Quantum Algorithms for Lattice-based Cryptography

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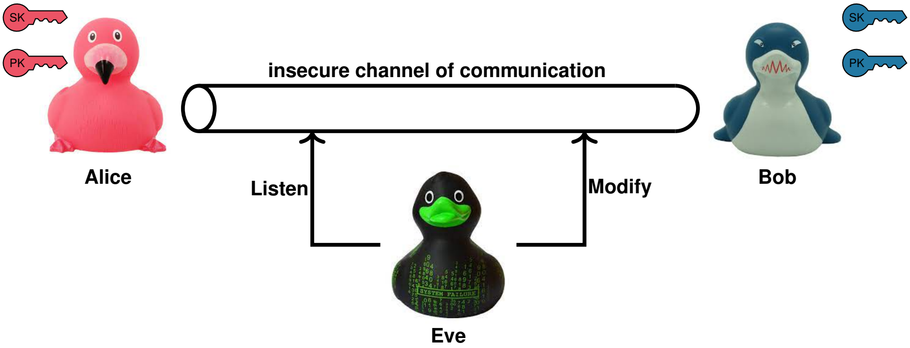
November 4, 2022

Hard Problems in Public Key Cryptography



factorisation, discrete logarithm

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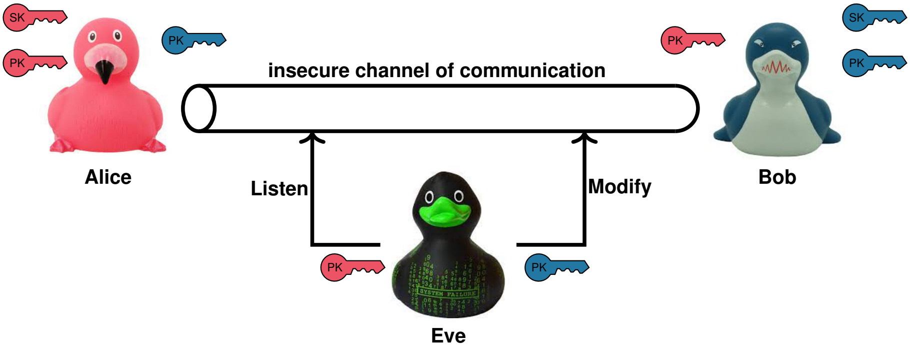

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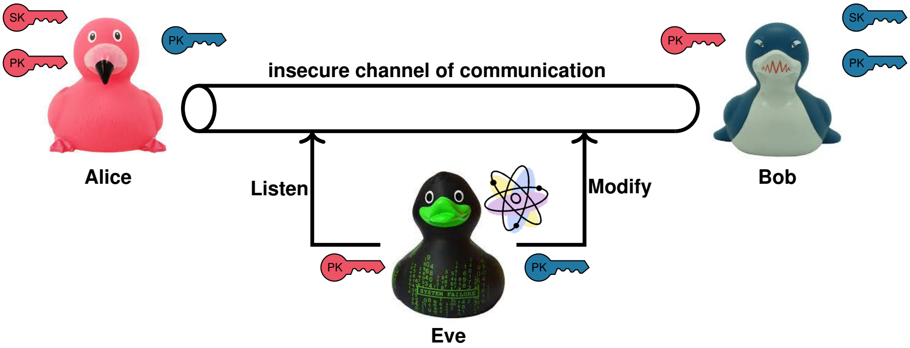

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lattices, codes, ...

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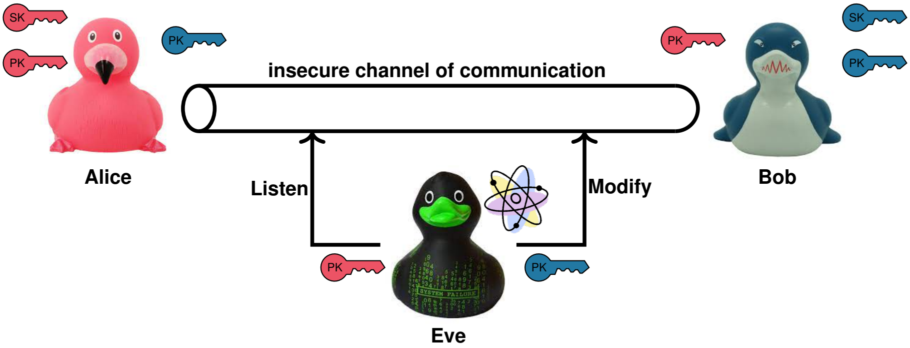

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lattices, codes, ...

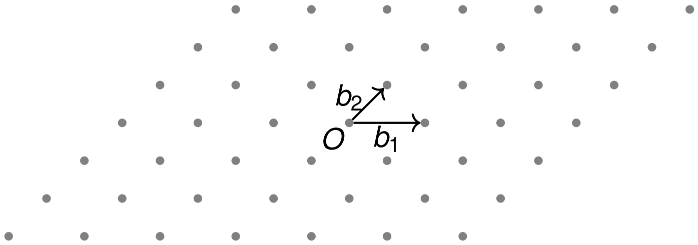
NIST selected algorithms:

- encryption: the only selected candidate is based on lattices
- signatures: 2 out of 3 based on lattices

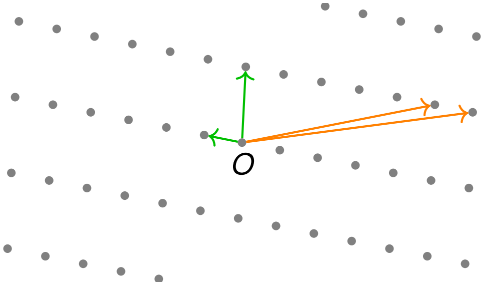
What is a (Euclidean) lattice?

Definition

$\mathcal{L}(\boldsymbol{b}_1, \ldots, \boldsymbol{b}_n) = \left\{\sum_{i=1}^{n} x_i \boldsymbol{b}_i : x_i \in \mathbb{Z}\right\}$ where $\boldsymbol{b}_1, \ldots, \boldsymbol{b}_n$ is a basis of $\mathbb{R}^n$.



Lattice-based cryptography: fundamental idea



- good basis: private information, makes problem easy
- bad basis: public information, makes problem hard

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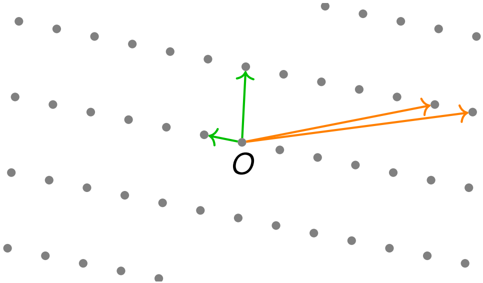

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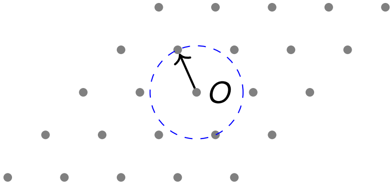
Basis reduction: transform a bad basis into a good one
Main tool: BKZ algorithm and its variants

Requires to solve the (approx-)SVP problem in smaller dimensions.

The Shortest Vector Problem



Shortest Vector Problem (SVP):
Given a basis for the lattice $\mathcal{L}$, find a shortest nonzero lattice vector.
$\lambda_1(\mathcal{L})$ = length of such a vector.

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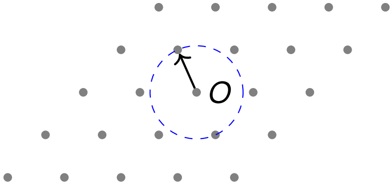

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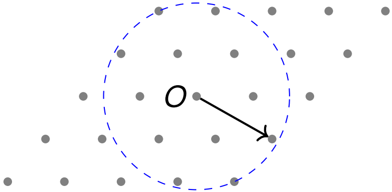

γ-approx-SVP ($\gamma > 1$):
Given a basis of $\mathcal{L}$, find a nonzero lattice vector of length at most $\gamma \cdot \lambda_1(\mathcal{L})$.
γ is approximation factor.

The Shortest Vector Problem

Depending on the dimension n:

- NP-Hardness (randomized reduction)
- NP $\cap$ co-NP
- Subexponential-time algorithms
- Poly-time algorithms

Approx factor:

- $O(1)$
- $\sqrt{n}$
- $2^{\sqrt{n}}$
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Main approaches for SVP:

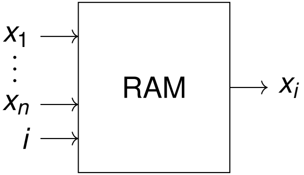
- Enumeration: $2^{O(n\log(n))}$ time and $\mathrm{poly}(n)$ space
- Sieving: $2^{O(n)}$ time and $2^{O(n)}$ space

Interlude: quantum memory models

classical access

quantum access

classical data



standard

quantum data

Assumption: $O(1)$ time cost

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classical access | quantum access

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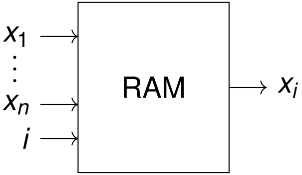

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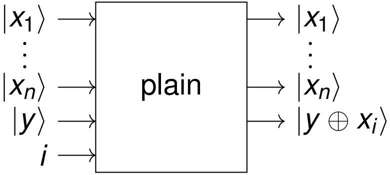

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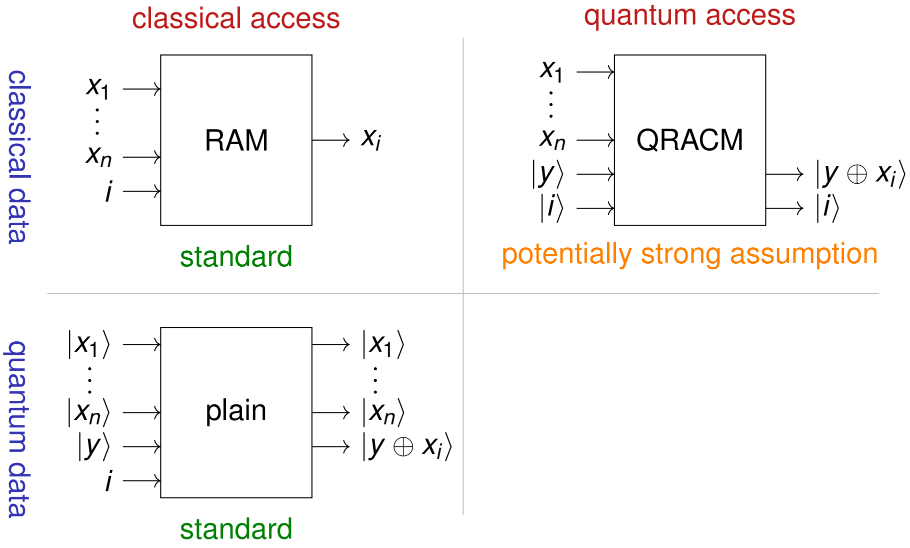

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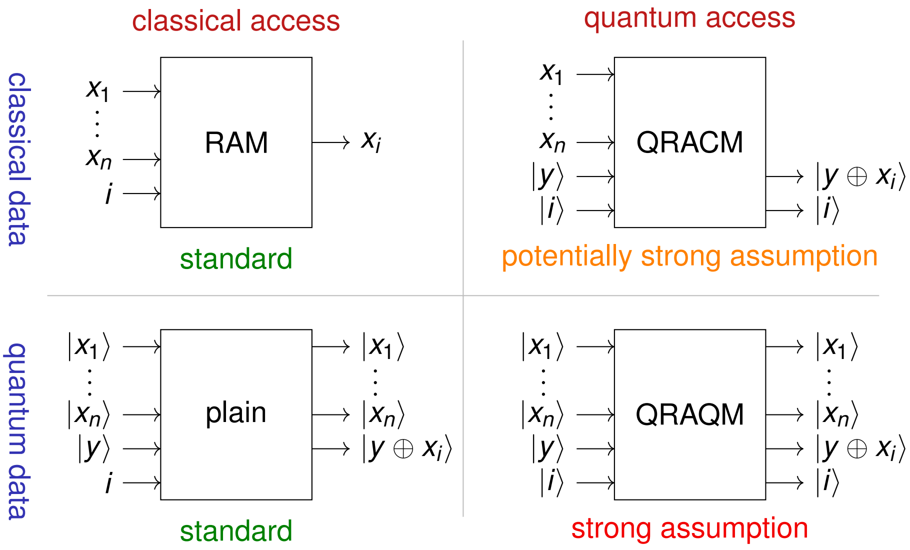

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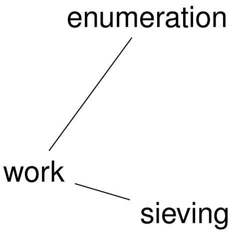

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- work
  - enumeration
    - quantum circuit [ANS18]
    - VQE [APSW22*]
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      - future work: quantum circuit?
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