Topics in Econometrics - Assignment 2

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Exercise 1d-e

Below are the mean of the estimated $\hat{\beta}$ using lasso, ridge,and OLS. Out of the 10,000 samples, $\hat{\beta^{lasso}}$ is always 0. This is because $\lambda = 20$ is too large. From class we know that the theoretical optimal lambda is $\lambda = 2\sigma\sqrt{\tau\frac{ln(p)}{N}} = 2\sqrt{\tau\frac{ln90}{1000}} << 20$.

We see that the ridge estimates do a better job, since ridge never shrinks estimators all the way to zero. But again, because λ is so big, it biases the absolute value of the estimates towards zero. Both do not perform as well as the OLs estimate.

	Lasso	Ridge	OLS
beta1	0	0.663	1.01
beta2	0	-0.661	-0.99

Exercise 1f

We see that for λ small enough, lasso performs quite well. As λ grows, it first filters out $\beta_3 - \beta_5$, which are not correlated with Y. The tradeoff though is that it biases β_1 and β_2 away from their true values of 1 and -1. We see that just before λ hits 1, it returns all zeros for the $\hat{\beta}$.

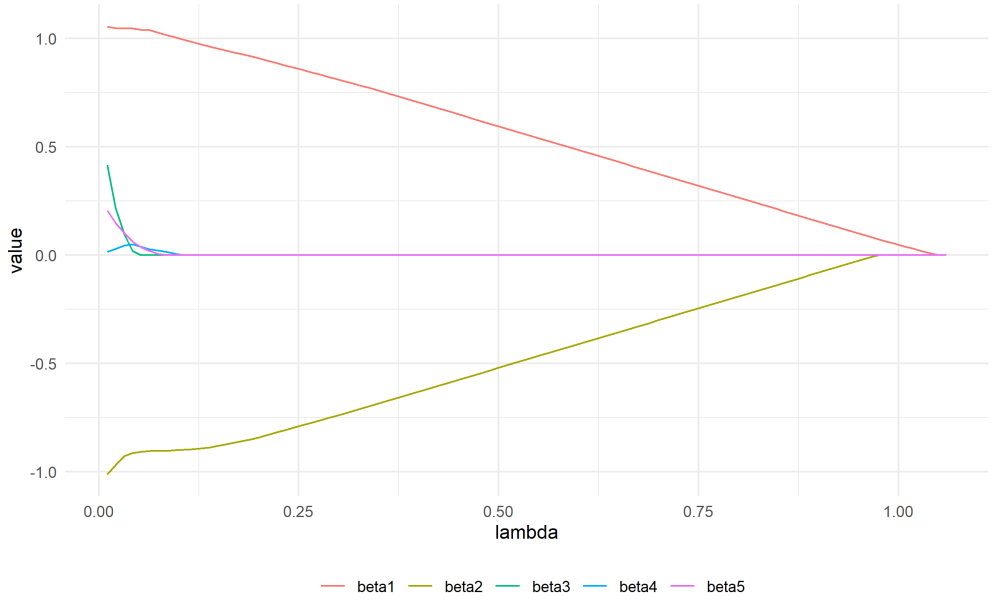

Figure 1: Value of coefficients with varying λ

Exercise 2b

The MSFE drops and reaches a minimum at 1.5.

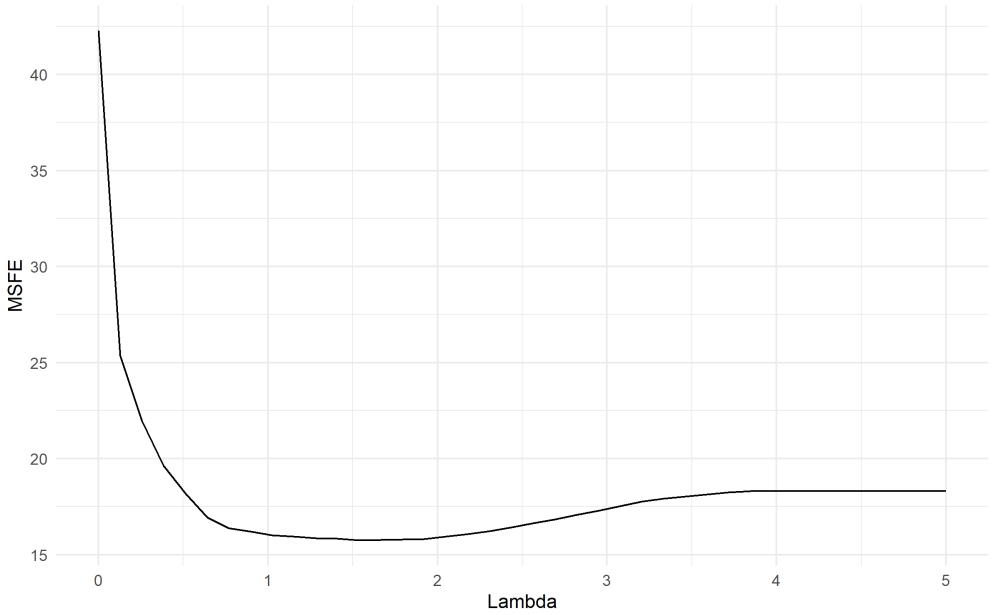

Figure 2: Exercise 2b - MSFE

I report the estimates of the optimal λ using the graph below, comparing the estimated

values to the true values. We see that β_1 is biased downward, which is to be expected as we have thrown in controls that steal variation away from β_1.

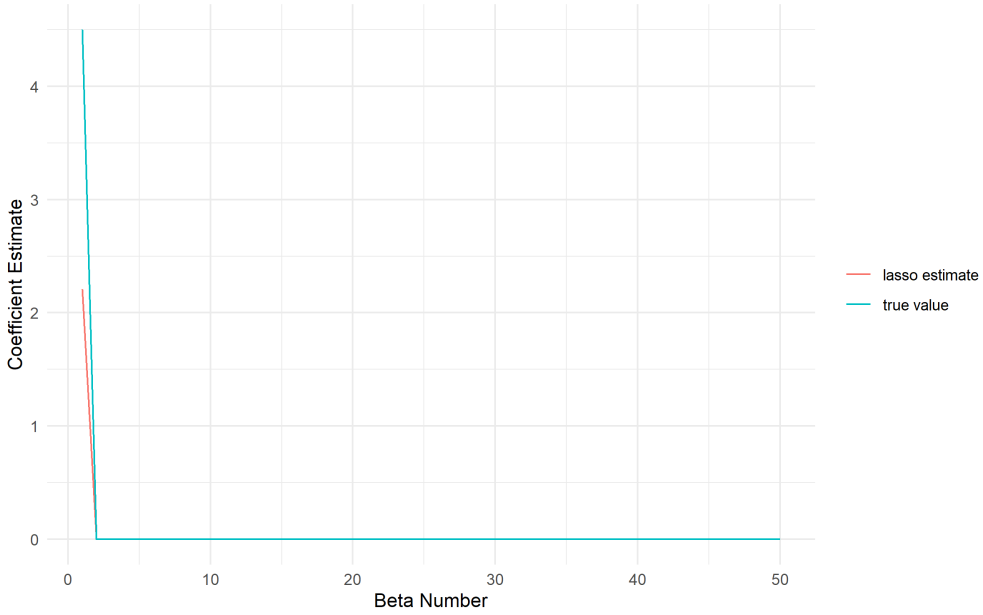

Figure 3: Exercise 2b - Estimates Using Optimal λ

Exercise 2c

We see larger values of MSFE compared to 2b. Lasso does not perform as well when there are many coefficients that are close to zero, which is the case here.

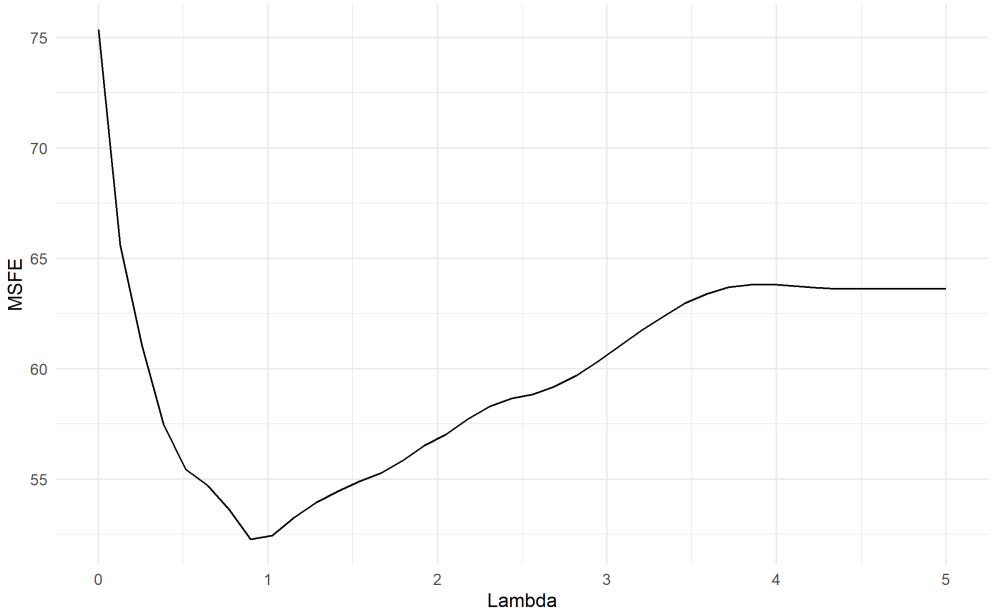

Figure 4: Exercise 2c - MSFE

Figure 5: Exercise 2c - Estimates Using Optimal λ

Exercise 2d

With correlation between the X terms, adding all the extra $X_2...X_{90}$ terms is stealing even more variation away from β_1, so we see that the lasso estimate for β_1 is smaller and the MSFE is higher when compared to 2b.

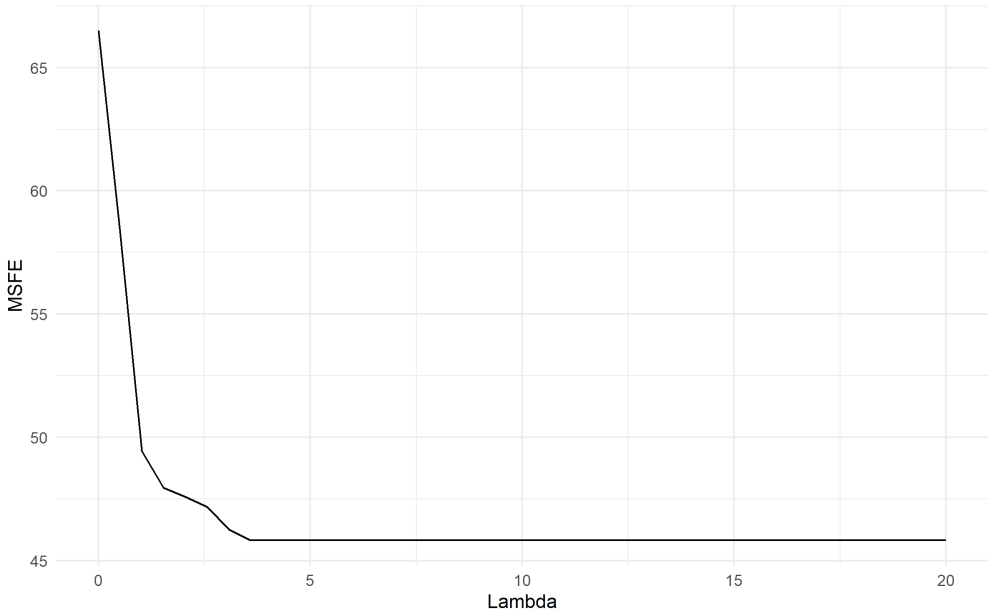

Figure 6: Exercise 2d - MSFE

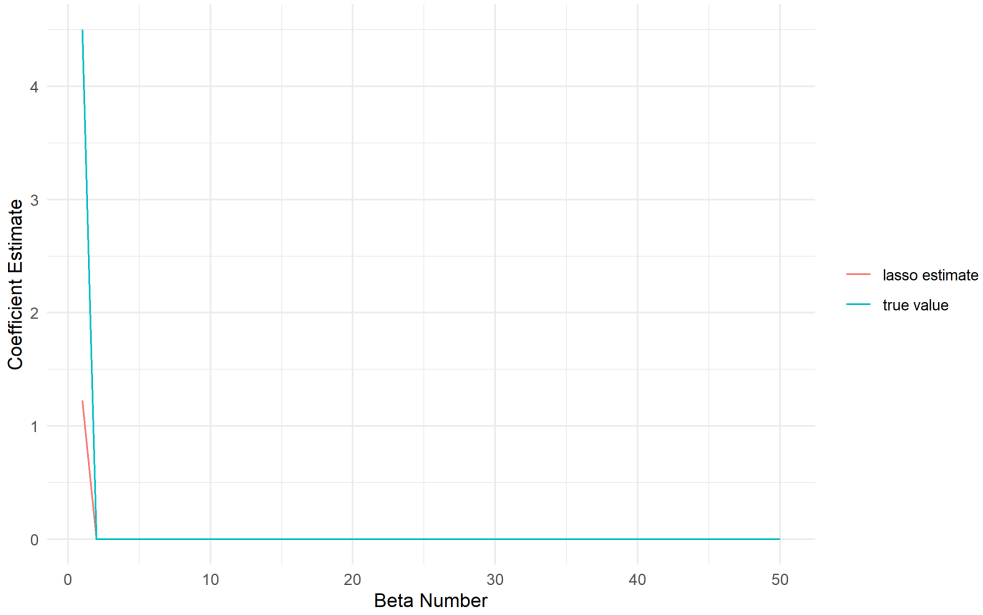

Figure 7: Exercise 2d - Estimates Using Optimal λ