8. [14 points] A coffee shop offers only one hour of free internet access to all its customers. The time t in hours a customer uses the internet at the coffee shop has a probability density function

$$p(t) = \begin{cases} at\sqrt{1-t^2} & 0 \le t \le 1. \\ 0 & \text{otherwise.} \end{cases}$$

where a is a constant.

a. [4 points] For what value of a is $p(t)$ a probability density function? Find its value without using your calculator.

b. [4 points] Find the cumulative distribution function $P(t)$ of $p(t)$. Make sure to indicate the value of $P(t)$ for all values of $-\infty < t < \infty$. Your final answer should not contain any integrals.

c. [3 points] Find the the probability that a customer is still using the internet after 40 minutes (without using your calculator).

d. [3 points] Find an expression for the mean of this distribution. Use your calculator to compute its value.