Problem 1. [16] Reasoning with a Joint Probability Table

The following FJPD table gives probabilities for three Boolean random variables, *X*, *Y*, and *Z*:

	Y		¬*Y*	
	Z	¬*Z*	*Z*	¬*Z*
X	0.70	0.015	0.10	0.02
¬*X*	0.08	0.01	0.07	0.005

(a) [4] What is *P*(*Y* | *X*)?

$P(Y|X)$ = (0.70+0.015)/(0.70+0.015+0.10+0.02) = 0.856

(b) [4] What is *P*(*Y*)?

$P(Y)$ = (0.70+0.015+0.08+0.01)/(0.805+0.195) = 0.805

(c) [4] What is *P*(*X*, *Z*)?

$P(X,Z)$ = (0.70+0.10)/(0.70+0.10+0.015+0.02+0.08+0.07+0.01+0.005)
= 0.80

(d) [4] Is the data consistent with *X* and *Z* being independent?

Since $P(X)$= 0.835, $P(Z)$= 0.95, and $P(X)*P(Z) \neq P(X,Z)$, the data is ***not*** consistent with *X* and *Z* being independent.

Problem 2. [20] Inference using a Bayesian Network

(a) [5] $P(A|B) = \frac{P(B|A)P(A)}{P(B)}$

$$= \frac{P(B|A)P(A)}{P(B,A) + P(B,\neg A)}$$

$$= \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|\neg A)P(\neg A)}$$

$$= \frac{0.2\times 0.1}{0.2\times 0.1 + 0.1\times 0.9} = \frac{2}{11} = 0.1818$$

(b) [5] $P(D) = P(D, A, B) + P(D, A, \neg B) + P(D, \neg A, B) + P(D, \neg A, \neg B)$

$$= P(D|A,B)P(A,B) + P(D|\neg A,B)P(\neg A,B) + P(D|A,\neg B)P(A,\neg B) + P(D|\neg A,\neg B)P(\neg A,\neg B)$$

$$= P(D|A,B)P(B|A)P(A) + P(D|\neg A,B)P(B|\neg A)P(\neg A) + P(D|A,\neg B)P(\neg B|A)P(A) + P(D|\neg A,\neg B)P(\neg B|\neg A)P(\neg A)$$

$$= 0.02 \times 0.2 \times 0.1 + 0.001 \times (1 - 0.1) \times (1 - 0.1) + 0.01 \times (1 - 0.2) \times 0.1 + 0.01 \times 0.1 \times (1 - 0.1)$$

$$= 0.0004 + 0.00081 + 0.0008 + 0.0009 = 0.0029$$

(c) [5] $P(E|A) = \frac{P(E,A)}{P(A)}$

$$= \frac{P(E,A,B,D) + P(E,A,\neg B,D) + P(E,A,B,\neg D) + P(E,A,\neg B,\neg D)}{P(A)}$$

$$= \frac{P(E|D)P(D|A,\neg B)P(\neg B|A)P(A) + P(E|\neg D)P(\neg D|A,B)P(B|A)P(A) + P(E|D)P(D|A,B)P(B|A)P(A) + P(E|\neg D)P(\neg D|A,\neg B)P(\neg B|A)P(A)}{P(A)}$$

$$= P(E|D)P(D|A,\neg B)P(\neg B|A) + P(E|\neg D)P(\neg D|A,B)P(B|A) + P(E|D)P(D|A,B)P(B|A) + P(E|\neg D)P(\neg D|A,\neg B)P(\neg B|A)$$

$$= 0.2 \times 0.02 \times 0.9 + (1 - 0.2) \times 0.01 \times 0.9 + 0.2 \times (1 - 0.02) \times 0.1 + (1 - 0.2) \times (1 - 0.01) \times 0.1$$

$$= 0.1096$$

(d) [5] $P(A, B, \neg C|D) = \frac{P(A,B,\neg C,D)}{P(D)}$

$$= \frac{P(D|A,B)P(B|A)P(\neg C|B)P(A)}{P(D)}$$

$$= \frac{0.02 \times 0.2 \times (1 - 0.5) \times 0.1}{0.00291}$$

$$= 0.0687$$