<<Introduction to Robotics>> 4^{th} edition

Yiyang Dong 2022.02.06.

§3.1. Introduction to Manipulator Kinematics

Kinematics is the science of motion that treats the object without regard to the forces that cause it.

- within the science of kinematics, one studies the position, the velocity, the acceleration, and all higher order derivatives of the position variables (w.r.t. time or any other variables.).
- this refers to all the geometrical and time-based properties of the motion.
- in this chapter: compute the position and orientation of the manipulator's end-effector (or linkages) relative to the base of the manipulator as a function of the joint variables.

§3.2. Link Description

A manipulator may be thought of as a set of bodies (Bodies = Links.) connected in a chain by joints

- Lower Pair: to describe the connection between a pair of bodies when the relative motion is characterized by two surfaces sliding over one another.

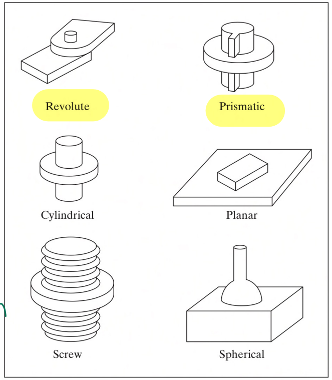

FIGURE 3.1: The six possible lower-pair joints.

Mechanical-design considerations favor manipulators that have joints with a single degree of freedom

- in order to position an end-effector generally in 3-space, a minimum of 6 joints is required.

⇒ 3 for position and 3 for orientation.

For the purpose of obtaining the kinematic equations of the mechanism, a link is considered ONLY as a rigid body that defines the relationship (between 2 neighboring joint axes) of a manipulator

- Joint Axis i is defined by a line in space or a vector direction, about which link i rotates relative to link i-1.
- A link can be specified with 2 numbers, which define the relative location of the two axes,
- A well-defined distance is measured along a line that is mutually perpendicular to both axes

⇒ figure 3.4 shows the mutually perpendicular line along which the link length, a_{i-1}

Link twist ⇒ defines the relative 'angle' of the two axes.

• Imagine a plance whose normal is the mutually perpendicular line just constructed, then we can project the axis i-1 and i onto this plane and measure the angle between them.

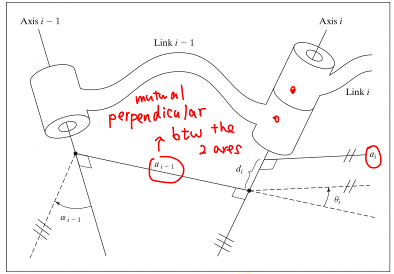

FIGURE 3.4: The link offset, d_i, and the joint angle, θ_i, are two parameters that may be used to describe the nature of the connection between neighboring links.

↳ This angle (α_{i-1}) is measured from axis $i-1$ ↷ axis i

※ Length a & Twist α can be used to define the relationship between any two axes in space

§3.3. Link-Connection Description.

① Intermediate Links in the Chain

• Link offset d_i at joint axis i : the distance along the common joint axis from one link to the next.

• Joint Angle θ_i : describes the amount of rotation about this common joint axis between one link and its neighbor.

- d_i is variable if joint i is Prismatic.
- θ_i is variable for a revolute joint.

The definition of Mechanisms by means of these quantities is a convention called the Denavit-Hartenberg Notation. (D-H)

② First and last links in the Chain

• At the ends of the chain, it is a convention to assign 0 to these quantities: $a_0 = a_n = 0$, $\alpha_0 = \alpha_n = 0$.

• If joint 1 is revolute, the zero position for θ_1 may be chosen arbitrarily and $d_1 = 0$.

• If joint 1 is prismatic, the zero position of d_1 is arbitrary, $\theta_1 = 0$.

③ Link Parameters.

Any Robot can be described kinematically by giving the values of 4 quantities for each link.

• Link Length a & Link Twist α describe the link itself.

• Link offset d & Joint Angle θ describe the link's connection to a neighboring link.

⇒ In the case of a Six-Jointed Robot with ALL revolute joints, 18 numbers in the form of 6 sets of (a_i, α_i, d_i) are fixed link parameters

θ_i : joint variable.

§3.4. Affixing frames to links

① Intermediate Links in the Chain

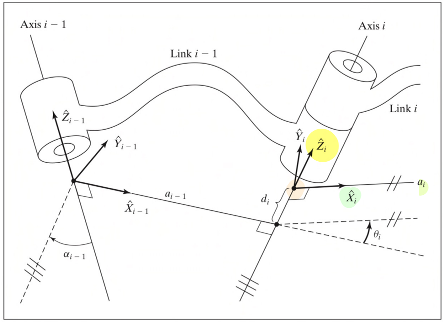

FIGURE 3.5: Link frames are attached so that frame $\{i\}$ is attached rigidly to link i.

- $\hat{Z}_i$: the $\hat{Z}$-axis of frame $\{i\}$ is coincident with the joint axis i
- The Origin of frame $\{i\}$ is located where the a_i perpendicular intersects the joint i axis.
  - $\hat{X}_i$ points along a_i in the direction from joint $i \to i+1$

② First and Last Links in the Chain:

We attach frame $\{0\}$ to the base of the Robot (link 0)

- arbitrary, fixed. simplifies matters to choose $\hat{Z}_0$ along axis 1, and to locate frame $\{0\}$ so that it coincides with frame $\{1\}$ when joint variable 1 is zero.

$\Rightarrow a_0 = 0,\ \alpha_0 = 0 \quad \begin{cases} d_1 = 0 & \text{if joint 1 is revolute} \\ \theta_1 = 0 & \text{if joint 1 is prismatic} \end{cases}$

- If joint n revolute, the direction of $\hat{X}_n$ is chosen so that it aligns with $\hat{X}_{N-1}$ when $\theta_n = 0$, and the frame $\{N\}$ is chosen so that $d_n = 0$;
- If joint n prismatic, the direction of $\hat{X}_N$ is chosen so that $\theta_n = 0$, and the Origin of $\{N\}$ at the intersection of $\hat{X}_{N-1}$ and joint axis n when $d_n = 0$.

③ Summary of the Link Parameters in terms of Link Frames

- a_i = the distance from $\hat{Z}_i$ to $\hat{Z}_{i+1}$ measured along $\hat{X}_i$
- α_i = the angle from $\hat{Z}_i$ to $\hat{Z}_{i+1}$ measured about $\hat{X}_i$
- d_i = the distance from $\hat{X}_{i-1}$ to $\hat{X}_i$ measured along $\hat{Z}_i$
- θ_i = the angle from $\hat{X}_{i-1}$ to $\hat{X}_i$ measured about $\hat{Z}_i$

The Convention outlined above does NOT result in a Unique attachment of frames to links:

- There are 2 choices of direction in which to point $\hat{Z}_i$, when we initially align the $\hat{Z}_i$ axis with joint axis i.
- There are 2 choices for the direction on $\hat{X}_i$, corresponding to the choice of signs, in the case of intersecting joint axes (i.e. $a_i = 0$)
- When axes i and $i+1$ are parallel, the choice of origin location for $\{i\}$ is arbitrary
  - (generally chosen to cause $d_i = 0$).
- When prismatic joints are present, there is quite a bit of freedom in frame assignment

④ Summary of Link-Frame Attachment Procedure

Given 2 axes i and $i+1$:

a) Identify the joint axes and Imagine (or draw) infinite lines along them. For (b) → (f), consider 2 of these neighboring lines

b) Identify the common perpendicular between i and $i+1$, or point of intersection, assign the ith link-frame origin.

c) Assign the $\hat{Z}_i$ axis pointing along the ith joint axis.

d) Assign the $\hat{X}_i$ axis pointing along the common perpendicular, or, if the axes intersect, assign $\hat{X}_i$ to be normal to the plane containing the two axes

e) Assign the $\hat{Y}_i$ axis to complete a right-hand coordinate system.

f) Assign {0} to match {1} when the first joint variables is zero. For {N}, choose an Origin location and $\hat{X}_N$ direction freely, but generally so as to cause as many linkage parameters as possible to become 0.

Example 3.3: Assign Link Frames to the Mechanism and give the D-H parameters

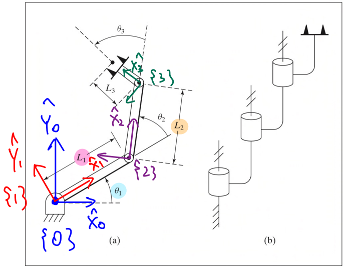

FIGURE 3.6: A three-link planar arm. On the right, we show the same manipulator by means of a simple schematic notation. Hash marks on the axes indicate that they are mutually parallel.

① Define the reference frame, {0}, it is fixed to the base and aligns with frame {1} when the first joint variable $\theta_1 = 0$.

② Because the arm lies in a plane with all $\hat{Z}$ axes parallel, there are NO Link offsets ⇒ all d_i, $d_i = 0$

③ When joint angle $\theta_1 = \theta_2 = \theta_3 = 0$ all $\hat{X}$ axes must align.

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	0	L_1	0	θ_2
3	0	L_2	0	θ_3

FIGURE 3.8: Link parameters of the three-link planar manipulator.

Example 3.4: Frame {0} & {1} are coincident because the robot is drawn for the position $\theta_1 = 0$

The rotational joint 1 & 3 rotate about $\hat{Z}_1$ & $\hat{Z}_3$.

※ The prismatic joint 2 slides along $\hat{Z}_2$ and the link offset d_2 is variable

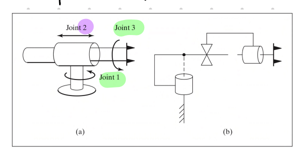

FIGURE 3.9: Manipulator having three degrees of freedom and one prismatic joint.

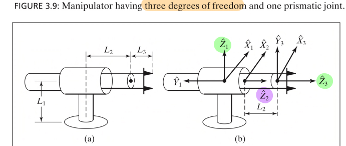

FIGURE 3.10: Link-frame assignments.

	i	α_{i-1}	a_{i-1}	d_i	θ_i	
R	1	0	0	0	θ_1	
Prismatic	2	90°	0	d_2	0	← fixed constant ($\hat{X}_1 // \hat{X}_2$)
R	3	0	0	L_2	θ_3	

FIGURE 3.11: Link parameters for the *RPR* manipulator of Example 3.4.

Example 3.5: In general, when $\hat{Z}_i$ and $\hat{Z}_{i+1}$ intersect, there are two choices for $\hat{X}_i$. There are 4 more possibilities, corresponding to the preceding 4 choices in Figure 3.13 + 14, but with $\hat{Z}_i$ pointing downward.

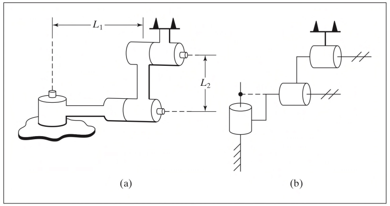

FIGURE 3.12: Three-link, nonplanar manipulator.

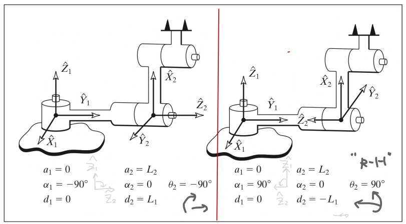

FIGURE 3.13: Two possible frame assignments.

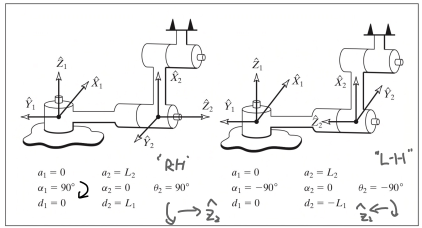

FIGURE 3.14: Two more possible frame assignments.

§3.5. Manipulator Kinematics

① Derivation of Link Transformations.

· For any given Robot, the transformation that defines frame $\{i\}$ relative to the frame $\{i-1\}$ will be a function of ONLY ONE variable, and the other three link parameters being fixed by mechanical design.

· By defining a frame for each link, we have broken the kinematics problem into n subproblems, and then break each subproblem ${}^{i-1}_{i}T$ into 4 subsubproblems. ⇒ each of these 4 transformations = a function of ONE link parameter only

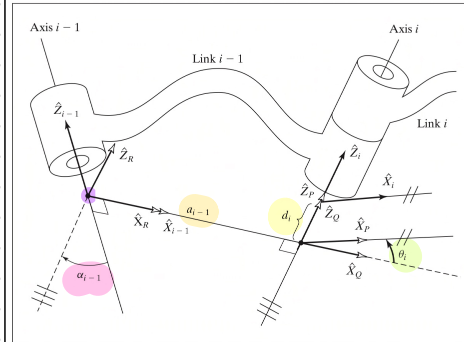

FIGURE 3.15: Location of intermediate frames $\{P\}$, $\{Q\}$, and $\{R\}$.

Frame $\{R\}$ differs from frame $\{i-1\}$ ONLY by a rotation of α_{i-1}

Frame $\{Q\}$ differs from $\{R\}$ by a translation a_{i-1}

Frame $\{P\}$ differs from $\{Q\}$ by a rotation θ_i

Frame $\{i\}$ differs from $\{P\}$ by a translation d_i

To transform vectors defined in $\{i\}$ to their description in $\{i-1\}$, we may write:

$${}^{i-1}P = {}^{i-1}_{R}T\,{}^{R}_{Q}T\,{}^{Q}_{P}T\,{}^{P}_{i}T\,{}^{i}P$$

$$\left({}^{i-1}_{i}T = R_X(\alpha_{i-1})\,D_X(a_{i-1})\,R_Z(\theta_i)\,D_Z(d_i) \right.$$

$$= \text{Screw}_X(a_{i-1}, \alpha_{i-1})\ \text{Screw}_Z(d_i, \theta_i)$$

$$\begin{cases} {}^{i-1}_{R}T = R_X(\alpha_{i-1}) \\ {}^{R}_{Q}T = D_X(a_{i-1}) \\ {}^{Q}_{P}T = R_Z(\theta_i) \\ {}^{P}_{i}T = D_Z(d_i) \end{cases}$$

$$\Downarrow$$

$${}^{i-1}_{i}T = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1}d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1}d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

individual link-transformation matrix

② Concatenating Link Transformations: ${}^{0}_{N}T = {}^{0}_{1}T\,{}^{1}_{2}T \cdots {}^{N-1}_{N}T$

${}^{0}_{N}T$ is a function of ALL n joint variables. (Kinematic equation)

· If the robot's joint-position sensors are queried, the Cartesian position and orientation of the Last Link can be computed by ${}^{0}_{N}T$, once the link frames $\{0\} \cdots \{N\}$ have been defined and the corresponding Link Parameters found.

§3.6. Actuator Space, Joint Space and Cartesian Space

- The position of ALL the links of a manipulator of n degree of freedom can be specified with a set of n joint variables. which is often referred to as the $n \times 1$ joint vector.
- when position is measured along orthogonal axes, and orientation is measured according to any of the conventions in Chapter 2, we can compute the Cartesian Space (or task-oriented / operational space) description from knowledge of the joint-space description.

The notion of Actuator Positions: the Sensors that measure the position of the manipulator are often located at the Actuators ⇒ some computations must be performed to realize the joint vector as a function of a set of actuator values (actuator vector)

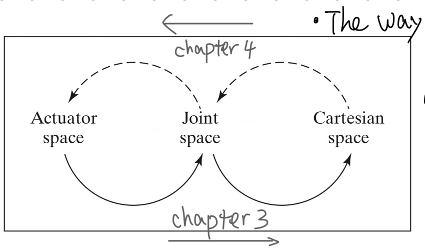

FIGURE 3.16: Mappings between kinematic descriptions.

- The ways in which actuators might be connected to move a joint are quite varied. For each robot we design or seek to analyze, the correspondence between Actuator Positions and Joint Positions must be solved.

§3.7. Kinematics of 2 industrial robots

①

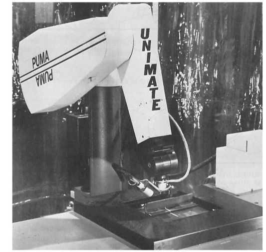

FIGURE 3.17: The Unimation PUMA 560. Courtesy of Unimation Incorporated, Shelter Rock Lane, Danbury, Conn.

- 6 DOF
- $\{0\}$ is coincident with $\{1\}$ when $\theta_1 = 0$.
- The joint axes of joints 4, 5, 6 all intersect at a common point and mutually orthogonal

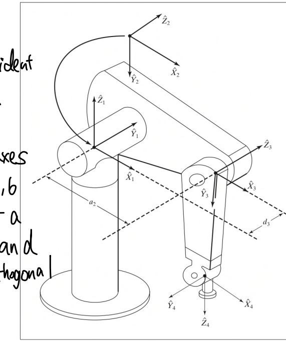

FIGURE 3.18: Some kinematic parameters and frame assignments for the PUMA 560 manipulator.

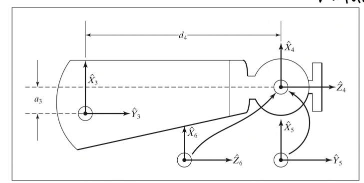

FIGURE 3.19: Kinematic parameters and frame assignments for the forearm of the PUMA 560 manipulator.

②

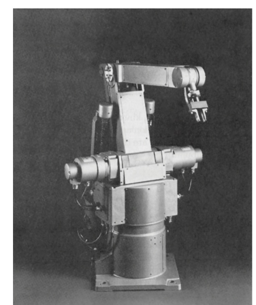

FIGURE 3.22: The Yasukawa Motoman L-3. Courtesy of Yasukawa.

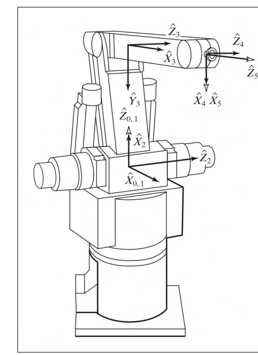

FIGURE 3.25: Assignment of link frames for the Yasukawa L-3.

5 DOF. not a simple open kinematic chain, but rather makes use of 2 linear actuators coupled to links 2 and 3 with four-bar linkages.

§3.8. Frames with Standard Names.

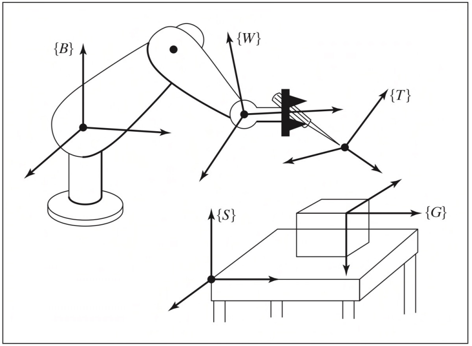

FIGURE 3.27: The standard frames.

ALL Robot Motions will be described in terms of these five frames:

① The Base Frame {B}. is located at the base of the Manipulator. nonmoving
· link 0.

② The Station Frame {S}: (task / world / universe frame)
· ALL actions of the Robot are performed relative to it.

③ The Wrist Frame {W}. is affixed to the last link of the manipulator. {N}. {W} has its origin fixed at a point called the wrist. of the manipulator.
· it is defined relative to its base frame. {W} = $^{B}_{W}T = {}^{0}_{N}T$

④ The Tool Frame {T}: {T} is affixed to the end of any tool the robot happens to be holding and is always specified w.r.t. {W}. {T} = $^{W}_{T}T$

⑤ The Goal Frame. {G} is a description of the location to which the Robot is to move the tool.