MSIN0180 Quantitative Methods for Business
MOCK Examination Paper

Academic Misconduct: Academic Misconduct is defined as any action or attempted action that may result in a student obtaining an unfair academic advantage. Academic misconduct includes plagiarism, obtaining help from/sharing work with others be they individuals and/or organisations or any other form of cheating. Definitions and penalties for Academic Misconduct are outlined in the Academic Manual.
It is forbidden to:

- Communicate or collaborate with other students or any other third parties in relation to this assessment.
- Discuss or share assessment content with other students or third parties.
- Copy or attempt to copy the work of another student(s).

Such actions will be referred to the Academic Misconduct Panel, the penalties of which may include exclusion from UCL.

Referencing: In an exam you demonstrate your learning from the module and communicate your learning in your own words and, where appropriate, through calculations. If you were to make direct use of somebody else's work, including making direct use of information generated by an artificial intelligent (AI) tool, either by memorising it or copying it word for word, you must reference and provide full citation for ALL sources used. This includes any direct quotes and paraphrased text. If in doubt, reference it. Failure to cite references correctly may result in your work being referred to the Academic Misconduct Panel.

Use of Artificial Intelligence (AI) Tools in your Assessment: Your module leader will explain to you if and how AI tools can be used to support your assessments. In some assessments, the use of generative AI is not permitted at all. In others, AI may be used in an assistive role which means students are permitted to use AI tools to support the development of specific skills required for the assessment as specified by the module leader. In others, the use of AI tools may be an integral component of the assessment; in these cases the assessment will provide an opportunity to demonstrate effective and responsible use of AI. See the introductory table on page 2 of this brief to check which category use of AI falls into for this assessment. Students should refer to the UCL guidance on acknowledging use of AI and referencing AI. Failure to correctly reference use of AI in assessments may result in students being reported via the Academic Misconduct procedure. Refer to the section of the UCL Assessment success guide on Engaging with AI in your education and assessment.

THIS QUESTION PAPER MUST NOT BE REMOVED FROM THE EXAM VENUE.

Exam Length:TWO (2) hours
Number of Sections: There is ONE (1) section to the examination paper.
Question/Mark Distribution: Section A consists of FIVE (5) questions. This section is worth ONE HUNDRED (100) marks.
Additional Materials: None
Artificial Intelligence (AI) category: Not permitted
Other notes: If you are unclear on any part of the question, state your assumptions in your answers.
Module Leader: Andrew Whiter
Internal Assessor: Matt Jones

SECTION A

This section consists of FIVE (5) compulsory question. Candidates should attempt ALL parts of the question. This section is worth ONE HUNDRED (100) marks.

A.1 (Total marks: 20)

A clothing manufacturer is under contract to deliver 77 units of garment x and garment y in any combination of its choosing.

a. Use the Lagrange multiplier method to find the combination that minimizes the cost of fulfilling the contract given the firm's total cost function

$$c = 7x^2 - 2xy + 5y^2 + 64$$ **[15 marks]**

b. Use your solution from (a) to estimate what happens to the cost if the constant of the constraint (77) were increased or decreased by one unit?
[5 marks]

A.2 (Total marks: 20)

Suppose that it costs a company $y = a + bx$ dollars to produce x units per week.
It can sell x units per week at a unit price of $P = c - 2ex$ dollars per week.
Each of a, b, c and e represents a positive constant.

a. What production level maximises the weekly profit? **[5 marks]**

b. What is the corresponding unit price? **[5 marks]**

c. What is the weekly profit at this level of production?
Simplify your answer. **[5 marks]**

d. At what price should each item be sold to maximise profits if the government imposes a tax of t dollars per item sold? Comment on the difference between this price and the price before tax.
[5 marks]

A.3 (Total marks: 20)

A company produces two goods, x units and y units, where the profit p is given by the following profit function:

$p = 194\,x - 3\,x^2 - 7\,x\,y - 4\,y^2 + 224\,y - 120$

a. Find the critical point(s) of this profit function. What is the profit at this/these point(s)? **[7 marks]**

b. Classify this/these critical point(s). **[6 marks]**

b. If the company's maximum production capacity is limited such that $x + 2\,y = 50$, use the Lagrange multiplier method to find the levels of x and y that maximise profit. What is this constrained maximum profit and how does it compare to the profit at the critical point(s)?
[7 marks]

A.4 (Total marks: 20)

Consider the interaction of three companies X, Y, and Z in a given market.
The changing market share (expressed as fractions of the market) for each of X, Y, Z is given by $x(t)$, $y(t)$, and $z(t)$, where t is measured in months.
The following dynamic relations have been identified for these interacting companies.

$x(t+1) = x(t) + 0.75\,z(t)$
$y(t+1) = 0.5\,y(t)$
$z(t+1) = 0.5\,y(t) + 0.25\,z(t)$

Let $\vec{s}(t) = \begin{pmatrix} x(t) \\ y(t) \\ z(t) \end{pmatrix}$ be the vector that contains each of the market share variables.

a. Express this system of discrete dynamic equations in the form $\vec{s}(t+1) = A\,\vec{s}(t)$ and find the eigenvalues of A.
[10 marks]

b. For any eigenvalues ≥ 1 find their associated eigenvectors.
[5 marks]

c. Describe the long term market shares of companies X, Y, Z. Why did we not need to find the eigenvectors of any eigenvalues < 1 to answer this question?
[5 marks]

A.5 (Total marks: 20)

a. Find the best least squares fit by a linear function to the following data:

x	0	1	2
y	1	3	9

[10 marks]

b. Find the equation of a circle ($r^2 = x^2 + y^2$)that gives the best least squares fit to the points $\left(-1, -\sqrt{2}\right), \left(0, \sqrt{3}\right), (1, 1)$.

[10 marks]