Applied Microeconometrics Problem Set 1

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Problem 1

Set Up:

- $Y(d) - Y(0) = \alpha$
- $Y = DY(d) + (1 - D)Y(0) = \tau D + Y(0)$
- Regression 1: $Y = a_1 + \beta D + \epsilon_1$
- Regression 2: $Y = a_2 + \gamma D + \psi X + \epsilon_2$

a)

Let $\hat{D} = D - \bar{D}$, and solve

$$
\begin{aligned}
\beta &= \frac{cov(\hat{D}), Y)}{var(\hat{D})} \\
&= \frac{cov(\hat{D}, \tau D + Y(0))}{var(\hat{D})} \\
&= \frac{cov(\hat{D}, \tau D)}{var(\hat{D})} + \frac{cov(\hat{D}, Y(0))}{var(\hat{D})} \\
&= \tau + \frac{cov(D, Y(0))}{var(D)}
\end{aligned}
$$

Where the last line uses the fact that $var(D-\bar{D}) = var(D)$ and $cov(D-\bar{D}, Y(0)) = cov(D, Y(0))$

Now let's solve for γ, using $\tilde{D} = D - \pi_0 - \pi_1 X$,

$$
\begin{aligned}
\gamma &= \frac{cov(\tilde{D}, Y)}{var(\tilde{D})} \\
&= \frac{cov(\tilde{D}, \tau D + Y(0))}{var(\tilde{D})} \\
&= \frac{cov(\tilde{D}, \tau D) + cov(\tilde{D}, Y(0))}{var(\tilde{D})} \\
&= \tau + \frac{cov(\tilde{D}, Y(0))}{var(\tilde{D})} \\
&= \tau + \frac{cov(D - \pi_0 - \pi_1 X, Y(0)}{var(\tilde{D})} \\
&= \tau + \frac{cov(D - \pi_1 X, Y(0))}{var(\tilde{D})}
\end{aligned}
$$

From this, we see that

$$|\alpha - \gamma| = \Big|\frac{cov(D - \pi_1 X, Y(0))}{var(\tilde{D})}\Big| \tag{1}$$

$$|\alpha - \beta| = \Big|\frac{cov(D, Y(0))}{var(D)}\Big| \tag{2}$$

and it is ambiguous which term is bigger, depending on the covariance between X and $Y(0)$.

b)

If D and X are uncorrelated, then $\pi_1 = 0$ and (1) would simplify to

$$\begin{aligned} |\alpha - \gamma| &= \Big|\frac{cov(D, Y(0))}{var(D)}\Big| \\ &= |\alpha - \beta| \end{aligned}$$

so the two would be equal.

c)

If X is uncorrelated with $Y(0)$ and $Y(d)$, then we have

$$\alpha - \gamma = \frac{cov(D, Y(0))}{var(\tilde{D})}$$

Expanding out the denominator,

$$\begin{aligned} var(\tilde{D}) &= var(D - \pi_0 - \pi_1 X) \\ &= var(D) + \pi_1 var(X) - 2\pi_1 cov(D, X) \end{aligned}$$

So whether $|\alpha - \gamma|$ is bigger or smaller than $|\alpha - \beta|$ is dependent on whether $\pi_1 var(X) - 2\pi_1 cov(D, X)$ is greater or less than zero.

d)

If $E[Y(0) \mid D = d, X = x] = E[Y(0) \mid X = x]$, then

$$\begin{aligned} \alpha - \gamma &= \frac{cov(D - \pi_1 X, Y(0))}{var(\tilde{D})} \\ &= \frac{cov(\pi_1 X, Y(0))}{var(\tilde{D})} \end{aligned}$$

which is still ambiguous when compared to $\alpha - \beta = \frac{cov(D,Y(0))}{var(D)}$

e)

Let $Y(0) = \delta_0 + \delta_1 X$ and plug in,

$$\begin{aligned}
\alpha - \gamma &= \frac{cov(D - \pi_1 X, \delta_0 + \delta_1 X)}{var(\tilde{D})} \\
&= \frac{cov(D, \delta_1 X) - cov(\pi_1 X, \delta_1 X)}{var(\tilde{D})} \\
&= \frac{\delta_1 cov(D, X) - \pi_1 \delta_1 var(X)}{var(\tilde{D})}
\end{aligned}$$

This is again ambiguous when compared to $\alpha - \beta$

Problem 2

a)

Note that with $X, Z \perp W$, we can rewrite $G(y)$ as:

$$\begin{aligned}
G(y) &= P(Y \leq a) \\
&= P(Y \leq a|W = 0)P(W = 0) + P(Y \leq a|W = 1)P(W = 1) \\
&= P(Z \leq a)P(W = 0) + P(X \leq a)P(W = 1) \\
&= F(y)(1 - \pi) + P(X \leq a)P(W = 1)
\end{aligned}$$

We know that

$$0 \leq P(X \leq a)P(W = 1) \leq P(W = 1) = \pi$$

So we can get both inequalities that we need:

$$\begin{aligned}
G(y) &\leq F(y)(1 - \pi) + \pi \\
&\Rightarrow F(y) \geq \frac{G(y) - \pi}{1 - \pi} \\
G(y) &\leq F(y)(1 - \pi) + 0 \\
&\Rightarrow F(y) \leq \frac{G(y)}{1 - \pi}
\end{aligned}$$

$F(y)$ must be in the interval [0,1] to be a probability, so we add the min and max operators to give us:

$$\max\left\{\frac{G(y) - \pi}{1 - \pi}, 0\right\} \leq F(y) \leq \min\left\{\frac{G(y)}{1 - \pi}, 1\right\}$$

In order to make lower bound sharp, for any y, fix X to be always larger than y, which will give us:

$$\begin{aligned}
P(Y \leq y) &= P(X \leq y)\pi + P(Z \leq y)(1 - \pi) \\
&= \pi + P(Z \leq y)(1 - \pi) \\
&\Rightarrow \frac{G(y) - \pi}{1 - \pi} = F(y)
\end{aligned}$$

To make the upper bound sharp, for any y, set X to be larger than y, so we have

$$
\begin{aligned}
P(Y \leq y) &= P(X \leq y)\pi + P(Z \leq y)(1-\pi) \\
&= P(Z \leq y)(1-\pi) \\
&\Rightarrow \frac{G(y)}{1-\pi} = F(y)
\end{aligned}
$$

b)

Let $a = G^{-1}(1-\pi)$.

$$
\begin{aligned}
E[Y \mid Y \leq a] &= E[WX + (1-W)Z \mid Y \leq a] \\
&= P(W=1 \mid Y \leq a)E[X \mid X \leq a] + P(W=0 \mid Y \leq a)E[Z \mid Z \leq a]
\end{aligned}
$$

Applying Bayes Rule, we have

$$
\begin{aligned}
P(W=1 \mid Y \leq a) &= \frac{P(Y \leq a|W=1)P(W=1)}{P(Y \leq a} \\
&= \frac{P(X \leq a)\pi}{P(Y \leq a)} \\
P(W=0 \mid Y \leq a] &= \frac{P(Y \leq a|W=0)P(W=0)}{P(Y \leq a} \\
&= \frac{P(Z \leq a)(1-\pi)}{P(Y \leq a)}
\end{aligned}
$$

Plugging this in, we have

$$
E[Y \mid Y \leq a] = \frac{\pi P(X \leq a)E[X \mid X \leq a] + (1-\pi)P(Z \leq a)E[Z \mid Z \leq a]}{P(Y \leq a)}
$$

By definition, $1-\pi = G(a) = P(Y \leq a)$ and $P(X \leq a)\pi = P(Y \leq a) - P(Z \leq a)(1-\pi)$, so we can plug this in to get

$$
\begin{aligned}
E[Y \mid Y \leq a] &= E[X \mid X \leq a]\frac{1-\pi-(1-\pi)P(Z \leq a)}{1-\pi} + E[Z \mid Z \leq a]\frac{P(Z \leq a)(1-\pi)}{1-\pi} \\
&\leq E[X \mid X \leq a](1 - P(Z \leq a)) + E[Z \mid Z \leq a]P(Z \leq a)) \\
&\leq E[Z \mid Z > a]P(Z > a) + E[Z \mid Z \leq a]P(Z \leq a)) \\
&= E[Z]
\end{aligned}
$$

The same argument holds for $E[Z] \leq E\left[Y \mid Y \geq G^{-1}(\pi)\right]$.

As we increase π, we are upweighting X, and the opposite happens as π approaches 0. When $\pi = 0$, the expectations hold with equality, making these bounds sharp.

Problem 3

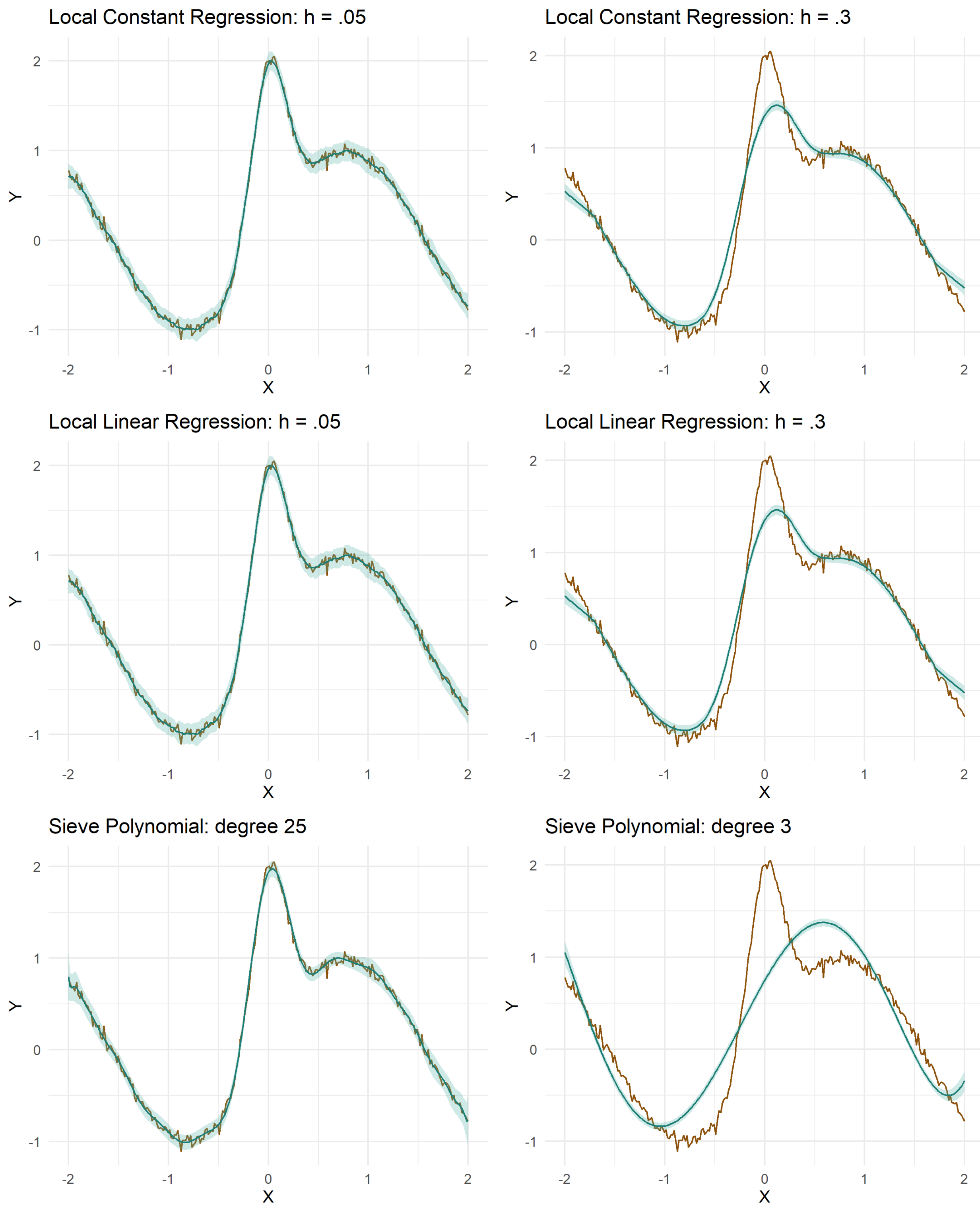
$$
\begin{aligned}
E\left[Y \mid D=d, P_d=p\right] &= E\left[E\left[Y \mid D=d, P_d=p, X\right]|X\right] \\
&= E\left[E[Y(d) \mid P_d=p, X] \mid X\right] \\
&= E\left[Y(d) \mid P_d=p\right]
\end{aligned}
$$

For the second part of the question, we have:

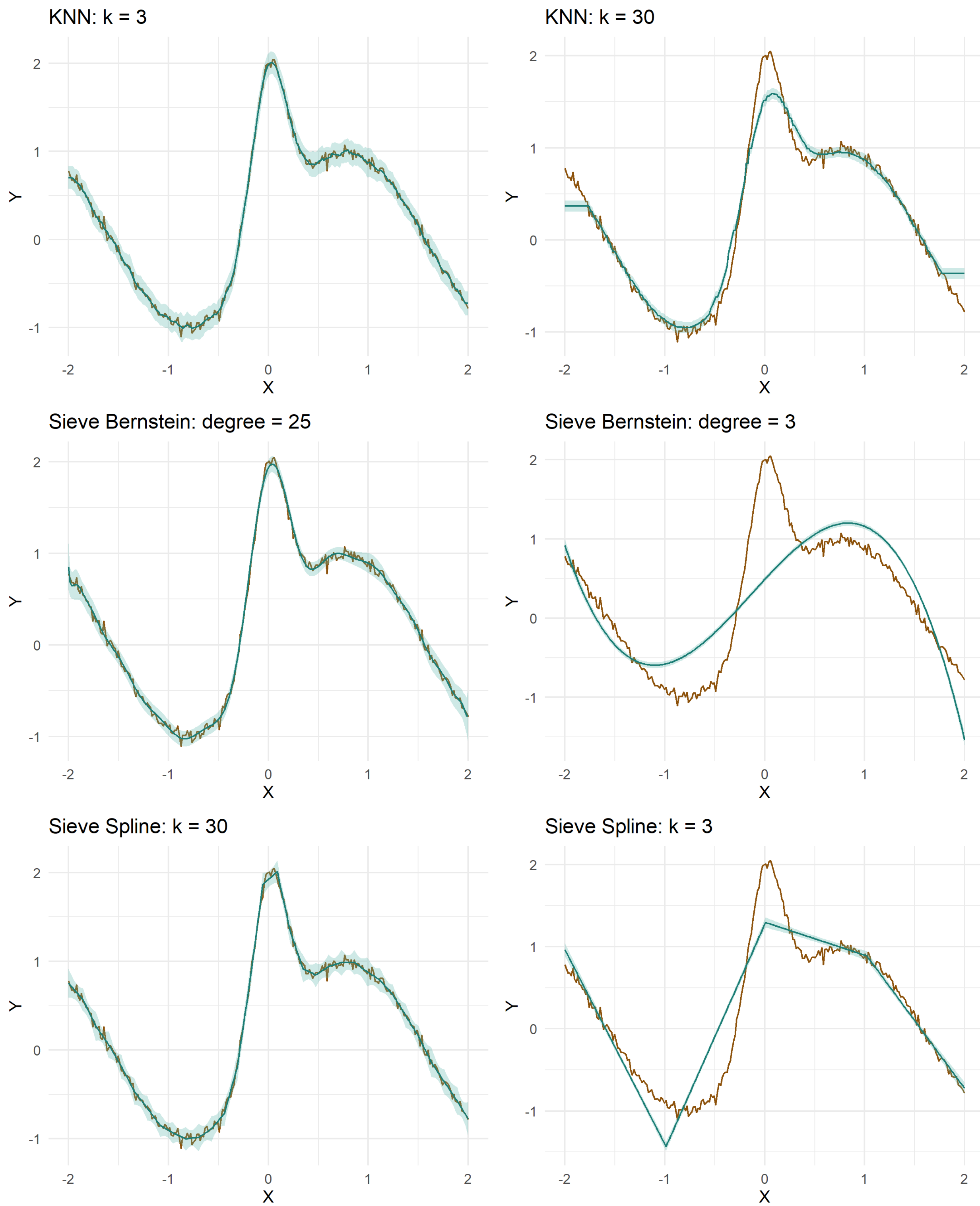
$$\begin{aligned}
E[Y(d)] &= E[E[Y(d)|X]] \\
&= E\left[\frac{1}{P}E[Y(d)|X]P(D=d|X)\right] \\
&= E\left[\frac{1}{P}E[Y\mathbb{1}\{D=d\}|X]\right] \\
&= E\left[E\left[\frac{1}{P}Y\mathbb{1}\{D=d\}|X\right]\right] \\
&= E\left[\frac{Y\mathbb{1}\{D=d\}}{P}\right]
\end{aligned}$$

Problem 4

Below I plot the results of a Monte Carlo simulation with $M = 50$ samples, each with $N = 250$ observations. The graphs below show the mean estimate (green), with a band for the ± 1 standard deviations, overlaid onto the true value of Y for each point (orange). For each method, I use two different tuning parameters to show the bias-variance trade-off, where the left hand side shows low bias and high variance, while the right side shows high bias and low variance.



Local Constant Regression: h = .05
Local Constant Regression: h = .3
Local Linear Regression: h = .05
Local Linear Regression: h = .3
Sieve Polynomial: degree 25
Sieve Polynomial: degree 3
Y
X
2
1
0
-1
-2


KNN: k = 3
KNN: k = 30
Sieve Bernstein: degree = 25
Sieve Bernstein: degree = 3
Sieve Spline: k = 30
Sieve Spline: k = 3
X
Y

Problem 5

a)

The authors include the percentage that is Protestant as a control, while the footnote states that "Protestants were more prone to vote for the Nazi Party". By controlling for this variable, you are potentially shutting off a path for the treatment effect.

b)

The replication of Panel A was straightforward, using simple OLS with clustered standard errors.

Table 1: Panel A: OLS

term	value
pog1349	0.0607
	(0.0224)
logpop25c	0.0390
	(0.0151)
perc_JEW25	0.0135
	(0.0113)
perc_PROT25	0.0003
	(0.0004)
N	320.0000
Adj. R^2	0.0514

I attempt to replicate Panel B and C, which reported ATT estimates using robust nearest neighbor estimation with the four closest matches. To replicate, I used the Mahalabonis distance method.

Table 2: Panel B: Matching

term	value
pog1349 - Matching	0.0722
	(0.0222)
pog1349 - Geographic Matching	0.0819
	(0.0295)

c)

I estimate the propensity score using logit, and then use nearest neighbor estimation with the four closest matches on the propensity score. I then use 100 bootstraps to find the standard error. Both the ATT and ATE estimates come quite close to the matching estimates.

Table 3: Propensity Score

term	value
pog1349 - Prop. Score ATT	0.0744 (0.0172)
pog1349 - Prop. Score ATE	0.0688 (0.0200)