Robust Extended Kalman filter for NLOS Mitigation and Sensor Data Fusion

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***Abstract*—This paper presents an algorithm based on the Robust Extended Kalman filter (REKF) for non-line-of-sight (NLOS) mitigation and sensor data fusion. The aim is to locate an off-the-shelf smartphone using only its speaker and its inertial measurement unit (IMU). The target emits inaudible sound signals which are detected by static receivers on the ceiling. Then, its position can be estimated using time differences of arrival (TDOA). The limiting factor of sound localization is the difficulty to identify the signals which travel directly from the target to the anchor nodes (line-of-sight) and the signals which are reflected to walls or other objects (non-line-of-sight). Nowadays, most of the smartphones are provided with an IMU which can be used for localization. However, its accumulative error deteriorates the result after a certain time. Then, a promising approach is to fuse the received sound timestamps and the IMU data in order to estimate the position of the target. The REKF reduces the effect of the non line-of-sight (NLOS) measurements by assigning them lower weights while the fusion with the IMU improves the accuracy, specially when only a reduced number of line-of-sight (LOS) signals are available.**

I. Introduction

Nowadays, GPS is a commonly adopted solution for localizing a target outdoors. However, it lacks connectivity indoors. In the recent years, many different approaches have been presented to locate a device indoors. However, it is still an open field of research. Using an off-the-shelf smartphone one can emmit inaudible sound signals from 18 kHz to 22 kHz [1]. These signals are received by receivers attached to the ceiling and can be used to estimate the position of the target using time difference of arrival (TDOA). However, in real life often the signals emitted from the smartphone are blocked by obstacles and only echoes are received. Then, it is challenging to identify the line-of-sight signals (LOS). In this paper we present a novel algorithm for localization and sensor data fusion in NLOS conditions by using a weighted Kalman filter which sets lower weights to unlikely measurements. Moreover, it fuses the data from the inertial measurement unit of the smartphone (IMU) to improve the accuracy in situations where a reduced number of LOS signals are available. The IMU has been proved to be capable of tracking pedestrians in indoor areas showing a maximum deviation of 1 m after a walk of 30 m [2]. However, it cannot be used as the only source of information due to its accumulative error. Recursive filters such as the extended Kalman filter have been proved to be capable of locating a target using TDOA measurements. Those algorithms assume Gaussian noise. However, the NLOS measurements are not Gaussian, which can lead to a high localization error in real environments. A common solution is the Robust Extended Kalman filter [3–6], which iteratively finds the weight of every measurement. In this paper we show how such algorithm can be used for sensor data fusion, weighting not only the TDOA measurements but also the IMU measurements according to their likelihood.

II. Problem formulation

We study the mathematical problem of finding the position of a target $\mathbf{S}_t$ in a two-dimensional space. The target $\mathbf{S}_t$ emits a signal at time t_s which is received by the receiver $\mathbf{M}_i$ as follows:

$$t_i = \frac{1}{c}|\mathbf{S}_t - \mathbf{M}_i| + t_s \tag{1}$$

where c is the signal velocity, t_i is the received timestamp and $|\cdot|$ denotes the Euclidean norm.

The sending time t_s is assumed to be unknown, therefore, we use a TDOA sensor model. Then, the measurement corresponding to the receivers i and j would be:

$$z_{ij} = \frac{1}{c}|\mathbf{S}_t - \mathbf{M}_i| - \frac{1}{c}|\mathbf{S}_t - \mathbf{M}_j| \tag{2}$$

In addition, the measurements from the IMU are used to estimate the position of the target S. We remove the effect of the gravity and extract the x and y components of the acceleration by combining the acceleration and the angular rate. Afterwards, a zero-velocity update approach [7] is used to estimate the position of the target.

Then, in our model the measurements are the reception times of the sound signals and the positions estimated by the IMU.

A. Extended Kalman filter

The Kalman filter fulfils a Bayesian filtering scheme, which is a probabilistic approach to recursive state estimation based on the Markov assumption, i.e. the assumption that the current state depends only on the previous state, not on the previous trajectory. The Extended Kalman filter [8] linearizes non-linear models and has been widely used in positioning due to its low complexity and high performance. The state of the filter $\mathbf{x}_t$

contains the position $\mathbf{S}_t$ and the velocity $\mathbf{V}_t$ of the target. We use the following acceleration model:

$$\begin{bmatrix}\mathbf{S}_t\\ \mathbf{V}_t\end{bmatrix} = \mathbf{B}\begin{bmatrix}\mathbf{S}_{t-1}\\ \mathbf{V}_{t-1}\end{bmatrix} + \mathbf{G}\mathbf{\Phi}_{t-1} \qquad \mathbf{\Phi}_{t-1} \sim \mathcal{N}(0,\mathbf{Q}) \tag{3}$$

where

$$\mathbf{B} = \begin{bmatrix}1 & 0 & \Delta t & 0\\ 0 & 1 & 0 & \Delta t\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix} \qquad \mathbf{G} = \begin{bmatrix}\frac{\Delta t^2}{2} & 0\\ 0 & \frac{\Delta t^2}{2}\\ \Delta t & 0\\ 0 & \Delta t\end{bmatrix} \tag{4}$$

where Δt is the elapsed time between two estimations. Then, the motion model is used to predict the next state $\widehat{\mathbf{x}}_t$ and its covariance matrix $\mathbf{P}_t^-$:

$$\begin{aligned}\widehat{\mathbf{x}}_t &= \mathbf{B}\mathbf{x}_{t-1}\\ \mathbf{P}_t^- &= \mathbf{B}\mathbf{P}_{t-1}\mathbf{B}^T + \mathbf{G}\mathbf{Q}\mathbf{G}^T\end{aligned} \tag{5}$$

The sensor model includes the measurements produced by sound localization and the measurements produced by the IMU. This relation is modeled as follows:

$$\mathbf{z}_t = h(\mathbf{x}_t) + \rho_t \qquad \rho_t \sim \mathcal{N}(0,\mathbf{R}_m) \tag{6}$$

As the relation is not linear, the extended Kalman filter makes use of the first-order Taylor expansion:

$$h(\mathbf{x}_t) = h(\widehat{\mathbf{x}}_t) + \mathbf{H}_t(\mathbf{x}_t - \widehat{\mathbf{x}}_t) \tag{7}$$

where $\mathbf{H}_t$ is the Jacobian matrix of the partial derivatives of h with respect to $\mathbf{x}_t$.

Then, the sensor model is used to correct the predicted state as follows:

$$\begin{aligned}\mathbf{K}_t &= \widehat{\mathbf{P}}_t^-\mathbf{H}_t^T(\mathbf{H}_t\widehat{\mathbf{P}}_t^-\mathbf{H}_t^T + \mathbf{R}_m)^{-1}\\ \mathbf{x}_t &= \widehat{\mathbf{x}}_t + \mathbf{K}_t\left(\mathbf{z}_t - h(\widehat{\mathbf{x}}_t)\right)\\ \mathbf{P}_t &= \left(\mathbf{I} - \mathbf{K}_t\mathbf{H}_t\right)\widehat{\mathbf{P}}_t\end{aligned} \tag{8}$$

where $\mathbf{z}_t$ is the measurement vector.

B. Weighted Least Squares

The problem of locating the target S_t can be formulated as an overdetermined system of equations such as $\mathbf{A}\mathbf{x} = \mathbf{b}$, being $\mathbf{A}$ a known matrix of size $M \times N$ and $\mathbf{b}$ a known vector of size M. Due to the noisy measurements, one has to find an approximation of the vector of variables $\mathbf{x}$.

Being $\mathbf{e}$ the vector of errors $\mathbf{e} = \mathbf{A}\mathbf{x} - \mathbf{b}$, one can minimize the squared error $\mathbf{e}^T\mathbf{e}$. This is the least squares approach and the result is:

$$\mathbf{x} = (\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathbf{b} \tag{9}$$

However, in scenarios such as indoor localization certain measurements have a high error (NLOS), while others have a limited error (LOS). The solution which minimizes $\mathbf{e}^T\mathbf{e}$ can be then highly affected by the NLOS measurements. Then, a better solution is to minimize a weighted version of the measurements $\mathbf{e}^T\mathbf{W}\mathbf{e}$, where $\mathbf{W}$ is a diagonal matrix with weights w_i in its diagonal. Then, $\mathbf{x}$ can be estimated as:

$$\mathbf{x} = (\mathbf{A}^T\mathbf{W}\mathbf{A})^{-1}\mathbf{A}^T\mathbf{W}\mathbf{b} \tag{10}$$

In order to calculate the weights, different approaches can be made:

1) Iteratively reweighted least squares: The weighted least squares can be generalized to an l_p approximation problem [9]. Then, the error function $||e||_p$ is:

$$||e||_p = \Big(\sum_n |e_n|^p\Big)^{1/p} \tag{11}$$

which can be rewritten as:

$$||e||_p = \Big(\sum_n |e_n|^{(p-2)}|e_n|^2\Big)^{1/p} \tag{12}$$

where e_n represents the nth term of the vector $\mathbf{e}$. Then, this has been shown to be equivalent to solve a weighted least squares problem with the following weights:

$$w_n = |e_n|^{(p-2)/2} \tag{13}$$

2) Huber's M-Estimator: Huber [10] introduced the *M-Estimators*, which minimize a function $\rho(e_n)$ instead of the squares:

$$\arg\min_{\mathbf{x}} \sum \rho(e_n) \tag{14}$$

Then, setting the partial derivatives with respect to the state $\mathbf{x}$ to zero would give the result. However, the derivative of ρ is not always linear. Therefore, one can define the derivative of the function ρ as

$$\rho'(e_n) = e_n w(e_n) \tag{15}$$

The weights are defined as:

$$\begin{cases} w_n(e_n) = 1 & |e_n| < a\\ w_n(e_n) = \frac{d\cdot\tanh(\frac{d(b-e_n)}{2})}{e_j} & a < |e_n| < b\\ w_n(e_n) = 0 & |e_n| > b\end{cases} \tag{16}$$

Where a and b are the *clipping points* and d is defined in order to have a continuous function. The value of a and b depends on the knowledge of the noise. A common approach is to take a=$\frac{\text{MAD}}{\theta}$ and b=$4a$, where MAD=$\text{median}_n(|e_n - \text{median}(e)|)$) and θ is a tuning parameter.

C. Robust Extended Kalman filter

The Extended Kalman filter (EKF) assumes Gaussian noise. However, in presence of NLOS measurements this assumption is not correct. One way to mitigate this is by weighting the measurements. The EKF can be formulated in the form of weighted least squares [6]. In order to do this, the equations can be reformulated as follows:

$$\begin{aligned}\mathbf{x}_t &= (\mathbf{N}^T\mathbf{W}_t\mathbf{N}_t)^{-1}\mathbf{N}_t^T\mathbf{W}_t\mathbf{Y}_t\\ \mathbf{N}_t &= \mathbf{V}_t^{-1}\begin{bmatrix}\mathbf{I}\\ \mathbf{H}_t\end{bmatrix}\\ \mathbf{Y}_t &= \mathbf{V}_t^{-1}\begin{bmatrix}\widehat{\mathbf{x}}_t\\ \widehat{\mathbf{z}}_t\end{bmatrix}\\ \widehat{\mathbf{z}}_t &= \mathbf{z}_t - h(\widehat{\mathbf{x}}_t) + \mathbf{H}_t\widehat{\mathbf{x}}_t\end{aligned} \tag{17}$$

where $\mathbf{W}_t$ is a diagonal matrix which contains the different weights. The matrix $\mathbf{V}_t\mathbf{V}_t^T$ is given by:

$$\mathbf{V}_t\mathbf{V}_t^T = \begin{bmatrix}\mathbf{P}_t^- & 0\\ 0 & \mathbf{R}_m\end{bmatrix} \tag{18}$$

Then, $\mathbf{V}_t$ can be calculated using the Cholesky decomposition. The weight j at the iteration k is calculated using Huber weights (Eq. 16), where:

$$e_n = |y_j - \mathbf{n}_j^T \mathbf{x}^{k-1}| \tag{19}$$

the variable y_j is the jth element of $\mathbf{Y}_t$ and $\mathbf{n}_j$ is the jth row of $\mathbf{N}_t$. Every iteration, the weights can be computed using one of the methods explained above (Eq. 13 or Eq. 16). In this approach we use Huber weights, which have been proved to be efficient against NLOS measurements [3]. Note that $\mathbf{Y}_t$ includes not only the measurement but the state. However, we are only interested in weighting the measurements. Therefore, we set constant weights to the rows of the state.

III. Experiments and Simulations

In order to prove the feasibility of the proposed approach, we test the algorithm with simulated and real data.

First we test the algorithm with synthetic data, assuming there are 10 receivers available. The sound measurements have a Gaussian noise of 0.3 ms. Furthermore, 40% of the measurements of the receivers are NLOS measurements. The NLOS measurements are simulated by adding a uniformly distributed error from 3 ms to 13 ms. The IMU data is simulated by assuming a constant angle error. We test the REKF and the EKF fusing the IMU data and the timestamps. In addition, we test the REKF using only the timestamps. The estimated and real positions can be seen in Fig.1. The resulting error is:

Algorithm	Mean Error (m)	Std. (m)
REKF with data fusion	0.1251	0.0829
REKF without data fusion	0.1641	0.1061
EKF with data fusion	0.3844	0.2040

Therefore, the REKF with data fusion has a lower error than the other alternatives.

In order to test the proposed approach in a real scenario, we use a smartphone to emit inaudible signals from 18 kHz to 22 kHz. These signals are received by eight static receivers attached to the ceiling (See Fig. 2). The target is located in an area with a high number of obstacles. The result can be seen in Fig. 3. The mean error achieved using a REKF without sensor data fusion is 0.19 m and the standard deviation is 0.19 m. The best result is achieved by the REKF with sensor data fusion: 0.16 m of mean error and 0.14 m of standard deviation. The cumulative distribution of the error can be seen in Fig. 4. The error at each position is calculated as the distance to the closest point of the real trajectory.

IV. Conclusions and Future Work

The algorithm shows promising results to avoid NLOS situations and the inertial measurements reduce the error of the estimated positions. We have shown that the REKF with data fusion outperforms the standard EKF and the REKF either using synthetic data or real data.

The presented approach shows the feasibility of sensor data fusion for locating a smartphone with a weighted algorithm. However, there is room for improvement. After a certain time, the positions of the IMU might become unusable due to the accumulative error. Therefore, the history of the estimated

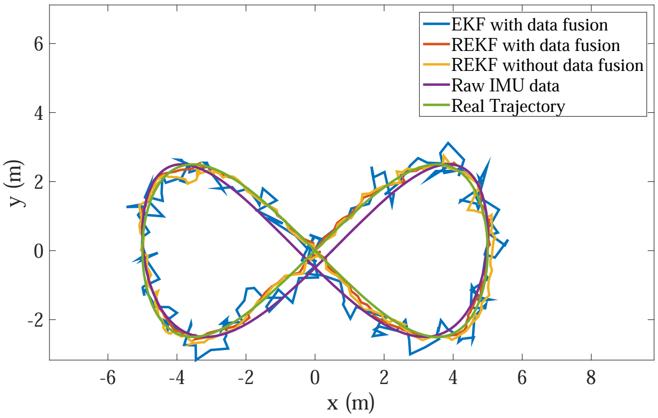

Fig. 1. Real and estimated positions of a target using synthetic data. In this simulation, 40% of the measurements are NLOS. The REKF which fuses the data from the receivers and the IMU achieves the lowest error: 0.13 m of mean error and 0.08 m of standard deviation.

Fig. 2. The receivers are mounted on metal bars at a median height of 4.84 m

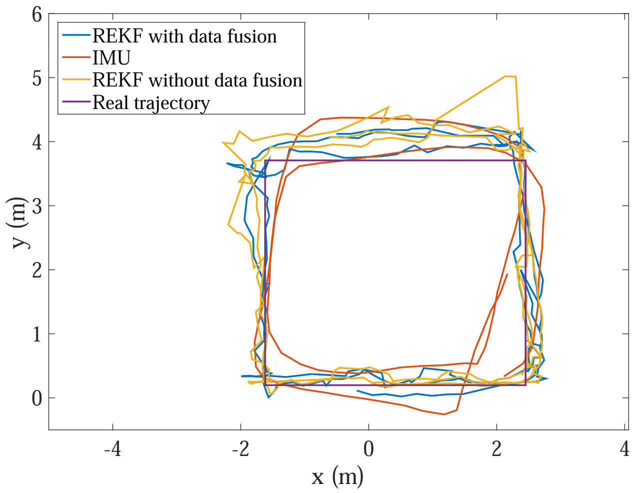

Fig. 3. Real and estimated positions of a smartphone emitting inaudible sound signals. The signals are detected by static receivers (see Fig. 2). The REKF which fuses the data from the receivers and the IMU achieves the lowest error: 0.16 m of mean error and 0.14 m of standard deviation.

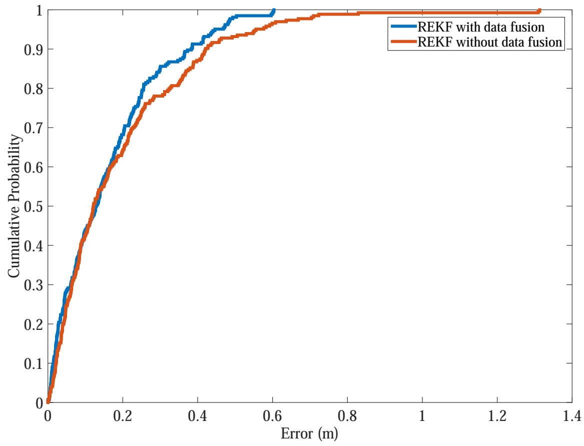

Fig. 4. Cumulative distribution function for the real experiment in Fig. 3. Fusing the IMU data, the error is below 0.37 m for 90% of the measurements. Without the IMU data, the error is below 0.42 m for 90% of the measurements.

positions by the data fusion algorithm would have to be used in order to realign both measurements. Furthermore, a multiple model Kalman filter [3] with a weighted and a non-weighted EKF would reduce the error in LOS conditions.

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