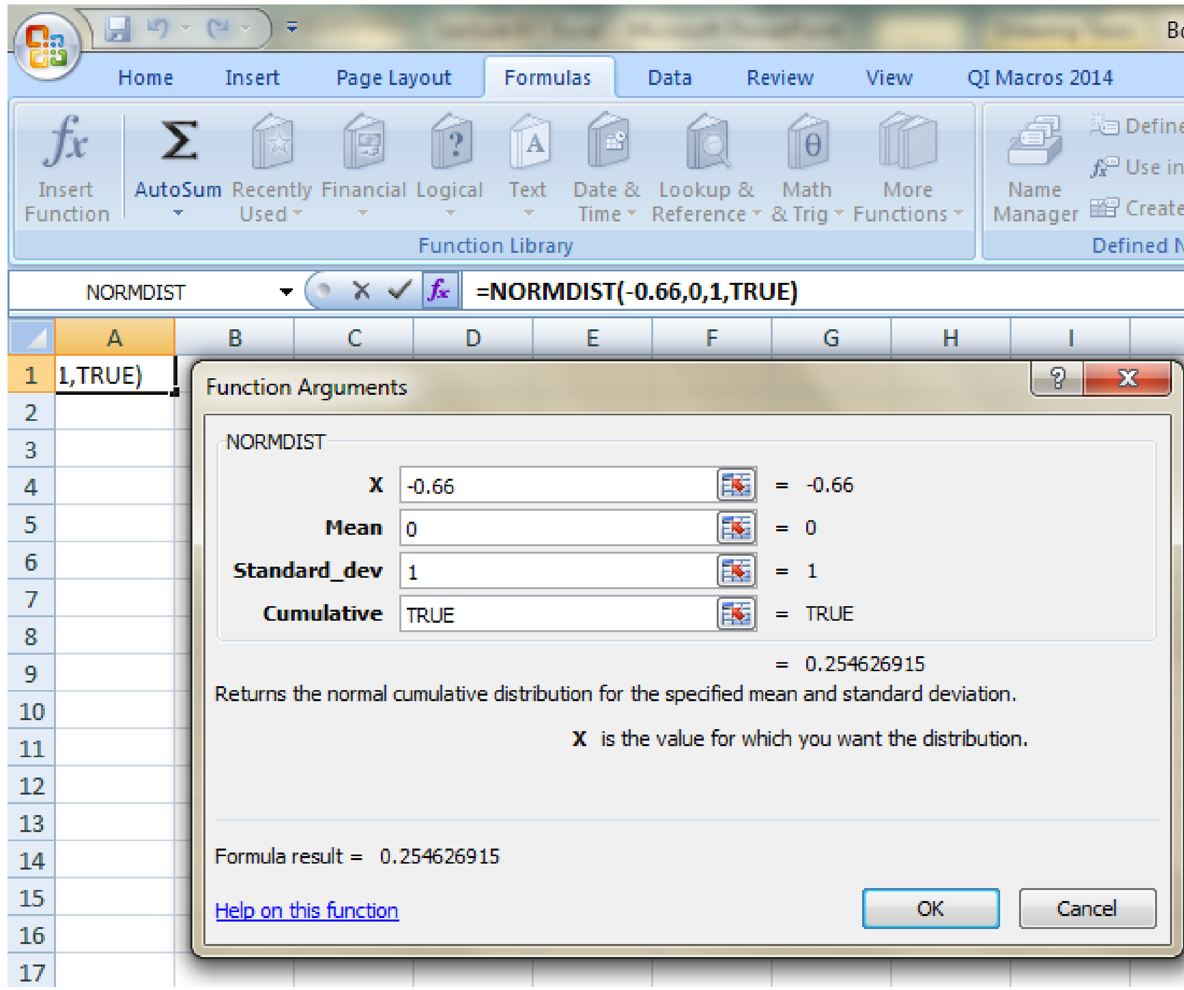
P(Z<0.66)=0.7454



Example

- Let Z be a random variable which have a standard normal distribution. Calculate P(Z<-0.66).

- Solution.
  P(Z<-0.66)=0.2546

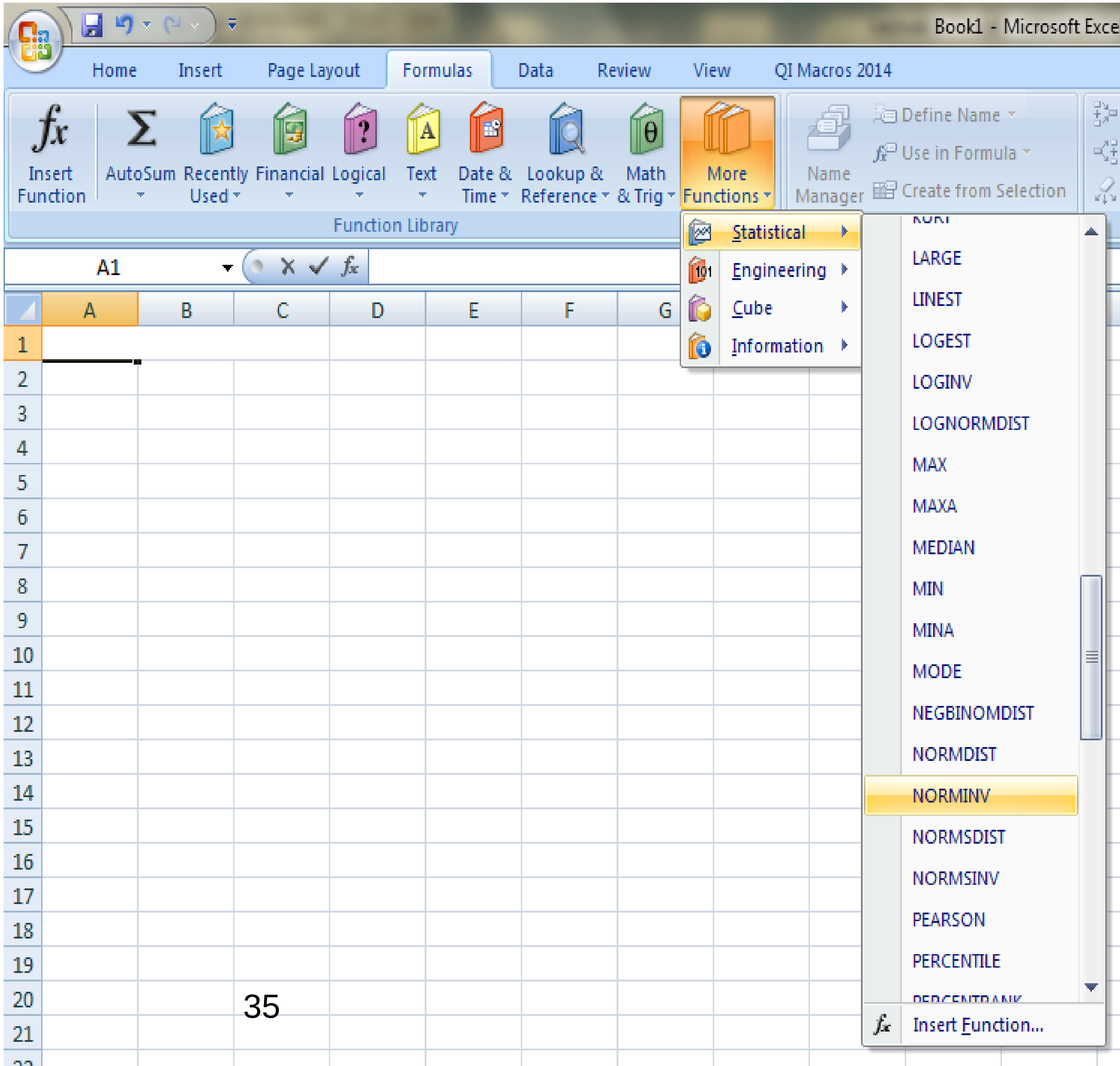


Inverse Approach

- Find the z value such that P(Z<z)=0.7454.
- Solution. Choose: Formulas

  ->More functions

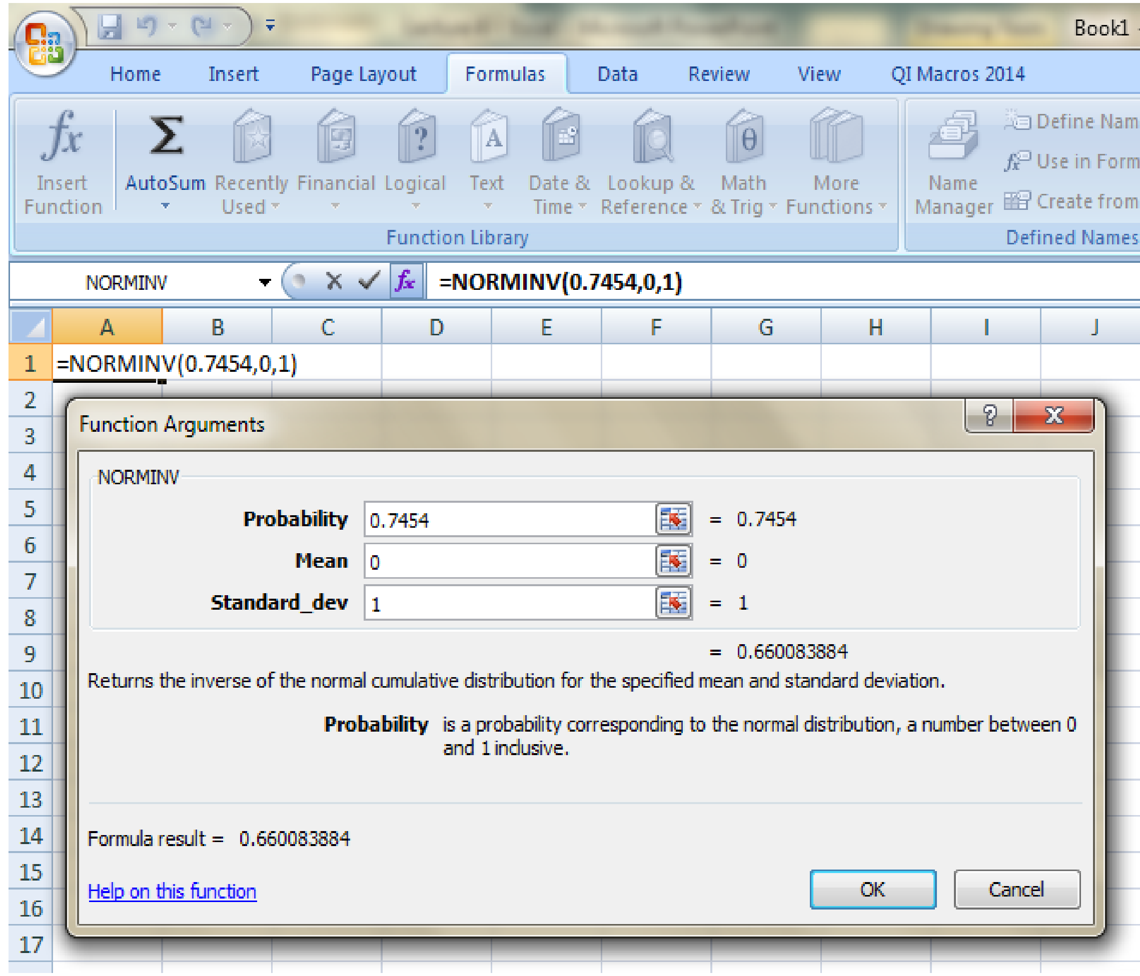
  ->Statistical
- Then choose NORM.INV



Example

- Find the z value such that $P(Z<z)=0.7454$

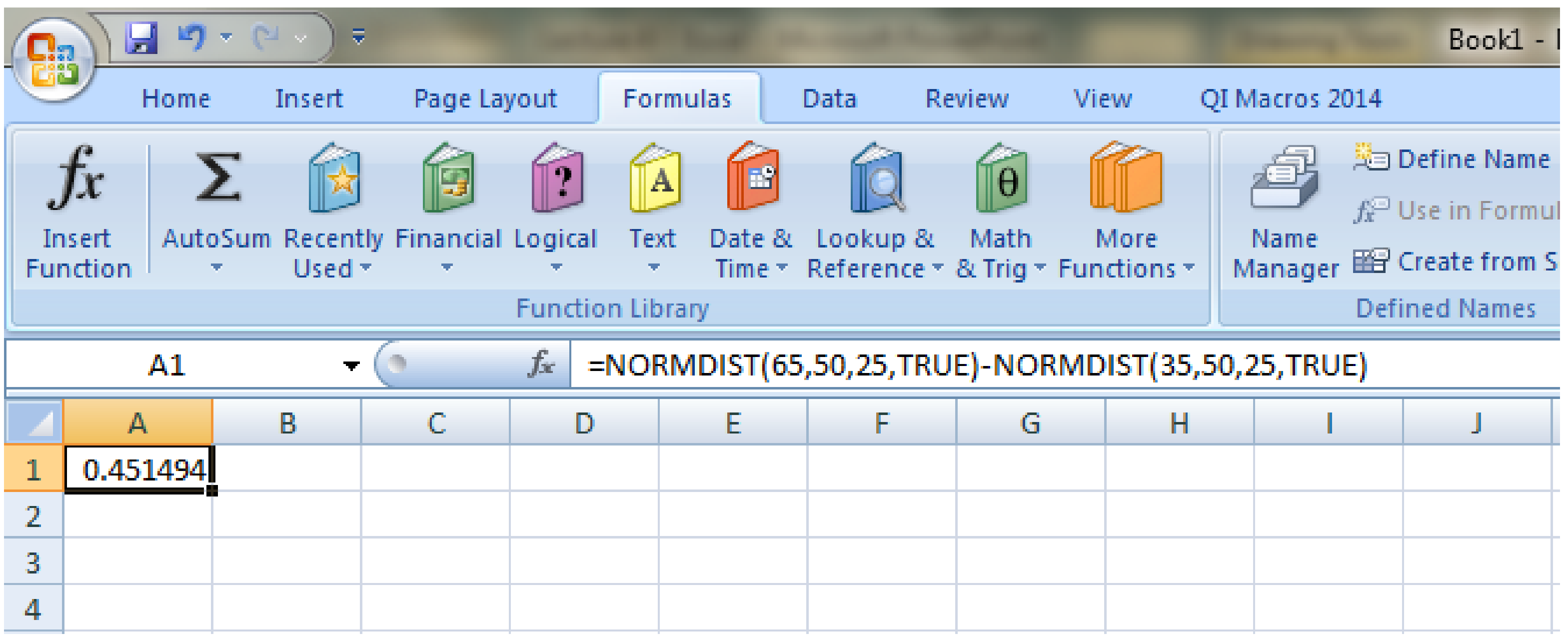
- Solution. $z=0.66$



Example

- Example. Let X be the amount of money spent by customers in a shop. Assume that $X \sim N(50,25^2)$. What is the probability that a customer spends between £35 and £65 in the shop?

- We get $P(35<X<65)=0.4514$



Recommended Reading

Chapter 4 of:

Aczel, A., & J. Sounderpandian. 2008. Complete Business Statistics. McGraw-Hill/Irwin, Seventh Edition.

*The course leader thanks Dr. Daphne Sobolev for her help in developing the course materials.

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