Consider a data set with three data points in $\mathbb{R}^2$:

$$X=\begin{bmatrix} 0 & 0 \\ 0 & -1 \\ -2 & 0 \end{bmatrix} \quad \boldsymbol{y}=\begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$$

Manually solve the support vector machine problem to obtain the optimal hyperplane (b^*, w^*) and its margin.

To gain the optimal hyperplane, we need to satisfy the following formula：

$$\begin{cases} \text{minimize：} \ \dfrac{1}{2}w^T w \\ \text{subject to：} \ y_n(w^T x_n + \mathrm{b}) \geq 1 \end{cases}$$

1. Add x and y to formula

$$X=\begin{bmatrix} 0 & 0 \\ 0 & -1 \\ -2 & 0 \end{bmatrix} \quad \boldsymbol{y}=\begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} \qquad \text{式子}: y_n(w_1x_1 + w_2x_2 + \mathrm{b}) \geq 1 \quad \mathrm{n} = 1,2$$

(i) $-\mathrm{b} \geq 1$

(ii) $-1(-w_2 + b) \geq 1 \ = \ w_2 - b \geq 1$

(iii) $-2w_1 + b \geq 1 \ = \ -2w_1 + b \geq 1$

2. 根據上述可以得出b、w_1、w_2三個值

$$\begin{cases} b \leq -1 \\ w_1 \leq -1 \\ w_2 \geq 0 \end{cases}$$

3. 另外w_1及w_2需要滿足下列式子

$$\frac{1}{2}w^T w(min) \geq 0.5$$

$$\frac{1}{2}({w_1}^2 + {w_2}^2) \geq 0.5$$

得出optimal hyperplane(b^*,w^*)

$$\begin{cases} b^* = -1 \\ {w_1}^* = -1 \\ {w_2}^* = 0 \end{cases}$$

Margin

$$\mathrm{g(x)} = \mathrm{sign}(-x_1 - 1)$$