

Option Greek

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Introduction

- Option is exposed to different type of market risk, for example: spot price, time to maturity, volatility and interest rate.
- Greeks measure the sensitivity of the value of a derivative or a portfolio to changes in parameter value(s) while holding the other parameters fixed.
- Mathematically, they are partial derivatives of the price with respect to the parameter values.
- Greeks are important not only in the mathematical theory of finance, but also for those actively trading. Financial institutions will typically set (risk) limit values for each of the Greeks that their traders must not exceed.
- Delta is the most important Greek since this usually confers the largest risk. Many traders will zero their delta at the end of the day if they are speculating and following a delta-neutral hedging approach as defined by Black-Scholes.

Aim

- Summarize and provide further insights of Greeks of each type of options.

Internal Link Referencing:

[1. Greeks Definition \(Done\)](#)

[2. European Plain Option\(Done\)](#)

- [a. Vanilla Option \(Done\)](#)
- [b. Regular Barrier Option \(Done\)](#)
- [c. Reverse Barrier Option \(Not in planned list\)](#)
- [d. Double Barrier Option \(Done\)](#)

[3. European Binary Option\(Done\)](#)

- [a. Binary Option \(Done\)](#)

[4. Path-Dependent Option \(American\)](#)

- [a. Binary One-Touch Option \(In Progress\)](#)
- [b. American Binary Knock-Out Option \(In Progress\)](#)
- [C. Asian Options \(In Progress\)](#)

1. Greeks Definition

- We are explaining up to **second derivatives**.

a. Delta (Δ)

- Measures rate of **option price** w.r.t. to **spot price**.

b. Theta (Θ)

- Measures rate of **option price** w.r.t. to **time to maturity**.

c. Vega (ν)

- Measures rate of **option price** w.r.t. to **volatility**.

d. Rho (ρ)

- Measures rate of **option price** w.r.t. to **interest rate**.

e. Gamma (γ)

- Measures rate of **Delta** w.r.t. to **spot price**.

f. Vanna ($\frac{\delta\nu}{\delta S}$)

- Measures rate of **Vega** w.r.t. to **spot price**.
- Alternately, measures rate of **Delta** w.r.t. to **volatility**.

g. Volga ($\frac{\delta\nu}{\delta\sigma}$)

- Measures rate of **Vega** w.r.t. to **spot price**.

2. European Plain Option

- Here we discuss about:
- [a. Vanilla Option](#)
- [b. Barrier Knock-In Option](#)
- [c. Barrier Knock-Out Option](#)
- [d. Double Knock-Out Option](#)

2a. Vanilla Option

Definition:

- A vanilla option is a financial instrument that gives the holder the right, but not the obligation, to buy or sell an underlying asset at a predetermined price **at maturity only (European)**.

- Note: Charts above assumes zero discount rate (except Rho).
- We can use the following charts to further intuitively illustrate **Vanna-Volga Pricing** as well.

Payoff Equations:

- Call Option:

$$\max(0, S_T - K)$$

- Put Option:

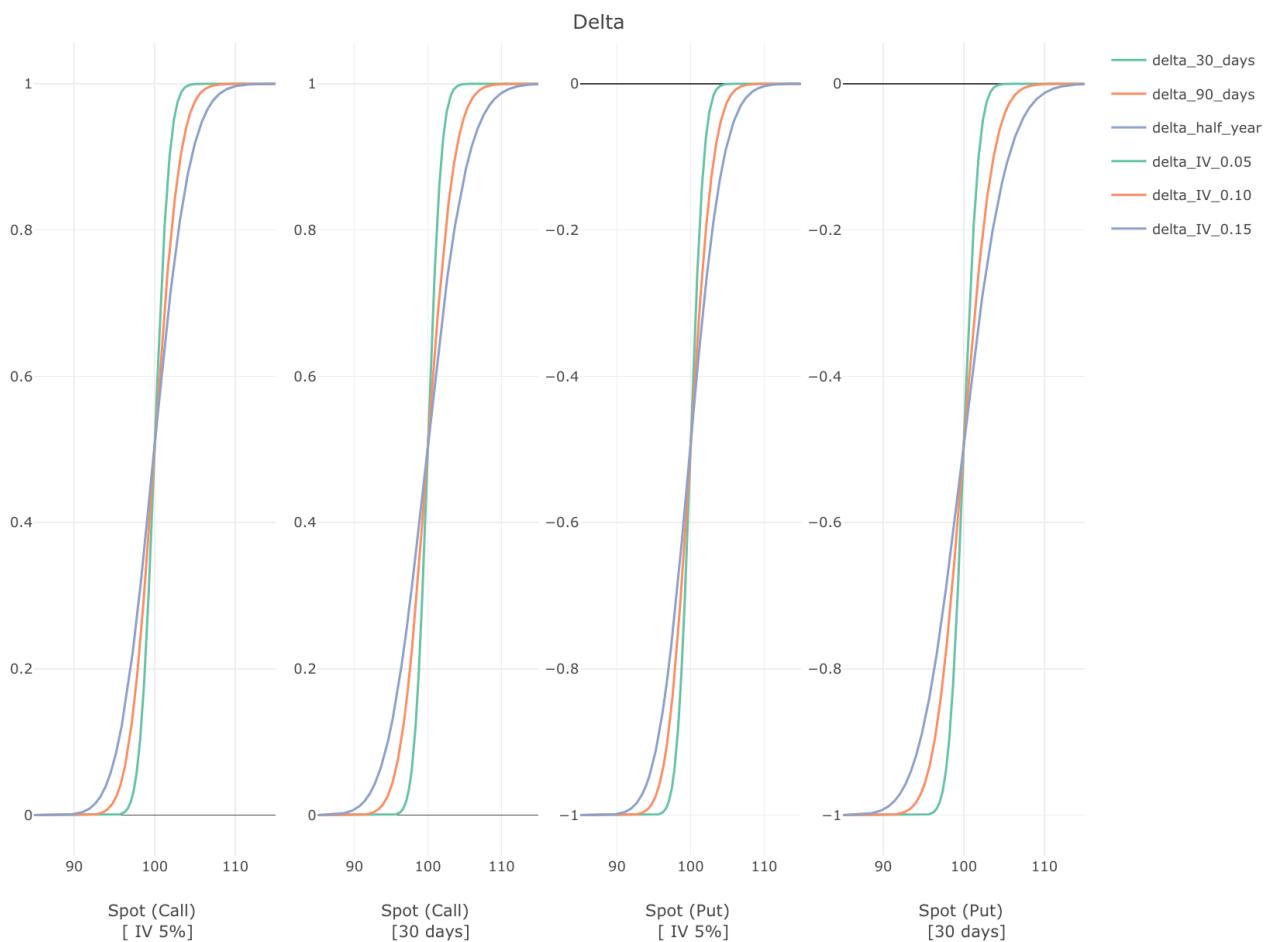
$$\max(0, K - S_T)$$

Closed Form Formula:

	Calls	Puts
fair value (V)	$S e^{-q\tau} \Phi(d_1) - e^{-r\tau} K \Phi(d_2)$	$e^{-r\tau} K \Phi(-d_2) - S e^{-q\tau} \Phi(-d_1)$
delta (Δ)	$e^{-q\tau} \Phi(d_1)$	$-e^{-q\tau} \Phi(-d_1)$
vega (\mathcal{V})	$S e^{-q\tau} \phi(d_1) \sqrt{\tau} = K e^{-r\tau} \phi(d_2) \sqrt{\tau}$	
theta (Θ)	$-e^{-q\tau} \frac{S \phi(d_1) \sigma}{2\sqrt{\tau}} - r K e^{-r\tau} \Phi(d_2) + q S e^{-q\tau} \Phi(d_1)$	$-e^{-q\tau} \frac{S \phi(-d_1) \sigma}{2\sqrt{\tau}} + r K e^{-r\tau} \Phi(-d_2) - q S e^{-q\tau} \Phi(-d_1)$
rho (ρ)	$K \tau e^{-r\tau} \Phi(d_2)$	$-K \tau e^{-r\tau} \Phi(-d_2)$
lambda (λ)		$\Delta \frac{S}{V}$
gamma (Γ)		$e^{-q\tau} \frac{\phi(d_1)}{S \sigma \sqrt{\tau}} = K e^{-r\tau} \frac{\phi(d_2)}{S^2 \sigma \sqrt{\tau}}$
vanna		$-e^{-q\tau} \phi(d_1) \frac{d_2}{\sigma} = \frac{\mathcal{V}}{S} \left[1 - \frac{d_1}{\sigma \sqrt{\tau}} \right]$
charm	$qe^{-q\tau} \Phi(d_1) - e^{-q\tau} \phi(d_1) \frac{2(r-q)\tau - d_2 \sigma \sqrt{\tau}}{2\tau \sigma \sqrt{\tau}}$	$-qe^{-q\tau} \Phi(-d_1) - e^{-q\tau} \phi(d_1) \frac{2(r-q)\tau - d_2 \sigma \sqrt{\tau}}{2\tau \sigma \sqrt{\tau}}$
vomma		$S e^{-q\tau} \phi(d_1) \sqrt{\tau} \frac{d_1 d_2}{\sigma} = \mathcal{V} \frac{d_1 d_2}{\sigma}$

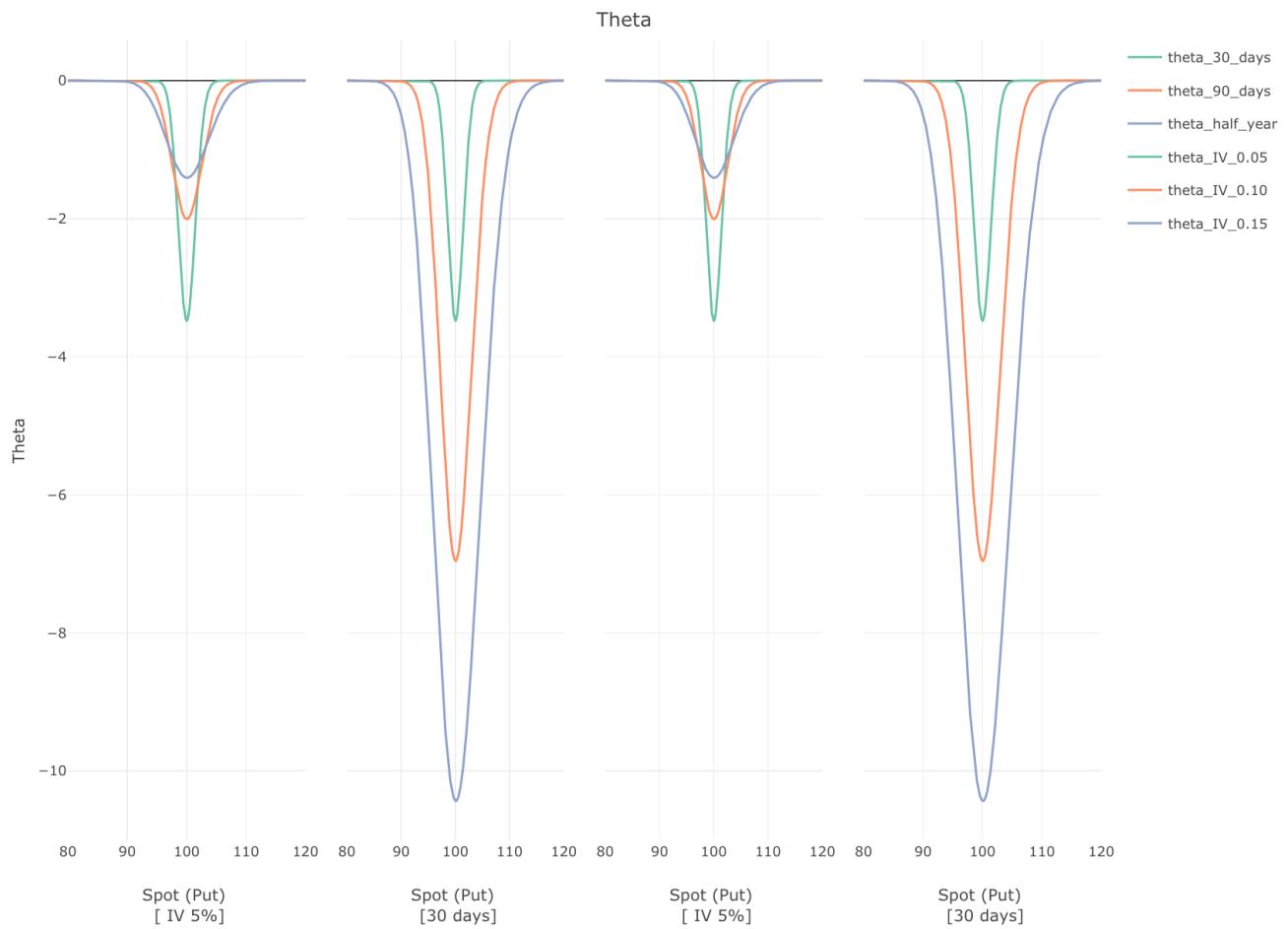
i. Delta:

- Delta's shape is like an **inverted Z curve**.
- Delta ranges from 0 to 1 for **CALL** and -1 to 0 for **PUT**.
- As the time remaining to expiration grows **shorter**, the time value of the option evaporates and correspondingly, the delta of in-the-money options increases while the delta of out-of-the-money options decreases.
- As volatility **rises**, the time value of the option **grows** as greater possibility of an option to be **ITM**.



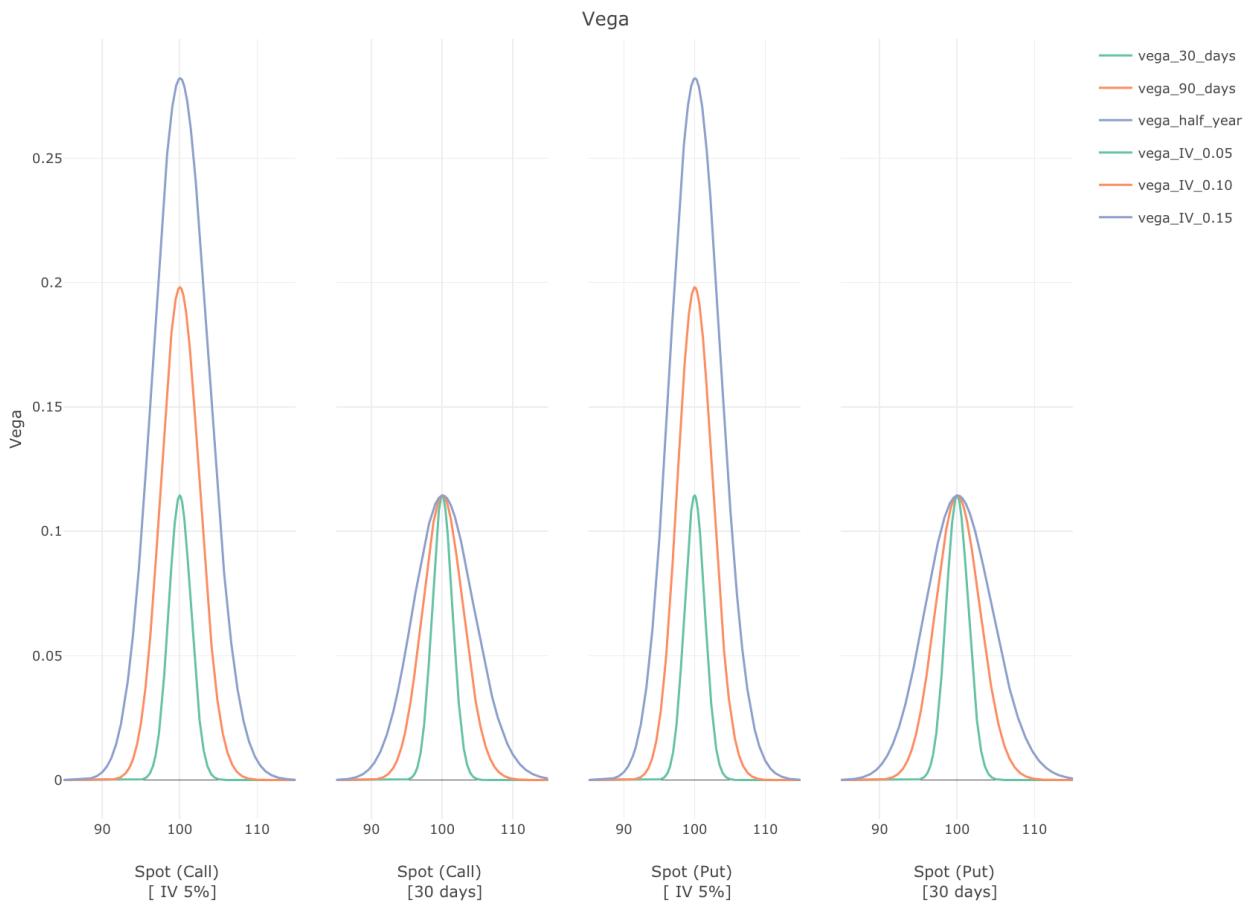
ii. Theta:

- Theta's shape is like a **downward bell curve**.
- Theta achieves its **lowest point at ATM**, and close to 0 for **deep ITM/OTM contract**.
- The theta is **highly sensitive around ATM**; This is pretty obvious as such options have the highest time value and thus have more premium to lose each day.
- The **shorter** the time to maturity, the higher the **theta**.
- The **higher** the volatility, the higher the **theta** because the time value on these options are higher and so they have more to lose per day.



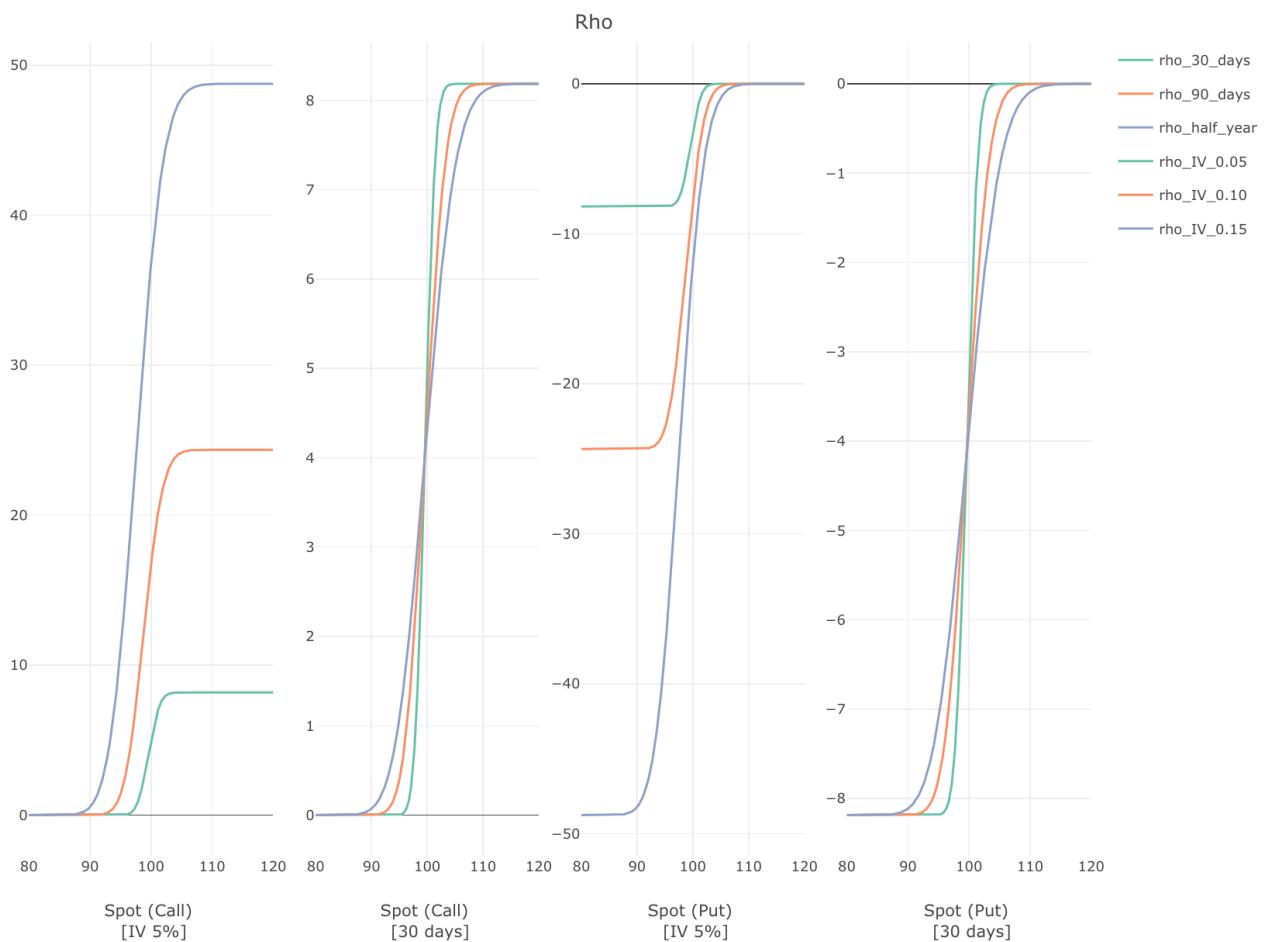
iii. Vega:

- Vega achieves the peak at **ATM**, like an **upward bell curve**.
- The **longer** time to maturity, the **higher** the vega.
- This makes sense as **time value makes up a larger proportion of the premium for longer term options** and it is the time value that is sensitive to changes in volatility.
- The **lower** the volatility, the **higher** the vega.



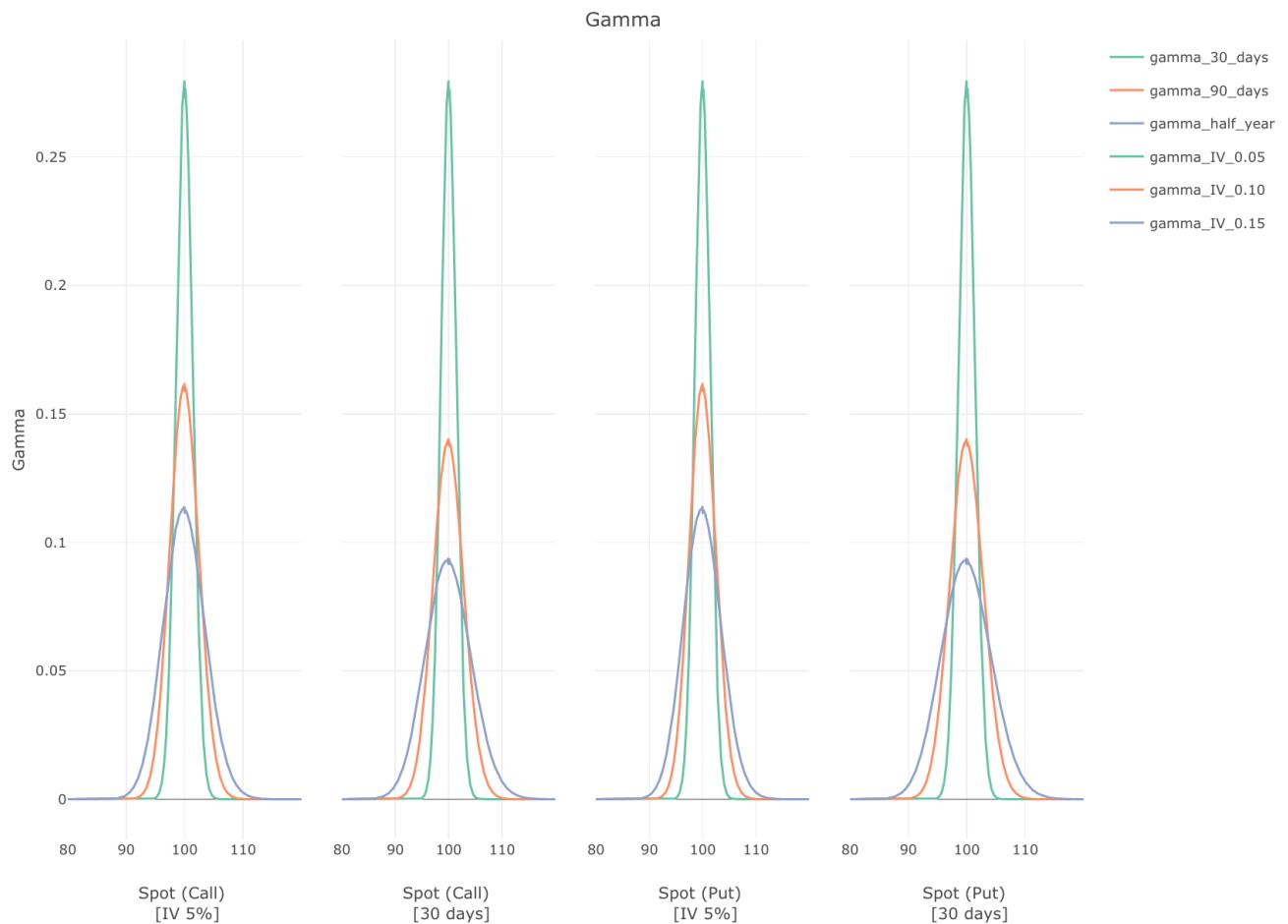
iv. Rho:

- Rho's shape is like **Delta** curve.
- Call and put option have positive and negative correlation with rho respectively.
- The **longer** the time to maturity, the **higher** the rho.
- The **lower** the volatility, the **higher** the rho* .



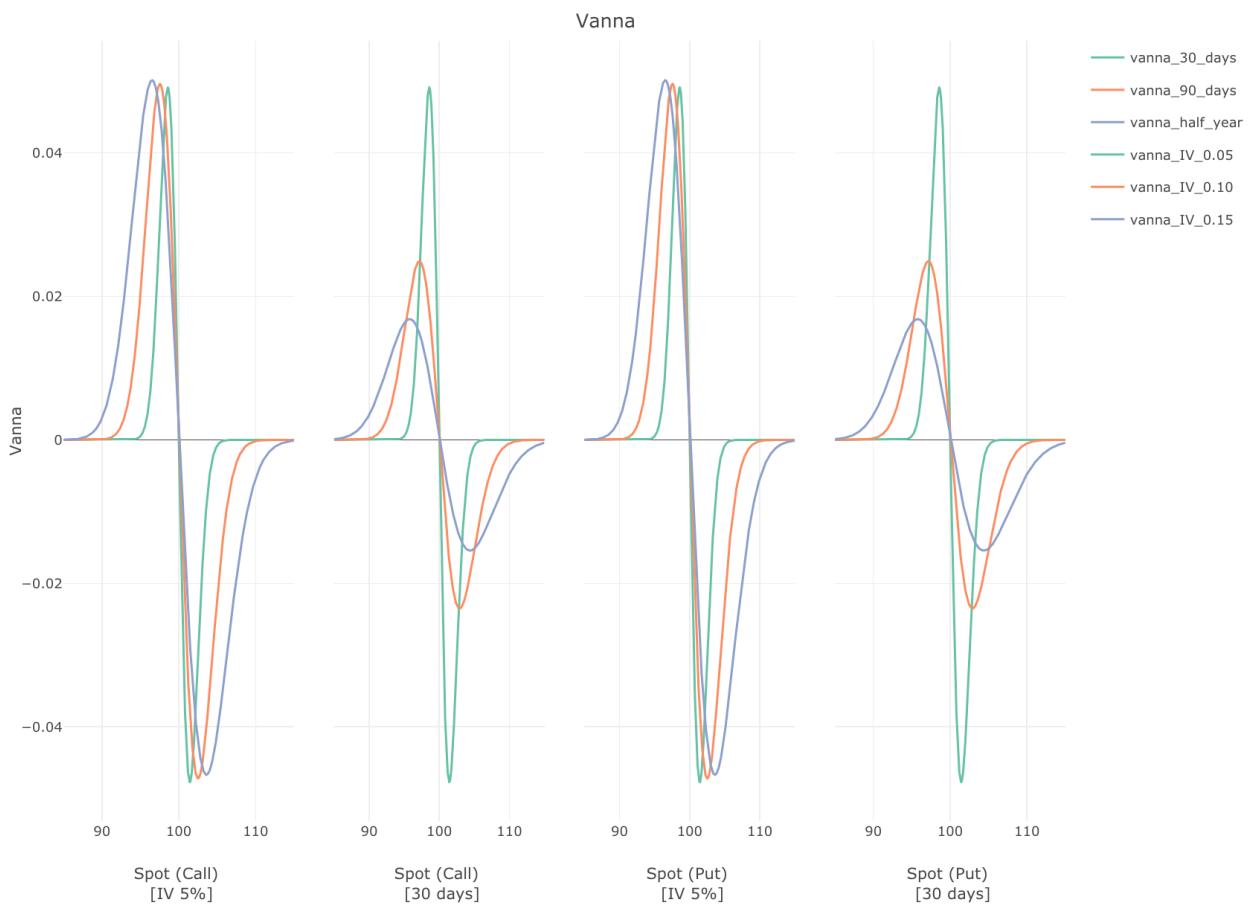
v. Gamma:

- Gamma's shape Like a **upward bell curve**, can be derived from Delta curve too.
- When **volatility is low**, the gamma of ATM is high while the gamma for deep ITM/OTM options approaches 0.
- This is due to the **time value** of such options are **low**, but it goes up dramatically as the underlying stock price approaches the strike price.
- When **volatility is high**, gamma tends to be **stable** across all strike prices.
- This can be explained that the time value of deep ITM/OTM options are already **quite substantial**.



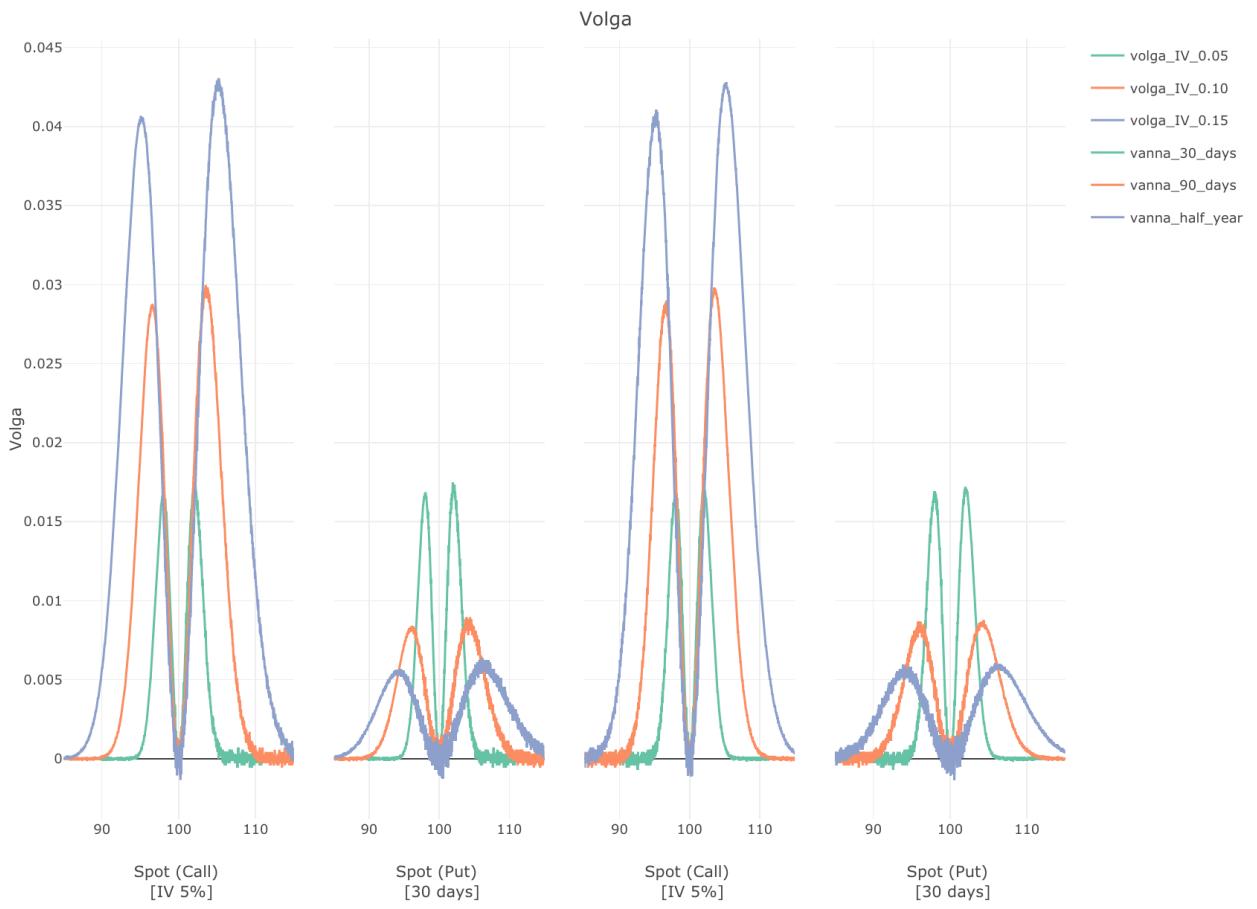
vi. Vanna:

- Vanna's shape is like a sin curve oscillating around **ATM**.
- Vanna is useful to maintain a **delta/vega hedged portfolio**.
- The **shorter** the time to maturity, the **higher** the vanna.
- The **lower** the volatility, the **higher** the vanna.



vii. Volga:

- Volga's shape is like a M-shaped curve/assymetrical absolute sin-cuve oscillating around **ATM**.
- Useful to maintain a **delta/vega hedged portfolio**.
- Volga is close to 0 for ATM option.
- Volga graph is assymetrical; observed a higher absolute magnitude of Volga for **higher strike price**.
- The **shorter** the time to maturity, the **higher** the vanna.
- The **lower** the volatility, the **higher** the vanna.



2b Regular Barrier Option

Definition:

- A regular barrier option is a **path-dependent** option which is **OTM** when the spot price reaches the barrier.
- In general, we should have theoretically:

$$\text{Knock - Out Option} + \text{Knock In Option} = \text{Vanilla Option}$$
- We have mainly two types of regular barrier options:
 - i. **Knock in:**
 - Position is worthless **until** barrier is reached.
 - ii. **Knock out:**
 - Position is worthless **if** barrier is reached.
- By the definition, **down&out or down&in** are for **calls**, and **up&out or up&in** are for **puts**.
 - Down/Up indicates the position of barrier compared to strike.
- Note: Charts above assumes zero discount rate (except Rho).
- In deal cancellation pricing on multiplier option, we used **down&out call** barrier option for **MULTUP** and **up&out put** barrier option for **MULTDOWN**.

Payoff Equations:

- Down & Out Call Option (cd0):

$$f(T) = \begin{cases} 0 & \text{if } S_{t^*} \leq B \text{ for } t^* \in [0, T], \\ S_T - K & \text{if } S_T > K \text{ and } S_T > B \text{ for all } t \in [0, T] \end{cases}$$

- Down & In Call Option (cio):

$$f(T) = \begin{cases} 0 & \text{if } S_t \leq B \text{ for all } t \in [0, T], \\ S_T - K & \text{if } S_T > K \text{ and } S_{t^*} > B \text{ for } t^* \in [0, T] \end{cases}$$

- Up & Out Put Option (puo):

$$f(T) = \begin{cases} 0 & \text{if } S_{t^*} \geq B \text{ for } t^* \in [0, T], \\ K - S_T & \text{if } S_T < K \text{ and } S_T < B \text{ for all } t \in [0, T] \end{cases}$$

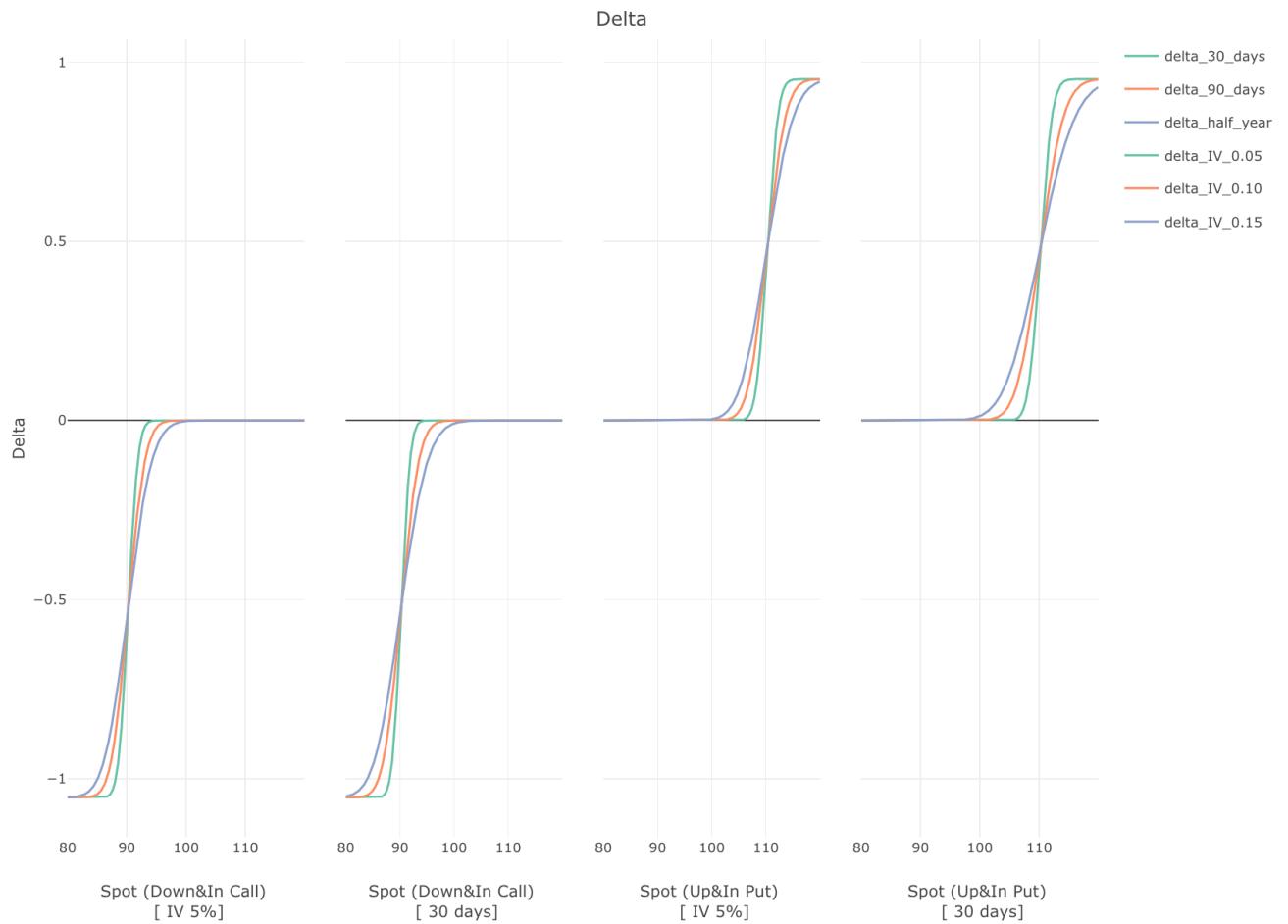
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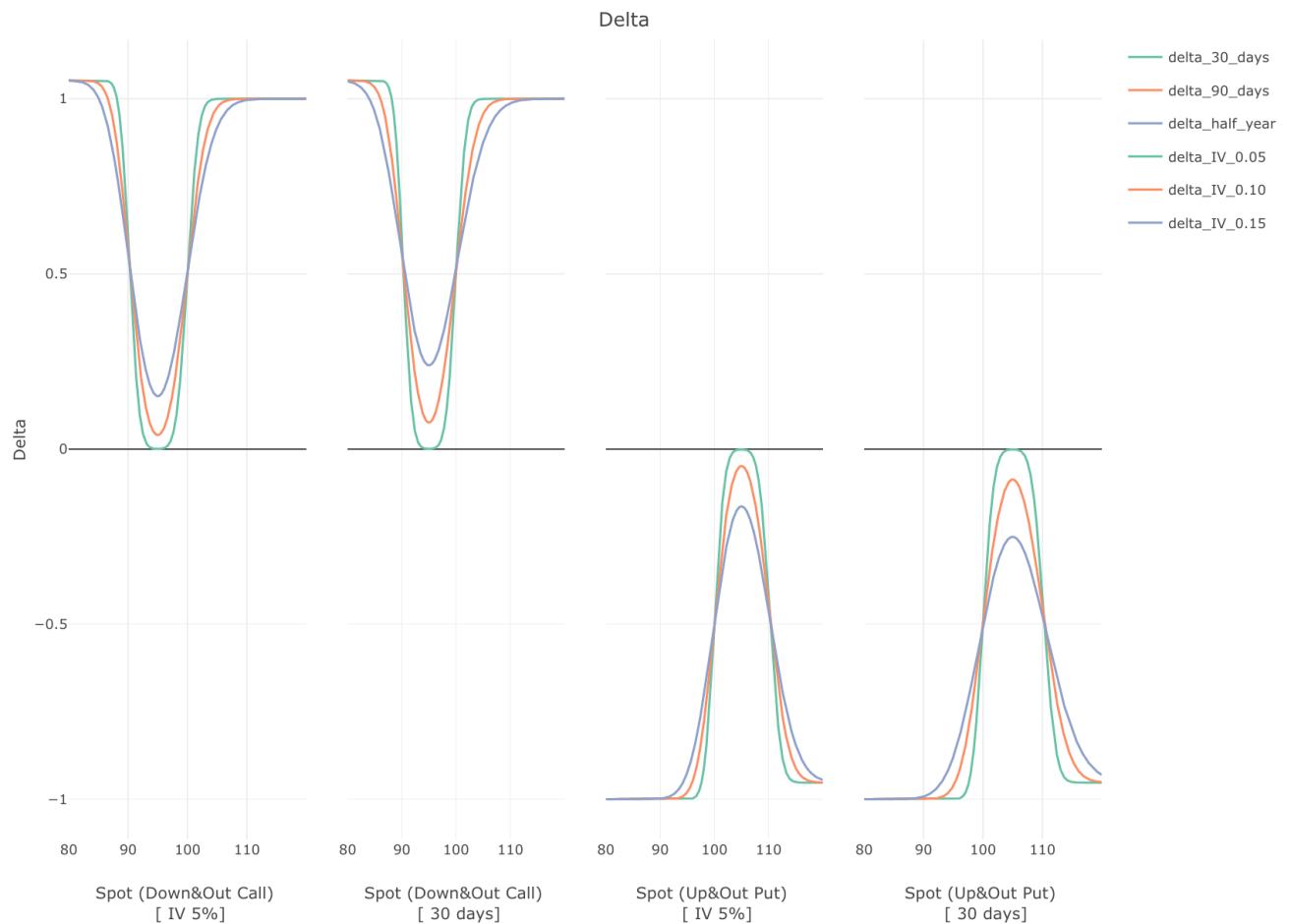
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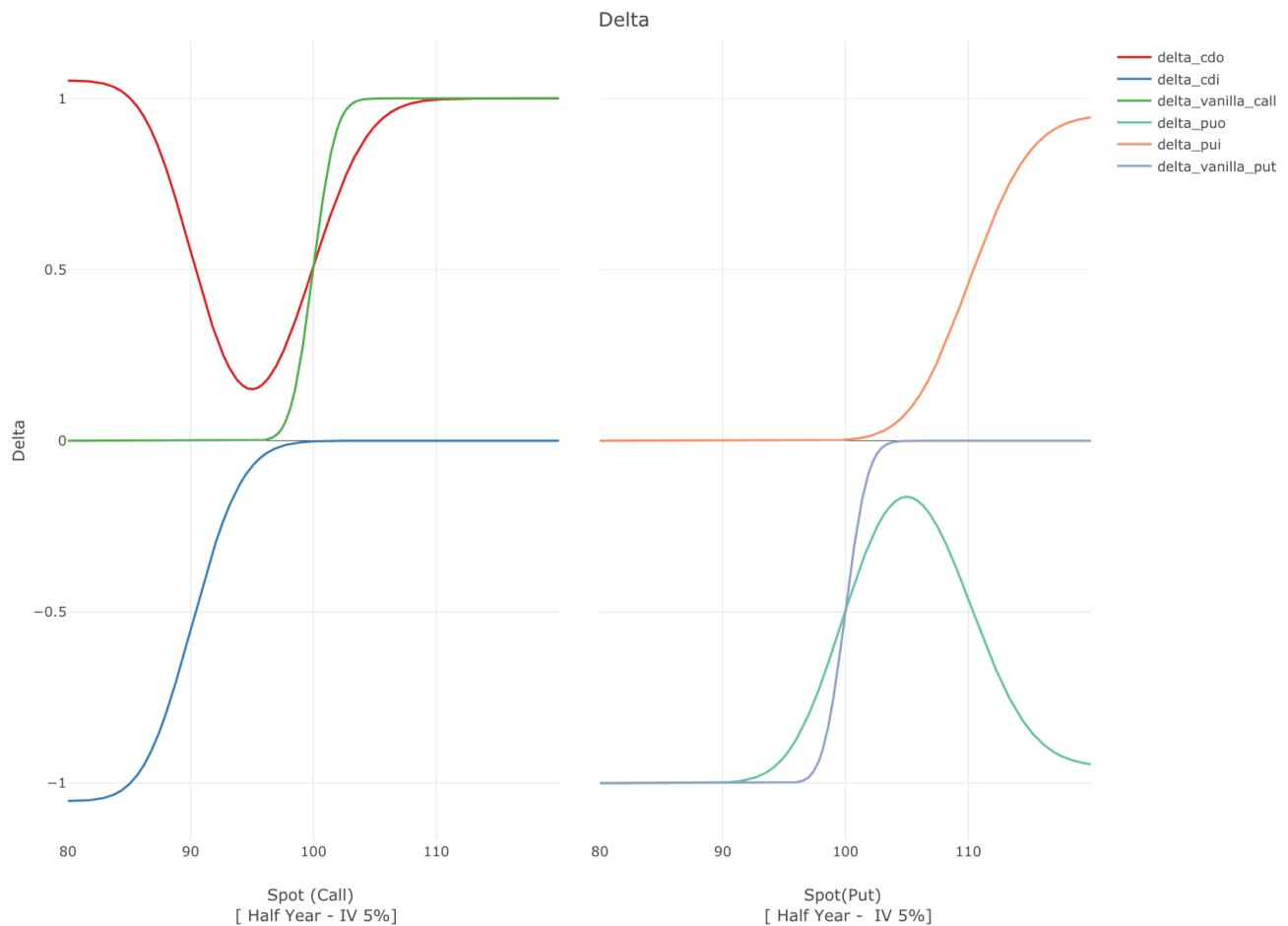
Knock - In:



Knock - Out:



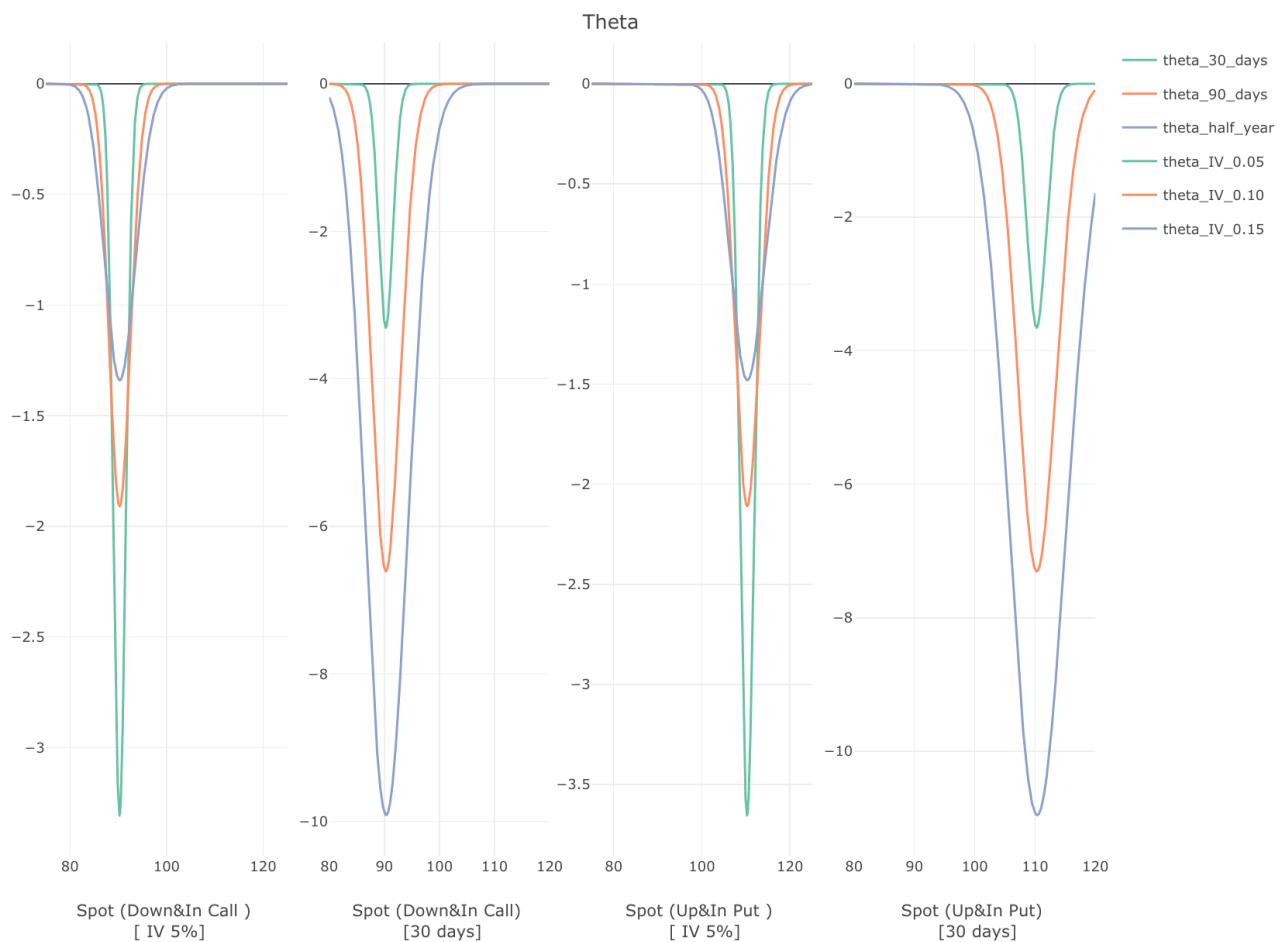
Compare to Vanilla:



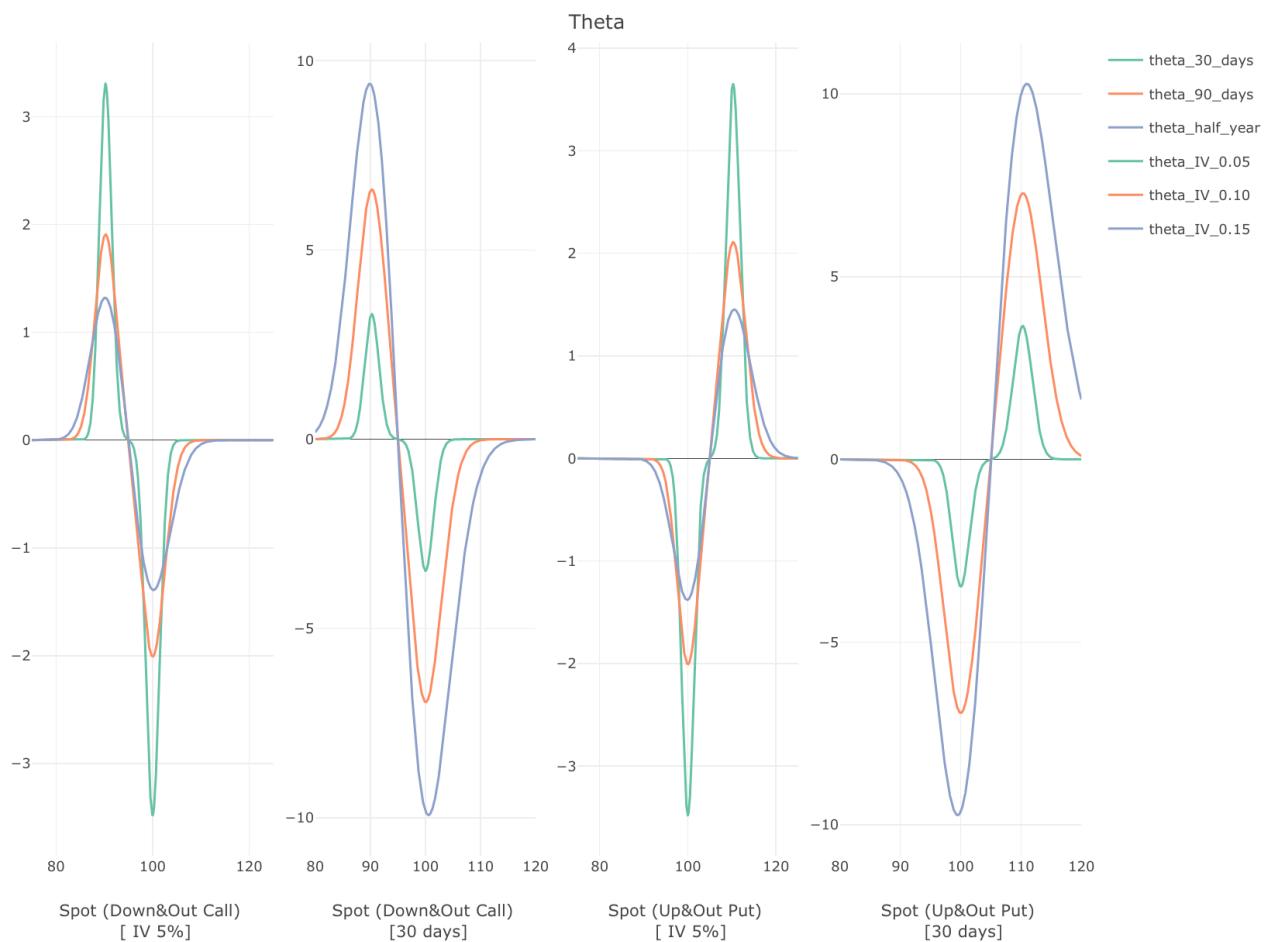
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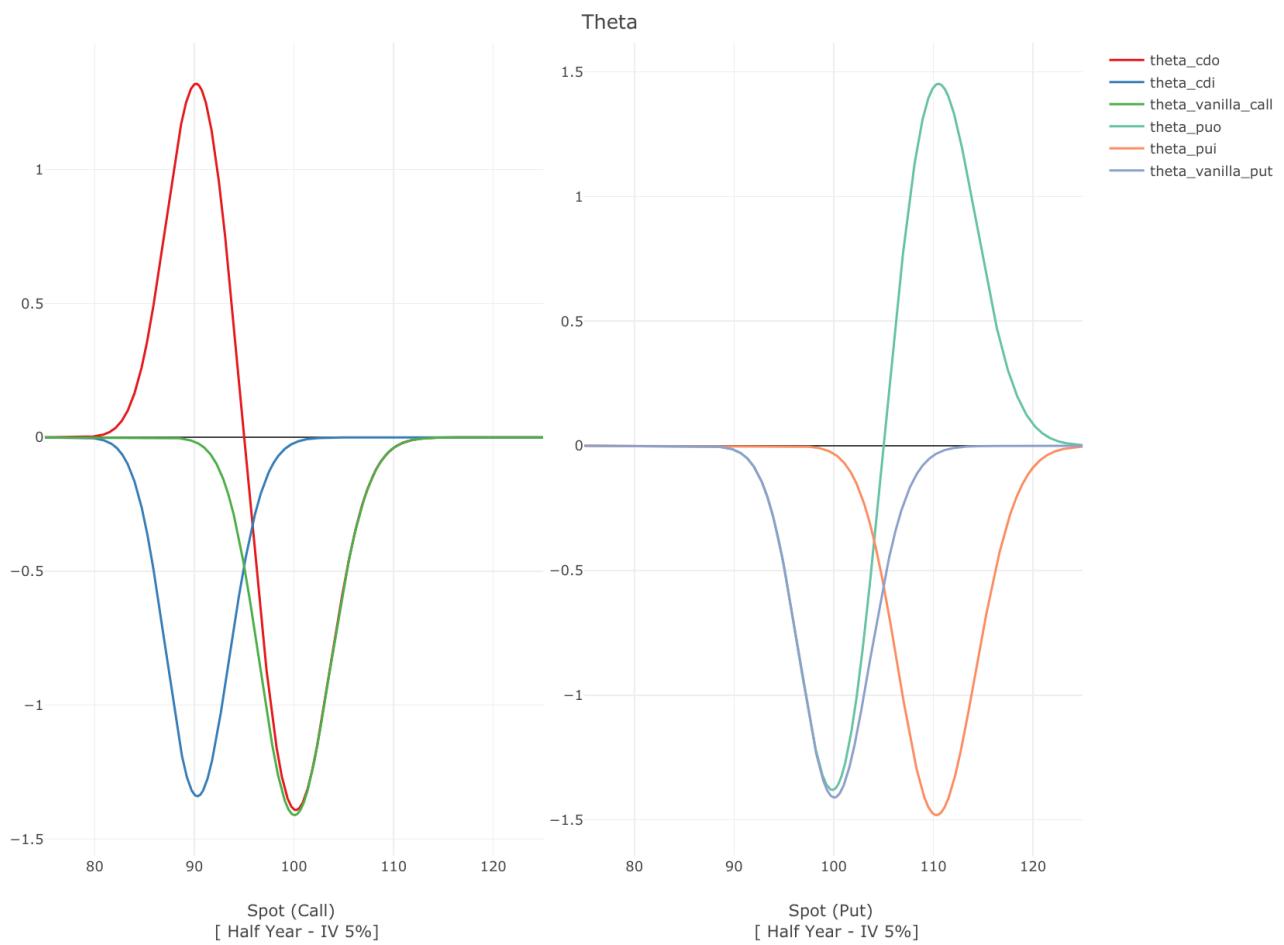
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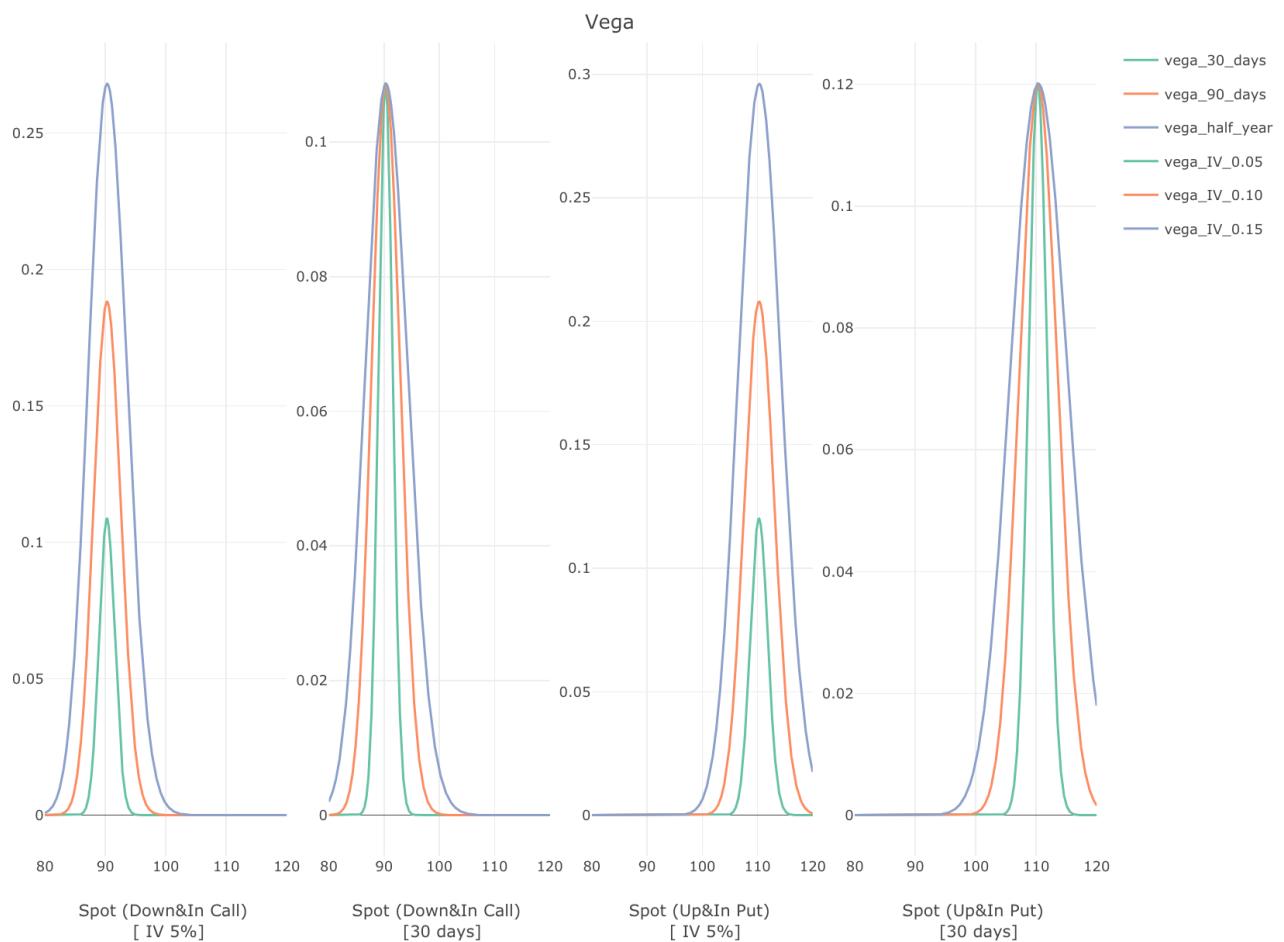
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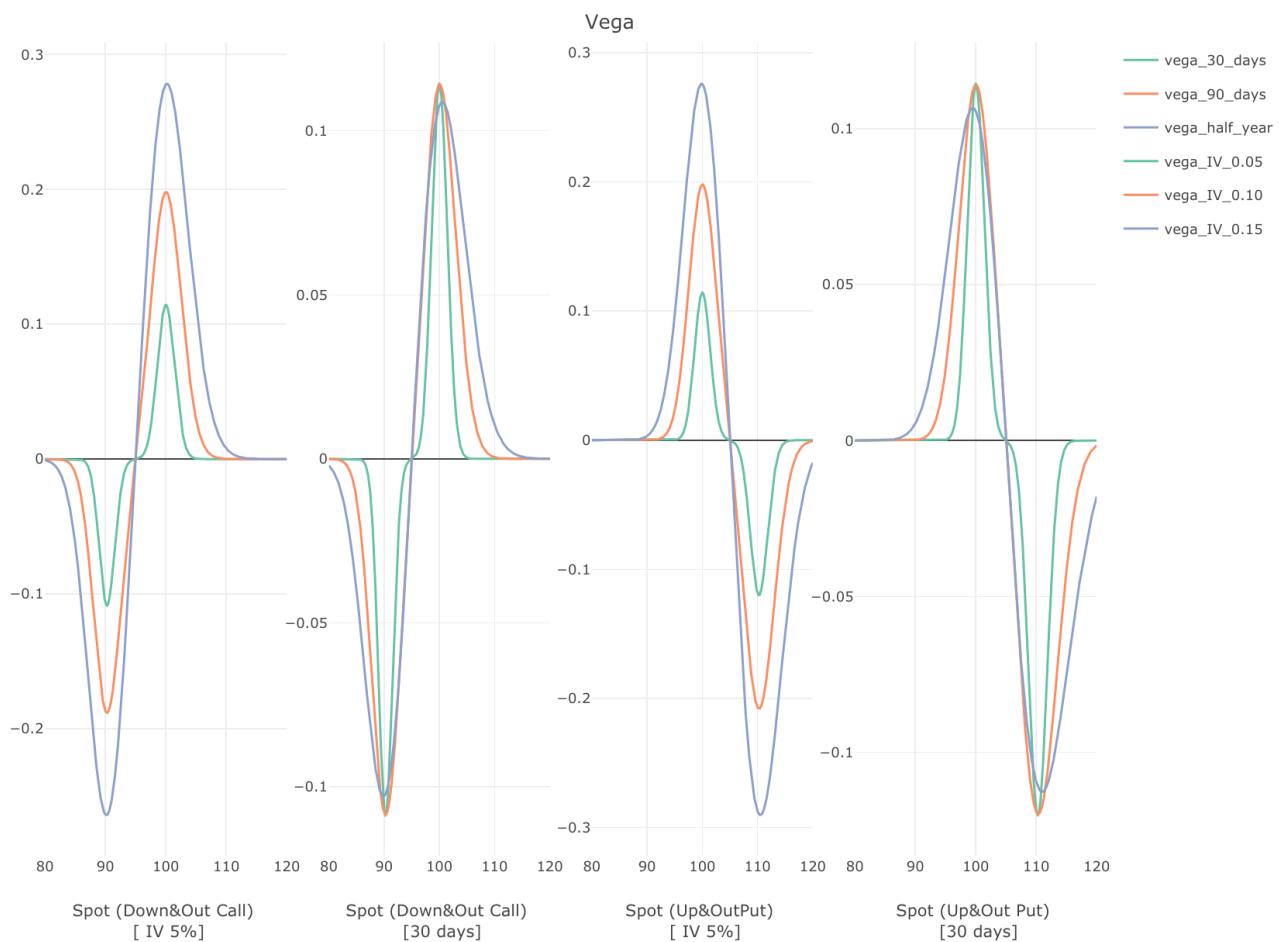
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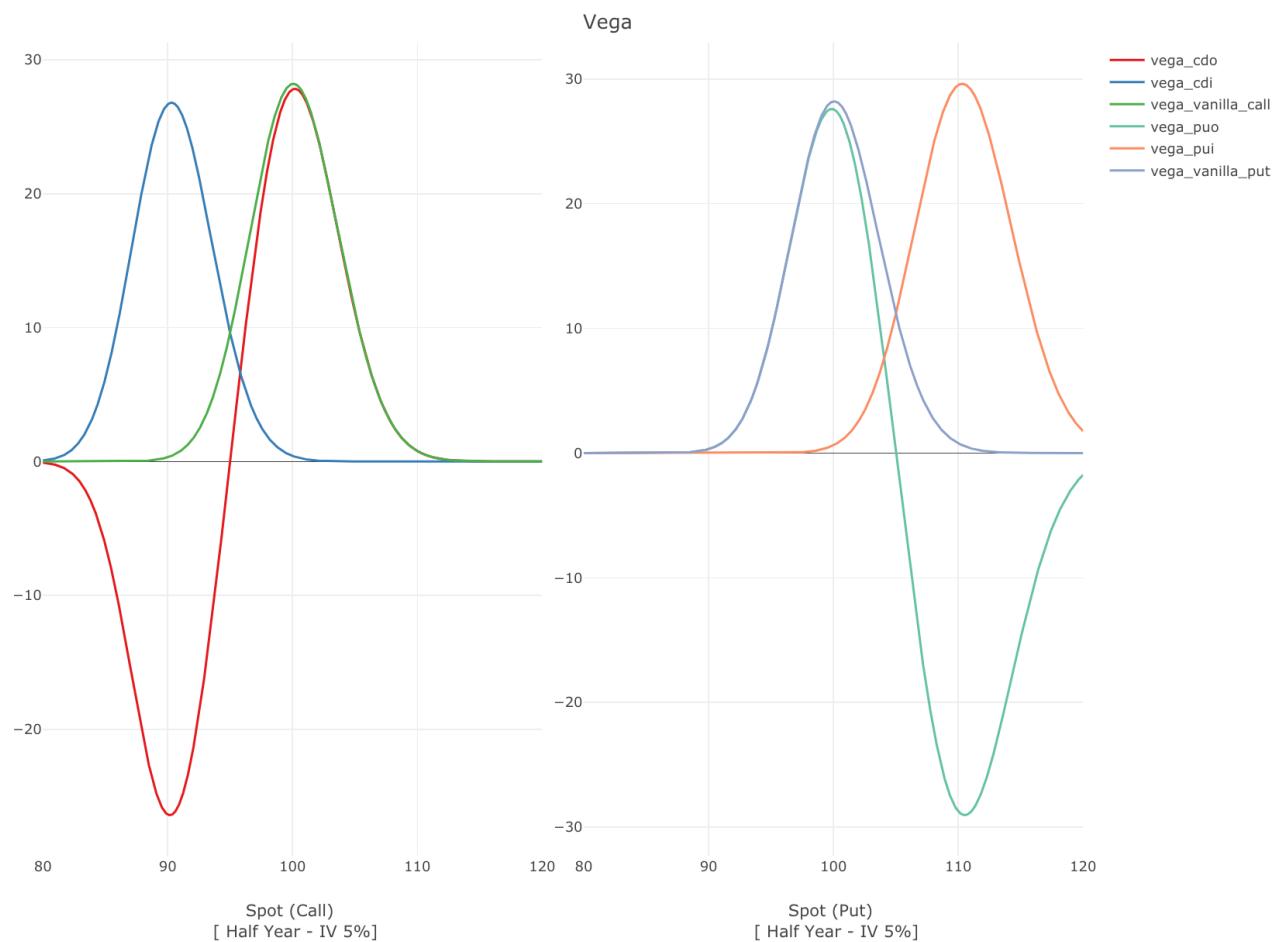
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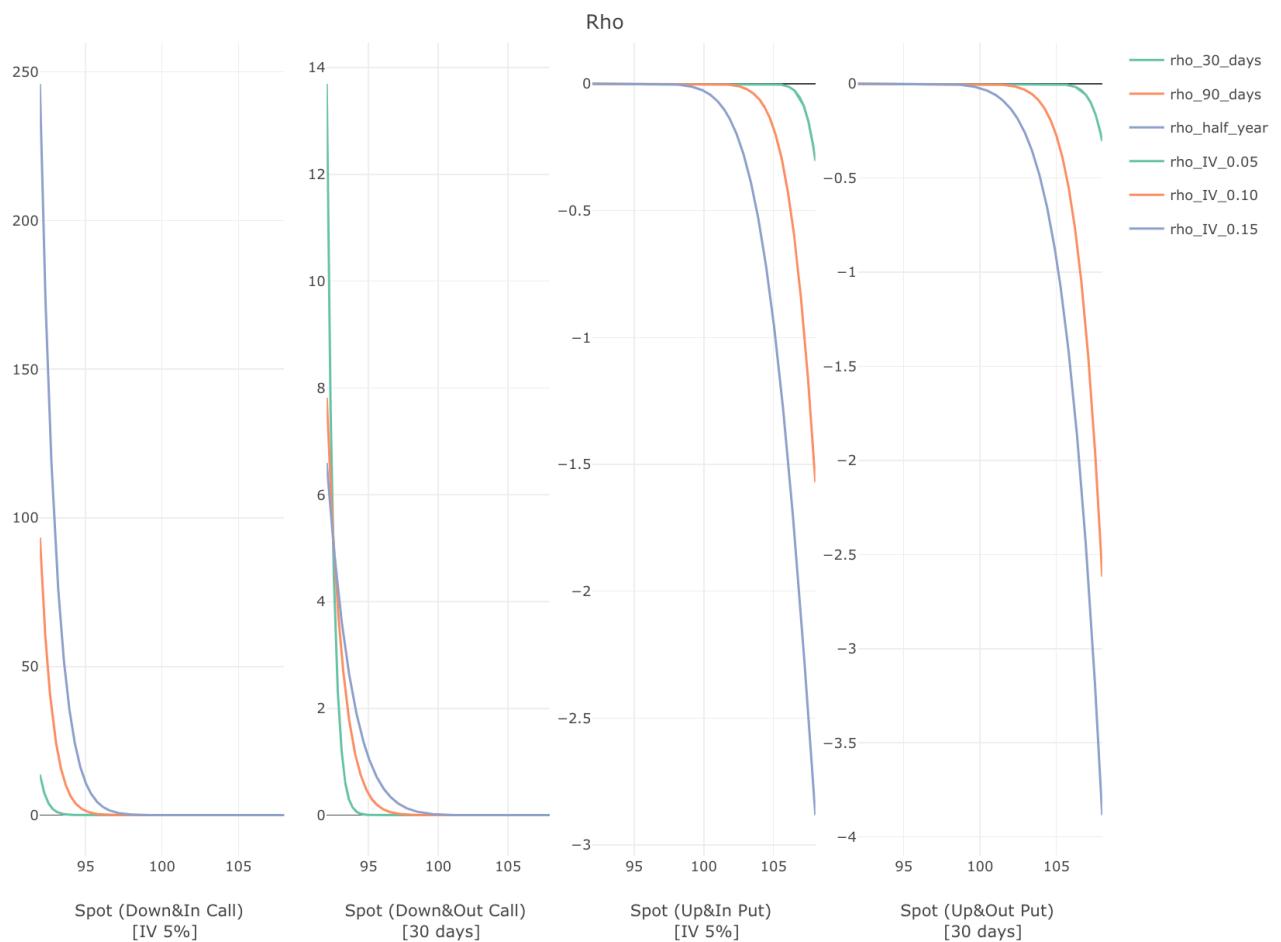
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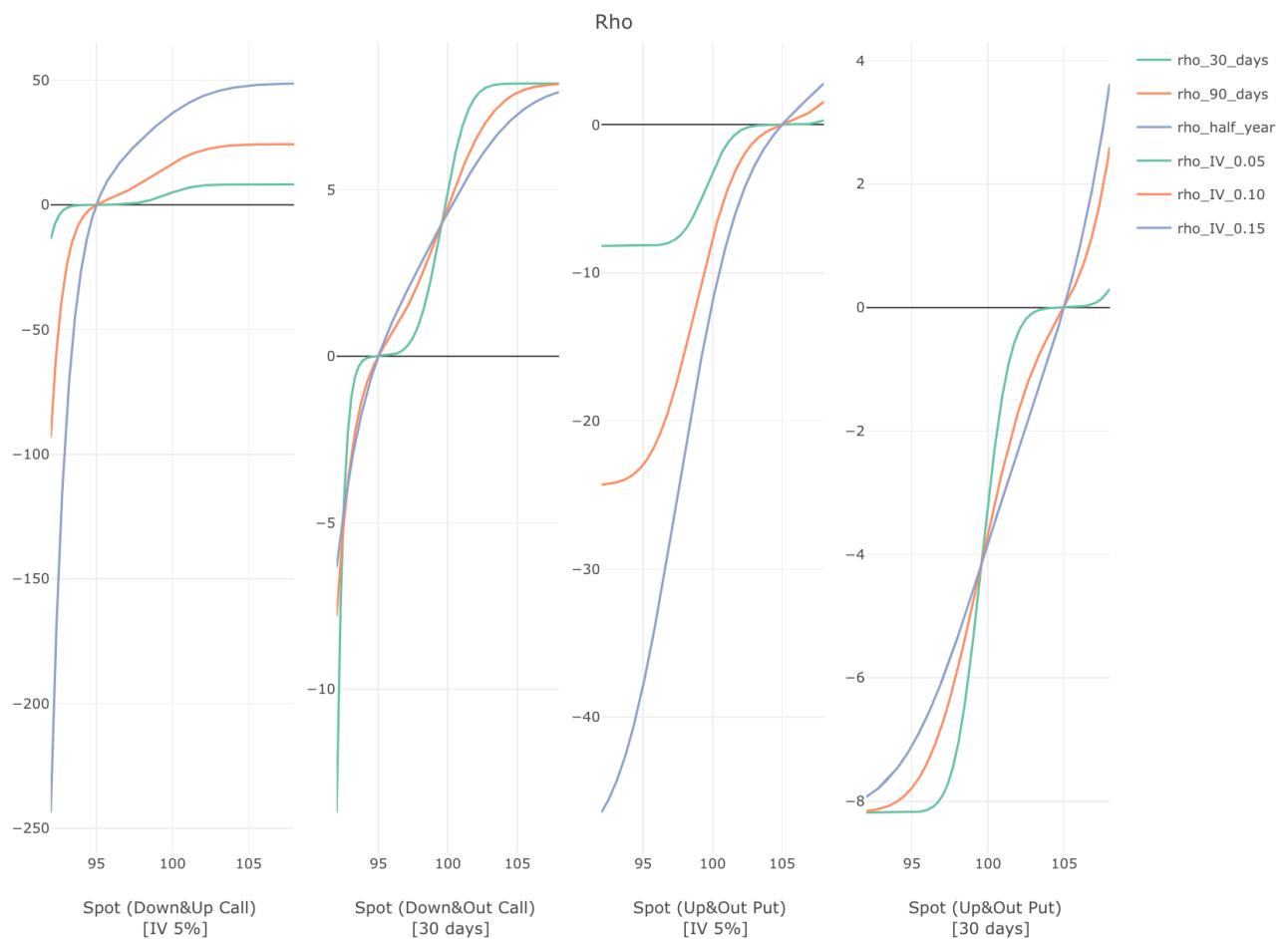
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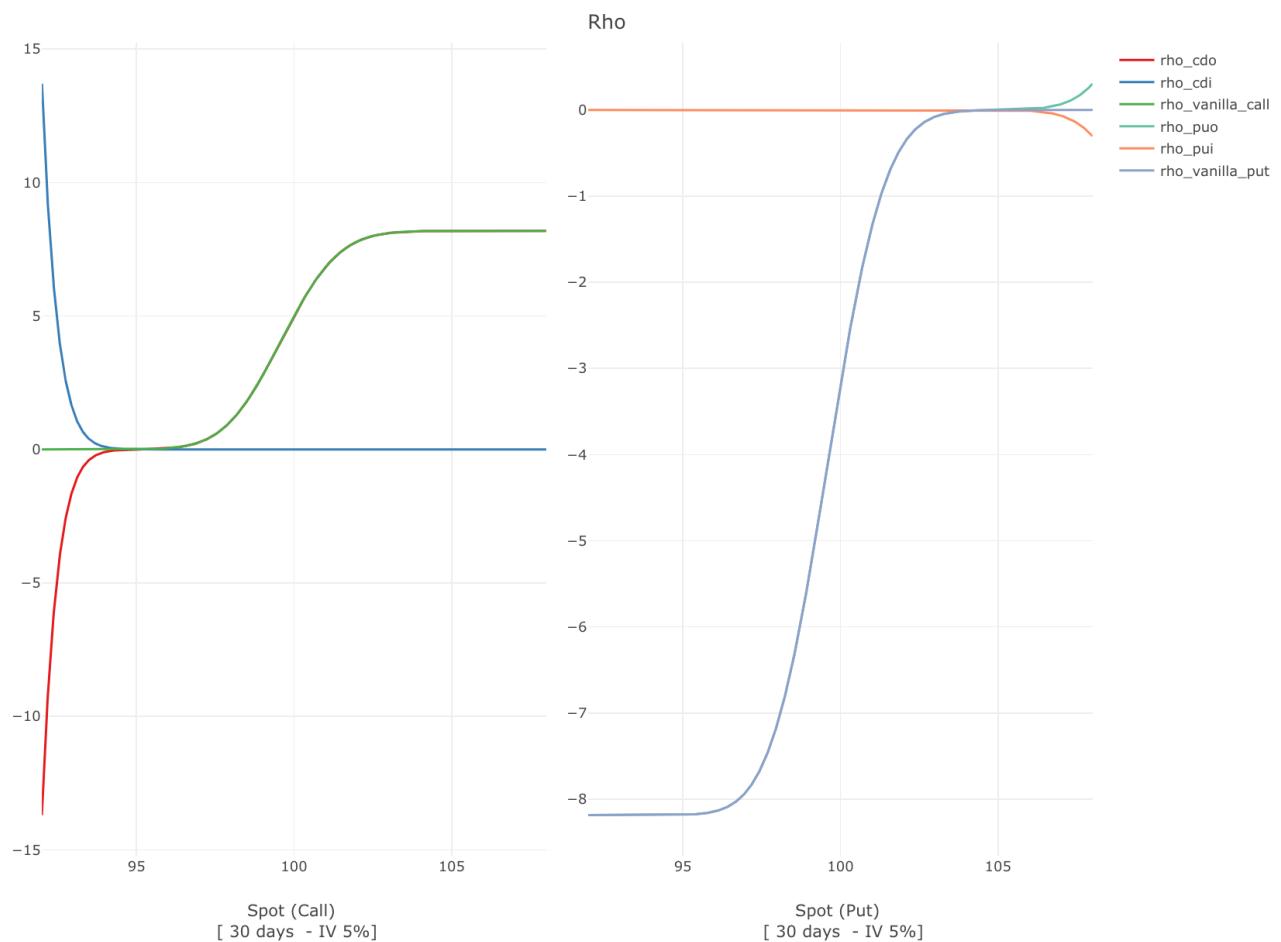
Knock - In:



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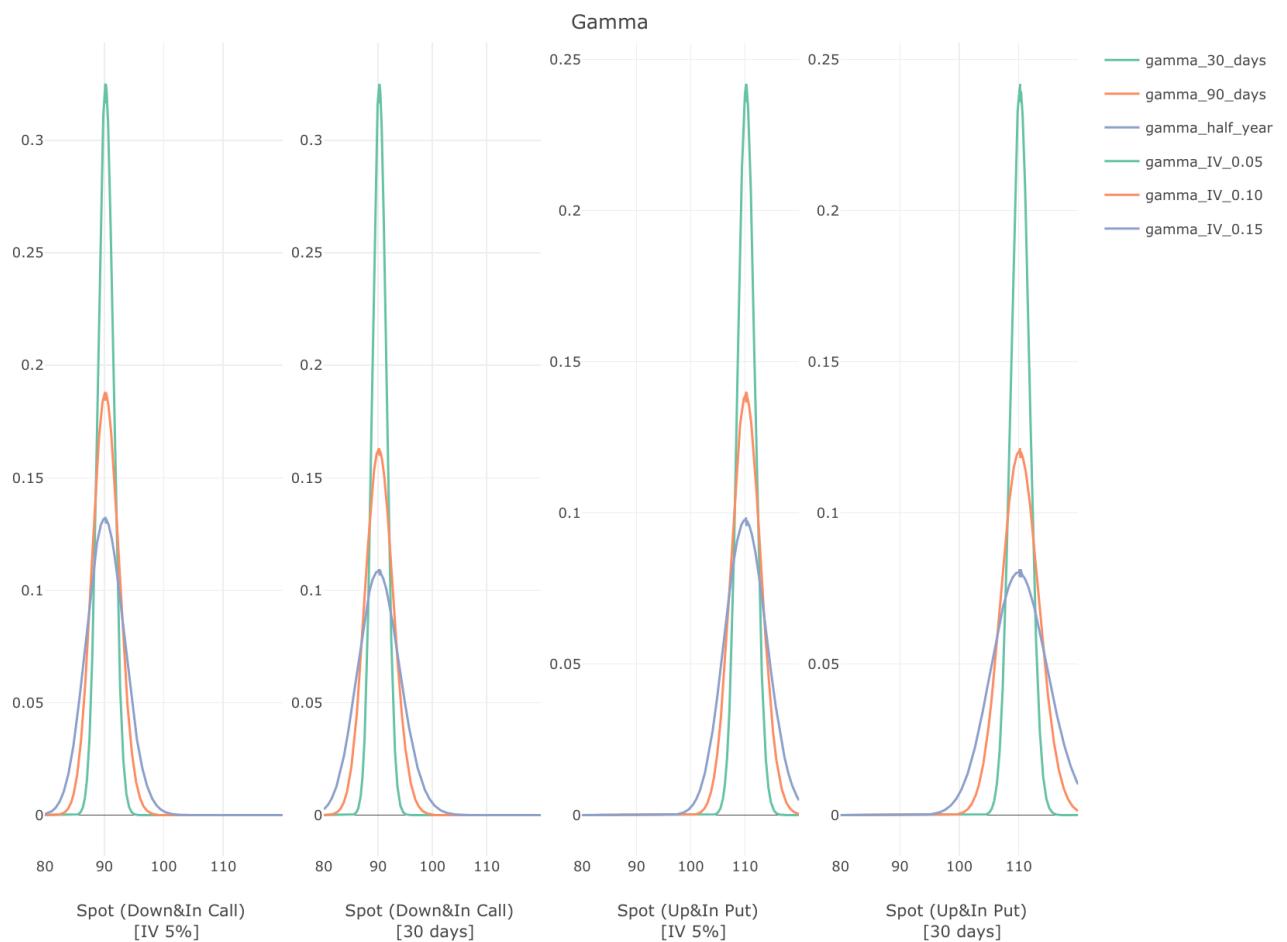
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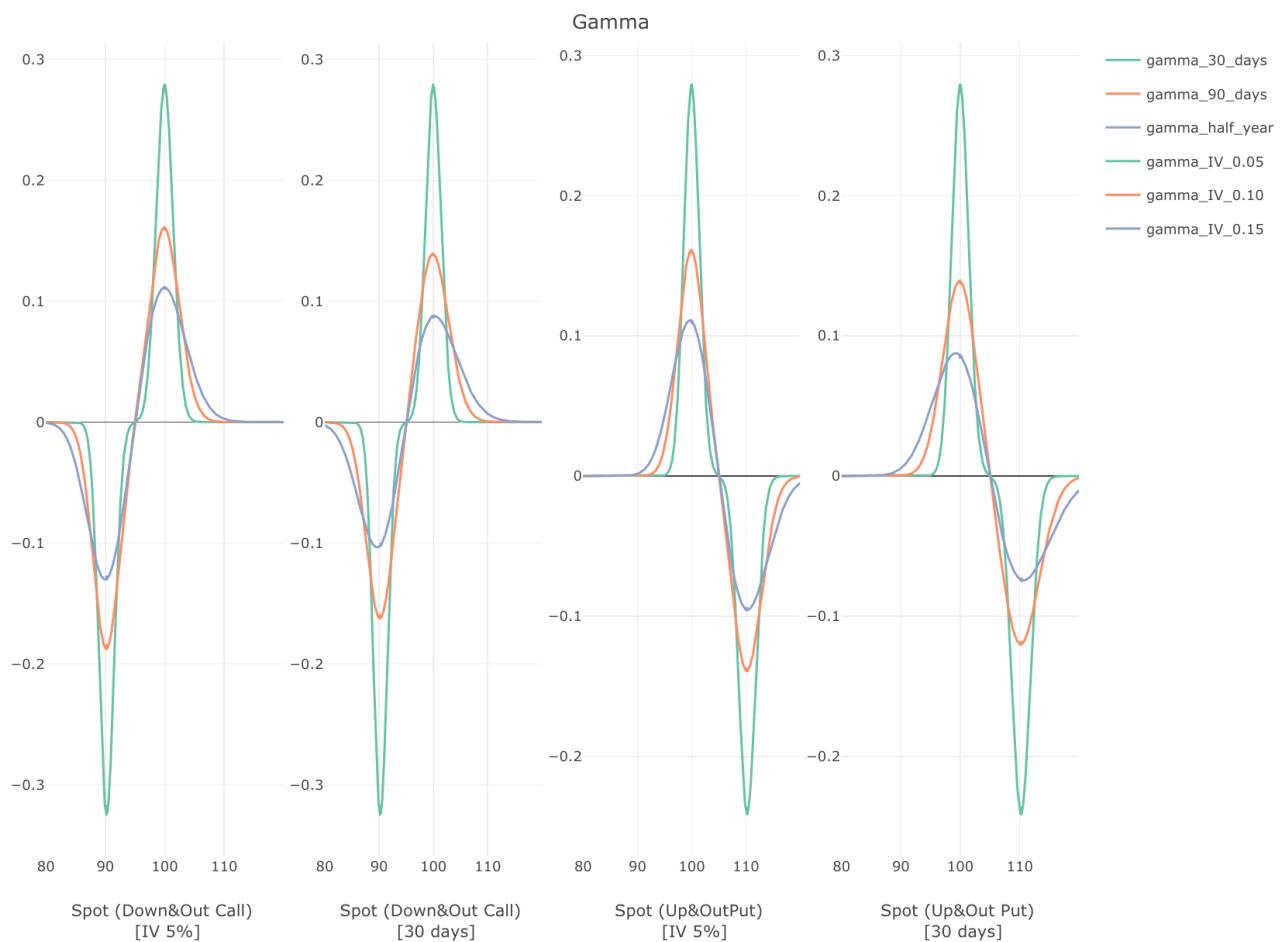
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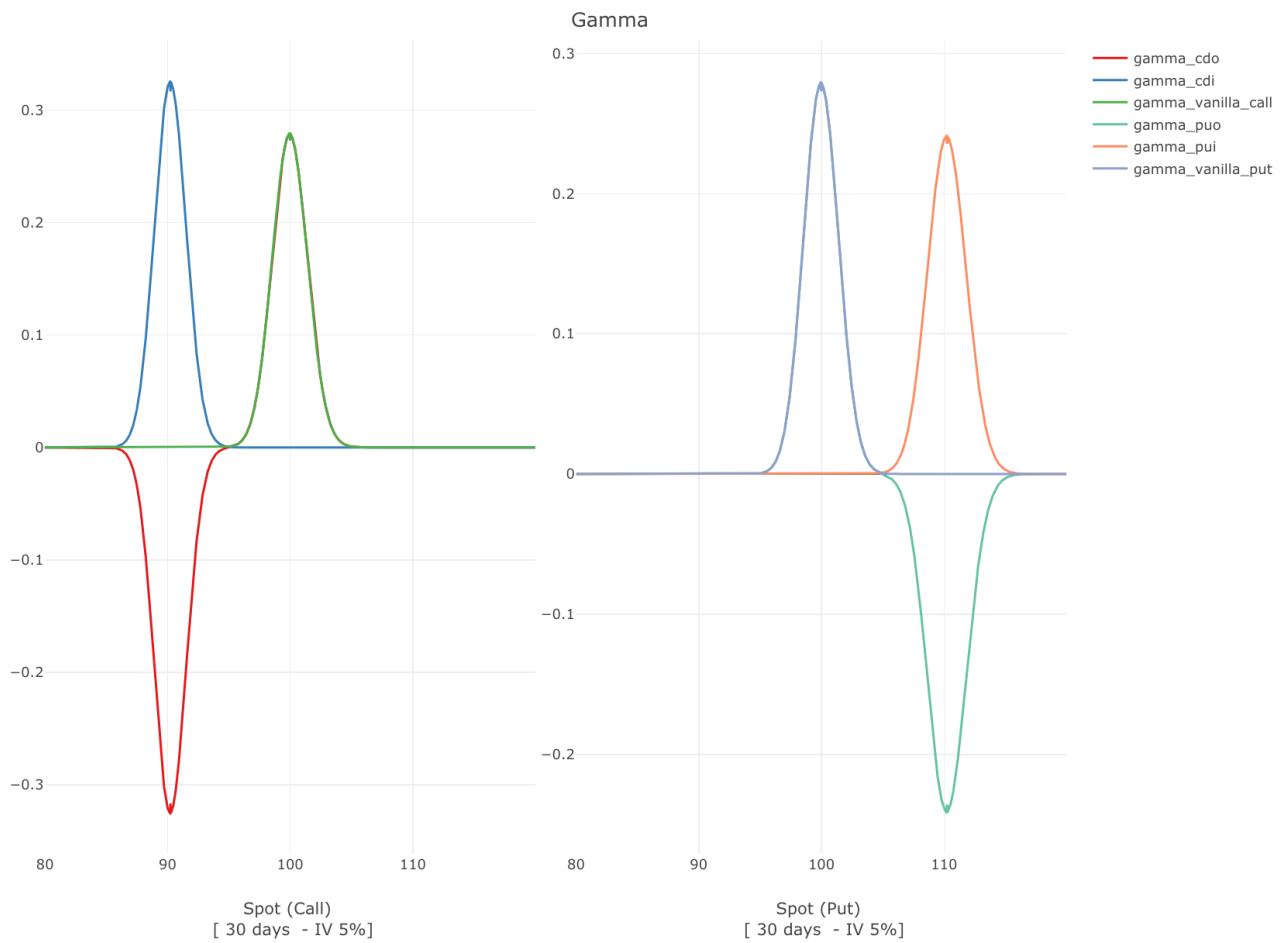
Knock - In:



Knock - Out:



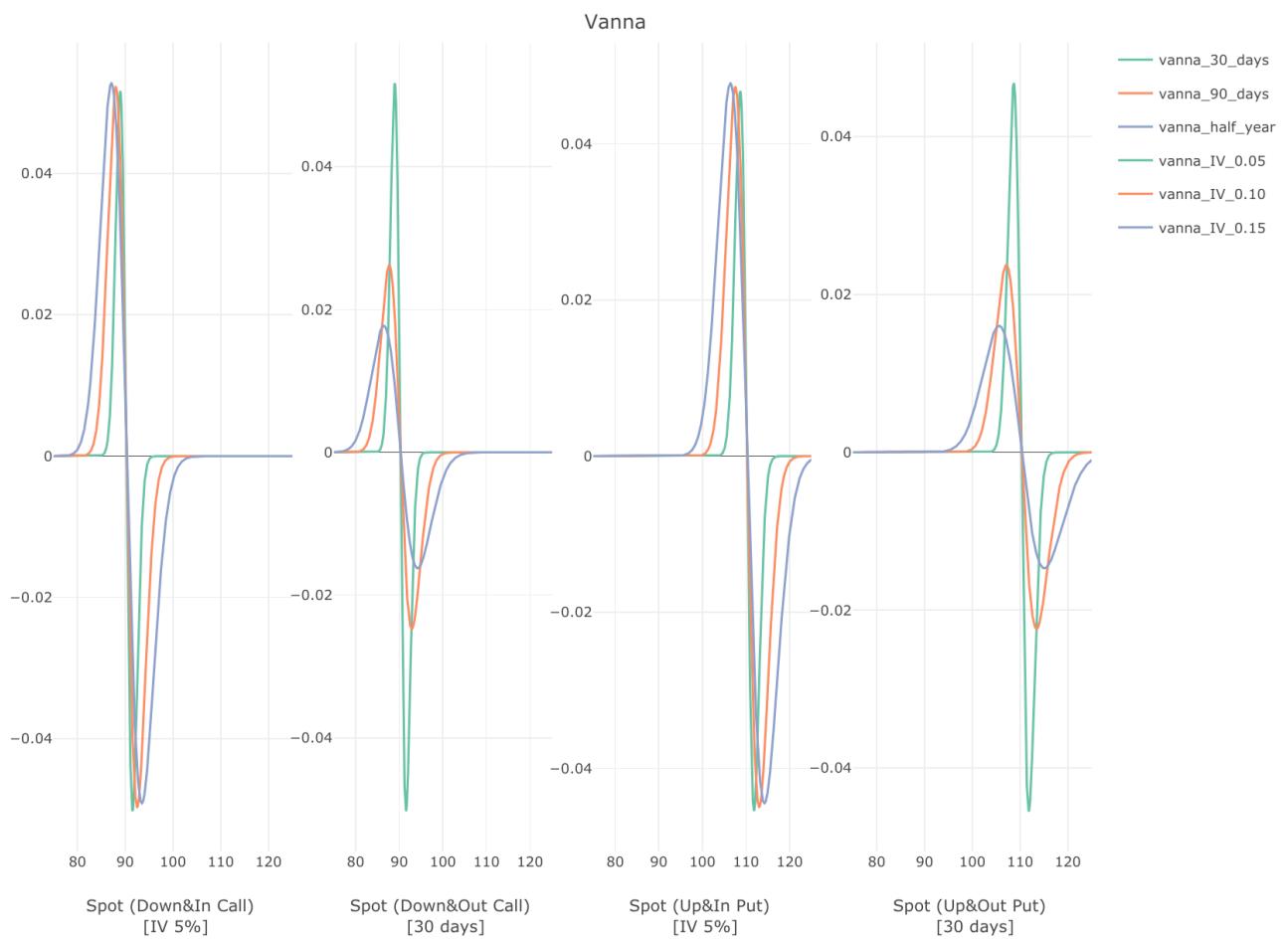
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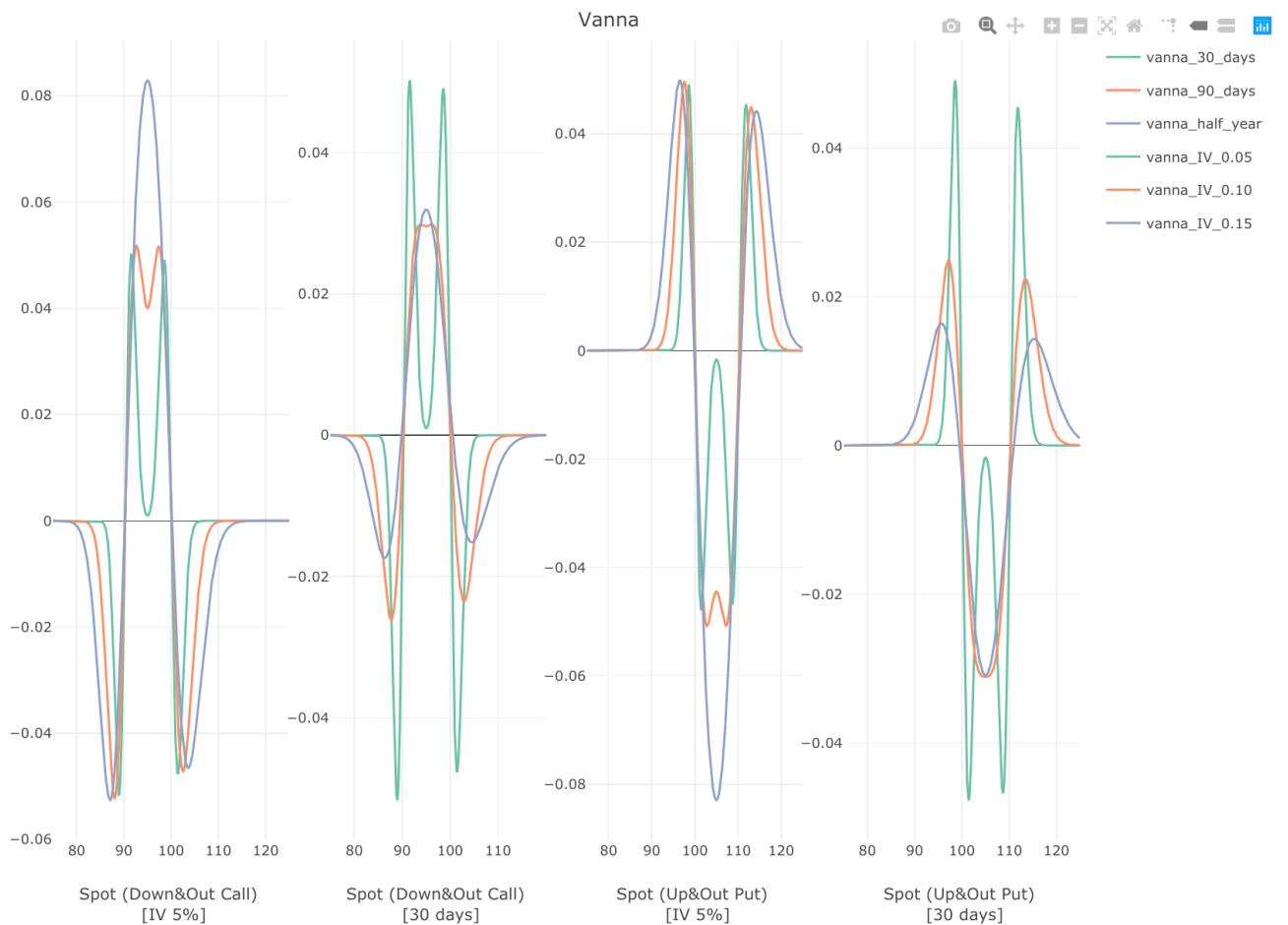
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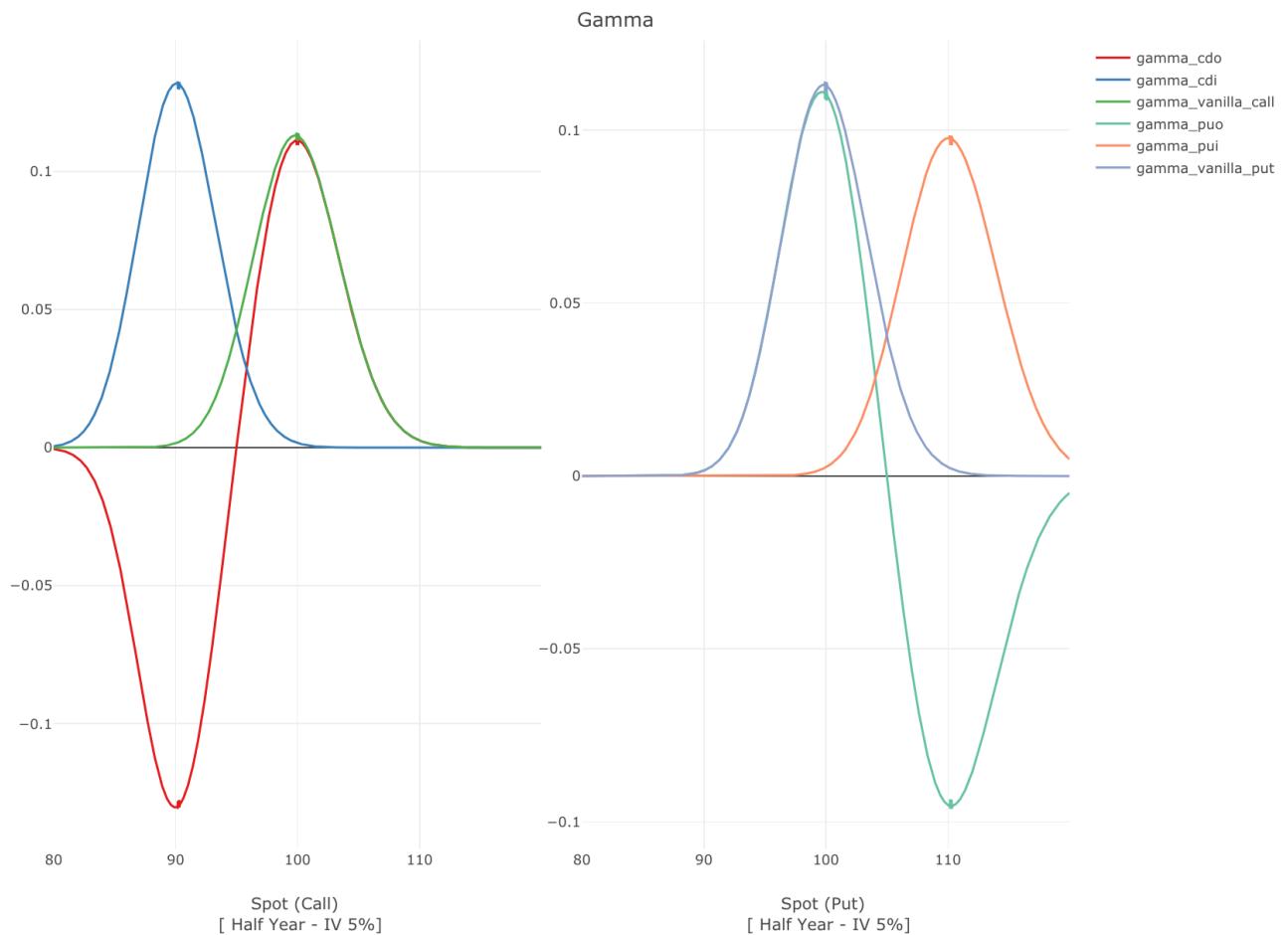
Knock - In:



Knock - Out:



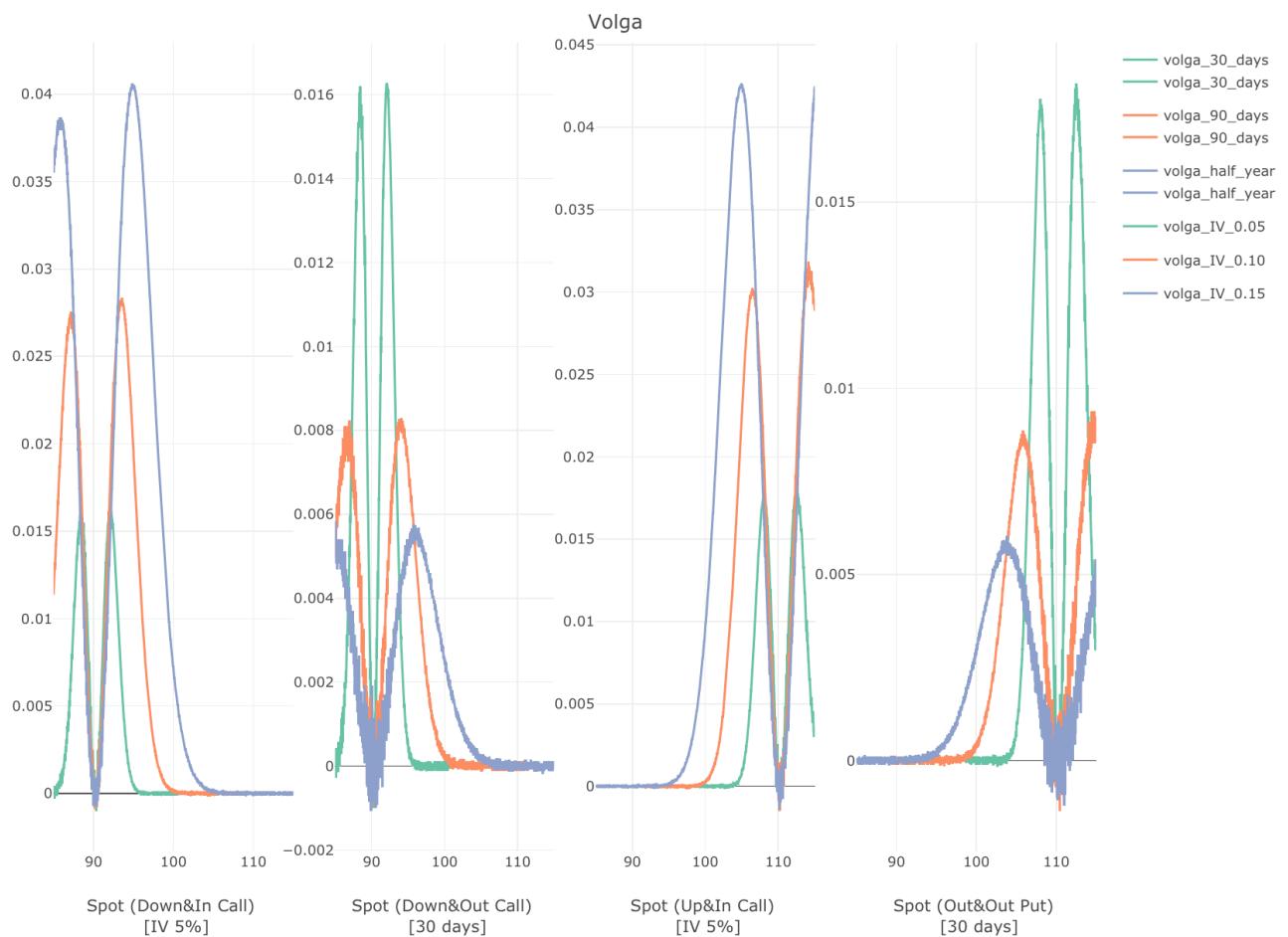
Compare to Vanilla:



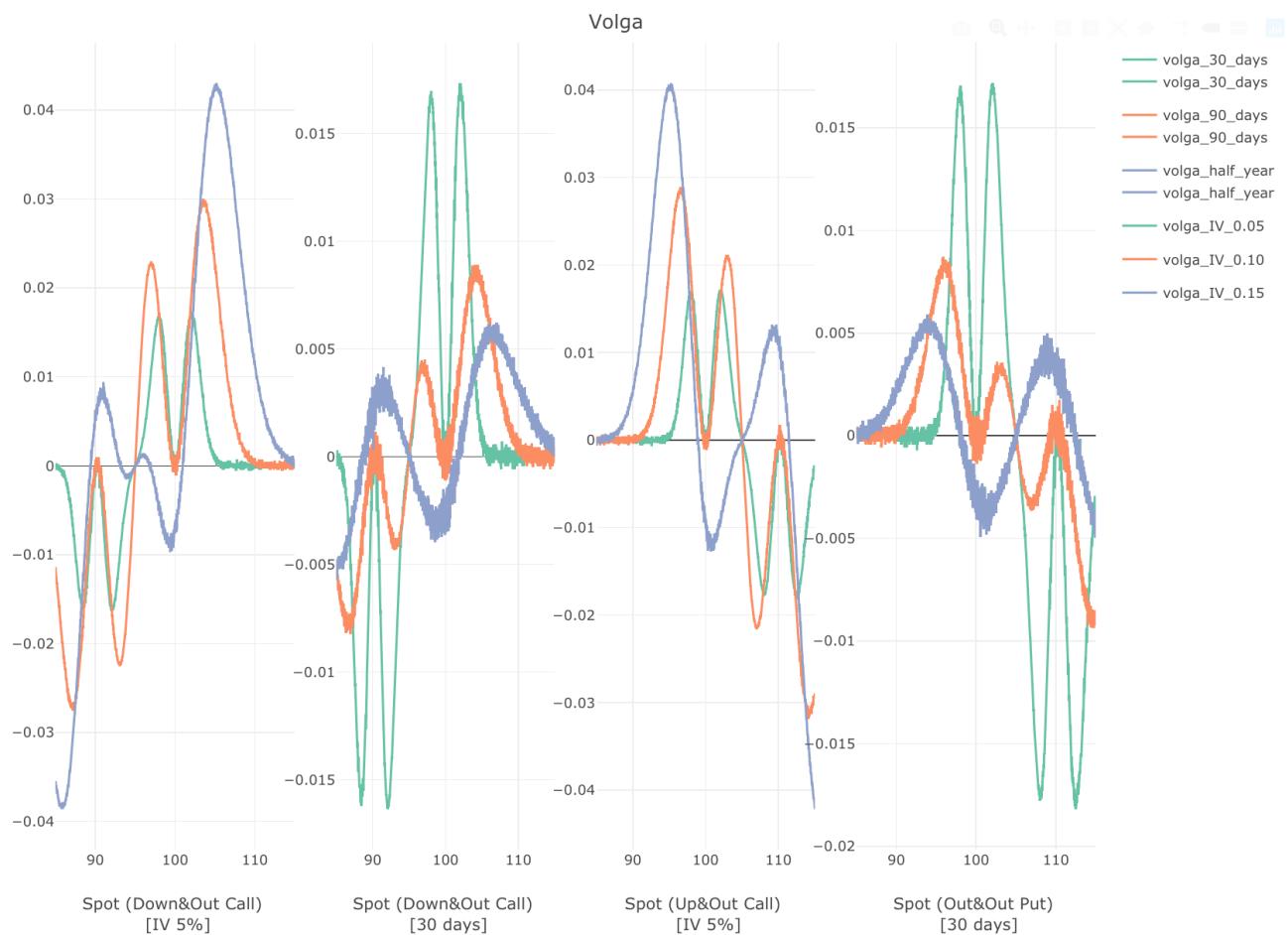
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- Volga is close to 0 for ATM option.
- Volga graph is assymetrical; observed a higher absolute magnitude of Volga for **higher strike price**.
- The **shorter** the time to maturity, the **higher** the vanna.
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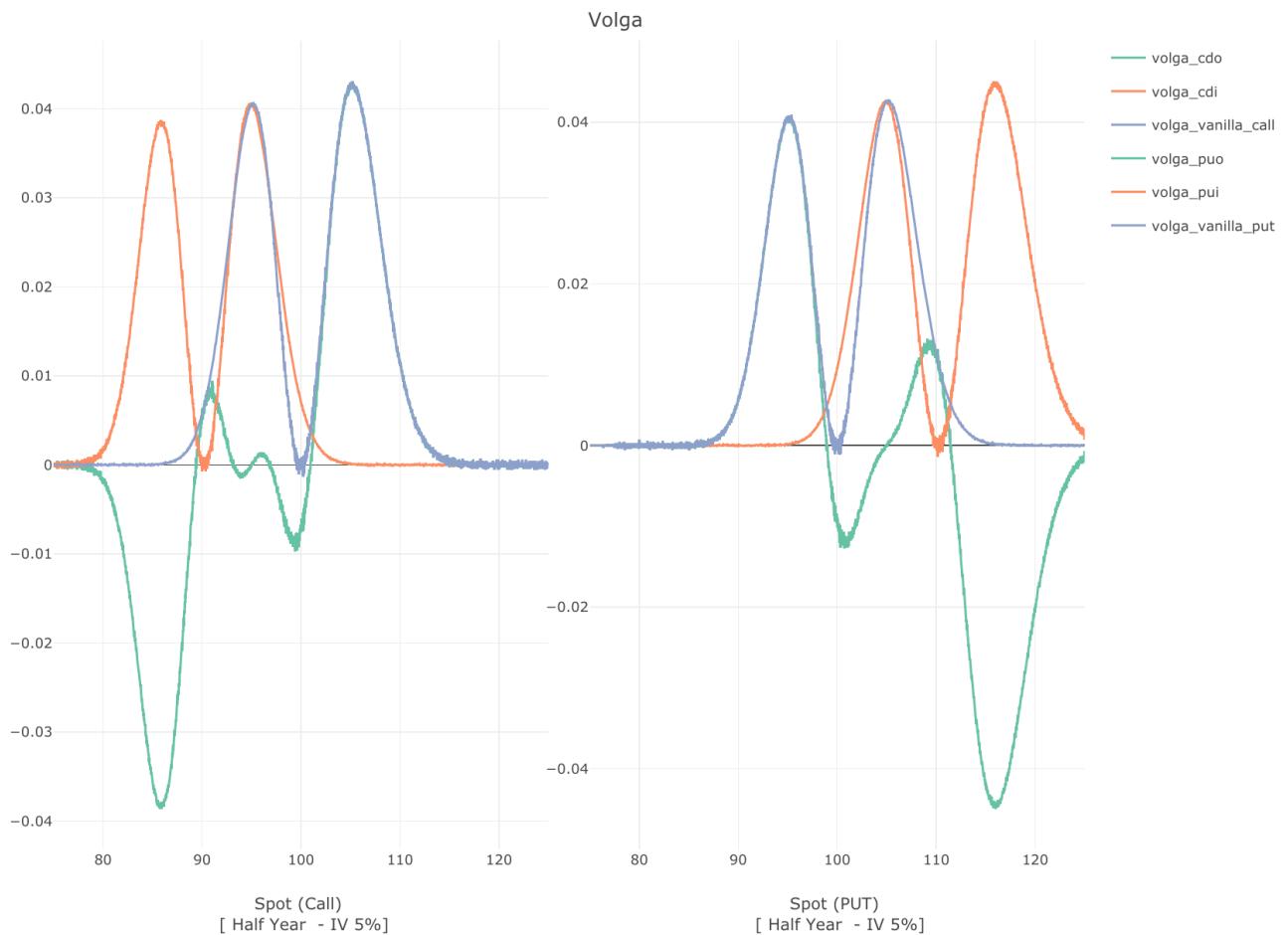
Knock - In:



Knock - Out:



Compare to Vanilla:



2d Double Barrier Option

Definition:

- A double barrier option is either **knocked in or knocked out** if the asset price touches one of **lower or upper barrier** during its lifetime.
- The definitions of flags are as below:
 - i. **"co"**: An up-and-out-down-and-out call,
 - ii. **"ci"**: An up-and-in-down-and-in call,
 - iii. **"po"**: An up-and-out-down-and-out put
 - iv. **"pi"**: An up-and-in-down-and-in call.
- Where **Down/Up** indicates the position of barrier compared to strike.
- Double barrier options can be priced analytically using a model introduced by **Ikeda and Kunitomo (1992)**.
- Reference: [Haug's Book, Chapter 2.10.2].
- In general, we should have theoretically:

$$\text{Knock - Out Option} + \text{Knock In Option} = \text{Vanilla Option}$$

- We have mainly two types of regular barrier options:
 - i. **Knock in**:
 - Position is worthless **until** barrier is reached.
 - ii. **Knock out**:
 - Position is worthless **if** barrier is reached.
- Note: Charts above assumes zero discount rate (except Rho).

- In deal cancellation pricing on multiplier option, we used **Double Knock-out Call** barrier option for **MULTUP** and **Double Knock-out Put** barrier option for **MULTDOWN**.

Payoff Equations:

- Double Knock-Out Call Option (co):

$$f(T) = \begin{cases} 0 & \text{if } S_{t^*} \leq L \text{ or } S_{t^*} \geq U \text{ for } t^* \in [0, T], \\ S_T - K & \text{if } S_T > K \text{ and } L < S_T < U \text{ for all } t \in [0, T] \end{cases}$$

- Double Knock-In Call Option (ci):

$$f(T) = \begin{cases} 0 & \text{if } S_t \leq B \text{ for all } t \in [0, T], \\ S_T - K & \text{if } S_T > K \text{ and } S_{t^*} > B \text{ for } t^* \in [0, T] \end{cases}$$

- Double Knock-Out Put Option (po):

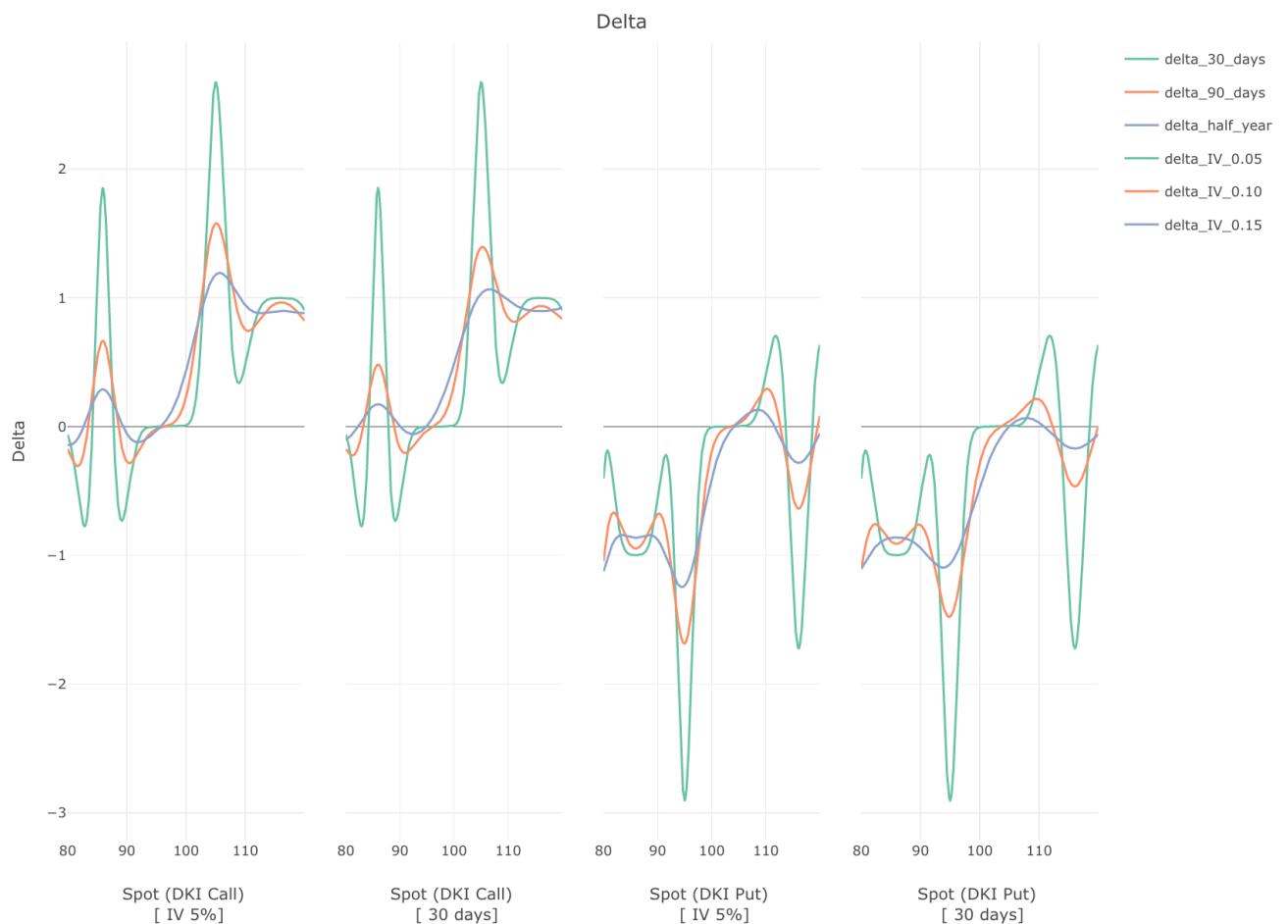
$$f(T) = \begin{cases} 0 & \text{if } S_{t^*} \leq L \text{ or } S_{t^*} \geq U \text{ for } t^* \in [0, T], \\ K - S_T & \text{if } S_T < K \text{ and } L < S_T < U \text{ for all } t \in [0, T] \end{cases}$$

- Double Knock-In Put Option (pi):

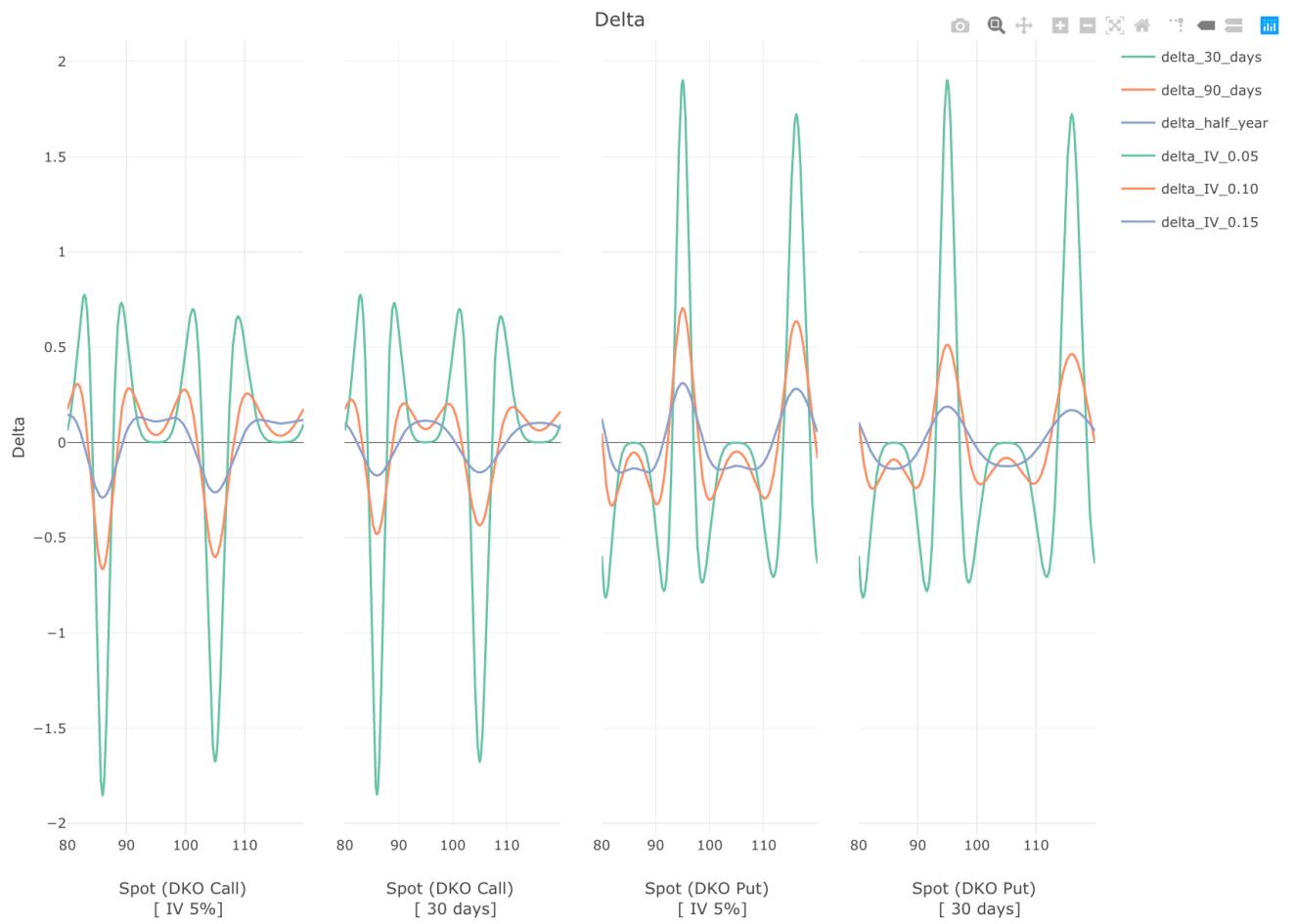
$$f(T) = \begin{cases} 0 & \text{if } S_t \geq B \text{ for all } t \in [0, T], \\ K - S_T & \text{if } S_T < K \text{ and } S_{t^*} < B \text{ for } t^* \in [0, T] \end{cases}$$

i. Delta:

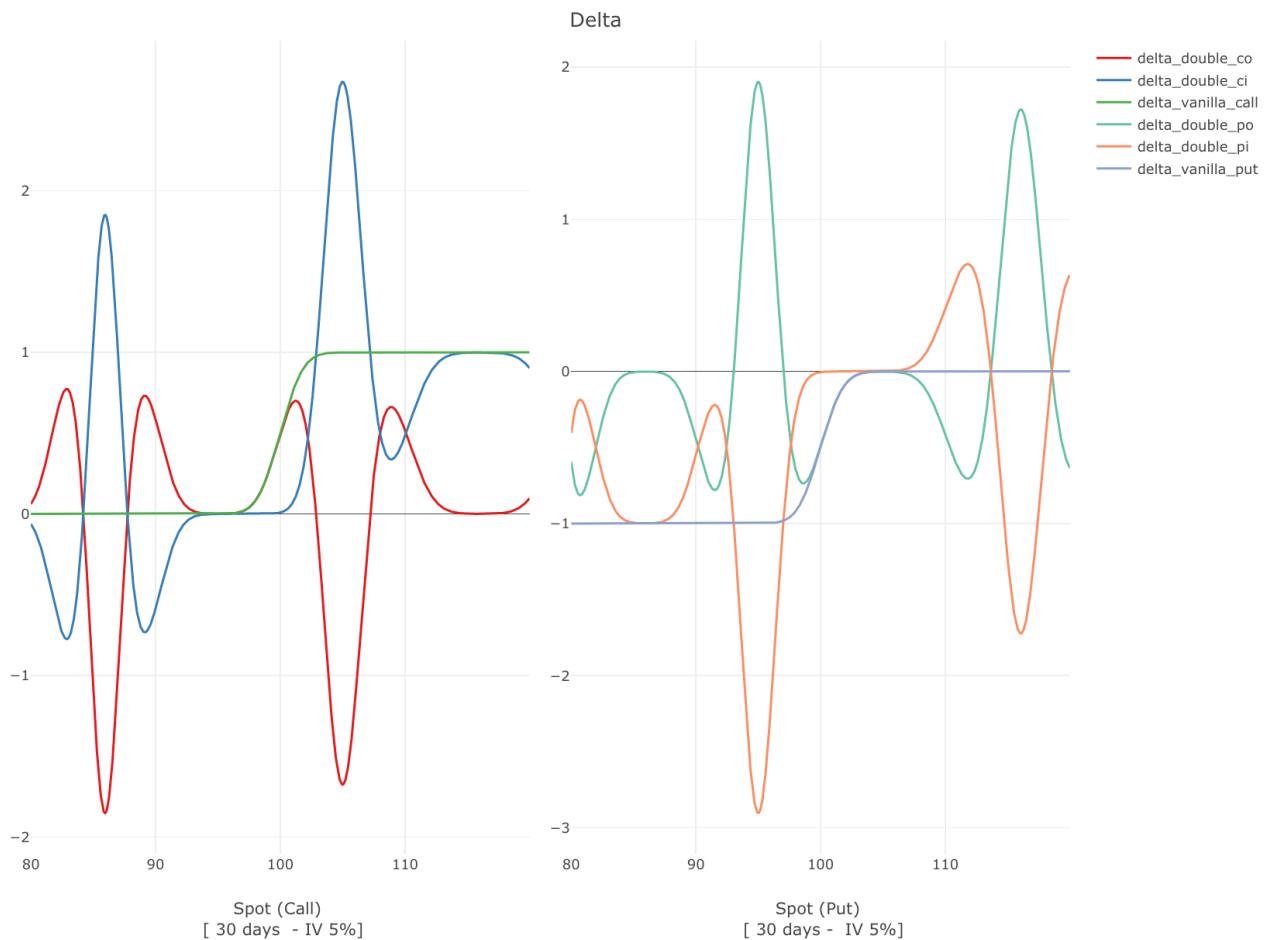
Knock - In:



Knock - Out:

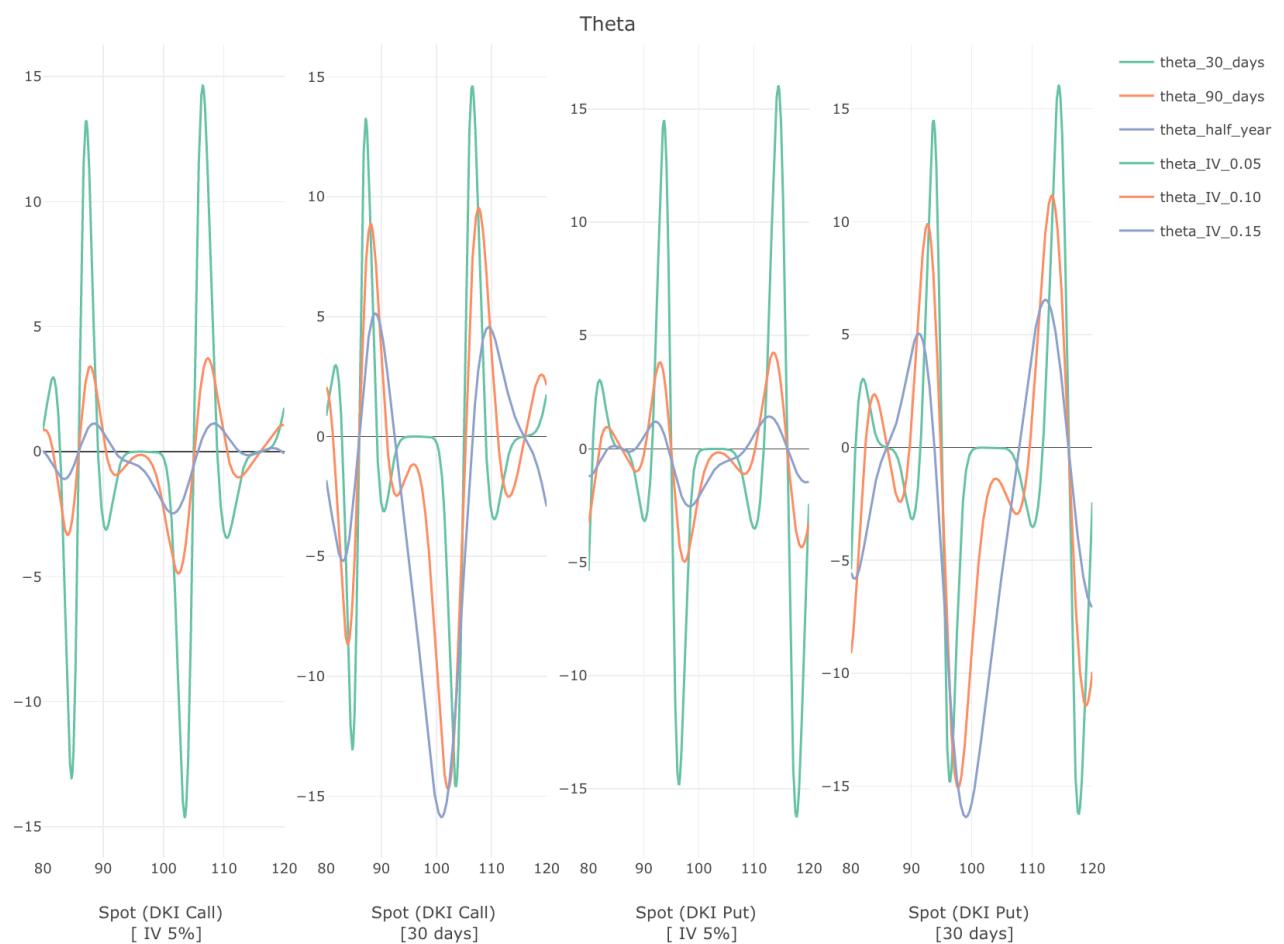


Compare to Vanilla:

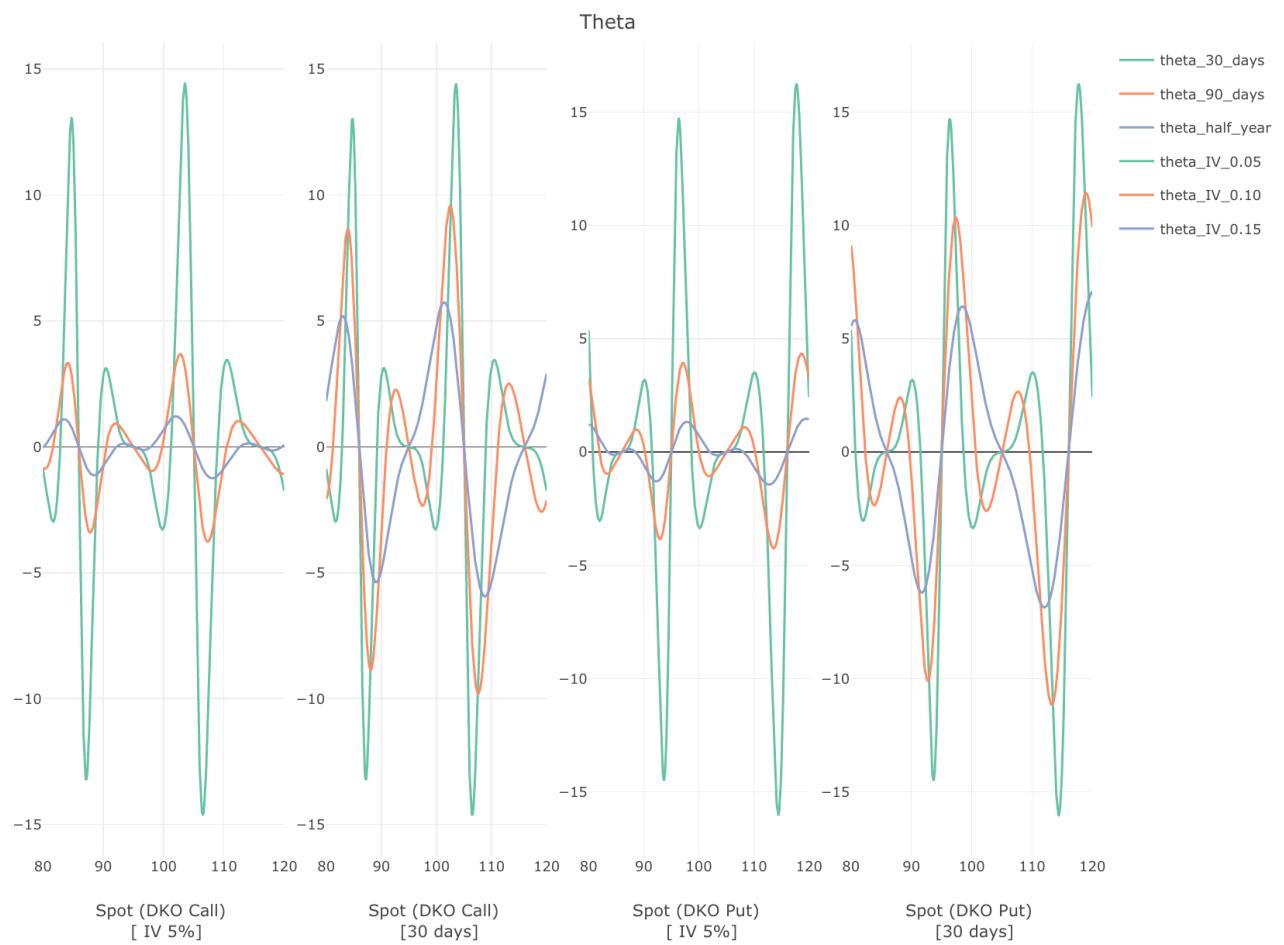


ii. Theta:

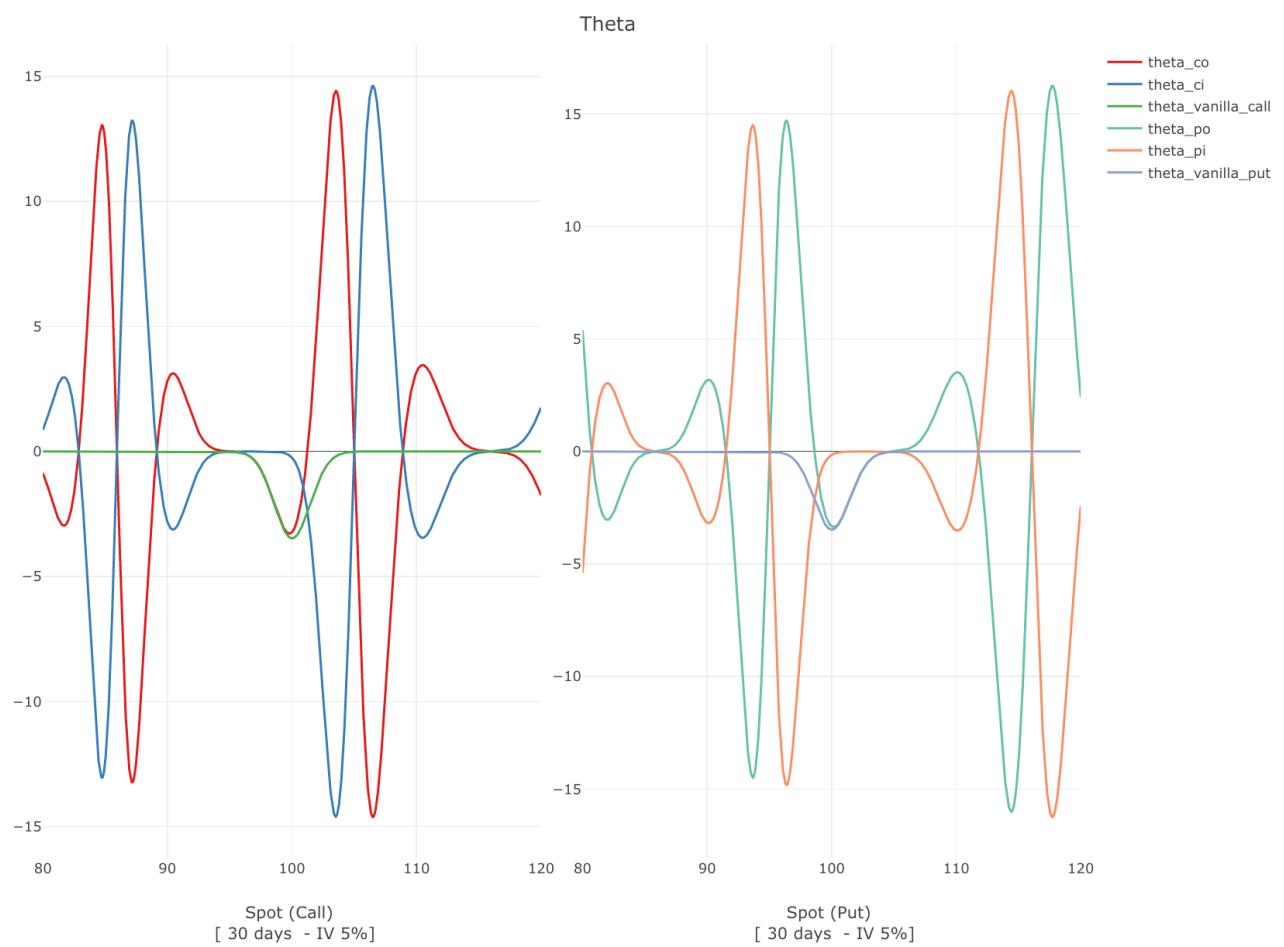
Knock - In:



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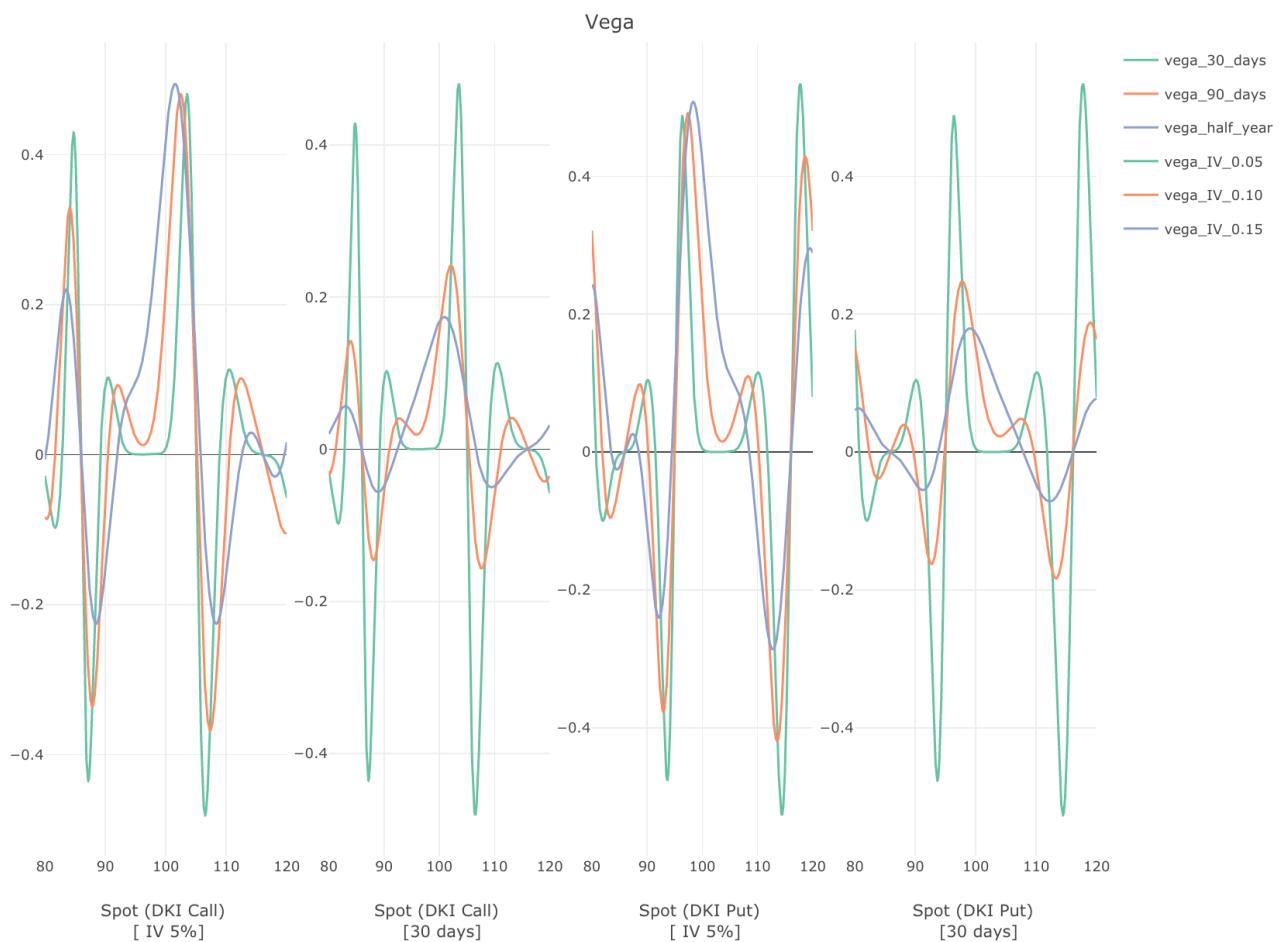


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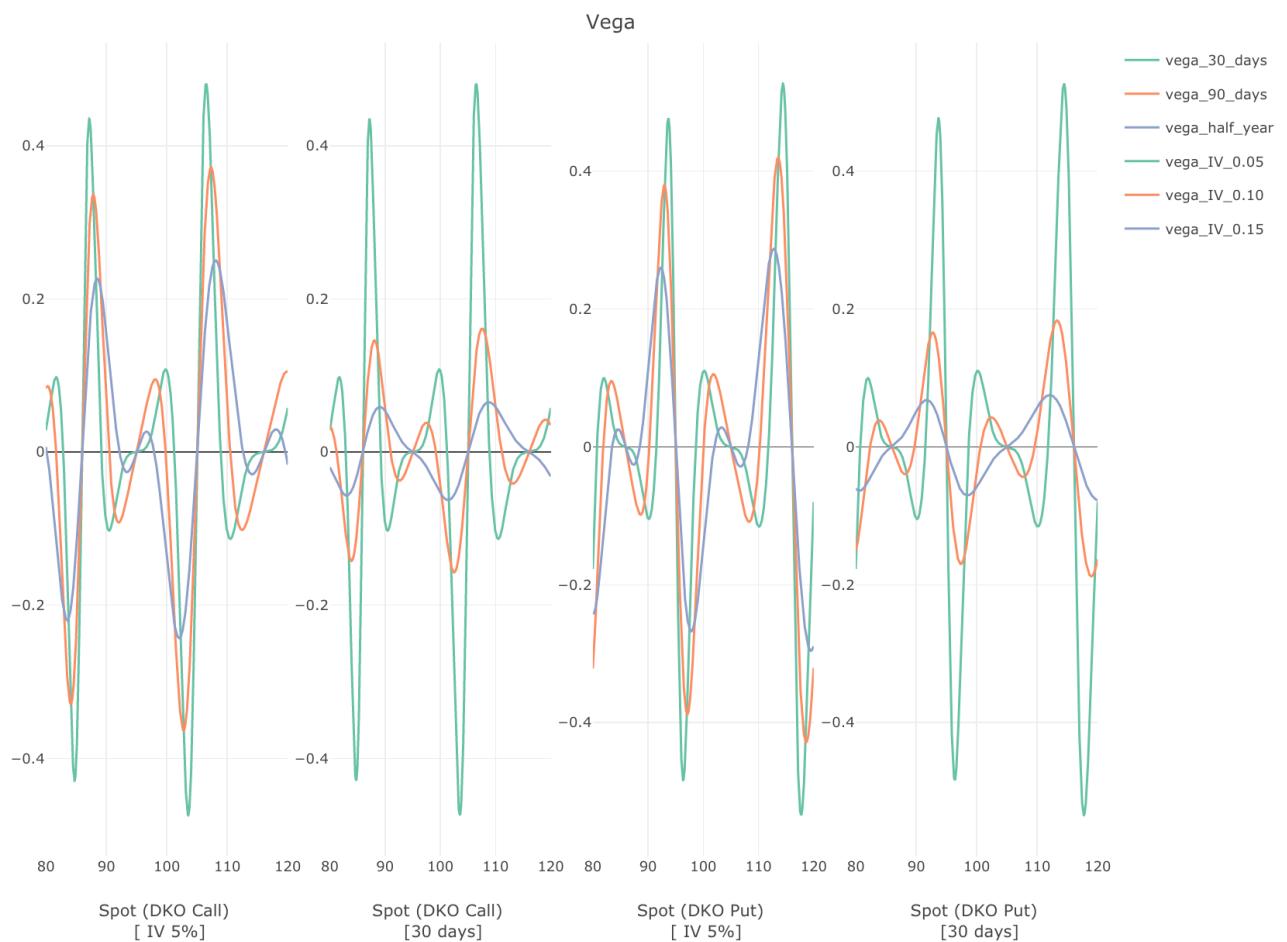


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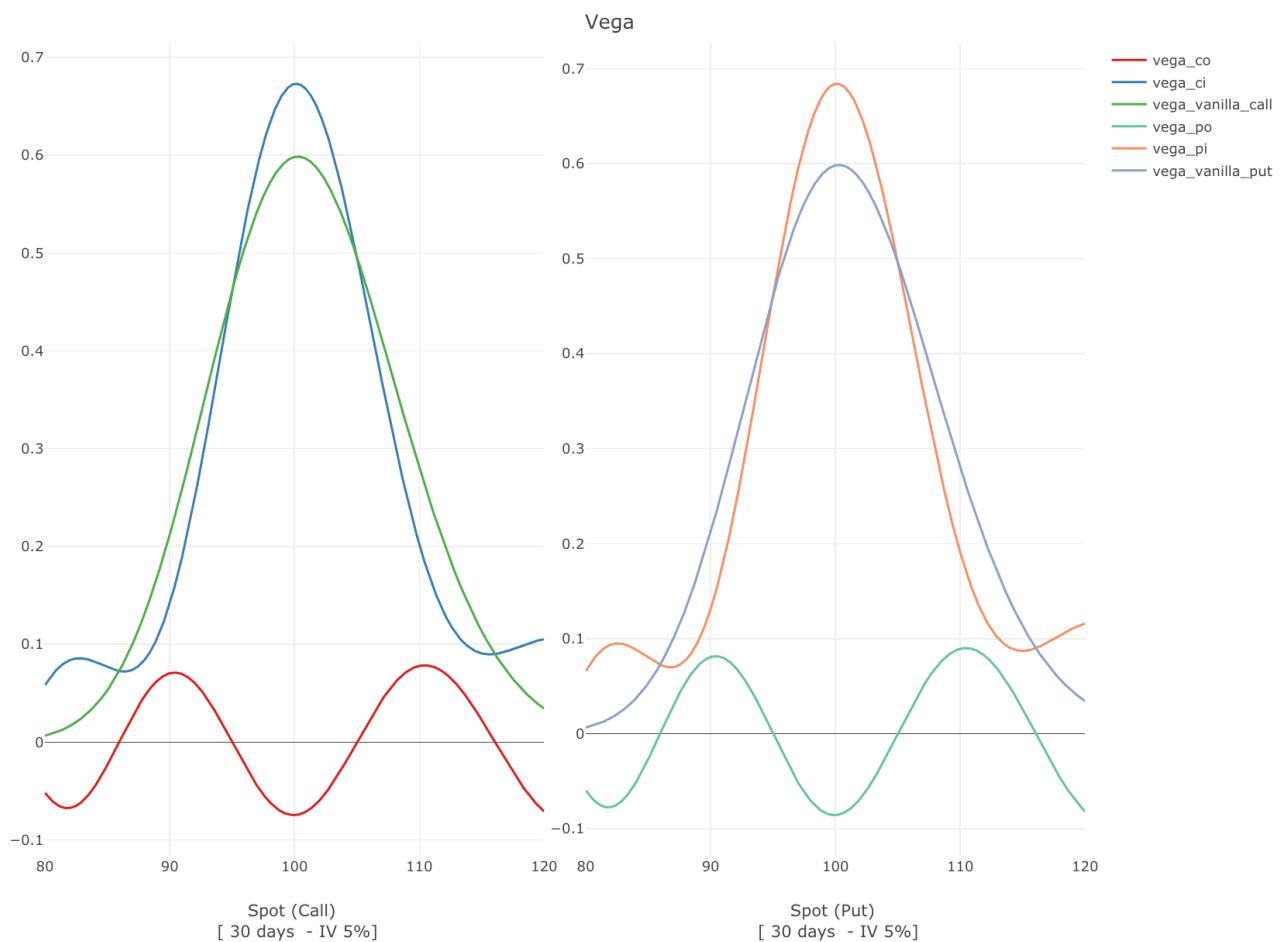
Knock - In:



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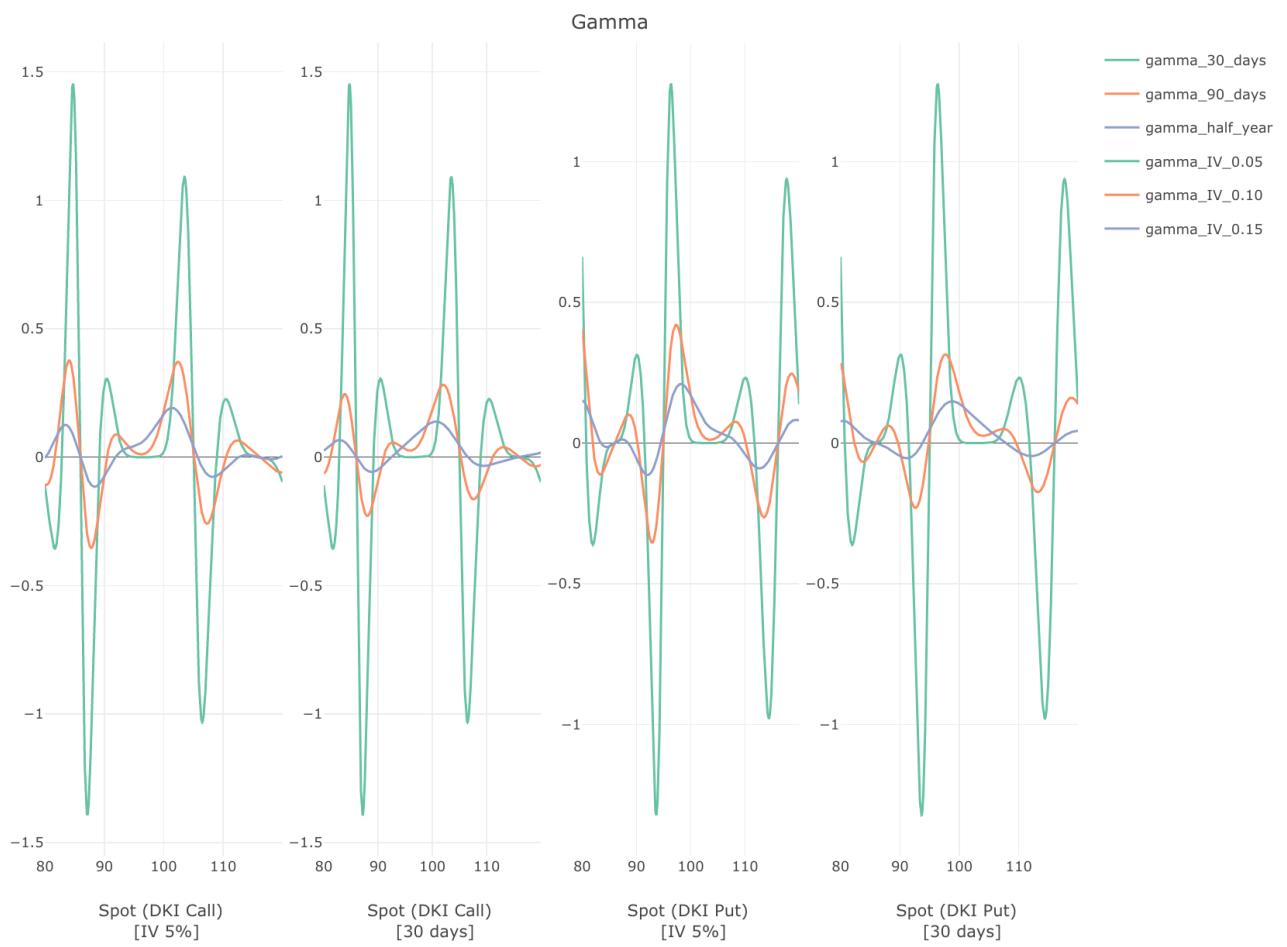


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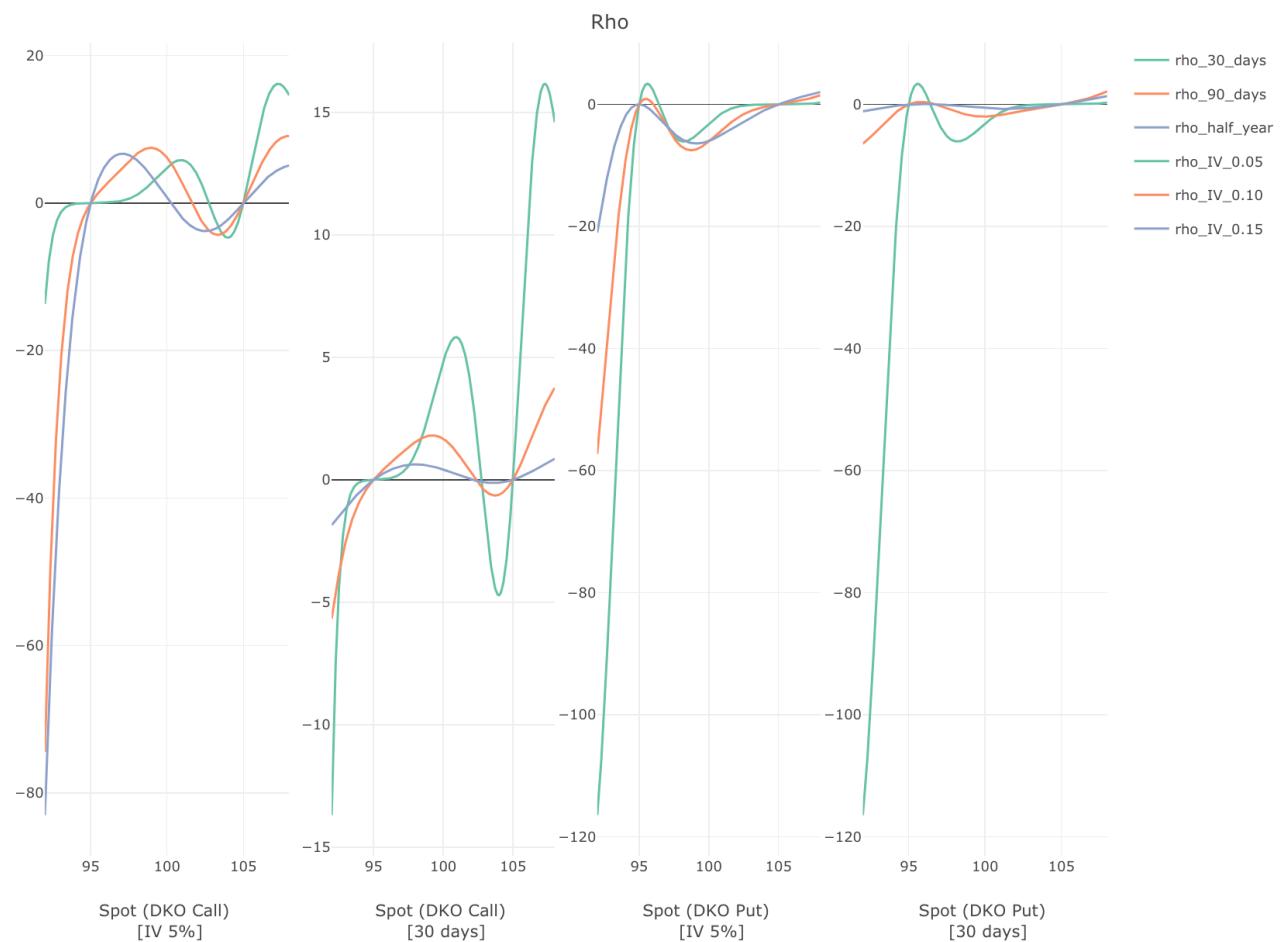


iv. Rho:

Knock - In:



Knock - Out:



Compare to Vanilla:

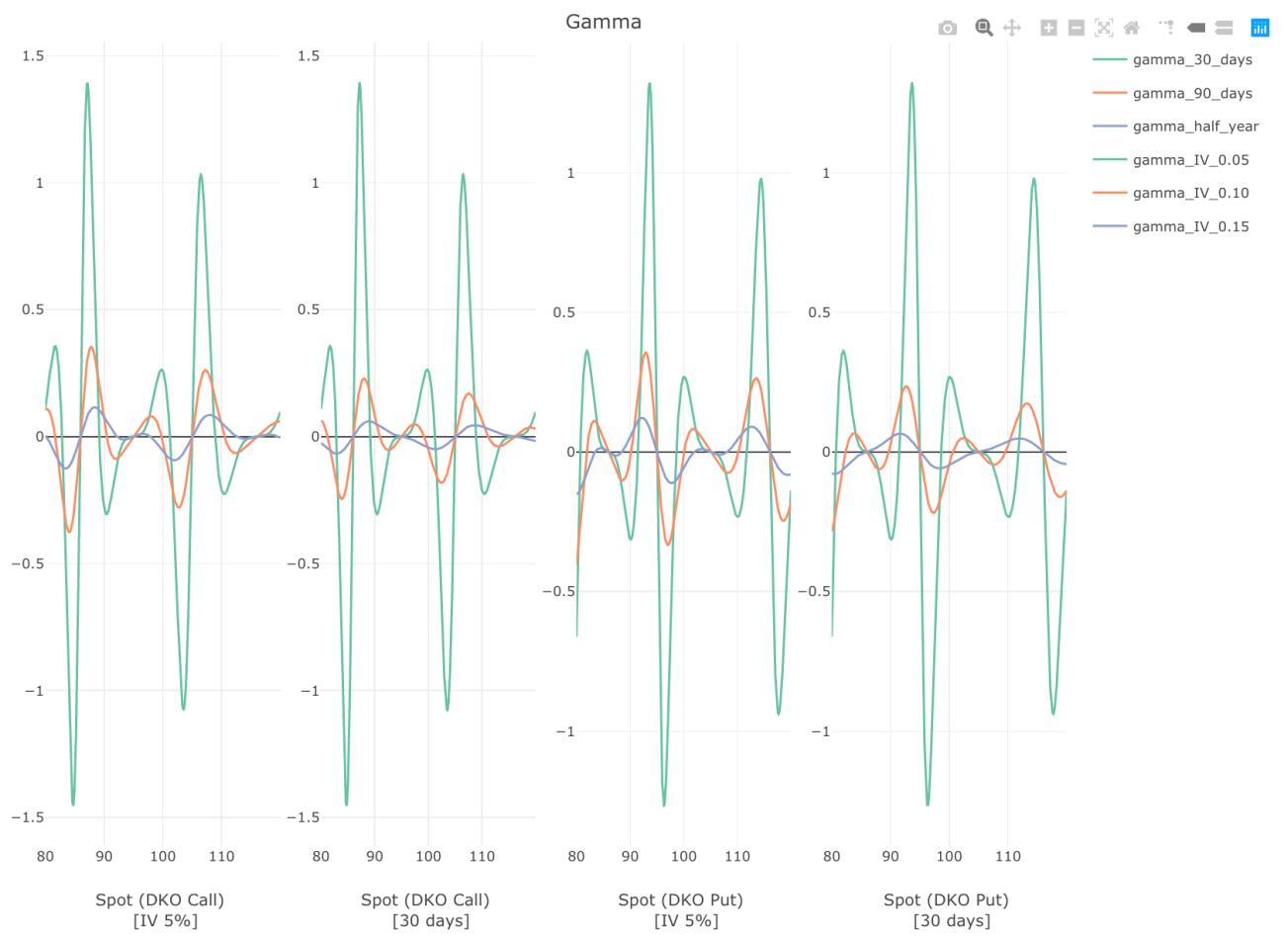


v. Gamma:

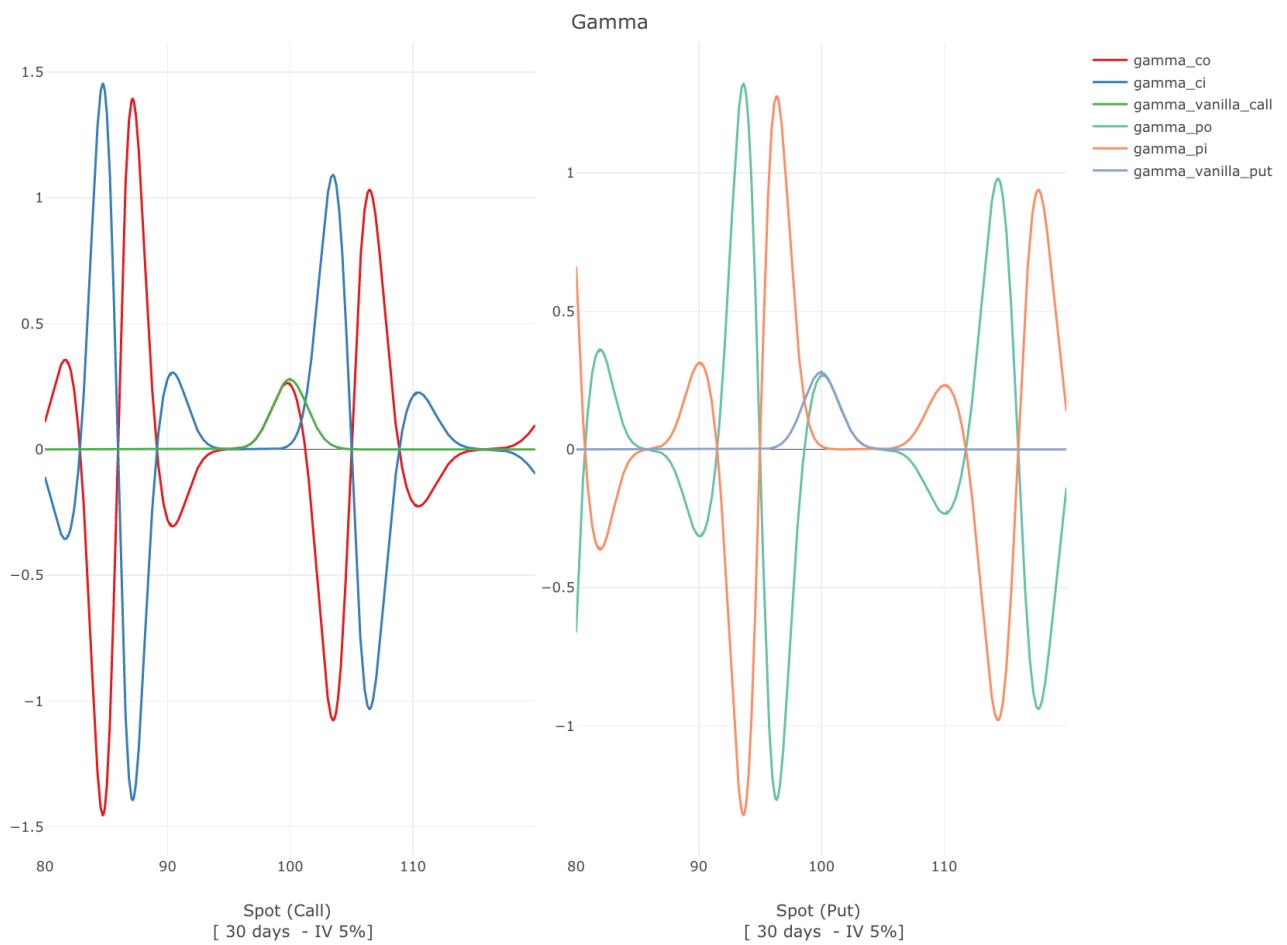
Knock - In:



Knock - Out:

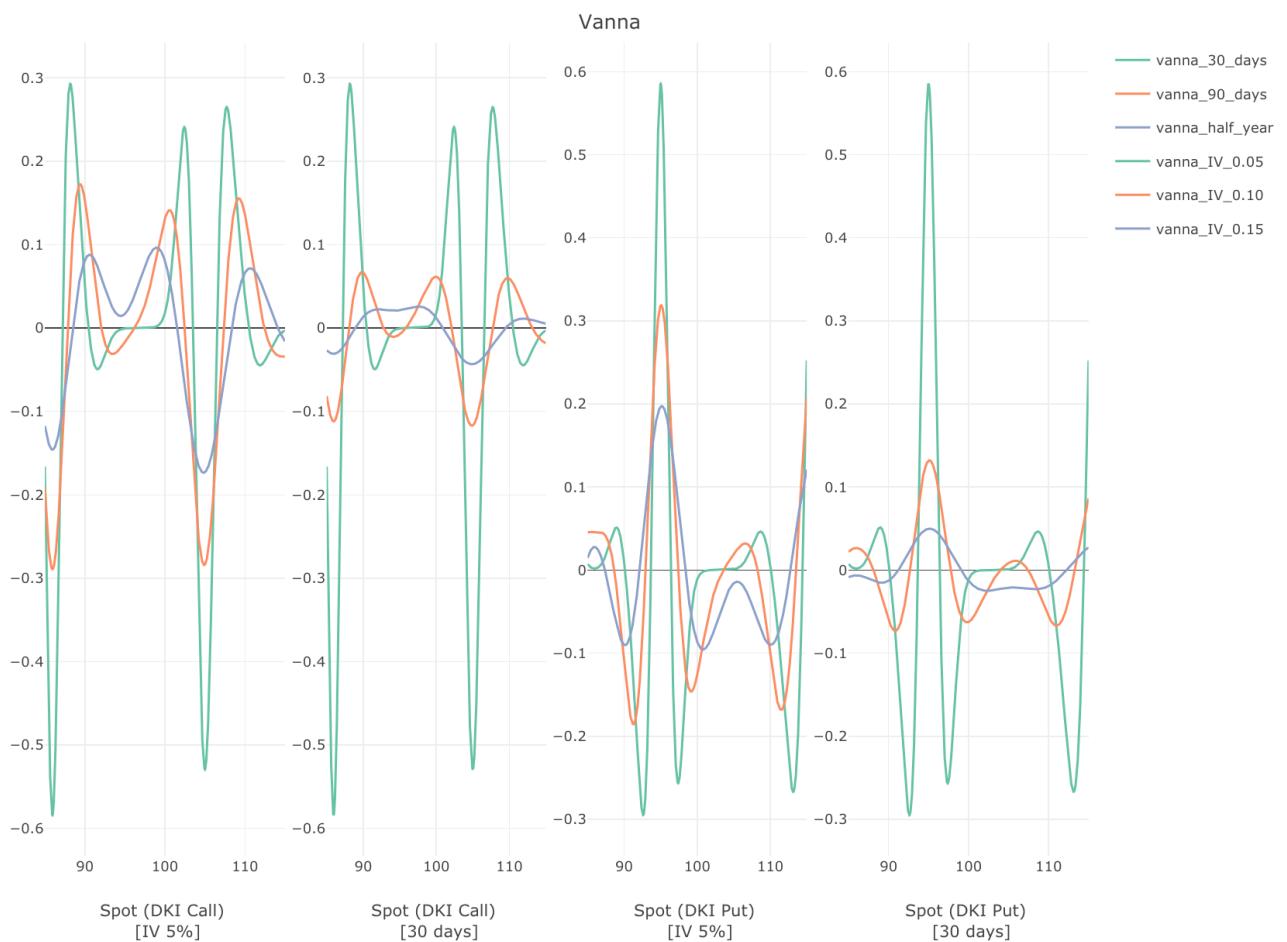


Compare to Vanilla:

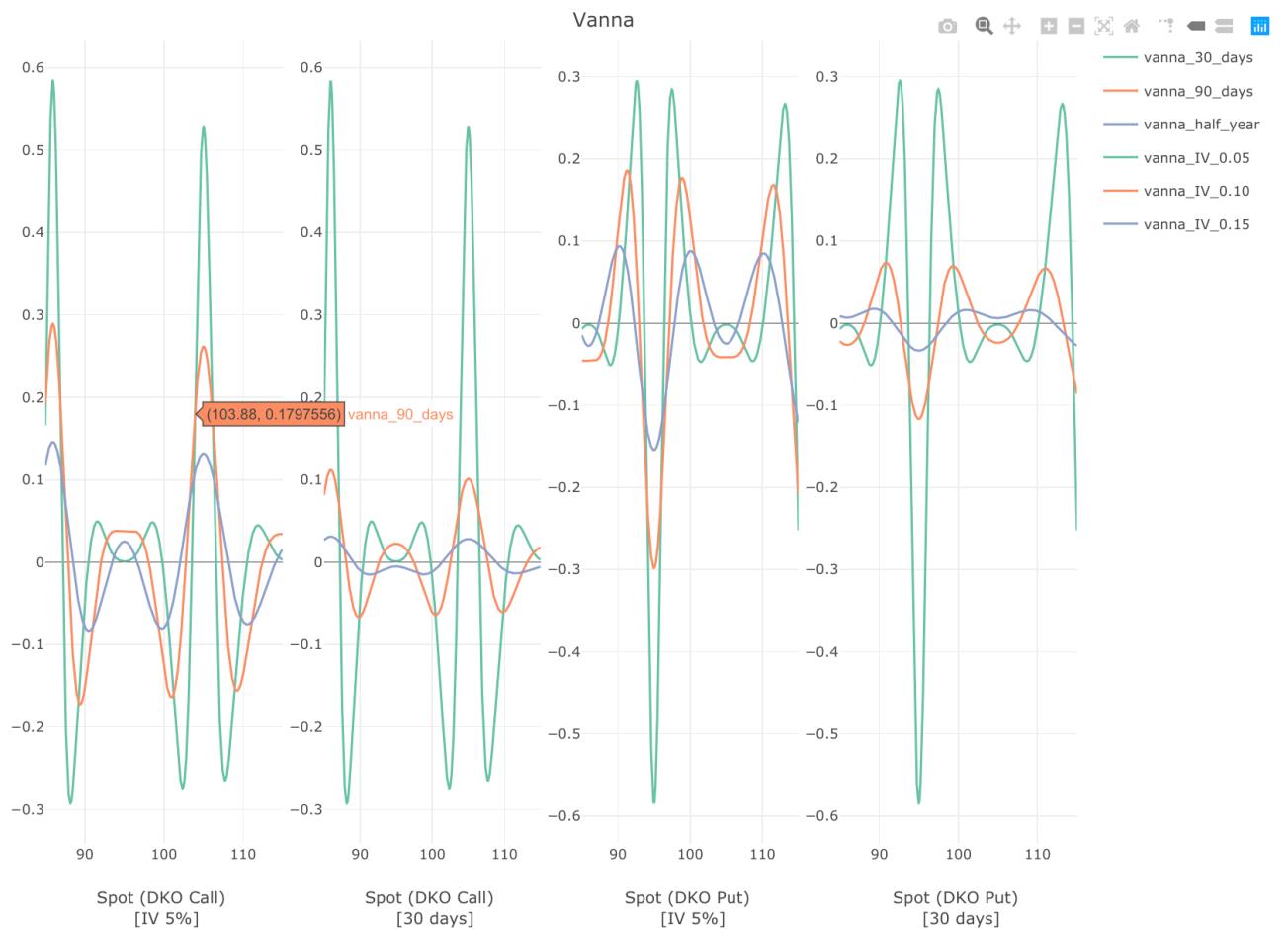


vi. Vanna:

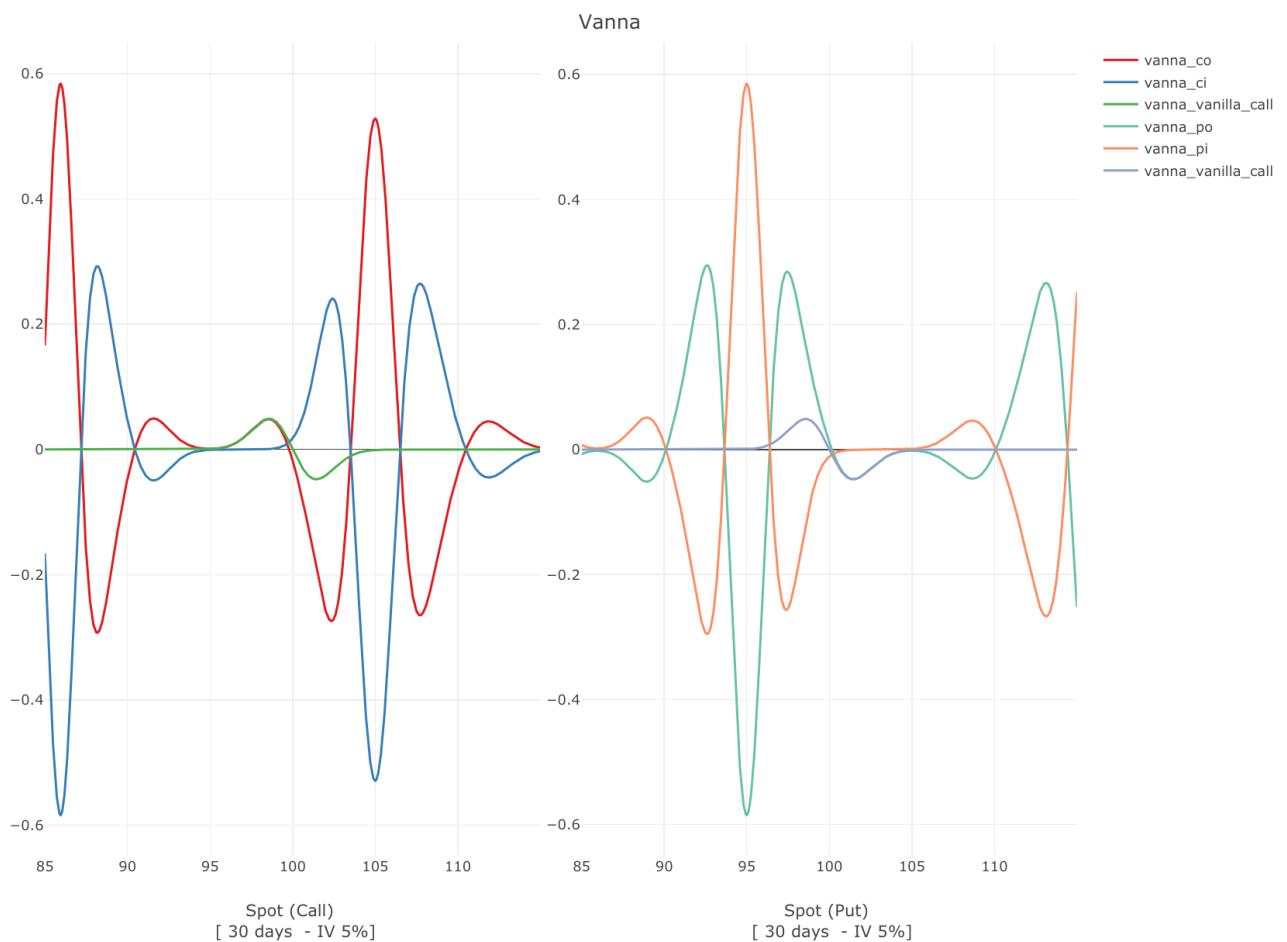
Knock - In:



Knock - Out:

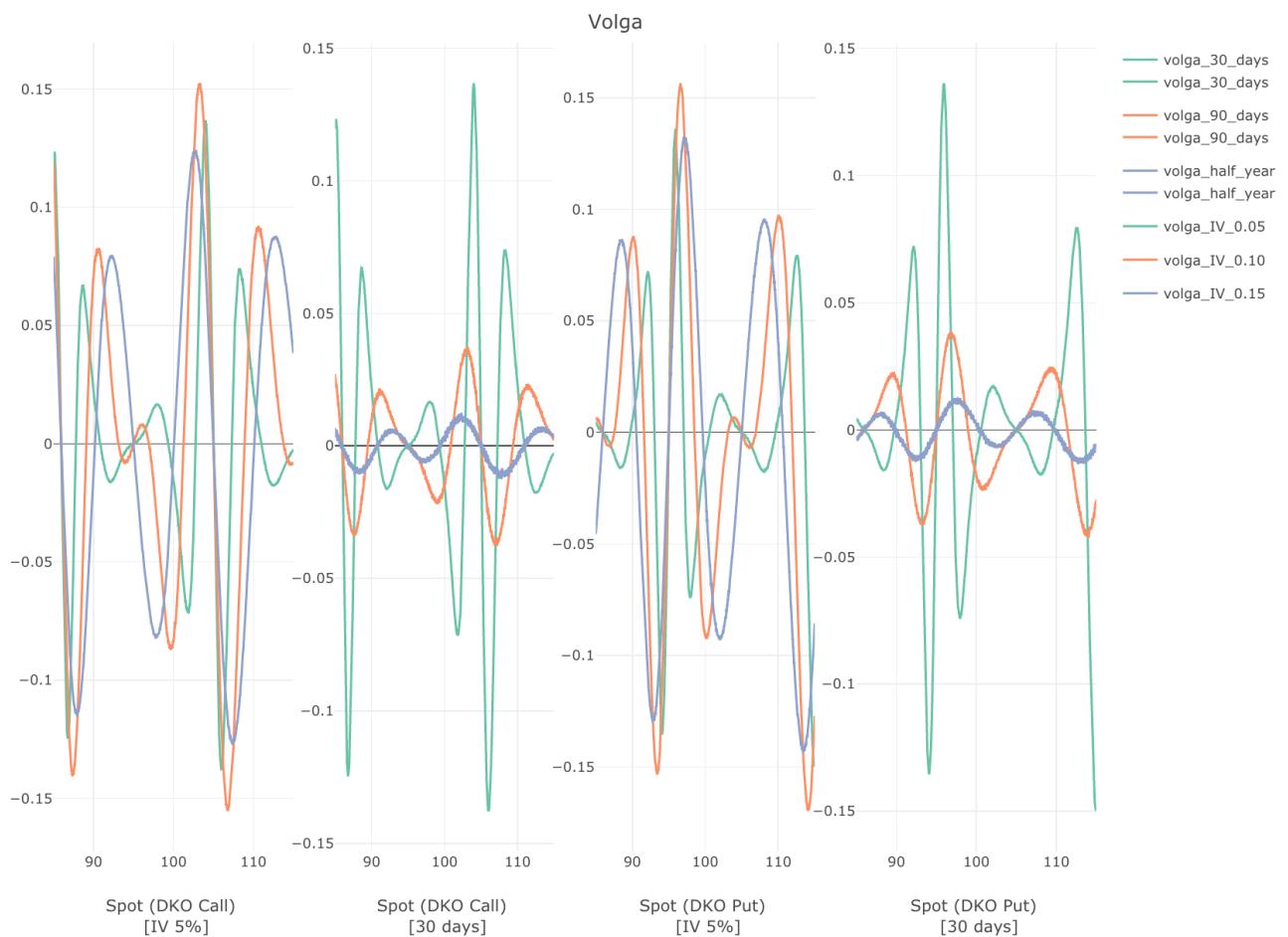


Compare to Vanilla:

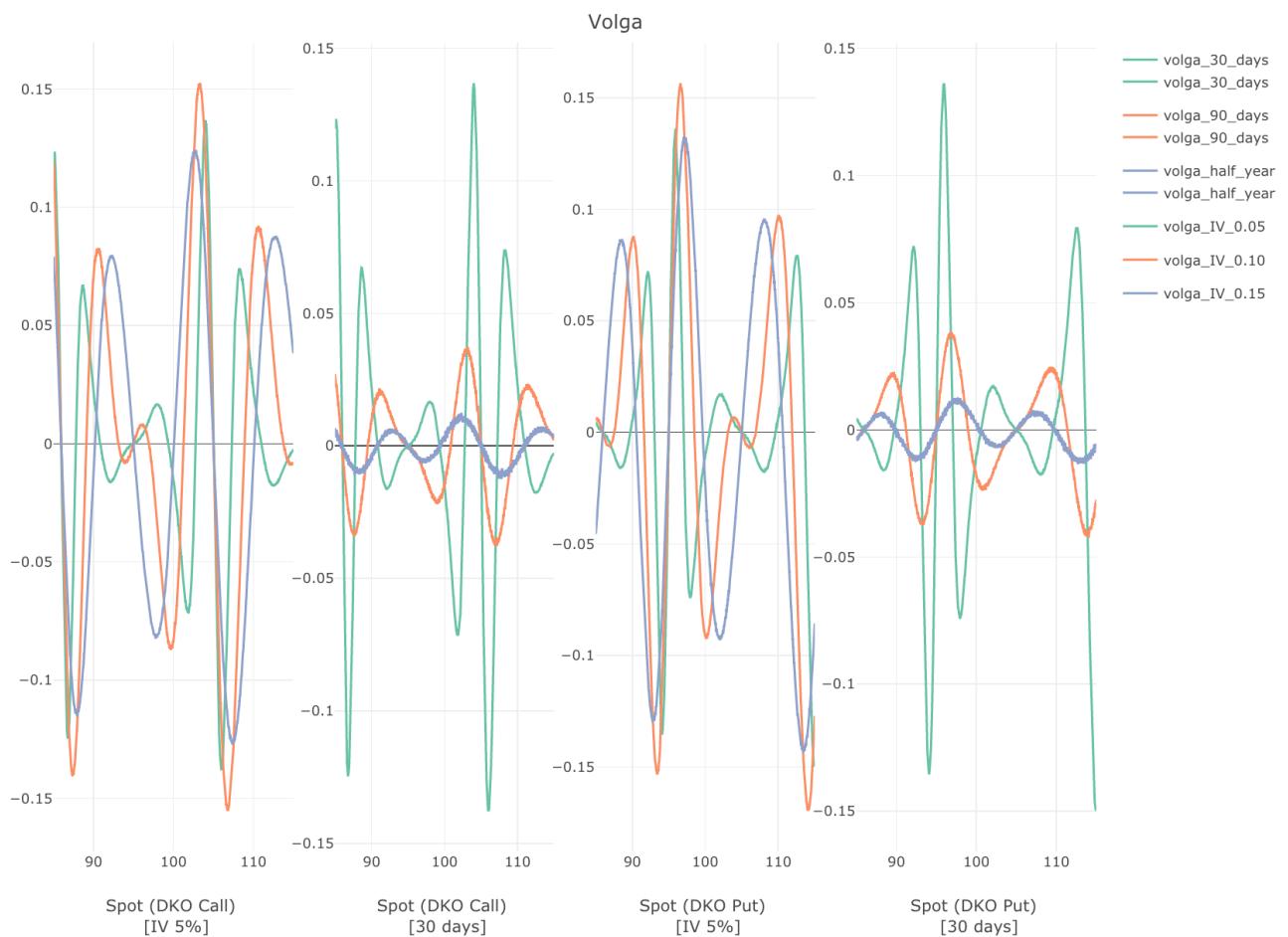


vii. Volga:

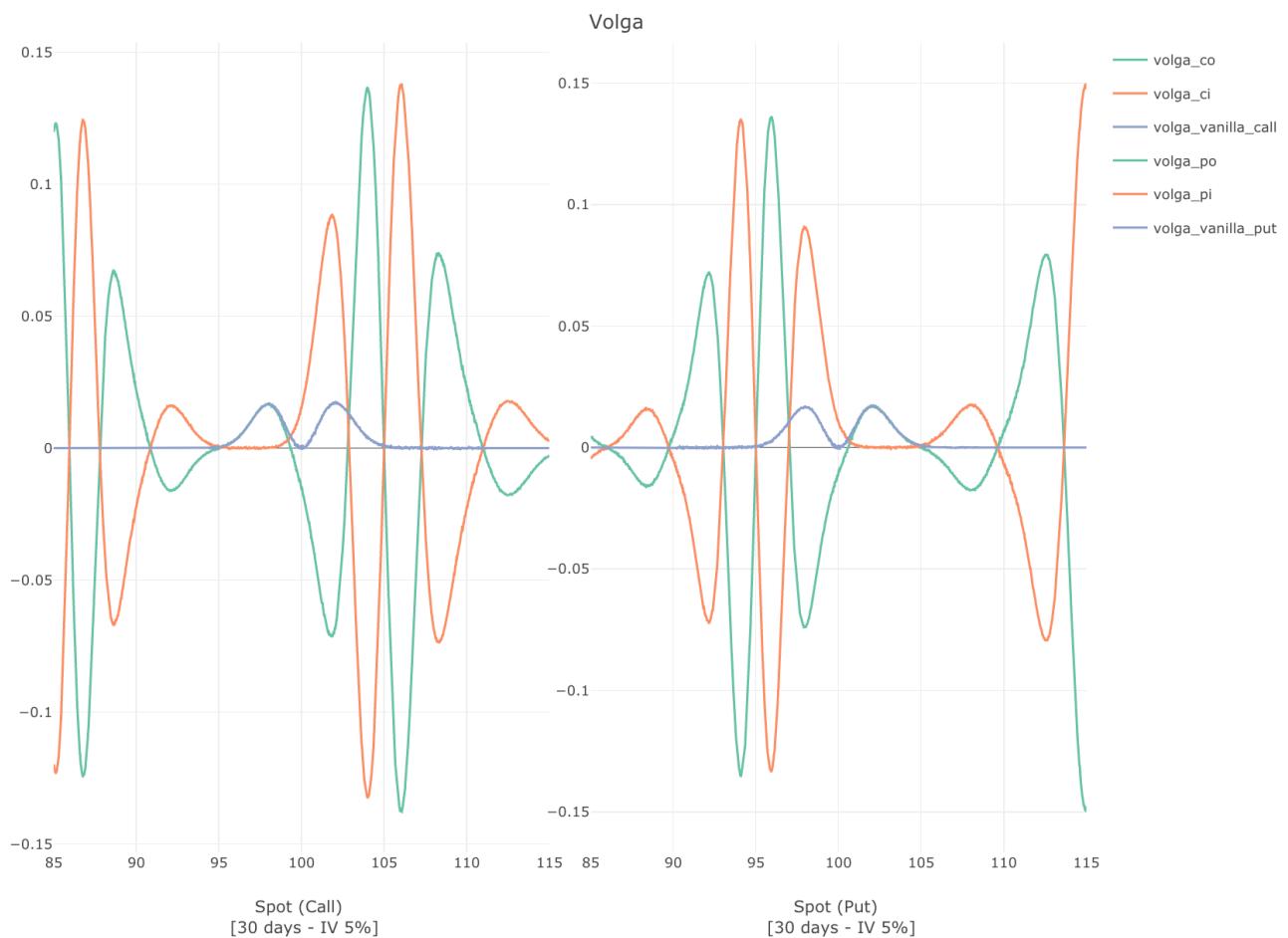
Knock - In:



Knock - Out:



Compare to Vanilla:



3a. Binary Option

Definition:

- A binary option is a financial product where the buyer receives a payout or loses their investment, based on if the option expires in the money.
- Binary options depend on the outcome of a "yes or no" proposition, hence the name "binary."
- Binary options have an expiry date and/or time.
- At the time of expiry, the price of the underlying asset must be on the **correct side of the strike price** (based on the trade taken) for the trader to make a profit.
- A binary option **automatically exercises**, meaning the gain or loss on the trade is automatically credited or debited to the trader's account when the option expires.

Type of Binary Option:

- We have four types of **european plain binary option**:
- i. Cash-or-nothing call
 - Pays out one unit of cash if the spot is above the strike at maturity.
$$C = e^{-r(T-t)} N(d_2)$$
- ii. Cash-or-nothing put
 - Pays out one unit of cash if the spot is below the strike at maturity.
$$P = e^{-r(T-t)} N(-d_2)$$

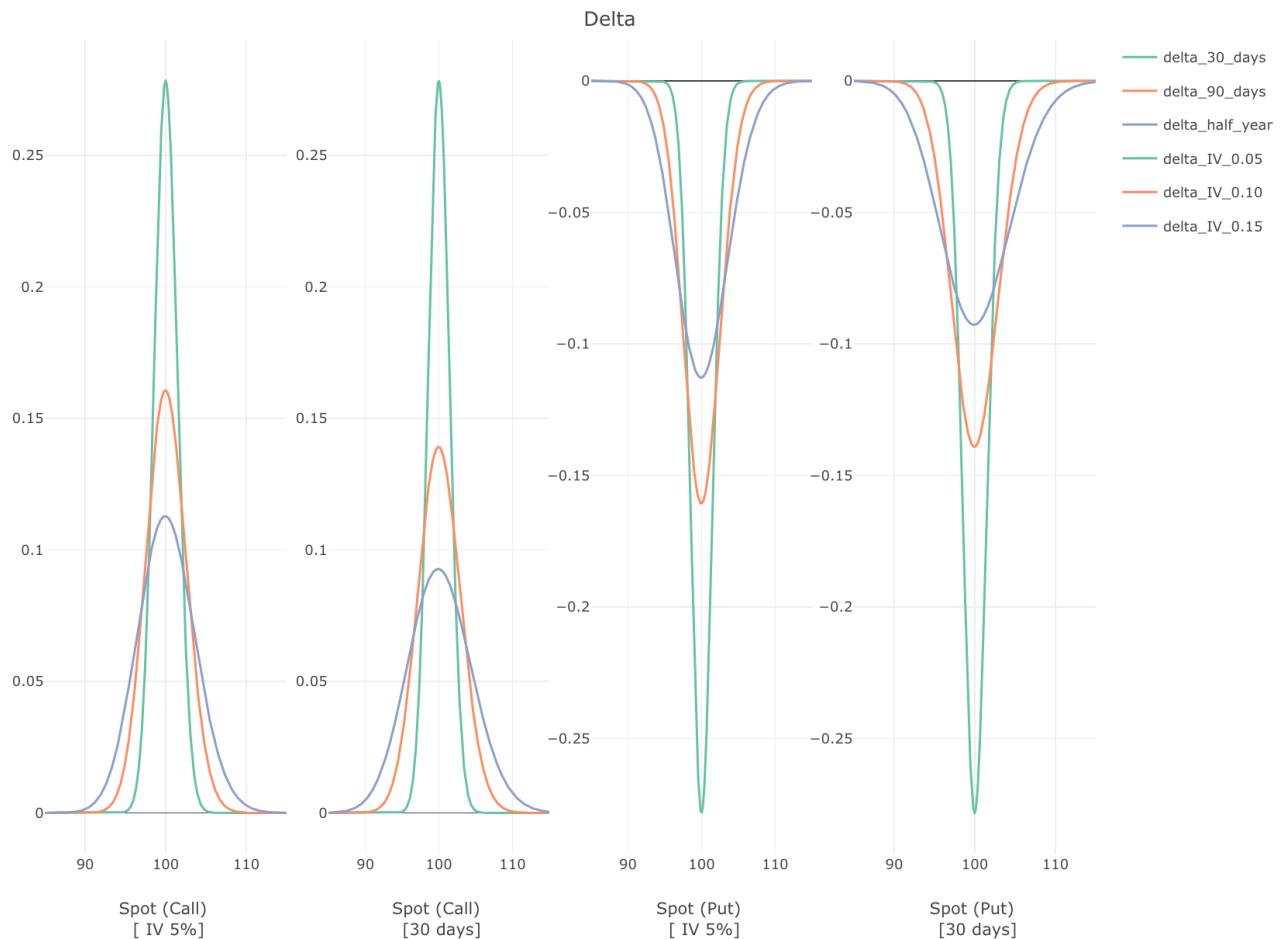
- iii. Asset-or-nothing call
 - Pays out one unit of asset if the spot is above the strike at maturity.
$$C = Se^{-q(T-t)} N(d_1)$$
- iv. Asset-or-nothing put
 - Pays out one unit of asset if the spot is below the strike at maturity.
$$C = Se^{-q(T-t)} N(-d_1)$$
- We will be studying greeks for Cash or Nothing Call/Put only

Payoff Equations:

- Call Option:
 - Payout if $(S_T > K)$ else 0.
- Put Option:
 - Payout if $(S_T < K)$ else 0.

i. Delta:

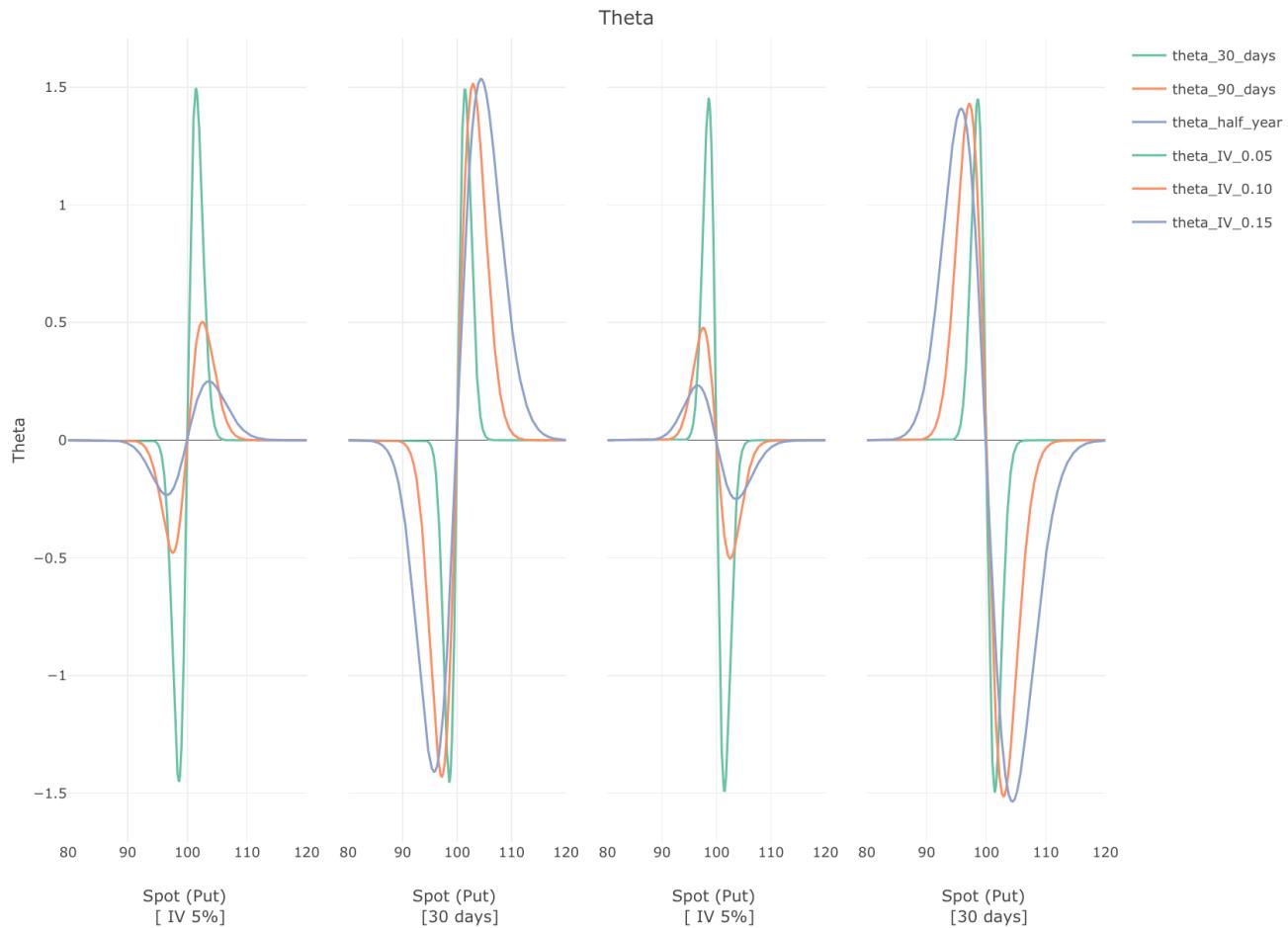
- Delta's shape is like an **upward and downward bell curve** for **CALL** and **PUT** respectively.
- Delta ranges from 0 to 1 for **CALL** and -1 to 0 for **PUT**.
- The **shorter** time to maturity/**lower** volatility, the higher the delta as **payoff is deterministic at ATM**.
- Bell curve shapes indicates when spot moves away from ATM, **shorter** time to maturity/**lower** volatility have lower delta.



ii. Theta:

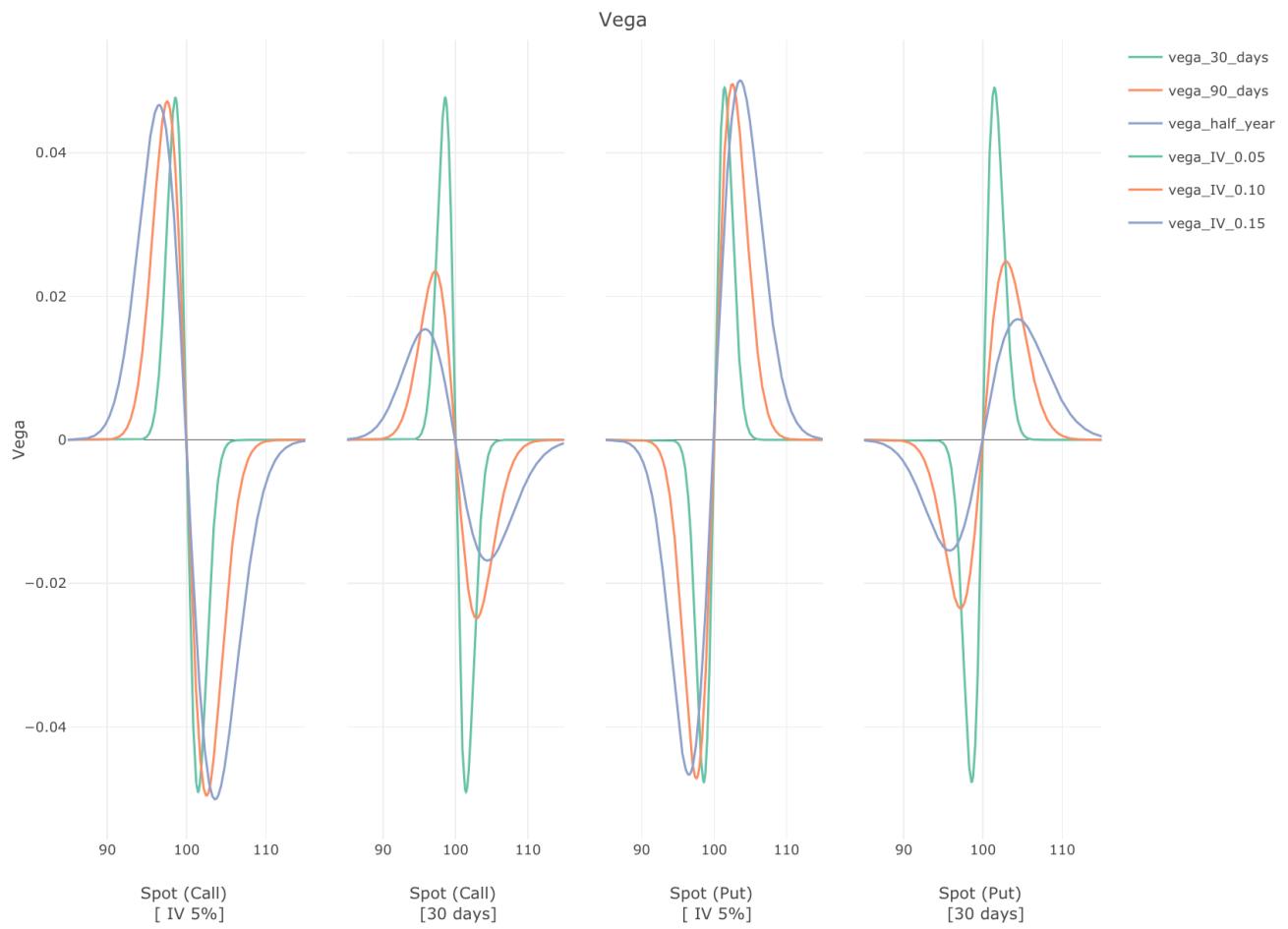
- Theta's shape is like an **negative and positive sine curve** for **CALL** and **PUT** respectively..

- Theta oscillates at ATM; close to 0 at **ATM**.
- The theta is **highly sensitive around ATM**; This is pretty obvious as such options have the highest time value and thus have more premium to lose each day.
- Theta **spreads across moneyness** as volatility **increases**.
- The **lower** time to maturity, the **higher sensitivity** of Theta.
- As moving to deep ITM/OTM, theta effects went to zero.



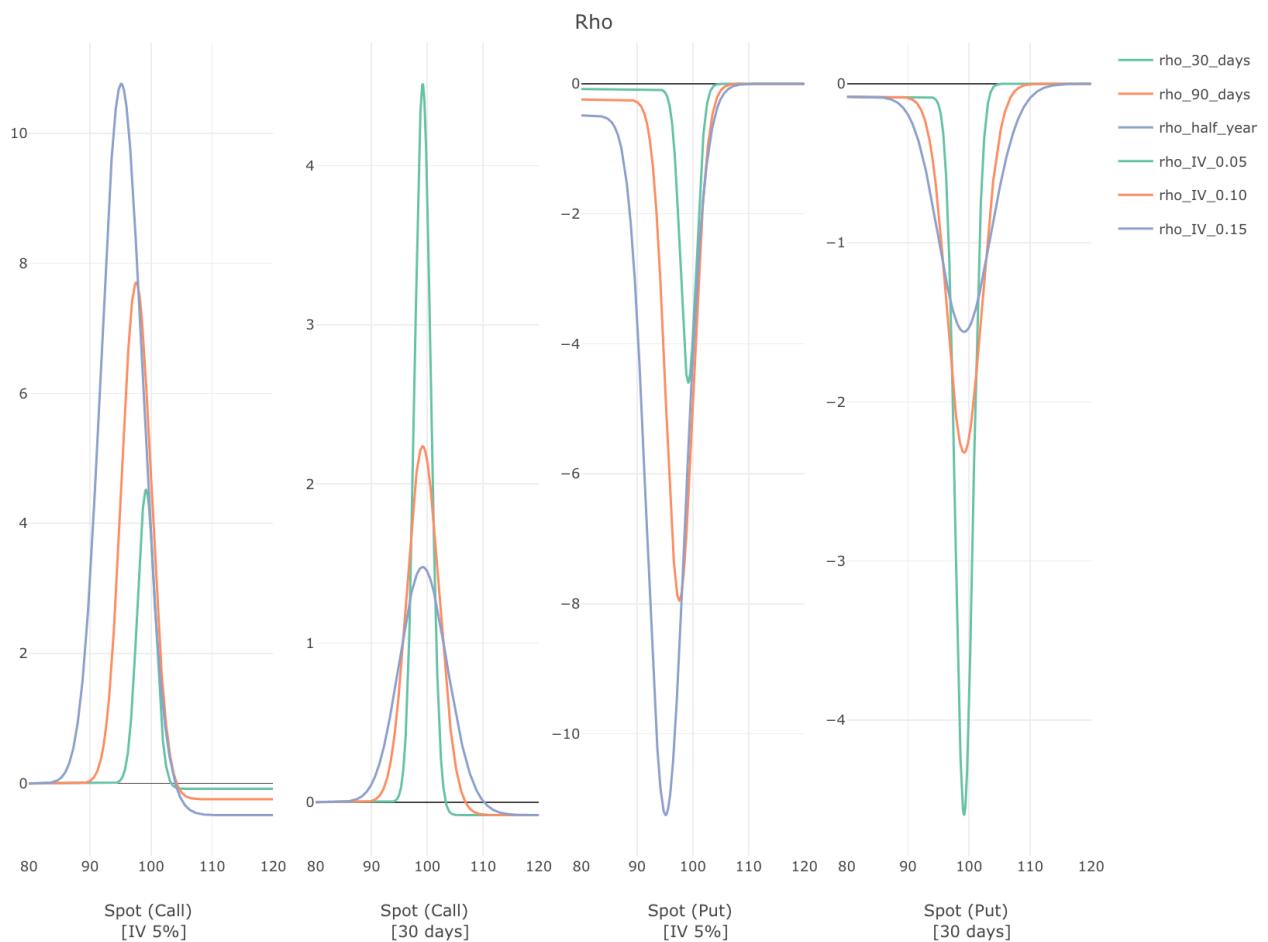
iii. Vega:

- Vega's shape is like an **positive and negative sine curve for CALL and PUT respectively..**
- Vega oscillates at ATM; close to 0 at **ATM**.
- The theta is **highly sensitive around ATM**; This is pretty obvious as such options have the highest time value and thus have more premium to lose each day.
- Vega **spreads across moneyness** as time to maturity **increases**.
- The **lower** volatility, the **higher sensitivity** of Vega.



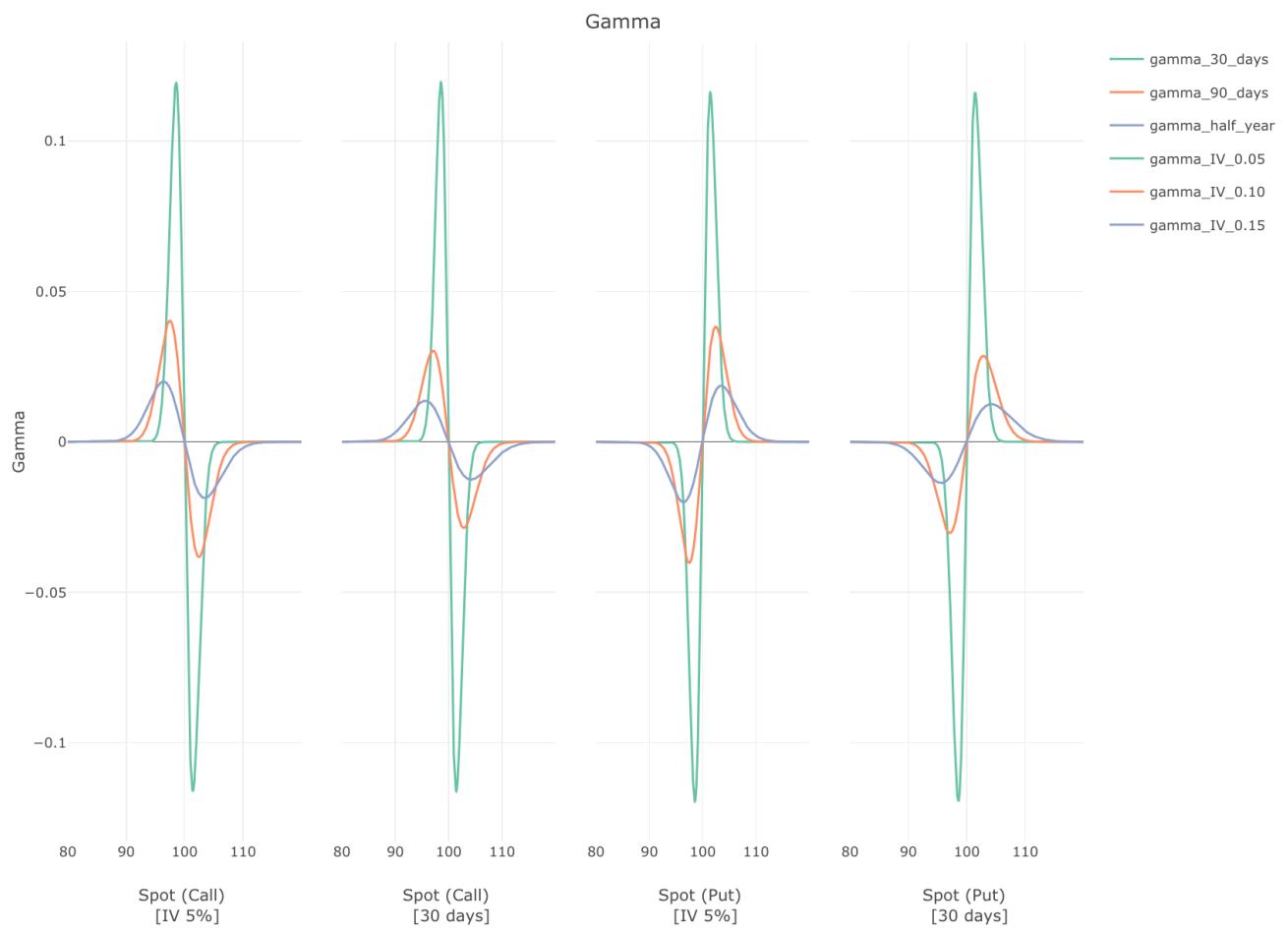
iv. Rho:

- Rho's shape is like **Delta** curve.
- Call and put option have positive and negative correlation with rho respectively.
- The **longer** the time to maturity, the **higher** the rho.
- The **lower** the volatility, the **higher** the rho* .



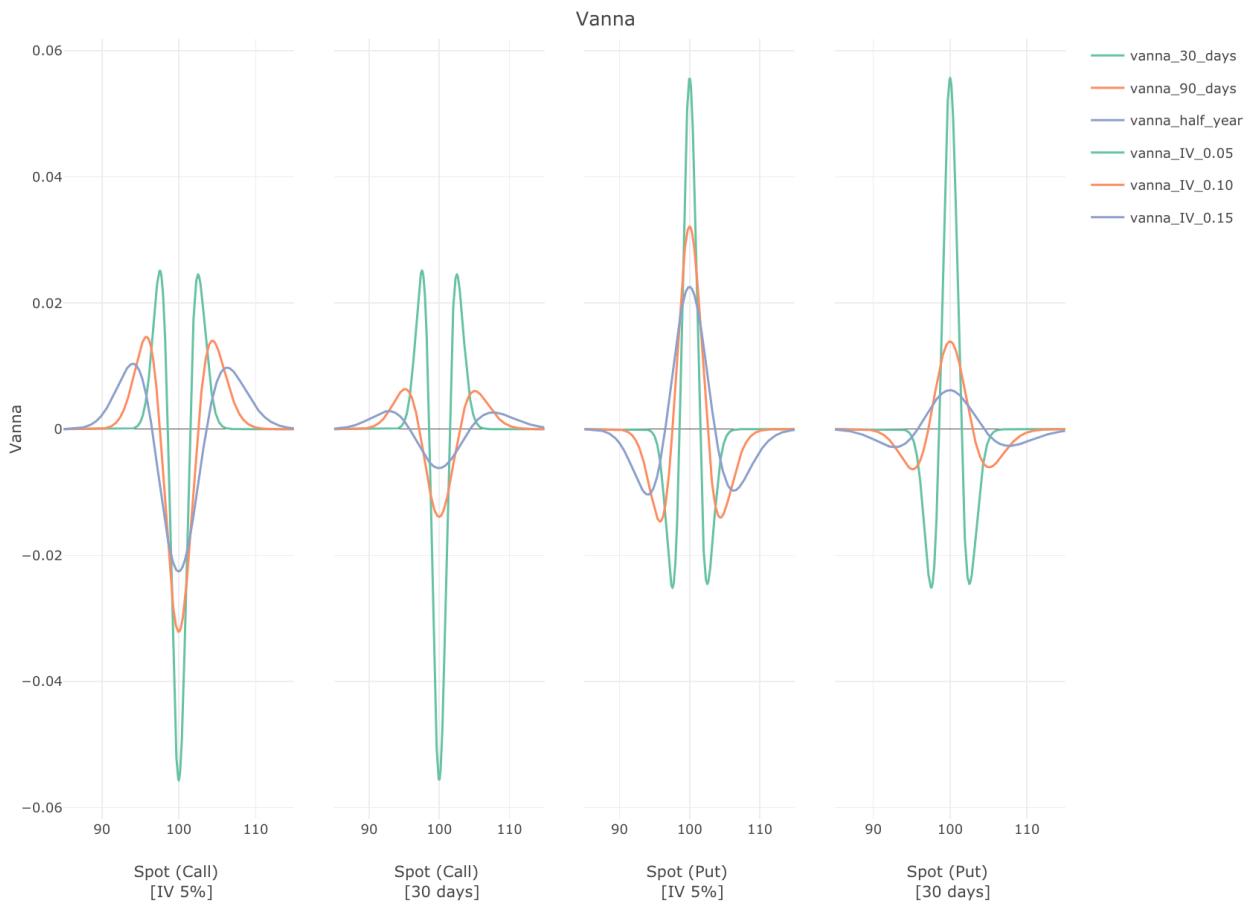
v. Gamma:

- Gamma's shape Like a **upward bell curve**, can be derived from Delta curve too.
- The **shorter** the time to maturity/**lower** the volatility, the **higher** the gamma.



vi. Vanna:

- Vanna's shape is like an **V** and **inverted V curve** for **CALL** and **PUT** respectively.
- The **shorter** the time to maturity/**lower** the volatility, the **higher** the vanna.



vii. Volga:

- Volga's shape is like an **negative and positive sine curve for CALL and PUT respectively..**
- Volga is close to 0 for ATM option.
- Like **Vega**, spreads across moneyness as time to maturity **increases**.
- The **lower** volatility, the **higher sensitivity** of Volga.

