

# Introduction to Model Checking



# Learning Objectives

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- Learn about formal verification and the basics of model checking
- Familiarise with transition systems
- Learn the type of properties that can be verified and how these could be defined

# Principal Methods for Verifying Complex Systems (1)

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- Testing
  - Performed on the system itself
  - How about testing distributed systems?
  - Proves the existence of bugs, but not their absence
- Simulation
  - A finite number of user-defined system trajectories meet the desired specification
  - How about completeness?

# Principal Methods for Verifying Complex Systems (2)

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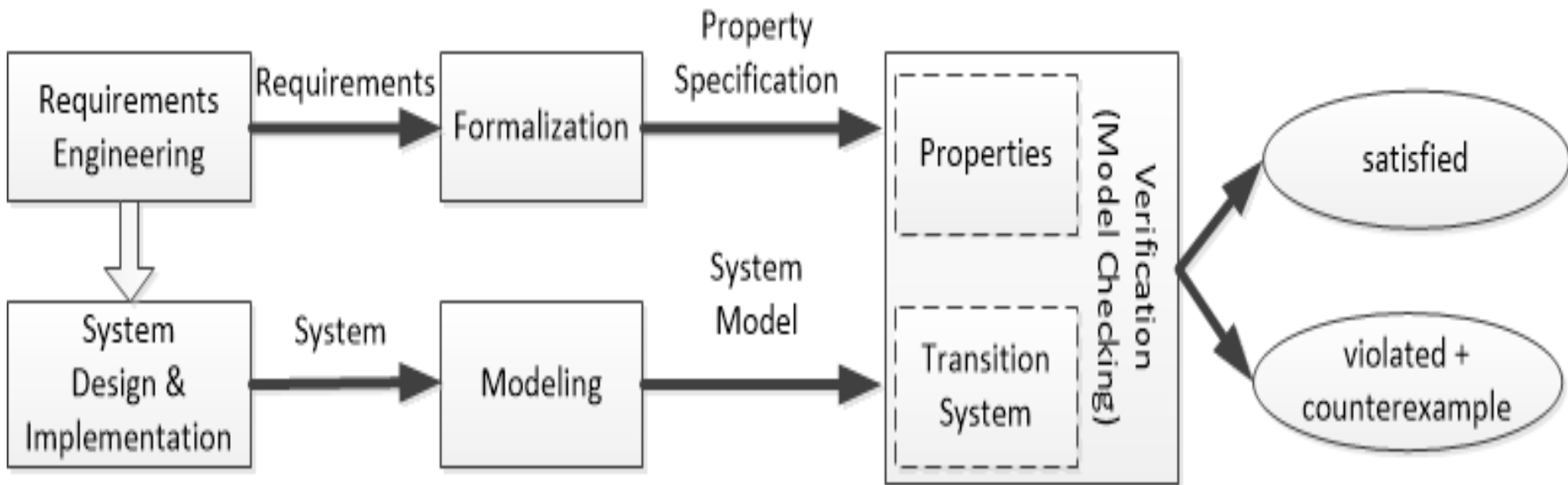
- Deductive verification
  - Manual mathematical proof of correctness of a model of a system
  - Cost?
  - Skills?
- Model checking
  - Automated technique
  - Given a finite-state model of a system and a formal property it checks systematically whether the property holds or not for that model

# The Model Checking Process (1)

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- Modelling phase
  - Model a system using a language
  - Define the formal properties to be checked
- Running phase
  - The model checker checks the validity of properties in the system model
- Analysis phase
  - Property satisfied ... check for next
  - Property violated ... look at counterexamples and refine the model

# The Model Checking Process (2)



# Pros of Model Checking

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- It is a general verification approach
- It supports partial verification
- It is not vulnerable to the likelihood that an error is exposed
- It provides diagnostic information
- Highly automated
- Has a sound and mathematical underpinning

# Cons of Model Checking

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- State-space explosion problem
- Not suited for infinite-state system or reasoning of abstract data types
- Verifies the system model and not the actual system
- Requires some expertise
- Model checkers are still software having defects



# A Few Tools...

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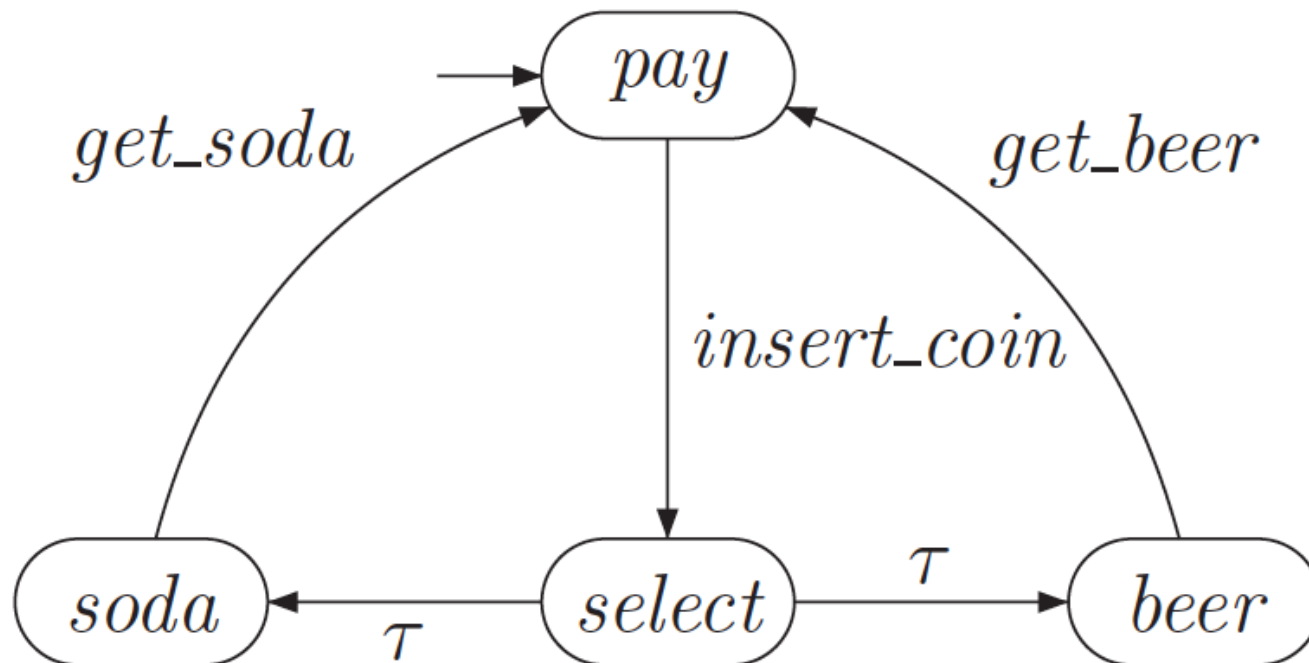
- nuXmv: a new symbolic model checker  
<https://nuxmv.fbk.eu>
- TLA+: a high-level language for modelling programs and systems  
<https://lamport.azurewebsites.net/tla/tla.html>
- Alloy: language and analyser for software verification  
<https://alloytools.org/>
- And many more...

# Transition System (TS)

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- A TS is a tuple  $(S, \text{Act}, \rightarrow, I, \text{AP}, L)$  where
  - $S$  is a set of states
  - $\text{Act}$  is a set of actions
  - $\rightarrow$  subset of  $S \times \text{Act} \times S$  is a transition relation
  - $I$  subset of  $S$  is the set of initial states
  - $\text{AP}$  is a set of atomic propositions for the states
    - Temporal characteristics to express simple know facts about the states of the system under consideration
  - $L: S \rightarrow 2^{\text{AP}}$  is a labelling function
- TS is called finite if  $S$ ,  $\text{Act}$  and  $\text{AP}$  are finite.

# A TS for a Vending Machine



# TS Elements

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- The state space  $S = \{\text{pay, select, soda, beer}\}$
- Initial state  $I = \{\text{pay}\}$
- Actions  $\text{Act} = \{\text{insert\_coin, get\_soda, get\_beer, } \tau\}$
- $\text{AP} = \{\text{paid, drink}\}$ 
  - $L(\text{pay}) = \{\}, L(\text{select}) = \{\text{paid}\}$
  - $L(\text{soda}) = L(\text{beer}) = \{\text{paid, drink}\}$

# What type of properties can we verify?

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# Safety properties

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- Safety characterised as:
  - *‘nothing bad should happen’*
- Examples of safety properties to verify
  - Mutual exclusion
  - Deadlock freedom
- Safety properties as *‘invariants’*
  - Considering a series of states in a system (behaviour) a given condition  $\Phi$  is required to hold for all reachable states.

# Liveness properties

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- Liveness characterised as:  
*‘something good will happen in the future’*
- Complementary properties to the safety ones
- Certain events occur infinitely often
- Examples
  - Each process will eventually enter its critical section
  - Each waiting process will eventually enter its critical section

# Fairness

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- Fairness assumptions rule out infinite behaviour (considered unrealistic)
- Often necessary to establish liveness properties
- Example on process fairness
  - Let's consider  $N$  processes  $P_1 \dots P_n$  that request a service
  - What's an unfair and fair strategy?



# Fairness constraints

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- Unconditional fairness  
*‘Every process gets its turn infinitely often’*
- Weak fairness  
*‘Every process that is continuously enabled from a certain time instant on gets its turn infinitely often’*
- Strong fairness  
*‘Every process that is enabled infinitely often gets its turn infinitely often’*

# Other type of properties

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- Functional correctness
  - Is the system behaving as it should?
- Reachability
  - Is it possible to reach to a certain state?
- Real-time properties
  - Is the system responding in a timely manner?

# How do we specify properties?

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- Use of logical formalisms, e.g., temporal logic
- Linear Temporal Logic (LTL)
  - Provides a logical formalism for specifying linear time properties
- Computation Tree Logic (CTL)
  - Branching temporal logic for specifying properties of a system

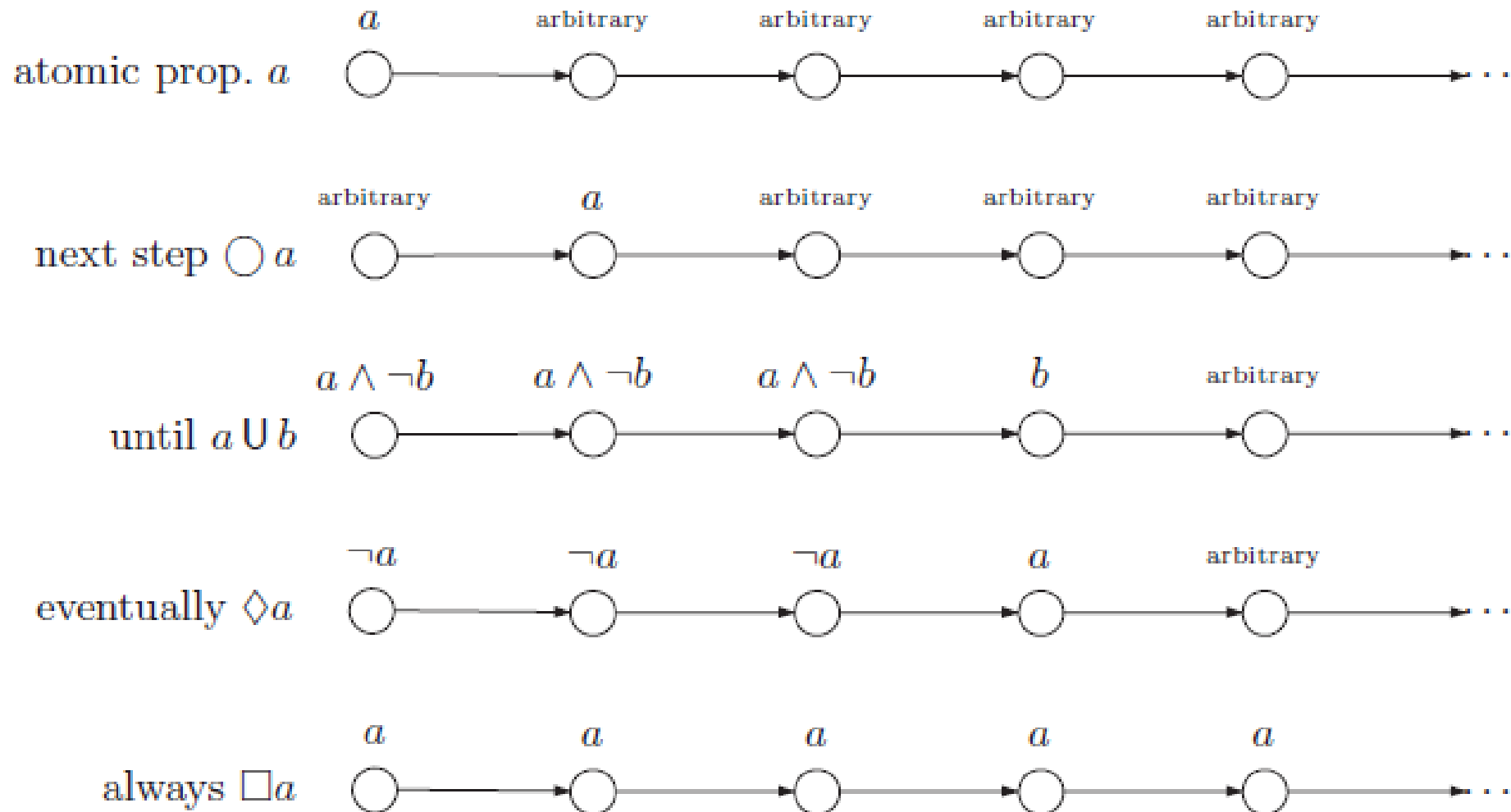
# Linear Time Logic

- Make use of atomic propositions:  $a \in AP$
- Boolean connectors:  $\wedge, \neg$
- Elementary temporal modalities
  - $\diamond$  ‘eventually’ in the future
  - $\Box$  ‘always’, now and forever in the future
- Unary prefix operator for ‘next’:  $O$ , e.g.,  $\circ \varphi$
- Binary infix operator for ‘until’:  $U$ , e.g.,  $\varphi_1 U \varphi_2$
- Syntax of LTL

$$\varphi ::= true \mid a \mid \varphi_1 \wedge \varphi_2 \mid \neg \varphi \mid \circ \varphi \mid \varphi_1 U \varphi_2$$

where  $a \in AP$

# Semantics of temporal modalities



# Hands on...

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- How to model the vending machine?
- What type of properties can we verify?

# Vending machine in nuXmv #1

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-- Define variables

VAR

state: {pay, select, soda, beer};

-- Indicates whether a coin is inserted

insert\_coin: boolean;

-- Indicates whether a soda is dispensed

get\_soda: boolean;

-- Indicates whether a beer is dispensed

get\_beer: boolean;

# Vending machine in nuXmv #2

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-- Initialize variables in the SM

ASSIGN

-- *Initial state is 'pay'*

init(state) := pay;

-- *Initially, no coin is inserted*

init(insert\_coin) := FALSE;

-- *Initially, no soda is dispensed*

init(get\_soda) := FALSE;

-- *Initially, no beer is dispensed*

init(get\_beer) := FALSE;



# Vending machine in nuXmv #3

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*-- Define the next state and output values for each state*

```
next(state) :=
  case
    state = pay & !insert_coin : pay;
    state = pay & insert_coin : select;
    state = select & !insert_coin : select;
    state = select & insert_coin : {soda, beer};
    state = soda : pay;
    state = beer : pay;
    TRUE : state;
  esac;
```

# Vending machine in nuXmv #4

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```
next(insert_coin) :=  
    case  
    -- A coin is inserted for the first time in 'select'  
    state  
        state = select & !insert_coin : TRUE;  
    -- Coin remains inserted  
        insert_coin : insert_coin;  
        TRUE : FALSE;  
    esac;
```

# Vending machine in nuXmv #5

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```
next (get_soda) :=
  case
    -- Soda is dispensed when in 'select' state with a coin
    inserted
      state = select & insert_coin : TRUE;
      TRUE : FALSE;
    esac;
next (get_beer) :=
  case
    -- Beer is dispensed when in 'select' state with a coin
    inserted
      state = select & insert_coin : TRUE;
      TRUE : FALSE;
    esac;
```

# Vending machine in nuXmv #6

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*-- Atomic propositions*

DEFINE

paid := insert\_coin;

drink := get\_soda | get\_beer;

*-- LTL properties*

LTLSPEC G ((state = soda | state = beer) -> F(paid &  
drink));

LTLSPEC G (!paid -> (F drink));

# nuXmv demo...

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# Questions?

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# References

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- Baier, C., & Katoen, J. P. (2008). Principles of model checking. MIT press. Chapters 1, 2, 3, 5, 6.
- <https://nuxmv.fbk.eu/downloads/nuxmv-user-manual.pdf>