Team HW 5

All answers need to be in exact values (**this will be the same for the final exam; no "0.333" for 1/3**). Explanation is needed for all questions unless otherwise specified.

1. Consider the power series

$$f(x) = \sum_{n=1}^{\infty} \frac{n \sin \frac{1}{n}}{4^n} (x+2)^{2n+1}.$$

   (a) Find the radius of convergence for the power series $f(x)$.

   (b) Find $f^{(100)}(-2)$ and $f^{(101)}(-2)$.

   (c) Consider another power series $\sum_{n=0}^{\infty} C_n (x-3)^n$. Given that this converges at $x = 1$ and diverges and $x = 7$, does it converge or diverge at each of the following x-values, or it cannot be determined? Provide explanation for each of the value.

$$-6 \quad -4 \quad -2 \quad 2 \quad 4 \quad 5 \quad 9$$

2. (a) Write down the first 4 non-zero terms and also the general term of the Taylor series expansion for $f(x) = \dfrac{d}{dx}\left(\dfrac{1}{1-x}\right) = \dfrac{1}{(1-x)^2}$ at $x = 0$.

   (b) What is the radius of convergence for your Taylor series in (a)? What is the interval of convergence?

   (c) Find a function which has the following Taylor series expansion centered at $x = 0$:

$$x + 2x^2 + 3x^3 + 4x^4 + \cdots = \sum_{n=1}^{\infty} n x^n.$$

   (d) Find the exact values of the following series. (Hint: use (c).)

$$1 + 2^2 \frac{1}{e} + 3^2 \frac{1}{e^2} + 4^2 \frac{1}{e^3} + \cdots = \sum_{n=1}^{\infty} n^2 \frac{1}{e^{n-1}}.$$