P2

(a)

$$P(R) = P(R,S) + P(R,\neg S)$$

$$= P(R|S)\cdot P(S) + P(R|\neg S)\cdot P(\neg S)$$

$$= 0.7 \times 0.25 + 0.3 \times 0.75$$

$$= 0.4 \#$$

(b)

$$P(We) = ①\,P(We,R,Wo) + ②\,P(We,\neg R,Wo) + ③\,P(We,R,\neg Wo) + ④\,P(We,\neg R,\neg Wo)$$

$$= ①\,P(We|R,Wo)\cdot P(R,Wo) + ②\,P(We|\neg R,Wo)\cdot P(\neg R,Wo)$$

$$+ ③\,P(We|R,\neg Wo)\cdot P(R,\neg Wo) + ④\,P(We|\neg R,\neg Wo)\cdot P(\neg R,\neg Wo)$$

$$= ①\,P(We|R,Wo)\cdot P(Wo|R)\cdot P(R) + ②\,P(We|\neg R,Wo)\cdot P(Wo|\neg R)\cdot P(\neg R)$$

$$+ ③\,P(We|R,\neg Wo)\cdot P(\neg Wo|R)\cdot P(R) + ④\,P(We|\neg R,\neg Wo)\cdot P(\neg Wo|\neg R)\cdot P(\neg R)$$

$$= ①\,0.12 \times 0.7 \times 0.4 + ②\,0.1 \times 0.2 \times 0.6 + ③\,0.25 \times 0.3 \times 0.4 + ④\,0.08 \times 0.8 \times 0.6$$

$$= 0.114 \#$$

(c)

$$P(M|S) = \frac{P(M,S)}{P(S)}$$

$$P(M,S) = ①\,P(M,S,R,Wo,We) + ②\,P(M,S,\neg R,Wo,We) + ③\,P(M,S,R,\neg Wo,We)$$

$$+ ④\,P(M,S,R,Wo,\neg We) + ⑤\,P(M,S,\neg R,\neg Wo,We) + ⑥\,P(M,S,\neg R,Wo,\neg We)$$

$$+ ⑦\,P(M,S,R,\neg Wo,\neg We) + ⑧\,P(M,S,\neg R,\neg Wo,\neg We)$$

$$= ①\,P(M|We)\cdot P(We|R,Wo)\cdot P(Wo|R)\cdot P(R|S)\cdot P(S)$$

$$+ ②\,P(M|We)\cdot P(We|\neg R,Wo)\cdot P(Wo|\neg R)\cdot P(\neg R|S)\cdot P(S)$$

$$+ ③\,P(M|We)\cdot P(We|R,\neg Wo)\cdot P(\neg Wo|R)\cdot P(R|S)\cdot P(S)$$

$$+ ④\,P(M|\neg We)\cdot P(\neg We|R,Wo)\cdot P(Wo|R)\cdot P(R|S)\cdot P(S)$$

$$+ ⑤\,P(M|We)\cdot P(We|\neg R,\neg Wo)\cdot P(\neg Wo|\neg R)\cdot P(\neg R|S)\cdot P(S)$$

$$+ ⑥\,P(M|\neg We)\cdot P(\neg We|\neg R,Wo)\cdot P(Wo|\neg R)\cdot P(\neg R|S)\cdot P(S)$$

$+ ⑦\ \underline{P(M|\neg We)\cdot P(\neg We|R,\neg Wo)\cdot P(\neg Wo|R)\cdot P(R|S)\cdot P(S)}$

$+ ⑧\ \underline{P(M|\neg We)\cdot P(\neg We|\neg R,\neg Wo)\cdot P(\neg Wo|\neg R)\cdot P(\neg R|S)\cdot P(S)}$

$\boxed{\text{cancelling } P(s)}$

$P(M|S) = \overset{①}{\underline{0.02\times 0.12\times 0.7\times 0.7}} + \overset{②}{\underline{0.02\times 0.1\times 0.2\times 0.3}}$

$+ \overset{③}{\underline{0.02\times 0.25\times 0.3\times 0.7}} + \overset{④}{\underline{0.42\times 0.88\times 0.7\times 0.7}}$

$+ ⑤\ \underline{0.02\times 0.08\times 0.8\times 0.3} + ⑥\ \underline{0.42\times 0.9\times 0.2\times 0.3}$

$+ ⑦\ \underline{0.42\times 0.75\times 0.3\times 0.7} + ⑧\ \underline{0.42\times 0.92\times 0.8\times 0.3}$

$= \overset{①}{\underline{0.001176}} + \overset{②}{\underline{0.00012}} + \overset{③}{\underline{0.00105}} + \overset{④}{\underline{0.181104}} + \overset{⑤}{\underline{0.000384}} + \overset{⑥}{\underline{0.02268}}$

$+ \overset{⑦}{\underline{0.06615}} + \overset{⑧}{\underline{0.092736}}$

$= 0.3654$ #

(d)

$$P(B|S) = \frac{P(B,S)}{P(S)}$$

$P(B,S) = \overset{①}{\underline{P(B,S,R,Wo)}} + \overset{②}{\underline{P(B,S,\neg R,Wo)}} + \overset{③}{\underline{P(B,S,R,\neg Wo)}} + \overset{④}{\underline{P(B,S,\neg R,\neg Wo)}}$

$= \overset{①}{\underline{P(B|S,Wo)\cdot P(Wo|R)\cdot P(R|S)\cdot P(S)}}$

$+ \overset{②}{\underline{P(B|S,Wo)\cdot P(Wo|\neg R)\cdot P(\neg R|S)\cdot P(S)}}$

$\boxed{\text{cancelling } P(S)}$

$+ \overset{③}{\underline{P(B|S,\neg Wo)\cdot P(\neg Wo|R)\cdot P(R|S)\cdot P(S)}}$

$+ \overset{④}{\underline{P(B|S,\neg Wo)\cdot P(\neg Wo|\neg R)\cdot P(\neg R|S)\cdot P(S)}}$

$P(B|S) = \overset{①}{\underline{0.8\times 0.7\times 0.7}} + \overset{②}{\underline{0.8\times 0.2\times 0.3}} + \overset{③}{\underline{0.4\times 0.3\times 0.7}} + \overset{④}{\underline{0.4\times 0.8\times 0.3}}$

$= 0.392 + 0.048 + 0.084 + 0.096$

$= 0.62$ #

(e)

$$P(S \mid B, \neg W_0)$$

$$= \frac{P(S, B, \neg W_0)}{P(B, \neg W_0)}$$

$$P(S, B, \neg W_0)$$

$$= P(S, B, \neg W_0, R) + P(S, B, \neg W_0, \neg R)$$

$$\overset{①}{=} \underline{P(B \mid S, \neg W_0) \cdot P(\neg W_0 \mid R) \cdot P(R \mid S) \cdot P(S)}$$

$$+ \overset{②}{} \underline{P(B \mid S, \neg W_0) \cdot P(\neg W_0 \mid \neg R) \cdot P(\neg R \mid S) \cdot P(S)}$$

$$\overset{①}{=} \underline{0.4 \times 0.3 \times 0.7 \times 0.25} + \overset{②}{} \underline{0.4 \times 0.8 \times 0.3 \times 0.25}$$

$$= 0.045$$

$$P(B, \neg W_0)$$

$$= P(B, \neg W_0, S, R) + P(B, \neg W_0, \neg S, R) + P(B, \neg W_0, S, \neg R) + P(B, \neg W_0, \neg S, \neg R)$$

$$\overset{①}{=} \underline{P(B \mid S, \neg W_0) \cdot P(\neg W_0 \mid R) \cdot P(R \mid S) \cdot P(S)}$$

$$+ \overset{②}{} \underline{P(B \mid S, \neg W_0) \cdot P(\neg W_0 \mid R) \cdot P(R \mid \neg S) \cdot P(\neg S)}$$

$$+ \overset{③}{} \underline{P(B \mid S, \neg W_0) \cdot P(\neg W_0 \mid \neg R) \cdot P(\neg R \mid S) \cdot P(S)}$$

$$+ \overset{④}{} \underline{P(B \mid S, \neg W_0) \cdot P(\neg W_0 \mid \neg R) \cdot P(\neg R \mid \neg S) \cdot P(\neg S)}$$

$$\overset{①}{=} 0.4 \times 0.3 \times 0.7 \times 0.25$$

$$+ \overset{②}{} 0.4 \times 0.3 \times 0.3 \times 0.75$$

$$+ \overset{③}{} 0.4 \times 0.8 \times 0.3 \times 0.25$$

$$+ \overset{④}{} 0.4 \times 0.8 \times 0.7 \times 0.75$$

$$= 0.021 + 0.027 + 0.024 + 0.168$$

$$= 0.24$$

$$P(S \mid B, \neg W_0)$$

$$= \frac{0.045}{0.24} = 0.1875 \ \#$$