# Package 'cknockoff'

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Title The cKnockoff Filter for FDR-controlled Variable Selection	
<b>Version</b> 0.0.0.9000	
<b>Date</b> 2022-01-13	
<b>Description</b> The cKnockoff method is a multiple testing procedure applied to the linear model for variable selection with FDR control. It almost surely dominates the fixed-X knockoff procedure.	
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cknockoff

The cKnockoff procedure

## **Description**

This function applies the cKnockoff procedure to the data (X, y) subjecting to the Gaussian linear model, selecting variables/features relevant for predicting the outcome with FDR control.

#### Usage

```
cknockoff(
   X,
   y,
   knockoffs = knockoff::create.fixed,
   statistic = stat.glmnet_coefdiff_lm,
   alpha = 0.05,
   Rstar_refine = F,
   n_cores = 1,
   prelim_result = NULL,
   X.pack = NULL
)
```

#### **Arguments**

X n-by-p matrix or data frame of features.

y response vector of length n.

knockoffs method used to construct the knockoff matrix for X. It should be a function tak-

ing a n-by-p matrix as input and returning a n-by-p matrix of knockoff variables. It is the same as the knockoffs argument in knockoff::knockoff.filter. By

default, knockoff::create.fixed is used.

statistic the knockoff feature statistics (W-statistics) function used to assess variable im-

portance. Any function in the family "statistics" in the R package knockoff that are suitable for the fixed-X setting can be supplied to this argument. But please be aware of the efficiency issue as this function will be called repeatedly in cknockoff. We suggest use the statistic functions provided by our package.

By default, a lasso statistic from our package is used.

alpha nominal false discovery rate (default: 0.05).

Rstar\_refine A logical value determining if we use a better estimation of the number of rejec-

tions in calibration. If TRUE, the procedure is cKnockoff\* and otherwise is the

vanilla cKnockoff. The default is FALSE.

n\_cores the number of cores to be used in computing cKnockoff in parallel. package

doParallel is required if n\_cores > 1. Otherwise it's computed sequentially.

prelim\_result either a knockoff.result object returned by knockoff::knockoff.filter

or a cknockoff.result object returned by cknockoff. cknockoff can read the information from a knockoff result or a cknockoff result and make possibly more

rejections with the same FDR control. See more details below.

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X.pack

An object of class cknockoff.X.pack returned by function process\_X. This is used only for simulations studies, where cknockoff is applied many times to the same fixed X, to accelerate computation. General users should ignore it and leave it as default = NULL. If X.pack is provided, the argument "knockoffs" is not needed and will be overwritten.

#### **Details**

If argument prelim\_result is supplied with a object, all the other parameters except n\_cores and Rstar\_refine will be overwritten by the information retrieved from it.

If a knockoff.result object is supplied to prelim\_result, cknockoff will return possibly more selections on the same problem with the same FDR control. To use, the function call of knockoff::knockoff.filter that returned such knockoff.result object must have arguments "statistic" and "fdr" explicitly provided (rather than relying on their defaults). See examples below.

It's possible that cknockoff cannot fetch the function "statistic" or value for "alpha" by their names from a knockoff.result object. If this is the case, please supply them to cknockoff explicitly via arguments.

It's possible to even make more rejections based on a cknockoff.result object. This is because cknockoff will only explore those promising features and discards the others, say those with large p-values. Calling cknockoff() on a cknockoff.result object would further explore some most promising ones among the discarded features and (rarely) possibly make several more rejections. Users can do this recursively and FDR is proved to be controlled whenever they decide to stop. The computational time of each call should be similar.

If the cknockoff.result object is obtained by setting Rstar\_refine = FALSE, more rejections may be made with the same FDR control by supplying cknockoff.result to prelim\_result and set Rstar\_refine = TRUE. See examples below.

#### Value

An object of class cknockoff.result. It is a list similar to the knockoff.result object, containing essentially the same information:

X matrix of original variables (scaled and possibly augmented)
Xk matrix of knockoff variables (cooresponding to the returned X)

y response vector (possibly augmented)

kn.statistic the knockoff feature statistics selected named vector of selected variables

sign\_predict the predicted signs of beta

record a list recording some information during the calculation. It aims to make com-

puting based on "prelim\_result" possible and easy. Users may ignore it.

## Examples

```
p <- 100; n <- 300; k <- 15
X <- matrix(rnorm(n*p), n)
nonzero <- sample(p, k)
beta <- 2.5 * (1:p %in% nonzero)</pre>
```

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```
y <- X %*% beta + rnorm(n)</pre>
print(which(1:p %in% nonzero))
# Basic usage
result <- cknockoff(X, y, alpha = 0.05, n_cores = 1)
print(result$selected)
# knockoff rejection
library("knockoff")
kn.result <- knockoff.filter(X, y,</pre>
                              knockoffs = create.fixed,
                              statistic = stat.glmnet_coefdiff_lm,
                              # must specify this argument explicitly
                              fdr = 0.05
                              # must specify this argument explicitly
print(kn.result$selected)
# improve knockoff result
result <- cknockoff(prelim_result = kn.result, n_cores = 2)</pre>
print(result$selected)
# improve previous cknockoff result
result <- cknockoff(prelim_result = result)</pre>
print(result$selected)
# improve previous cknockoff result with cknockoff*
result <- cknockoff(prelim_result = result, n_cores = 2, Rstar_refine = TRUE)</pre>
print(result$selected)
```

create.fixed.MRC

Creates fixed-X knockoff variables by MRC

## Description

This function is a R wrapper of the knocproduce\_FX\_knockoffskpy function in the Python package knockpy. It creates fixed-X knockoff variables by Minimizing Reconstructability (see <a href="https://arxiv.org/abs/2011.14625">https://arxiv.org/abs/2011.14625</a>). To use this function, Users should have the R package reticulate and the python package knockpy installed.

## Usage

```
create.fixed.MRC(
   X,
   method = c("mvr", "maxent", "mmi", "sdp", "equicorrelated", "ci"),
   solver = NULL,
   knockpy = NULL,
   num_processes = 1
)
```

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#### **Arguments**

X n-by-p matrix of the features.

method Same argument as the "method" in the smatrix.compute\_smatrix function in

knockpy package. Take value from "mvr", "maxent", "mmi", "sdp", "equicorre-

lated", and "ci". The default is "mvr".

solver Same argument as the "solver" in the smatrix.compute\_smatrix function in

knockpy package. Take value from "cd", "sdp", "psgd". Can be leave as NULL.

knockpy The R object wrapping the Python module "knockpy". Users may provide their

own wrapper using "reticulate::import" if "knockpy" is not installed in the default python environment (e.g. in some environments created by anaconda).

tion in knockpy package. Number of parallel process to use in computing the

matrix approximately.

#### Value

A list containing the following components:

X n-by-p matrix of original variables (normalized).

Xk n-by-p matrix of knockoff variables.

process\_X Compute and store matrices needed by cknockoff() from on X

## **Description**

Compute and store matrices needed by cknockoff() from on X

## Usage

```
process_X(X, knockoffs = knockoff::create.fixed)
```

## **Arguments**

X n-by-p matrix of original variables.

knockoffs either knockoff matrix of X or a knockoffs function that can generate it. If the

knockoff matrix is supplied, both X and knockoffs should be properly normalized, e.g. using the returned X and Xk of the knockoff::create.fixed function. By

default, knockoff::create.fixed is used.

#### Value

An object of class "cknockoff.X.pack". This object is a list containing many matrices like the knockoff matrix and a basis of the linear space spanned by X, etc. Users don't need to work on the object themselves but pass it to the "X.pack" parameter in cknockoff() if needed.

#### **Examples**

stat.glmnet\_coefdiff\_lm

Lasso Coefficient-Difference feature statistics with tiebreaker

#### **Description**

Fit Lasso on the augmented model y  $[X,\widetilde{X}]$  with regularization parameter  $\lambda$ . Then, compute the difference statistic

$$W_j^0 = |Z_j| - |\tilde{Z}_j|$$

where  $Z_j$  and  $\tilde{Z}_j$  are the coefficient estimates for the jth variable and its knockoff, respectively. For those variables that both themselves and their knockoffs are not selected by Lasso, we break the tie by their correlation with the residue, by defining

$$\rho_j = |X_i^T residue| - |\tilde{X}_i^T residue|$$

for them and  $\rho_j=2\lambda$  for the others. The final feature statistics are

$$W_j = W_j^0 + \rho_j.$$

 $\lambda$  is set to be  $2\tilde{\sigma}$ , where  $\tilde{\sigma}$  is an estimator of the noise level that is independent of  $[X, \tilde{X}]^T y$ .

#### Usage

```
stat.glmnet_coefdiff_lm(X, X_k, y, sigma_tilde = NULL)
```

#### **Arguments**

X	n-by-p matrix of original variables.
X_k	n-by-p matrix of knockoff variables.
у	y vector of length n, containing the response variables.
sigma_tilde	An estimator of the noise level that is independent of $[X, \widetilde{X}]^T y$ . If not provided, it will be computed inside the function based on X, X_k, and y, in which case we must have $n > 2p$ .

#### **Details**

If sigma\_tilde is not provided and n=2p, sigma\_tilde will be computed from the residue of the OLS fitting y X. The resulting estimator is thus not independent of  $[X,\widetilde{X}]^Ty$ . Users should avoid this case if they want a guaranteed FDR control. Though in practice it shouldn't really make a difference.

Details of the calculation of this feature statistics can be found in (our paper).

The implementation of this function is modified from the knockoff::stat.glmnet\_coefdiff() function.

#### Value

A vector of knockoff feature statistics W of length p.

#### See Also

```
Other statistics: stat.glmnet_lambdasmax_lm()
```

## **Examples**

stat.glmnet\_lambdasmax\_lm

Efficient Lasso Signed-Max feature statistics

## **Description**

This function provides a computationally efficient feature statistic that behave similarly to the Lasso Signed-Max feature statistics produced by the knockoff::stat.glmnet\_lambdasmax() function. We compute a  $\hat{\lambda}_j$  as an approximation of the maximal regularization parameter  $\lambda$  at which the jth variable enters the Lasso model. And obtain the feature statistics

$$W_j = (\hat{\lambda}_j \vee \hat{\lambda}_{j+m}) \cdot sgn(\hat{\lambda}_j - \hat{\lambda}_{j+m})$$

#### Usage

stat.glmnet\_lambdasmax\_lm(X, X\_k, y, sigma\_tilde = NULL, nlambda = 10)

## Arguments

X n-by-p matrix of original variables.X\_k n-by-p matrix of knockoff variables.

y vector of length n, containing the response variables.

sigma\_tilde An estimator of the noise level that is independent of  $[X, \widetilde{X}]^T y$ . If not provided,

it will be computed inside the function based on X, X\_k, and y, in which case

we must have n > 2p.

nlambda the number of grid points in computing the Lasso path. A larger value will make

the calculation more precise but more expensive. The default value is 10.

## **Details**

If sigma\_tilde is not provided and n=2p, sigma\_tilde will be computed from the residue of the OLS fitting y X. The resulting estimator is thus not independent of  $[X,\widetilde{X}]^Ty$ . Users should avoid this case if they want a guaranteed FDR control. Though in practice it shouldn't really make a difference.

Details of the calculation of this feature statistics can be found in (our paper).

The implementation of this function is modified from the knockoff::stat.glmnet\_lambdasmax() function.

#### Value

A vector of knockoff feature statistics W of length p.

#### See Also

Other statistics: stat.glmnet\_coefdiff\_lm()

## Examples

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