Comprehension Check 1

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1 Free-trade Equilibrium

Set Up:

- Balanced Budgets: $Y_i = \sum_j X_{ij}$
- Free trade: $\tau_{ij} = 1, \forall i, j$
- $\psi = 1$
- Preferences: $C_j = \left(\sum_{i=1}^{n} C_{ij}^{(\sigma-1)/\sigma}\right)^{\sigma/(\sigma-1)}$, where C_{ij} is the demand for good i in country j
- Price Index: $P_j = \left(\sum_{i=1}^{n} P_{ij}^{1-\sigma}\right)^{1/(1-\sigma)}$
- With free trade, $P_{ij} = P_{ii}$

Trade Flows:

$$X_{ij} = \left(\frac{P_{ij}}{P_j}\right)^{1-\sigma} \sum_i X_{ij}$$
$$X_{ij} = \frac{Y_i^{1-\sigma} Q_i^{\sigma-1}}{\sum_l Y_l^{1-\sigma} Q_l^{\sigma-1}} \sum_i X_{ij}$$

Summing up imports over j, we have

$$\sum_j X_{ij} = Y_i = \sum_j \frac{Y_i^{1-\sigma} Q_i^{\sigma-1}}{\sigma_l Y^{1-\sigma} Q_l^{\sigma-1}} Y_j$$
$$= \left(\frac{Y_i}{Q_i}\right)^{1-\sigma} \sum_j \frac{Y_j}{\sum_l (Y_l/Q_l)^{1-\sigma}}$$

Now plug in $Y_i = Q_i^{\frac{\epsilon}{\epsilon-1}}$

$$Q^{\frac{\epsilon}{\epsilon-1}} = \left(Q^{\frac{1}{\epsilon-1}}\right)^{1-\sigma} \sum_j \frac{Q_j^{\frac{\epsilon}{\epsilon-1}}}{\sum_l Q_l^{\frac{1-\sigma}{\epsilon-1}}}$$
$$\Rightarrow Q_i^{\frac{\varepsilon}{c-1} - \frac{1-\sigma}{\varepsilon-1}} = \frac{\sum_{j=1}^{N} Q_j^{\frac{\varepsilon}{\varepsilon-1}}}{\sum_{\ell=1}^{N} Q_\ell^{\frac{1-\sigma}{e-1}}}$$

We see that setting $\epsilon = 1 - \sigma$, both sides of the equation would equal 1.

2 Welfare

$$
\begin{aligned}
\frac{Y_i}{P_i} &= \frac{Q^{\frac{\epsilon}{\epsilon-1}}}{\left(\sum_j P_{ij}^{1-\sigma}\right)^{\frac{1}{1-\sigma}}} \\
&= \frac{Q^{\frac{\epsilon}{\epsilon-1}}}{\sum_j \left(Q_i^{\frac{1-\sigma}{\epsilon-1}}\right)^{\frac{1}{1-\sigma}}} \\
&= \frac{Q^{\frac{\epsilon}{\epsilon-1}}}{\left(n \times Q_i^{\frac{1-\sigma}{\epsilon-1}}\right)^{\frac{1}{1-\sigma}}} \\
&= n^{\frac{1}{\sigma-1}} \times Q^{\frac{\epsilon}{\epsilon-1}-\frac{1}{\epsilon-1}} \\
&= n^{\frac{1}{\sigma-1}} Q
\end{aligned}
$$

3 Immiserizing Growth

In order for there to be immiserizing growth, it must be that at some point, $\frac{\partial \frac{Y_i}{P_i}}{\partial Q_i} < 0$. Using the welfare result above, we have

$$
\frac{\partial \frac{Y_i}{P_i}}{\partial Q_i} = n^{\frac{1}{\sigma-1}}
$$

This is always greater than 0, so there can never be immiserizing growth in the Armington model.