Assignment 3 Writeup

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Urban Theory

In Henderson (1974) there are a continuum of equilibria. In Behrens, Duranton, Robert-Nicoud (2014), Proposition 4 says there is a unique talent-homogeneous equilibrium. Is Proposition 4 correct? If it is correct, very clearly explain why talent heterogeneity makes the equilibrium unique when the homogeneous-worker case in Henderson (1974) yields multiple equilibria. If it is incorrect, identify the error and state the correct claim.

Proposition 4 from Behrens, Duranton, Robert-Nicoud (2014): If γ/ε is close to unity, the talent-homogeneous equilibrium is unique and such that

$$L(t) = \left(\frac{1+\gamma}{1+\varepsilon}\xi t^{1+a}\right)^{1/(\gamma-\varepsilon)}$$

where

$$\xi \equiv \frac{(\varepsilon\sigma)^{1+\varepsilon}S^{1+a}}{\gamma\theta}$$

Following their proof, each individual solves a constrained optimization problem that consists of picking the city with talent t that maximizes her expected indirect utility from the menu of possible cities. They solve for the equation that determines the menu of talents and populations that supports a talent-homogenous equilibrium:

$$\gamma\theta L(t)^{\varepsilon}\left[\frac{\xi t^{1+a} - L(t)^{\gamma-\varepsilon}}{L(t)}dL(t) + \frac{1+a}{1+\varepsilon}\xi t^{a} dt\right] = 0 \tag{1}$$

Rearranging, we get

$$\frac{dL(t)}{dt} = \frac{1+a}{1+\epsilon}\frac{\xi t^a L(t)}{L(t)^{\gamma-\epsilon} - \xi t^{1+a}}$$

The solution to this differential equation is

$$\xi t^{a+1}L(t)^{1+\epsilon} - \frac{(1+\epsilon)L(t)^{\gamma+1}}{\gamma+1} = c_1$$

The paper sets the initial condition, c_1, to 0, which allows them to derive the functional form for $L(t)$ shown in the proposition. Thus, individuals with talent t's utility optimization maps to an $L(t)$ curve, there is a unique talent homogeneous equilibrium. However, if we do not specify c_1, then there is a continuum of equilibria. We can interpret Berhens, Duranton, Robert-Nicoud (2014) to say that conditional on knowing c_1, there is a unique talent-homogenous equilibrium.

In addition, if we were to take $c_1 = 0$ and solve for $L(t)$, we get

$$\begin{aligned}
\xi t^{a+1} L\left(t\right)^{1+\varepsilon} &= \frac{(1+\varepsilon) L\left(t\right)^{\gamma+1}}{\gamma+1} \\
\Rightarrow L\left(t\right)^{\varepsilon-\gamma} &= \frac{1+\varepsilon}{(\gamma+1)\xi t^{a+1}} \\
L\left(t\right) &= \left(\frac{\gamma+1}{1+\varepsilon}\xi t^{a+1}\right)^{\frac{1}{\gamma-\varepsilon}}
\end{aligned}$$

So $z = \frac{\gamma+1}{1+\varepsilon}$, and the formula for z given in the paper is incorrect.

Trade and Urban Theory

Look at Behrens and Robert-Nicoud's "Agglomeration Theory with Heterogeneous Agents" chapter in the Handbook of Urban and Regional Economics. On page 204, they propose a theory of metropolitan specialization in which different cities are home to different industries. What is their prediction? How can this be investigated empirically? What is the role of comparative advantage? Is comparative advantage sufficient for the existence of a specialized equilibrium?

Behrens and Robert-Nicoud predict that cities specialize based on their comparative advantage, and the source of this comparative advantage is a combination of technology and external scale economies. Local city governments create cities in order to maximize the utility of their residents. In this set up, this means the local governments choose to specialize their city in the industry in which they have comparative advantage. Because of the log supermodularity of utility, there is positive assortative matching between industries and cities. Agglomeration economies and comparative advantage enter as complements in the utility function.

Davis and Dingel (2020) investigates this empirically based on a model that predicts larger cities specialize relatively in skill-intensive activities. They test the relationship by regressing log sectoral employment on its log total population. Their results show that more skilled groups and more skill-intensive sectors have higher population elasticities of employment.

Comparative advantage drives specialization in the handbook, but the model in the handbook is contingent on the presence of local governments and their ability to control migrant flows. When agents are free to migrate across cities, migration would equalize utilies across cities and industries until $u_{ci} = u_{dj}$ for $\forall c, d$ cities and $\forall i, j$ industries. This then gives us that

$$\begin{aligned}
p_i A_{ci} &= p_j A_{dj} \\
\frac{p_i}{p_j} &= \frac{A_{dj}}{A_{ci}}
\end{aligned}$$

Taking the usual comparative advantage equation, we have

$$\begin{aligned}
&\frac{A_{cj}}{A_{ci}} < \frac{p_i}{p_j} < \frac{A_{dj}}{A_{di}} \\
\Rightarrow &\frac{A_{cj}}{A_{ci}} < \frac{A_{dj}}{A_{ci}} < \frac{A_{dj}}{A_{di}}
\end{aligned}$$

This chain gives us two results:

$$A_{di} < A_{ci}$$
$$A_{cj} < A_{dj}$$

Thus, when there is free mobility, it is actually absolute advantage that drives specialization.