

P₁

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$$a) P(\text{Word} = \text{great}) = \frac{13+17+15}{12+12+1+1+17+17+14+14+12+13+17+15} = \frac{45}{137} \approx 0.3285$$

$$b) P(\text{Document} = 1 | \text{Word} = \text{we}) = \frac{12}{12+17+14} = \frac{12}{43} \approx 0.2791$$

$$c) P(\text{Document} = 2 | \text{Word} = \text{am or Word} = \text{are}) = \frac{17}{12+1+17+14+2} = \frac{17}{46} \approx 0.3696$$

$$d) P(\text{Word} = \text{great}) = \frac{13}{12+12+1+1+13} \times \frac{1}{6} + \frac{17}{17+17+17} \times \frac{1}{3} + \frac{15}{14+14+12+13} \times \frac{1}{2}$$

$$= \frac{1}{18} + \frac{1}{9} + 0.15957 \approx 0.05556 + 0.11111 + 0.15957 \approx 0.3262$$

$$e) P(D=1 | W=\text{we}) = \frac{P(W=\text{we} | D=1) P(D=1)}{P(W=\text{we})}$$

$$P(W=\text{we}) = \frac{1}{39} \times \frac{1}{6} + 0 + \frac{2}{47} \times \frac{1}{2} = \frac{1}{234} + \frac{1}{47} \approx 0.00427 + 0.02128 \approx 0.026$$

$$P(D=1 | W=\text{we}) = \frac{\frac{1}{39} \cdot \frac{1}{6}}{0.026} \approx 0.1644$$

$$f) P(D=2 | W=\text{am or } W=\text{are}) = \frac{P(W=\text{am or } W=\text{are} | D=2) \cdot P(D=2)}{P(W=\text{am or } W=\text{are})}$$

$$P(W=\text{am or } W=\text{are} | D=2) = \frac{17+0}{17+17+17} = \frac{1}{3}$$

$$P(W=\text{am or } W=\text{are}) = \left(\frac{12}{39} \times \frac{1}{6} + \frac{1}{3} \times \frac{1}{3} + \frac{14}{47} \times \frac{1}{2} \right) + \left(\frac{1}{39} \times \frac{1}{6} + 0 + \frac{2}{47} \times \frac{1}{2} \right) \approx 0.0513 + 0.1111 + 0.1489$$

$$+ 0.026 \approx 0.337$$

$$P(D=2 | W=\text{am or } W=\text{are}) = \frac{\frac{1}{3} \times \frac{1}{3}}{0.337} \approx 0.3297$$