MSIN0180 Formula Sheet

Differentiation Rules

$\frac{d}{dx} x^n = n\, x^{n-1}$ $\quad\quad$ $\frac{d}{dx}(c\, u) = c\, \frac{du}{dx}$ $\quad\quad$ $\frac{d}{dx}(u + v) = \frac{du}{dx} + \frac{dv}{dx}$

$\frac{d}{dx}(u\, v) = u\, \frac{dv}{dx} + v\, \frac{du}{dx}$ $\quad\quad$ $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\,\frac{du}{dx} - u\,\frac{dv}{dx}}{v^2}$

$\frac{d}{dx} f(g(x)) = \frac{d\, f(u)}{du} \cdot \frac{d\, g(x)}{dx}$ $\quad\quad$ $\frac{d}{dx} e^u = e^u\, \frac{du}{dx}$, $\;u$ is a differentiable function of x

The Gradient Vector

$\nabla f = \frac{\partial f}{\partial x}\, i + \frac{\partial f}{\partial y}\, j$ $\quad$ where i is the unit vector in the x direction and j is the unit vector in the y direction

Directional Derivative

$$\left(\frac{df}{ds}\right)_{u,P_0} = (\nabla f)_{P_0} \cdot \frac{u}{|u|}$$

Extreme values of single variable functions

If $f''(c) < 0$ then a local maximum, if $f''(c) > 0$ then a local minimum, if $f''(c) = 0$ then an inflection point

Extreme values of functions of two variables

If $f_{xx} < 0$ and $f_{xx}\, f_{yy} - {f_{xy}}^2 > 0$ then a local maximum, if $f_{xx} > 0$ and $f_{xx}\, f_{yy} - {f_{xy}}^2 > 0$ then a local minimum, if $f_{xx}\, f_{yy} - {f_{xy}}^2 < 0$ then a saddle point, if $f_{xx}\, f_{yy} - {f_{xy}}^2 = 0$ then test is inconclusive,

Lagrange Multiplier method

To find the local extreme values of $f(x, y)$ subject to the constraint $g(x, y) = 0$ solve
$\nabla f = \lambda \nabla g$ $\quad$ and $\quad$ $g(x, y) = 0$.

Basic integration formulae

$\int k\, dx = k\, x + C$ $\quad\quad$ $\int x^n\, dx = \frac{x^{n+1}}{n+1} + C$, $\;n \neq -1$

$\int \frac{1}{x}\, dx = \ln|x| + C$ $\quad\quad$ $\int e^x\, dx = e^x + C$ $\quad\quad$ $\int a^x\, dx = \frac{a^x}{\ln a} + C$, $\;a > 0,\; a \neq 1$

Integration using the Substitution Rule

$\int f(g(x))\, g'(x)\, dx = \int f(u)\, du$ $\;$ where $u = g(x)$

Integration by Parts

$$\int f(x)\, g'(x)\, dx = f(x)\, g(x) - \int f'(x)\, g(x)\, dx$$

alternatively this can be expressed as : $\int u\, dv = u\, v - \int v\, du$

Solving Separable Differential Equations of the form $\frac{\mathrm{dy}}{\mathrm{dx}} = \frac{g(x)}{h(y)}$

$\int h(y)\,\mathrm{dy} = \int g(x)\,\mathrm{dx}$

Solving Linear Differential Equations of the form $\frac{\mathrm{dy}}{\mathrm{dx}} + P(x)\,y = Q(x)$

$y = \frac{1}{v(x)} \int v(x)\,Q(x)\,dx, \quad \text{where } v(x) = e^{\int P(x)\,dx}$

Linearization

$L(x) = f(a) + f'(a)(x - a)$

Taylor Series

$f(x) \approx f(x_0) + \sum_{n=1}^{\infty} \frac{1}{n!} f^{(n)}(x_0)(x - x_0)^n$

Maclaurin Series

$f(x) \approx f(0) + \sum_{n=1}^{\infty} \frac{1}{n!} f^{(n)}(0)\,x^n$

The Normal Equation

$A^T A \vec{x}^* = A^T \vec{b}$

Eigenvalues and eigenvectors definitional equation

$A \vec{v} = \lambda \vec{v}$

Diagonalisation

$A = S B S^{-1}$

To find eigenvalues solve the Characteristic Equation

$\det(A - \lambda I_n) = 0$

To find eigenvectors solve

$(A - \lambda I_n)\vec{v} = \vec{0}$

Discrete linear dynamical systems

$\vec{x}(t) = A^t \vec{x}_0$

$\vec{x}(t) = c_1 \lambda_1{}^t \vec{v}_1 + \ldots + c_n \lambda_n{}^t \vec{v}_n, \quad \text{where } \vec{x}_0 = c_1 \vec{v}_1 + \ldots + c_n \vec{v}_n$