Important Resources:

1. My google drive folder/webpage
2. "Resources" from course website:
http://www.math.lsa.umich.edu/courses/116/Resources/index.html

Expectations for students regarding series:

- Nth term test (if the terms of a series don't go to zero, the series diverges).
    - Students can call it the **nth term test (for divergence), or the divergence test.**
    - Students can write that **"the terms of the series don't go to zero, so the series diverges."**
- For alternating series test, students should write the hypotheses, when using it, but students do not have to justify them (if they're right, of course). If students are writing the hypotheses in general terms, a_n, they need to state what a_n they are referring to.
- Justifications for Comparison and LCT should have the same components as justifications for integrals.
- Students must write that the SERIES converges/diverges. This shows an understanding of the difference between sequences and series.
- The following abbreviations will be accepted:
    - (D)CT=comparison test
    - LCT=limit comparison test
    - AST=alternating series test
    - IT=integral test

"sum of e^(-x)" or geometric series:
1. justification using geometric series with ratio less than 1 is better
2. you can do integral test, and then exponential decay test.

Summary

4.7

- Check whether you can use L'hopital before you actually do it, so limit has to be in the form "$\frac{0}{0}$", "$\frac{\infty}{\infty}$", "$0 \cdot \infty$"

- "0 * infinitiy": Convert it into a fraction

Example: $\lim_{x \to \infty} x \cdot e^{-x} = \lim_{x \to \infty} \frac{x}{e^{x}}$

or $\lim_{x \to \infty} \frac{e^{-x}}{x^{-1}}$

- NEVER divide it by 0, nor do +,-,*,/,power with infinity

- NEVER NEVER NEVER plug in infinity to a function

Example: $\lim_{t \to \infty} \int_0^t -e^{-x}\,dx = \lim_{t \to \infty} e^{-x}\Big|_0^t = \lim_{t \to \infty} e^{-t} - e^{-0} = -1$

Non Example: $\lim_{t \to \infty} \int_0^t -e^{-x}\,dx = \cancel{\lim_{t\to\infty} e^{-x}\Big|_0^{\infty}}$

$= -1$

7.6,7.7

"Improper Integral Expectation" from "Resources"
"p-test and exp decay test" from "Resources"

7.6
- ALWAYS replace problematic point(infty/zeros of denominator) by a LIMIT before any integration(antiderivative/substitution/by parts)
- CANNOT evaluate at the problematic point, NEVER write $e^{-x}\Big|_0^{\infty}$, $\frac{1}{x}\Big|_0^1$, $\frac{1}{x^2}\Big|_0^1$

7.7
- p test or exp decay test, you NEED to write "integral of xxx"(what you are integrating)
- p-test:
    - NEED "by p-test with p=?"
    - WARNING: p-test works differently for x around 0.
- Exponential decay test:
    - NEED "by exponential decay test"

Comparison Test (for both integral and series):

Resources: "Comparison Test for integral Expectation"

NEED:
1) Inequality
2) New integral/series
3) Old integral/series

1) Inequality:
- NO integral/summation
- NEED range of x: example "for x >10^100", or "0<x<1/10^100"
  (for series similar, but you want n instead of x)
- Want conv: 0<= OLD <= NEW
  Want div: 0<= NEW <= OLD

Make sure you inequality is CORRECT

2),3) NEED integral/summation(say what you are integrating or summing up explicitly)
NEED "by... test", you can use abbre for "Comparison Test" or "Limit Comparison Test".

For (Limit Comparison Test)LCT:
Basically same as comparison test

1)Limit of ratio is a positive finite number
2)New Integral/Series
3)Old Integral/Series

DO NOT us LCT if you have similar to

$$\int_1^\infty \frac{5+\cos^2(x)}{x^2}\,dx$$

Basically, constant+cosine/sine is a bad sign for LCT.

8.7, 8.8

Done

PDF:
-Area under whole graph =1
-Area under graph from a to b= probability P(a<=X<=b)
-Interpretation of p(x): (For example, use p(3) here)The probability of (given context) between 2.99 to 3.01 is !!Approximately!! p(3)*(3.01-2.99)
-p(x)>=0

CDF:
-Increasing from 0 to 1
-Writing down the formula: integral of pieces, PLUS the area of previous part.
-Values give probability of X<=t directly, let P(t) be the CDF.
P(t)=P(X<=t)=probability of X at most t.
1-P(t)=P(X>=t)=probability of X at least t.

Median:
- Look for the piece that median lies in
- then area on the left=1/2=area on the right (use the easier one)

Mean: $E[X] = \int_{-\infty}^{\infty} x f(x) dx$

Normal Distribution PDF: $f(x) = \frac{e^{-(x-\mu)^2/(2\sigma^2)}}{\sigma\sqrt{2\pi}}$

μ = mean = median
σ = standard deviation

9.1/9.2

Series = limit of partial sum
Series is not limit of terms

Possible for series to diverge, but the lim of terms convergs.

Sequence:
-Conv=>bounded
 Unbounded=>div
-(Monotone Convergence theorem)
 Monotone+bounded=> convergence

Geometric Series:

How to approach problems:

1) write down S_1,S_2,S_3...

   NEVER over simplifies (add/multiply numbers), KEEP track of pattern

   EXPAND all Parentheses. Rearrange the general term to have increasing power of common ratio, Wrap up the geometric series part with parenthesis.

2) Write S_n, write as a long sum first with "+...+" by observing the pattern you find, then use the formula

$a_n = S_n - S_{n-1}$

9.3,9.4

IMPORTANT remark:
For "..." the terms, NEED a formula, CANNOT write "a_n".

n-th term test:
-Compute Limit "terms" as a number, +/- infty or DNE, CANNOT just say not equal to 0
-"Sum"of "terms"divergent by "n-th term test"

Integral test:
NEED:
-"function">0 and decreasing(no justification)
-Justification for integral as in the improper integral part
- "Sum of (insert formula)" is conv/div by integral test.
WARNING:
If integral converges to e.g. 10, series converges, but does NOT EQUAL to 10.

p-test/CT/LCT:
similar to earlier

Warning: It has to be alternating sign!

AST: (alternating series test)
NEED:

1) $\boxed{\text{-"(insert formula after taking absolute value)" decreasing or "(insert formula for n+1 after taking absolute value)<(... for n...)"}}$

2) - "lim (insert formula after abs value)=0"

3) -"Sum (insert formula) converges by AST"

NEVER "div by AST", try n-term test instead.

Example

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$$

$$\boxed{\begin{array}{l} 1)\ \frac{1}{n} \text{ is decreasing} \\ \text{or } \frac{1}{n+1} < \frac{1}{n} \end{array}}$$

2) $\lim_{n \to \infty} \frac{1}{n} = 0$

3) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$ converges by AST.

Ratio Test:
- Limit of ABSOLUTE VALUE of ratio
- write "2n+2" instead "2(n+1)" to help yourself
- to evaluate limit:
1) Group similar terms
2) Simplify each group of similar terms
3) Take limit

"Sum (inserted terms) conv/div by ratio test"

Absolute convergence:
1) Check $\sum |a_n|$
   if Convergent: done, the series converges absolutely
   if Divergent: Next step
2) Check the sum of original series, $\sum a_n$
   if Convergent: Series converges conditionally
   if Divergent: Series divergent

Example

$$\sum_{n=0}^{\infty} \frac{n!}{5^n}$$

$$L = \lim_{n\to\infty} \left| \frac{(n+1)!}{5^{n+1}} \frac{5^n}{n!} \right|$$

1) $= \lim_{n\to\infty} \left|\frac{(n+1)!}{n!}\right| \left|\frac{5^n}{5^{n+1}}\right|$

2) $= \lim_{n\to\infty} \left|\frac{n+1}{1}\right| \left|\frac{1}{5}\right|$

3) $= \infty > 1$

So, $\sum_{n=0}^{\infty} \frac{n!}{5^n}$ diverges by ratio test.

9.5

Radius of Convergence:
Ratio Test, set < 1 after getting limit and solve for "|X-center| < radius"

Interval of convergence:
1) Ratio test as above
2) Plug in end points to check.
(Ratio test never works for the end points).

Example

$$\sum_{n=1}^{\infty} \frac{(-1)^n n}{4^n}(x+3)^n$$

$$L = \lim_{n\to\infty} \left| \frac{\frac{(-1)^{n+1} n}{4^{n+1}}(x+3)^{n+1}}{\frac{(-1)^n}{4^n}(x+3)^n} \right| = \lim_{n\to\infty} \left| \frac{(n+1)(x+3)}{4n} \right| = \frac{1}{4}|x+3|$$

By Ratio Test $\frac{1}{4}|x+3| < 1$ series converges
$\Leftrightarrow |x+3| < 4$ series converges
and $\frac{1}{4}|x+3| > 1 \Leftrightarrow |x+3| > 1$ seres diverges

So radius of convergence is 4.

General Hint

You are NOT asked to simplify answer, so don't waste time calculating 3^6 or 13*57.

You are the best!