Simulating London’s Underground with Graph Theory and Spatial Interaction Model

Word count: 3258

The code and data has been pushed to GitHub.

1 London’s Underground Resilience

This part evaluates the resilience of London's underground system by analyzing the impact of removing specific stations that may present vulnerabilities. We focus on both the topological network and a weighted model that accounts for passenger flows.

1.1 Topological Network

1.1.1 Centrality Measures

Centrality metrics provide a quantitative measure of a node's significance within the network (Latora and Marchiori, 2007). To pinpoint the most topologically important stations in the underground system, we employ three distinct centrality measures. These metrics help identify potential congestion areas, crucial transfer points, or vulnerabilities within the transit network. Three indices, degree, closeness, and betweenness centrality, are used to capture a node’s importance as being directly connected to others, being accessible to others, and being the intermediary between others (Wang *et al.*, 2011; Bloch, Jackson and Tebaldi, 2023).

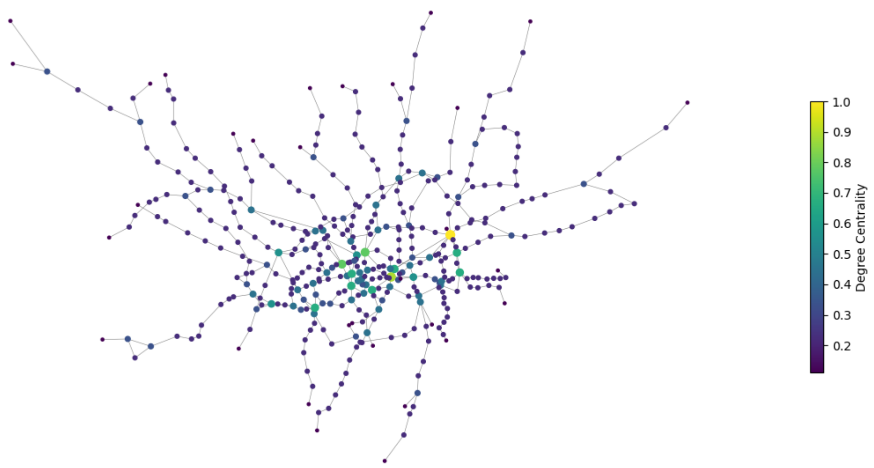


Figure 1. Degree Centrality of Tube Stations

Table 1. Centrality Measures

	Definition	In London Underground context	Equation
Degree Centrality	Degree centrality quantifies how many direct connections a station has, revealing nodes that serve as major hubs in the network. This measure is particularly straightforward in undirected networks, where it counts the total number of links a node has, and the importance would be considered limited when traffic flow is considered (Freeman, 2002).	Stations exhibiting high degree centrality typically serve as major interchange hubs where multiple lines converge. These pivotal nodes enable passengers to transfer seamlessly between different routes, enhancing the connectivity and efficiency of the system.	$$k_i = \sum_{j=1}^{N} A_{ij}$$ Where k_i is the degree centrality of node i in the network of N nodes, and A_{ij} is an element of the adjacency matrix.
Betweenness Centrality	Betweenness centrality is a critical measure in network analysis that identifies nodes positioned on the most direct paths between others within a network. This centrality indicates a node's potential to influence and control the movement or flow within the network because it acts as a necessary passageway between various node pairs. (Latora and Marchiori, 2007; Newman, 2005).	Disconnecting stations with high betweenness centrality might isolate certain areas or substantially increase travel times, reflecting their role in connecting disparate sections or clusters within the network (Brandes, 2001; Barthélemy, 2004). Therefore, identifying and understanding the role of these high-betweenness stations can be essential for effective transport planning and resilience strategies, ensuring the network can handle disruptions without major losses in service or efficiency.	$$C_{b(i)} = \sum_{s \neq i \neq t} \frac{\sigma_{st}(i)}{\sigma_{st}}$$ Where σ_{st} represents the total number of shortest paths from node s to node t, $\sigma_{st}(i)$ is the number of those paths that pass through node i.
Closeness centrality	Closeness centrality reflects the proximity between on node and other nodes in the network (Zhang and Luo, 2017). Nodes that exhibit high closeness centrality are typically more accessible and centrally located within the network, making them vital connectivity points.	These stations are strategically important for ensuring more efficient passenger routes throughout the network. High closeness centrality implies that a station is not only well-connected but also optimally positioned to offer multiple, efficient routing options, enhancing the overall resilience of the transport system. In situations where a network faces potential disruptions, such as station closures or line malfunctions, stations with high closeness centrality can provide necessary alternative paths, maintaining network performance and minimizing inconvenience to passengers.	$$C_i = \frac{1}{l_i} = \frac{n}{\sum_j d_{ij}}$$ Where d_{ij} represents the shortest path between nodes i and j, and n is the total number of nodes. l_i is the average geodesic distance from one node to all others (Evans and Chen, 2022)

Table 2. Top Ten Stations by Centrality

Rank	Station/ Degree	Degree Centrality	Station/ Betweenness	Betweenness Centrality	Station/ Centrality	Closeness Centrality
1	Stratford	9	Stratford	0.297846	Green Park	0.114778
2	Bank and Monument	8	Bank and Monument	0.290489	Bank and Monument	0.113572
3	King's Cross St. Pancras	7	Liverpool Street	0.270807	King's Cross St. Pancras	0.113443
4	Baker Street	7	King's Cross St. Pancras	0.255307	Westminster	0.112549
5	Earl's Court	6	Waterloo	0.243921	Waterloo	0.112265
6	Oxford Circus	6	Green Park	0.215835	Oxford Circus	0.111204
7	Liverpool Street	6	Euston	0.208324	Bond Street	0.110988
8	Waterloo	6	Westminster	0.203335	Farringdon	0.110742
9	Green Park	6	Baker Street	0.191568	Angel	0.110742
10	Canning Town	6	Finchley Road	0.165085	Moorgate	0.110314

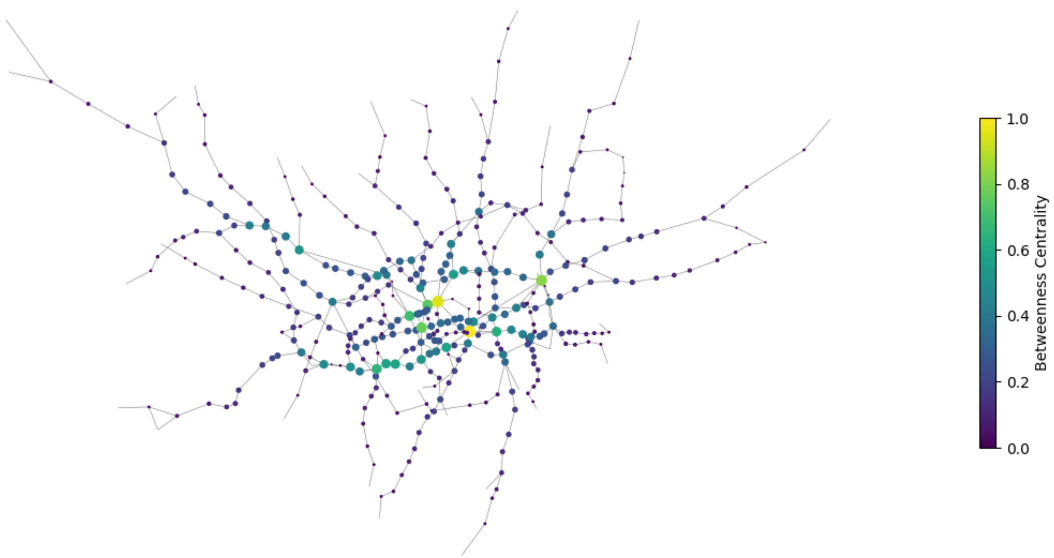


Figure 2. Betweenness Centrality of Tube Stations

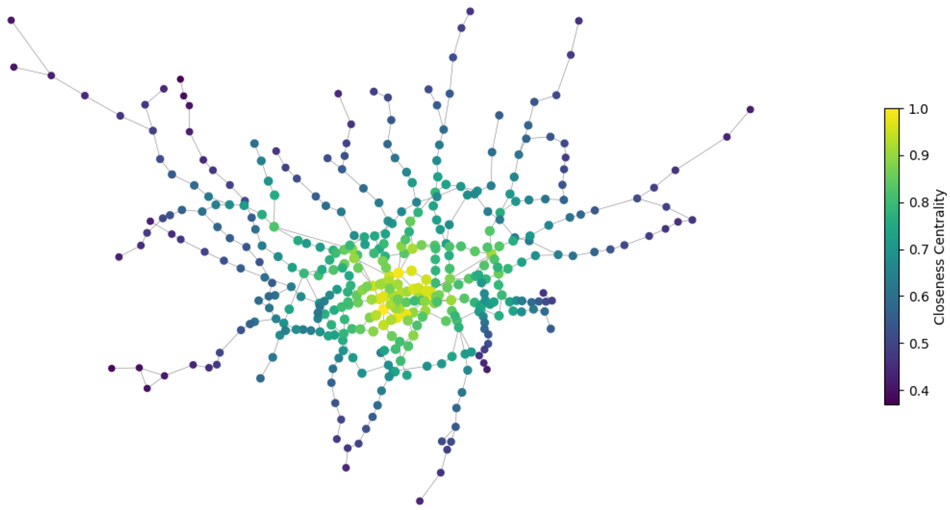


Figure 3. Closeness Centrality of Tube Stations

1.1.2 Impact Measures

(a) Average Degree

The average degree of a network is the mean value of the degrees of all nodes in the network, showing how interconnected the nodes within the network are. If D is the degree of each node, then:

$$Average\ Degree = \frac{\sum_i D_i}{n}$$

Where n is the total number of nodes in the network.
Although straight-forward, it is a useful tool to analyze the reality. However, it does not make sense in a directed network, because the direction of the ties is likely to be meaningful (Danila *et al.*, 2007).
In London underground network, the average degree would represent the average number of direct connections each station has. A higher average degree generally indicates a more robust network, with many direct connections between stations, which in turn suggests greater redundancy (Wang *et al.*, 2011). In practical terms, this redundancy means that if one part of the network encounters a disruption, there are multiple alternative routes that can help maintain the overall functionality of the system. This makes the network more resilient to individual failures, whether due to maintenance issues, accidents, or other unexpected events.
The original average degree for the London underground is 2.329, which means each station has two directly connecting stations on average.

(b) Global Efficiency

Global efficiency quantifies the overall effectiveness of a network in facilitating the flow of information or traffic between its nodes. It's the average of the inverse shortest path length between every pair of nodes (Lu and Shi, 2007; Stamos, 2023).

$$Global\ Efficiency = \frac{1}{n(n-1)}\sum_{i \neq j} \frac{1}{d_{ij}}$$

Where d_{ij} is the shortest path distance between nodes i and j, and n is the total number of nodes.

In the context of the London Underground, a global efficiency value of 0.101256, although seemingly low, reflects the characteristics typical of a large and complex transit system. Such a value indicates that, on average, routes between stations involve several transfers or stops. However, it also suggests that despite the size of the network and the longer paths, there are enough interchanges available to keep the system functional and relatively efficient.

This measure could be applied to any network. For social or organizational networks, high global efficiency means that information can be disseminated quickly and widely across connections, enhancing collaboration and the spread of ideas (Anderson and Yotov, 2016).

1.1.3 Node Removal

The results from the two strategies and various centrality measures reveal distinct patterns and varying degrees of impact on the underground network when stations are removed.

(a) Non-Sequential Removal Strategy

Randomly removing stations from the top ten centrality rankings offers insights into the fundamental importance of each node, as determined by their initial centrality scores.

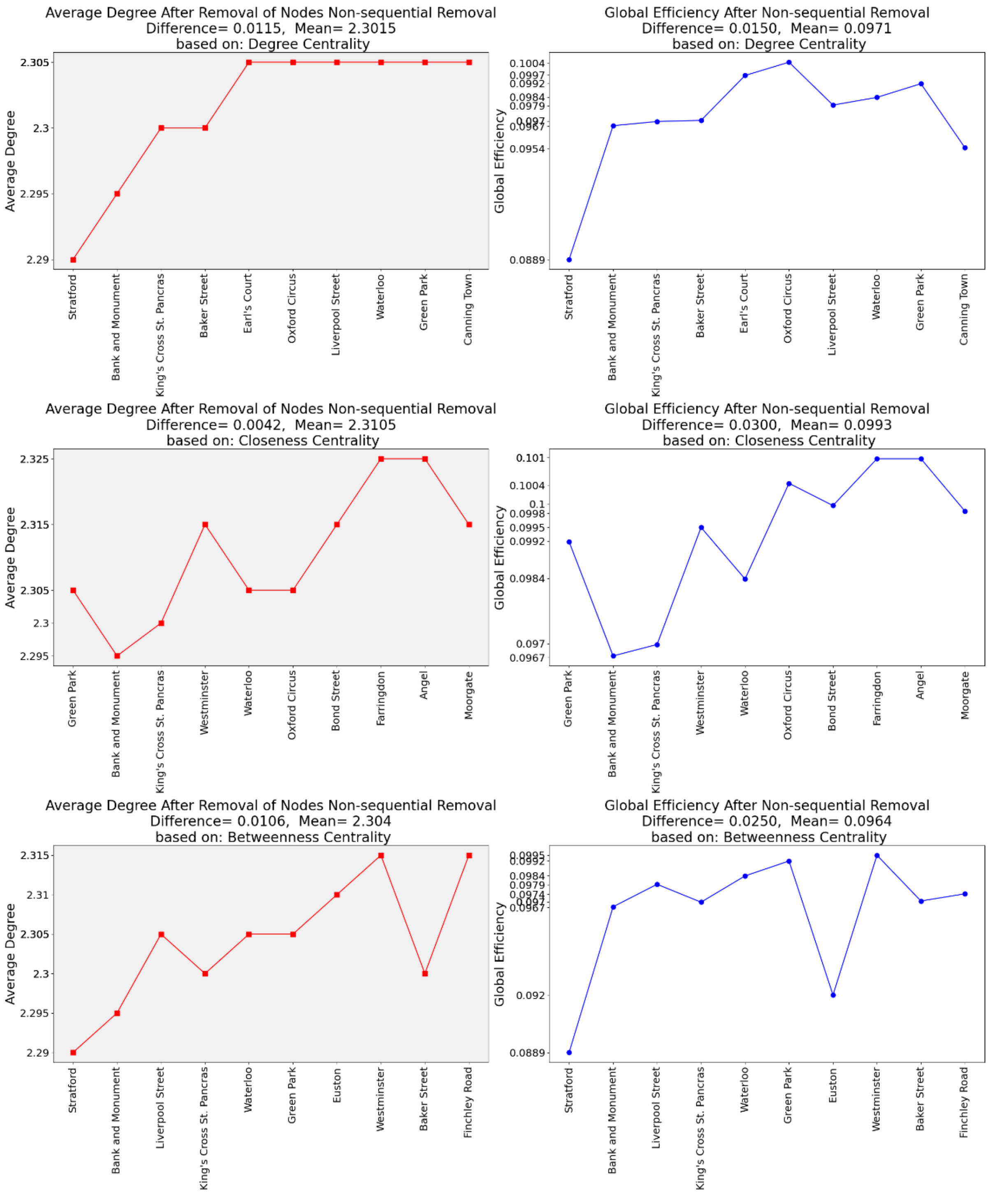


Figure 4. Impact measures after non-sequential removal.

The increase in average degree after certain removals based on betweenness centrality might be due to the redistribution of edges as the network structure adjusts, as the betweenness metric focuses on nodes that act as bridges in the shortest paths across the network, which means the network reorganizing itself to maintain connectivity. London tube networks may be more resilient to node failures if the stations have low betweenness centrality, as their removal doesn't significantly alter global efficiency.
In addition, stations with high closeness centrality are integral to maintaining short path

lengths throughout the network, and their removal could disproportionately increase travel times or information path lengths.

Bank and Monument station is a critical node in terms of both degree and betweenness centrality, highlighting its role as a major interchange point. Removing this station not only diminishes connectivity but also significantly disrupts the efficient transit across the network. Conversely, the network shows some resilience when Liverpool Street and Waterloo are removed, thanks to alternative pathways or nearby stations capable of handling the redirected passenger flow. On the other hand, while the removal of Green Park affects the average degree, it has a minimal impact on global efficiency, suggesting that although it is a well-connected station, other parts of the network are able to compensate for its absence.

(b) Sequential Removal Strategy

The sequential removal strategy involves iteratively eliminating the most significant node from the network one at a time, based on the selected centrality measure, until all targeted nodes are removed. This approach helps to reveal the network's resilience to disruptions, pinpointing potential bottlenecks where the removal of a node could cause substantial losses in efficiency. It mirrors real-world scenarios where the importance of nodes can shift due to changes in network dynamics, thereby offering a realistic view of the network's vulnerability under stress.

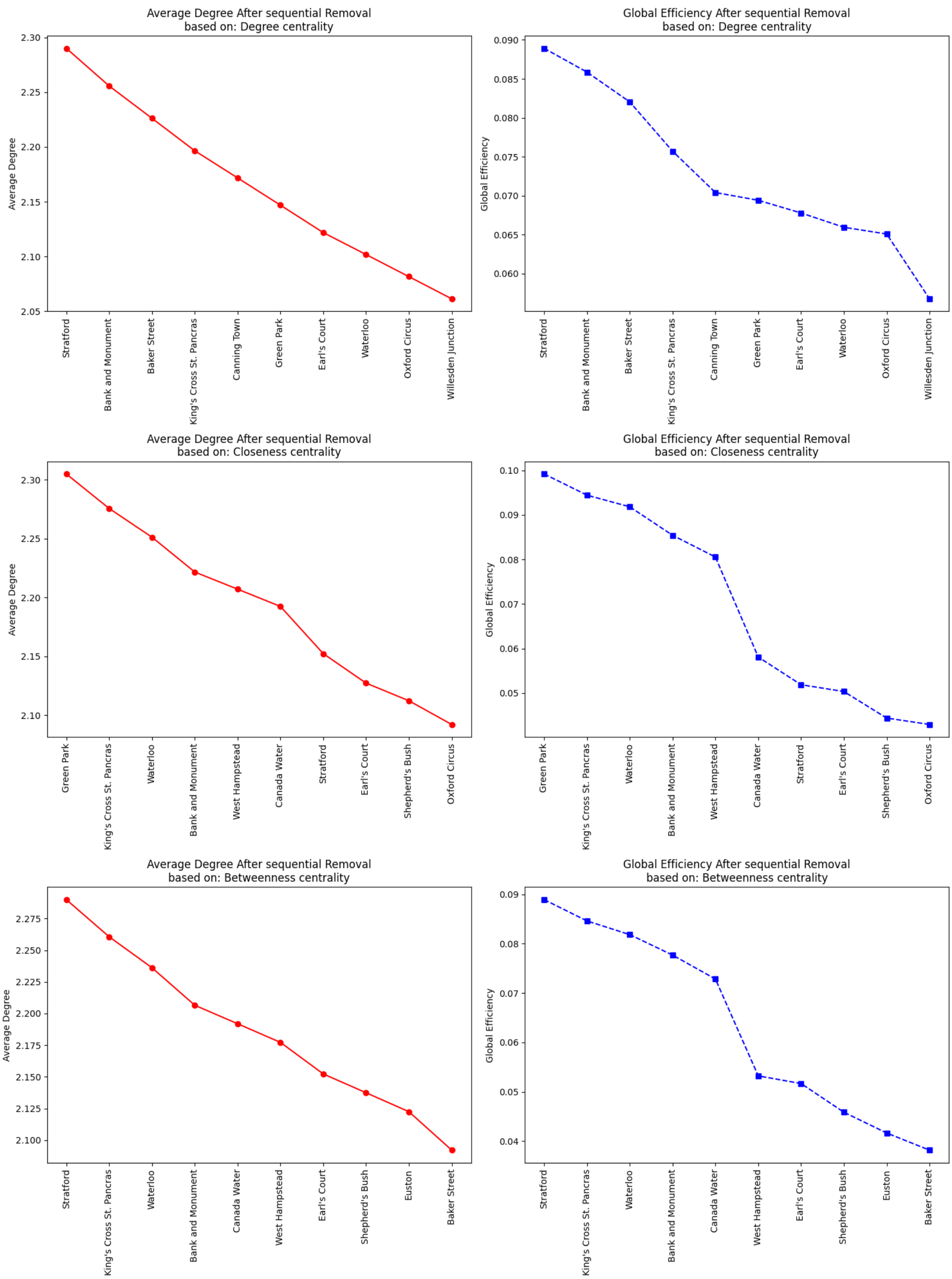


Figure 5. Impact measures after sequential removal.

The graph indicates that removing Waterloo station impacts the network's global efficiency more significantly than its average degree, highlighting its importance in facilitating efficient travel. Similarly, despite having fewer connections, Earl's Court exhibits a noteworthy effect on the network's performance upon its removal.

The sequential removal of stations shows a clear trend that both the average degree and global efficiency of the network decline as these pivotal nodes are removed. The consistent results across different centrality measures affirm the effectiveness of these metrics in pinpointing essential nodes within the network. This analysis helps in understanding the potential impacts of planned disruptions, such as the closure of the Piccadilly line, and emphasizes the critical role these central stations play in sustaining network resilience.

1.1.4 Node Removal Analysis

Betweenness centrality appears to more accurately capture a station's importance in the functioning of the Underground, as evidenced by graphs showing that removing stations based on this metric results in the largest drop in global efficiency. This measure identifies stations that serve as essential connectors or bridges along travel routes throughout the network.

The Sequential Removal Strategy is more effective at studying resilience, as it accounts for the dynamic nature of the network and how it adapts after each removal. Particularly in scale-free networks, which are vulnerable to targeted attacks but resistant to random ones, this strategy leads to significant reductions in global efficiency. It simulates a real-world scenario where the network has to continually adjust to changes, demonstrating of the network's true resilience.

Table 3. Global Efficiency Decrease after removing 10 stations

Centrality Measure	Global Efficiency Decrease (%)
Degree (nonseq)	32.800370
Closeness (nonseq)	26.777002
Betweenness(nonseq)	40.230861
Degree (seq)	46.167198
Closeness (seq)	60.229742
Betweenness (seq)	65.059324

Table 4. Average Degree Decrease after removing 10 stations

Centrality Measure	Average Degree Decrease (%)
Degree (nonseq)	24.788579
Closeness (nonseq)	37.564842
Betweenness(nonseq)	33.017454
Degree (seq)	21.093367
Closeness (seq)	19.383763
Betweenness (seq)	18.773910

Global Efficiency is a better option for evaluating the impact of node removal on a network because it directly correlates with the network's core function of efficiently transporting passengers. The greater the reduction in global efficiency, the more significant the impact on the network's performance. In the context of public transportation, this measure directly translates to the passenger experience, as it reflects increased travel times and reduced connectivity.

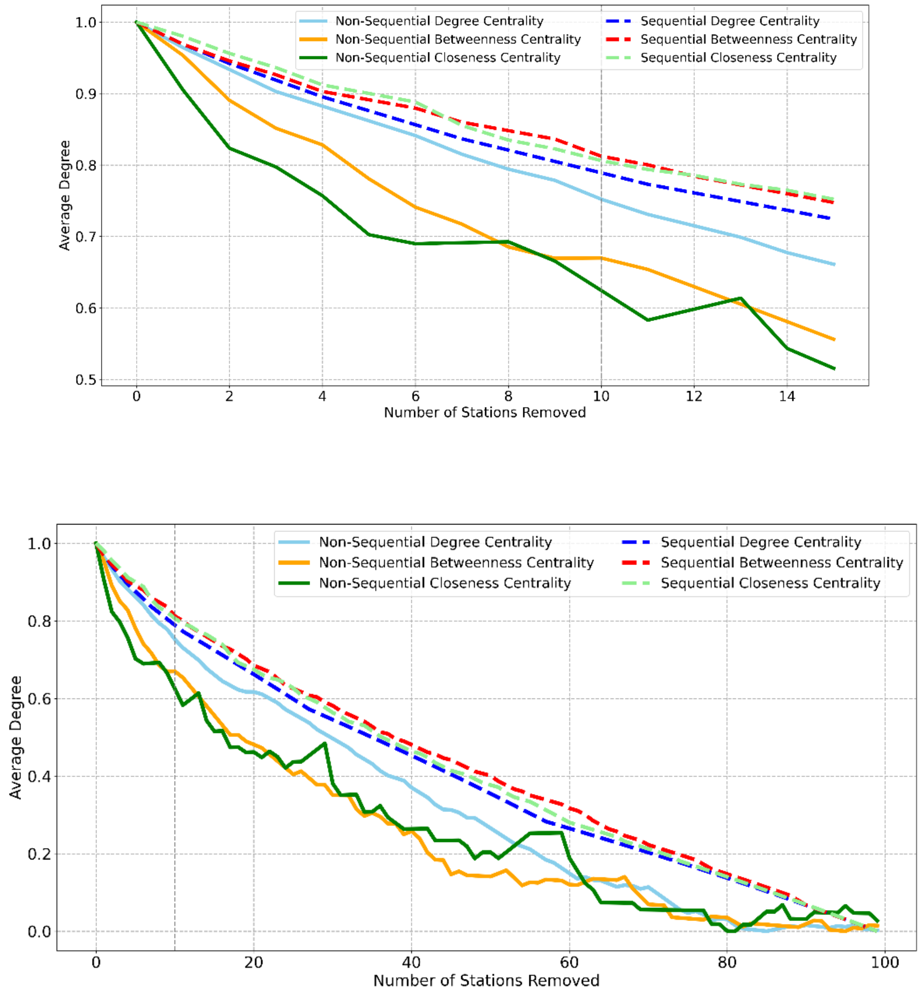
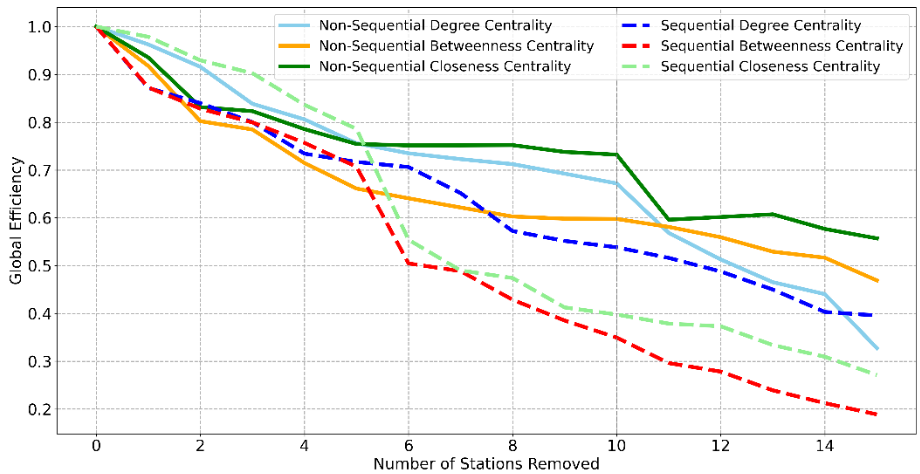


Figure 6. Average degree in station removal.



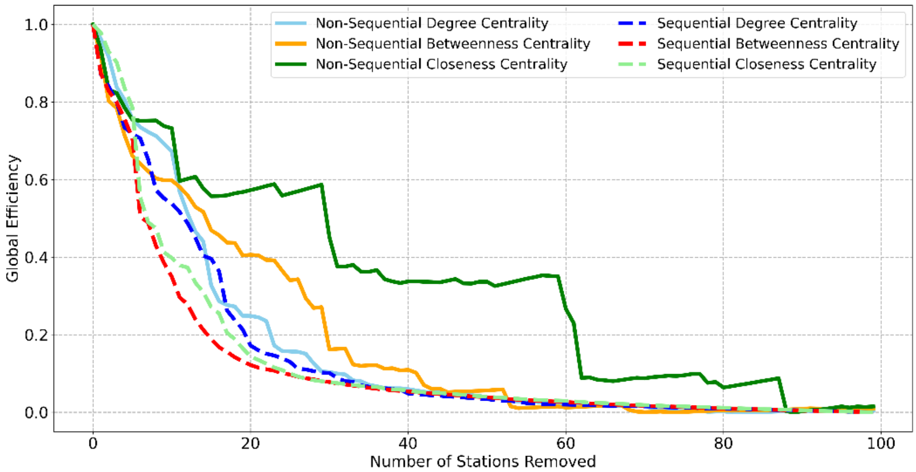


Figure 7. Global Efficiency in station removal.

In summary, the betweenness centrality measure, when used in conjunction with the sequential removal strategy, provides a robust method for evaluating the resilience of the underground. Global efficiency is the most comprehensive measure for assessing the consequences of node removals, reflecting how such changes affect the network's primary function.

Transport for London (TFL) recently claimed the closure of the Piccadilly line for updates, which accounts for 10% of the total length of the London tube network, would have significant impacts on the network's resilience. Thus, future study should consider practical influence on whole underground system by the closure.

1.2 Flows in the Weighted Network

The tube network is regarded as a directed weighted network where the flows of passengers are assigned to the links between stations.

1.2.1 Centrality in the Weighted Network

Betweenness centrality indicates the most relevant stations in detecting the vulnerability of the underground system, and it is re-computed by taking flows into account as weight instead of number of geodesics passing through a node.

$$C_{b(i)} = \sum_{s \neq i \neq t} \frac{p_{st}(i)}{p_{st}}$$

Where p_{st} is the total flows between node s and t, $p_{st}(i)$ is the number of flows that pass through node i.

Table 7. Top Ten Stations by Weighted Betweenness Centrality

Station	Weighted Betweenness Centrality
Green Park	0.572556
Bank and Monument	0.505288
Waterloo	0.416429
Westminster	0.381366
Liverpool Street	0.336817
Stratford	0.331291
Bond Street	0.29183
Euston	0.284236
Oxford Circus	0.270764
Warren Street	0.254286

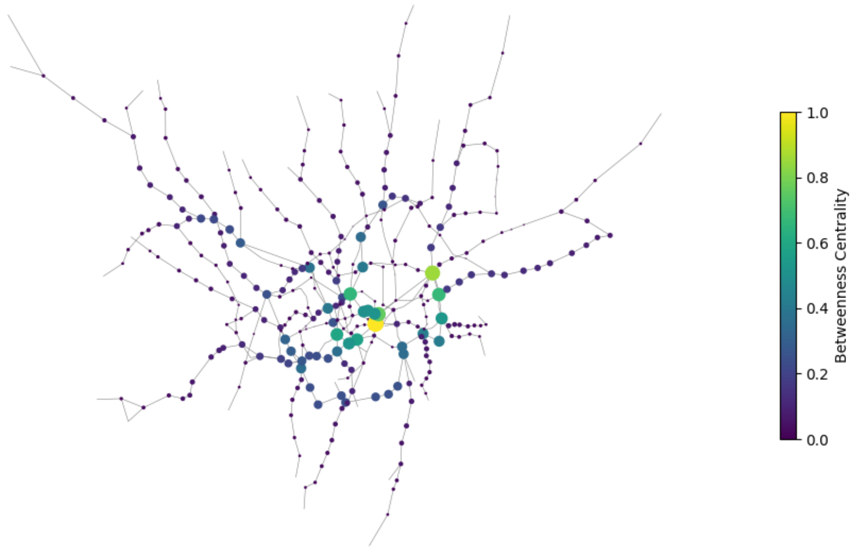


Figure 8. Betweenness centrality with weighted network

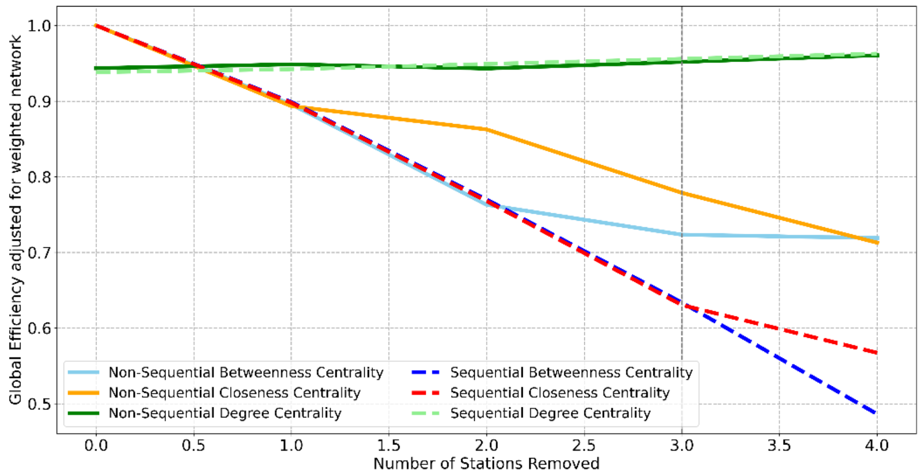
In both topological and weighted network analyses, Bank and Monument consistently emerge as crucial, with its significance even more accentuated in scenarios where passenger flows are taken into account. In the weighted analysis, stations like West Ham and Canning Town rise into the top ten, highlighting their high passenger volumes. Conversely, Finchley Road, which ranks in the top ten in the unweighted scenario, drops out in the weighted analysis, likely due to relatively lower passenger flow despite its structural significance.

The variation in station rankings underscores how incorporating passenger flows shifts the focus from merely the structural centrality of stations to their significance based on traffic volume. In a weighted network, the criticality of a station extends beyond its number of direct connections to the impact of those connections on passenger traffic. This could influence decisions regarding network enhancements, maintenance schedules, and strategies for emergency response, prioritizing stations differently based on their role in passenger dynamics rather than just their topological placement.

1.2.2 Impact Measures in a Weighted Network

To adjust the global efficiency for a network where weights represent capacities, invert the weights so higher capacities become lower costs, then calculate global efficiency using the shortest paths derived from these inverted weights. This method ensures that paths with greater flows contribute more to the network's overall efficiency, accurately reflecting the network's capacity for traffic.

However, the average degree cannot indicate flows by calculationg average directed connections. Therefore, we introduce Characteristic Path Length (Strang *et al.*, 2018) to calculate average length of shortest path with weight, which focuses on the most heavily trafficked paths, thus providing a more realistic measure of connectivity and flow across the network.



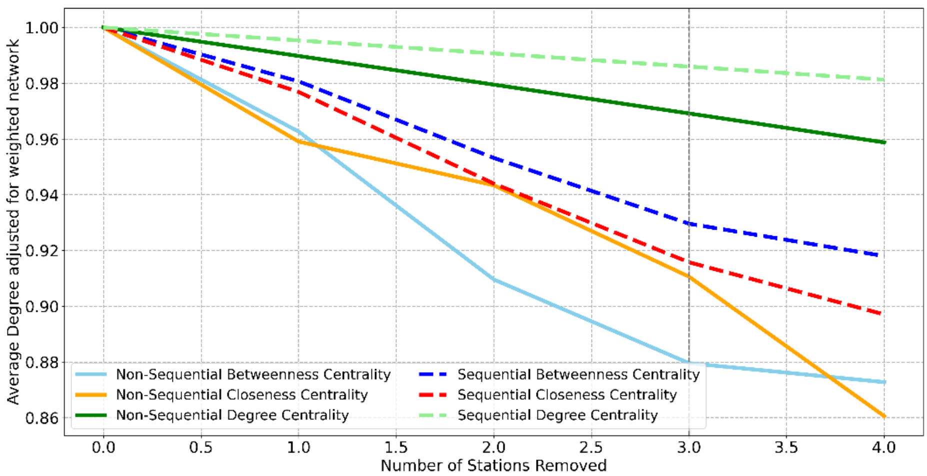


Figure 9. Impact measure for weighted network

1.2.3 Re-experiment with flows

According to the betweenness centrality, we recalculate the characteristic path length as a new impact measure.

Table 8. Recalculate the impact measure for between centrality

Removals	Station removed(seq)	Characteristic Path Length (seq)	Global Efficiency (seq)	Station removed(non-seq)	Characteristic Path Length(non-seq)	Global Efficiency (non-seq)
1	Stratford	14.496	0.0982	Bank and Monument	14.872	0.0948
2	King's Cross St. Pancras	15.310	0.0934	Liverpool Street	15.031	0.0938
3	Waterloo	16.790	0.0903	King's Cross St. Pancras	16.030	0.8892

Table 8. Recalculate the impact measure for weighted between centrality

Removals	Station removed	Weighted Characteristic Path Length	Weighted Global Efficiency
1	Bank and Monument	18.789	0.0968
2	King's Cross St. Pancras	18.789	0.0899
3	Canada Water	18.789	0.0879

The removal of Bank and Monument, King's Cross St. Pancras, and Canada Water shows no change in the weighted characteristic path length, but a noticeable decline in weighted global efficiency. While the most efficient paths remain the same length, their efficiency and functionality decrease, highlighting the importance of these nodes in maintaining optimal flow.

Bank and Monument station's closure will have the largest impact on passengers. With a high adjusted betweenness centrality, the station serves as a central hub for multiple routes, including the Central, Northern, Waterloo & City, and District and Circle lines at Bank, alongside the DLR at Bank and Monument. This extensive connectivity is vital for linking substantial parts of the Greater London area.
Given its central position, any disruptions at Bank and Monument are likely to affect several lines, thereby impacting overall network traffic flow and potentially creating bottlenecks on alternative routes.

2 Spatial Interaction Models

The spatial interaction model is based on the gravity model of Newton's law of gravity. It considers the attraction between two locations is directly proportional to their respective masses (or importance) and inversely proportional to the distance or deterrent factor between them, effectively simulating the intensity of activity between these points (Fotheringham and O'Kelly, 1989; Haynes and Fotheringham, 2020).

$$T_{ij} = K\frac{P_i^{\alpha} P_j^{\gamma}}{d_{ij}^{\beta}} = KO_i^{\alpha} D_j^{\gamma} d_{ij}^{-\beta}$$

T_{ij} is the flow between origin *i* and destination j. P_i and P_i have been updated to O_i and D_j to more accurately reflect the roles of each location. O_i denotes the origin and is associated with the production measure, such as the population size, while D_j is the variable relating to attraction of destination, which could be the level of salaries or the number of available jobs.

K is a scaling constant that relates flows to the total volume of the system in different scenarios, which can be calculated as:

$$K = \frac{T}{\sum_i \sum_j O_i^{\alpha} D_j^{\gamma} d_{ij}^{-\beta}}$$

where T is the sum of the observed flows.

α modifies the original production measure O_i, enhancing its influence on the interactions, while γ adjusts the attraction measure D_j at the destination, reflecting its appeal and ability to draw flows.

β is a parameter relating to the deterrence effects associated with distance, such as travel costs and transportation efficiency between locations.

Together, K, α, β, and γ are calibrated to tailor the model for various scenarios, allowing it to reflect diverse geographical and economic conditions accurately (Openshaw, 1976; Haynes and Fotheringham, 2020).

2.1 Models and Calibration

2.1.1 Introduction to models

Alan Wilson developed a suite of spatial interaction models derived from the gravity model, each incorporating various constraints or additional factors to address different analytical needs (Wilson, 1971).

In the unconstrained model, there are no fixed parameters for the flows. Each flow between nodes must be estimated independently, providing a flexible but computationally intensive approach.

$$T_{ij} = KO_i^{\alpha} D_j^{\gamma} f(c_{ij}) \text{ subject to } \sum_{i=1}^{n} \sum_{j=1}^{m} T_{ij} = T$$

The origin-constrained model assumes that the total outflow from each origin node is fixed. This setup allows the model to focus on how these fixed amounts are distributed across various destinations based on the attractiveness and distance of each destination.

$$T_{ij} = A_i O_i D_j^{\gamma} f(d_{ij}) \text{ subject to } O_i = \sum_i T_{ij}$$

Conversely, the destination-constrained model fixes the total inflow to each destination node. This model primarily examines how different origins contribute to these fixed inflows, influenced by the production capabilities of the origins and the intervening costs.

$$T_{ij} = B_j O_i^{\alpha} D_j f(d_{ij}) \text{ subject to } D_j = \sum_i T_{ij}$$

The doubly constrained model applies fixed constraints on both inflows and outflows at each node, making it the most structured of the models. This approach provides a detailed examination of how fixed volumes of flows are distributed across the network, requiring calibration of parameters to align the model closely with real-world data (Fotheringham and O'Kelly, 1989).

$$T_{ij} = A_i B_j O_i D_j f(c_{ij}) \text{ subject to } O_i = \sum_i T_{ij} \; and \; D_j = \sum_i T_{ij}$$

Each of these models serves a specific purpose, from allowing complete flexibility in estimating flows to providing strict controls on either the supply side (origin) or the demand side (destination), or both.
Two common cost functions, Inverse Power-law and Negative Exponential (Wilson, 1971) are:

Inverse Power: $f(d_{ij}) = d_{ij}^{-\beta}$

Negative Exponential: $f(d_{ij}) = exp(-\beta d_{ij})$

2.1.2 Model calibration

The Spatial Interaction Model is particularly suited for examining transport flows that are influenced by the distribution of population sizes and job opportunities, as evidenced by Origin-Destination (OD) matrix data. This model represents the commuting behaviors, especially in the utilization of subway stations where these flows are predominantly shaped by the socioeconomic traits of the neighboring regions.

In scenarios where the residential population remains stable, it is logical to use an origin-constrained model. This model presumes that the total amount of trips originating from each site remains constant. Given the stability of residential populations, this assumption of unchanging commuter numbers from each origin point is typically reliable, thus providing a stable foundation for the model. With the origin figures held constant, this approach allows researchers and urban planners to focus on how variations in the destinations, such as fluctuations in job availability, affect commuting trends.

A Poisson log-linear regression technique is used. This approach is particularly suitable for managing data on commuting flows, which consist of non-negative integers. Moreover, it aids in the precise calibration of model parameters, ensuring they accurately depict the observed patterns in commuter behavior.

In this model, the negative exponential law is applied to the distance cost function, based on observations that in urban underground transport systems, the influence of distance on commuter behavior is less marked. Thus, the decay effect of distance on interactions does not require the sharper decline that would be typical of an inverse power law.

Origin-constrained Model can be written as:

$$T_{ij} = A_i O_i D_j^{\gamma} f(d_{ij}) \text{ subject to } O_i = \sum_i T_{ij}$$

The regression equation for calibrating and estimating is (Batty and Mackie, 1972):

$$\lambda_{ij} = exp(\alpha_i + \gamma ln D_j - \beta ln d_{ij})$$

```
                 Generalized Linear Model Regression Results
==============================================================================
Dep. Variable:                  flows   No. Observations:                61413
Model:                            GLM   Df Residuals:                    61013
Model Family:                 Poisson   Df Model:                          399
Link Function:                    Log   Scale:                          1.0000
Method:                          IRLS   Log-Likelihood:            -9.0994e+05
Date:                Sat, 20 Apr 2024   Deviance:                   1.6477e+06
Time:                        23:14:23   Pearson chi2:                 2.40e+06
No. Iterations:                     8   Pseudo R-squ. (CS):              1.000
Covariance Type:            nonrobust
===============================================================================
                                                 coef    std err          z
-------------------------------------------------------------------------------
station_origin[Abbey Road]                    -2.9143      0.041    -70.509
station_origin[Acton Central]                 -1.1621      0.029    -39.960
station_origin[Acton Town]                    -1.6131      0.017    -92.801
station_origin[Aldgate]                       -2.9430      0.020   -150.138
station_origin[Aldgate East]                  -2.8548      0.019   -151.960
station_origin[All Saints]                    -2.8783      0.037    -77.219
station_origin[Alperton]                      -1.6542      0.026    -64.731
station_origin[Amersham]                       1.0008      0.030     33.747
station_origin[Anerley]                       -1.0369      0.040    -26.044
station_origin[Angel]                         -2.5875      0.017   -156.011
station_origin[Archway]                       -1.7164      0.015   -117.258
...
distance                                      -0.0002   1.88e-07   -814.175
===============================================================================
nega beta 1.53e-04
nega gamma -7.55e-01
```

Figure 10. Regression result

The result of the calibration return a 0.755 for γ and 0.000153 for β.
Thus, our final spatial interaction model as the negative exponential origin-constrained model is:

$$T_{ij} = A_i O_i D_j^{0.755} exp(-0.000153 d_{ij})$$

Table 7. Model performance

Model	Decay	R-squared	RMSE
Unconstrained model	Inverse power	0.321	108.355
	Negative exponential	0.362	105.722
Origin-constrained Model	Inverse power	0.388	102.893
	Negative exponential	**0.468**	**96.263**
Destination-constrained Model	Inverse power	0.350	106.012
	Negative exponential	0.399	102.168
Doubly-constrained Model	Inverse power	0.355	110.283
	Negative exponential	0.372	99.576

To ensure the selecting accuracy of spatial interaction model, we calculate the performance index. The model we calibrate performs the best overall, having the highest R^2 value (0.468) and the lowest RMSE (96.263), which is more effective at capturing the actual patterns of spatial interactions.

2.2 Scenarios

2.2.1 Scenario A: Decrease in Jobs at Canary Wharf

Following Brexit, Canary Wharf experiences 50% decrease in available jobs, dropping from 58772 to 29386. Utilizing the calibrated parameter β, we recalculated the commuter flows to reflect this change. To maintain consistency in the total number of commuters departing from each origin station before and after the job reductions at Canary Wharf, we recalculate the set of origin balancing factors A_i with the basis of calibrated spatial interaction model.

$$A_i = \frac{1}{\sum_j D_j^{\gamma} d_{ij}^{-\beta}}$$

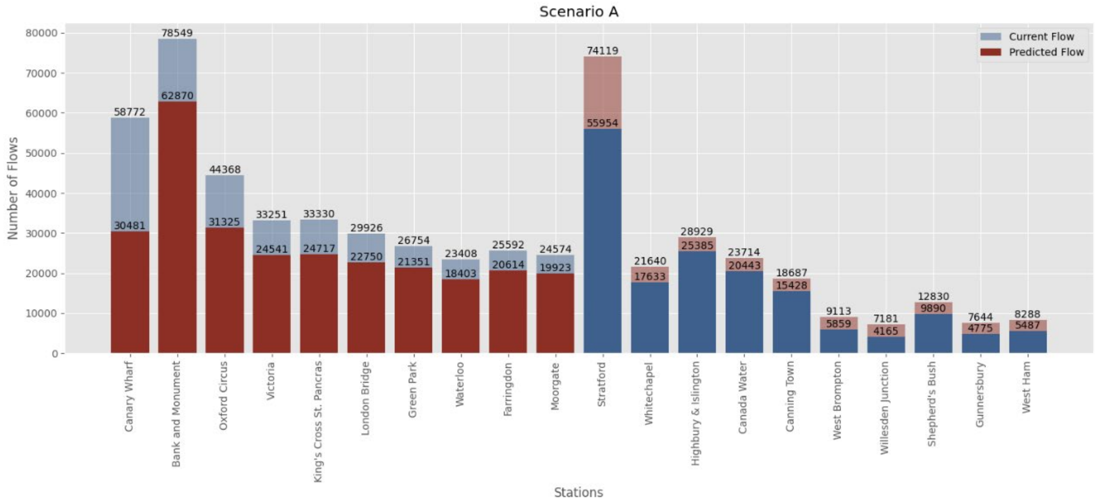
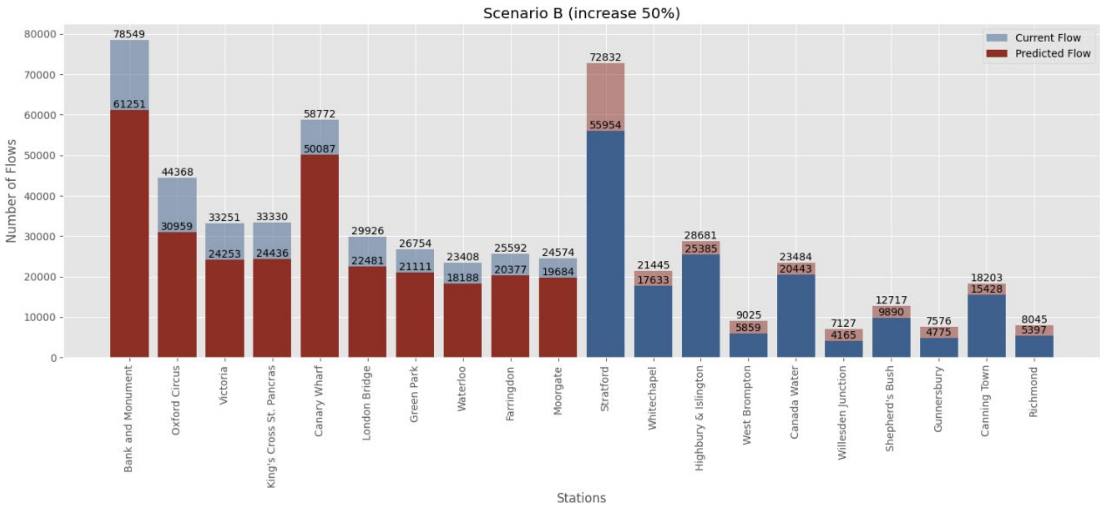


Figure 11. ScenarioA

Figure 11 shows the current and predicted flow of commuters at various stations, especially when external economic changes like Brexit can lead to significant shifts in commuter patterns. It also highlights the importance of flexible transit systems that can adapt to changing economic landscapes.

2.2.2 Scenario B: Increase in Transport Cost

In Scenario B, we adjust for a significant rise in transportation costs, using distance as a proxy for these costs. Consequently, we modify our distance scaling parameter β, increasing it by 50% and 100% to better capture the increased sensitivity to distance. This scenario is likely to affect not only commuting patterns but also employment decisions. Higher transportation costs could make certain job locations less appealing due to the added expense and time of commuting, potentially leading individuals to seek jobs closer to home or in more accessible locations. As with Scenario A, we will ensure that the total number of commuters remains unchanged when recalculating the origin balancing factors A_i.



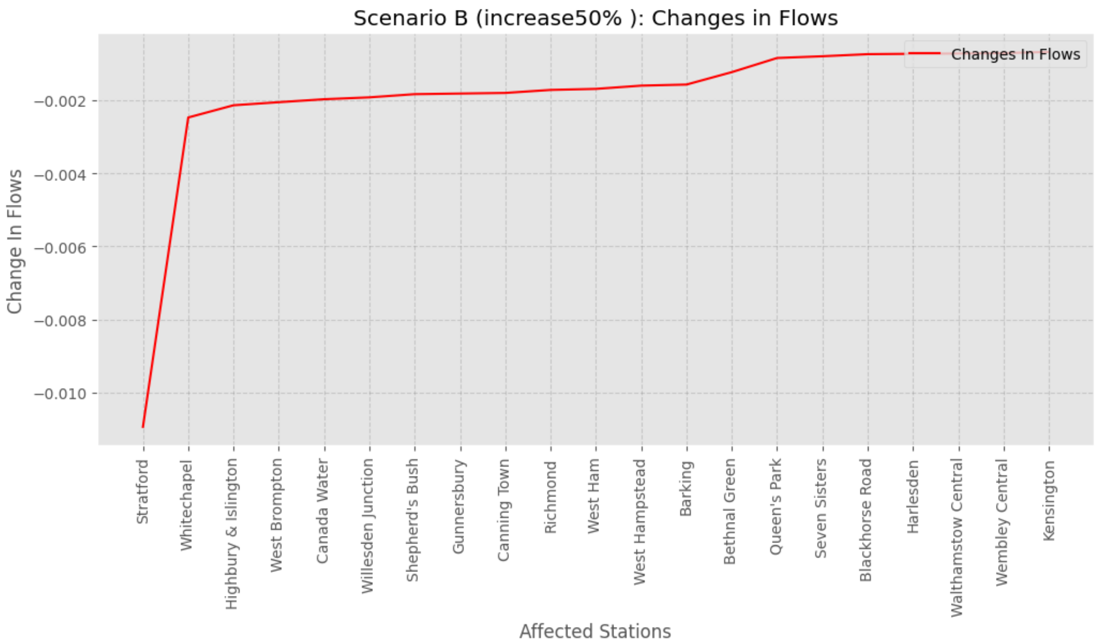
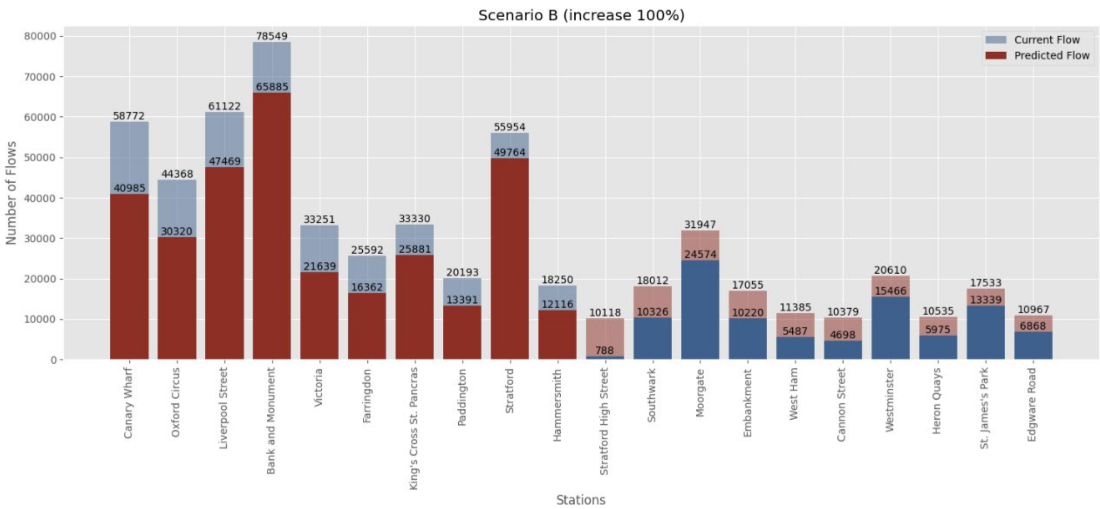


Figure 12. **ScenarioB1**

Canary Wharf exhibits a marked reduction in commuter flow, a clear sign of how heightened costs are making the commute less appealing. Similarly, central stations like Bank and Monument and Oxford Circus, which are highly connected, also see a significant decrease in predicted flow. This might suggest that commuters are looking for alternatives to costly or lengthy commutes.

The drop in flow isn't consistent across all stations. Some stations experience a less severe decrease or even maintain their existing flow levels, indicating a possible shift in commuter destinations or changes in commuter behavior, such as choosing job locations closer to home or opting for different modes of transportation.



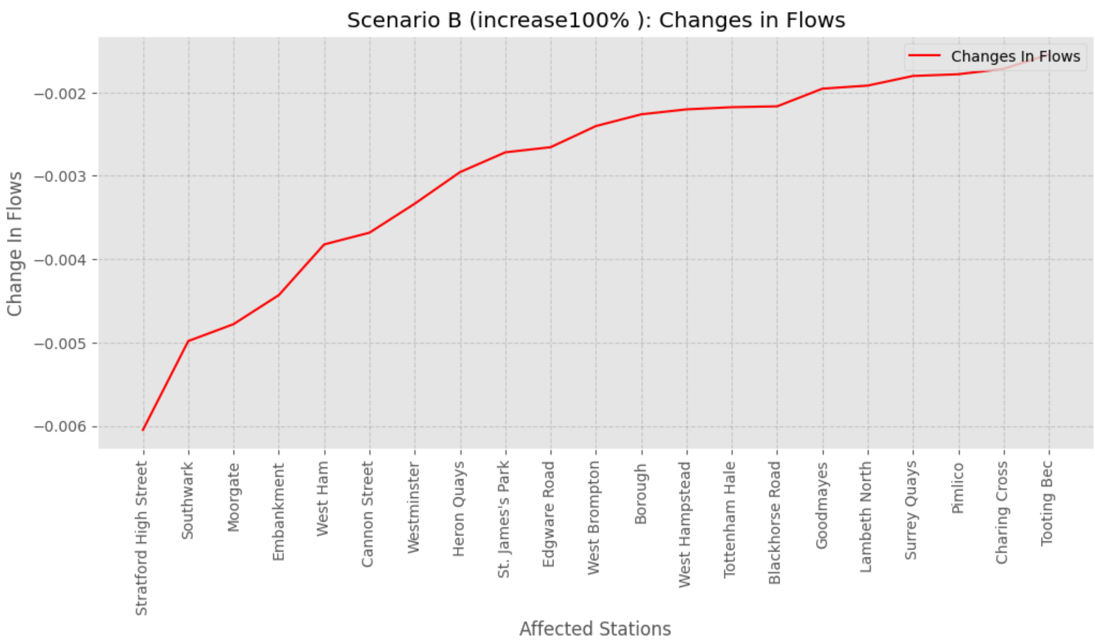


Figure 13. ScenarioB2

2.2.3 Changes in Flows from Scenarios

The three scenarios present different circumstances and their respective impacts on commuter flows, each driven by distinct changes in either job availability or transportation costs.

In post-Brexit Scenario A, the impact of job reduction is direct and significant at Canary Wharf, but the redistribution of flows might not be as extensive across the network, as the overall commuting population hasn't changed, just the attractiveness of one destination.

In Scenario B, two different parameter selections indicate a 50% and then a 100% increase in transport costs. These changes impact commuter behavior across the entire network due to the universal rise in commuting expenses, not just a single destination.

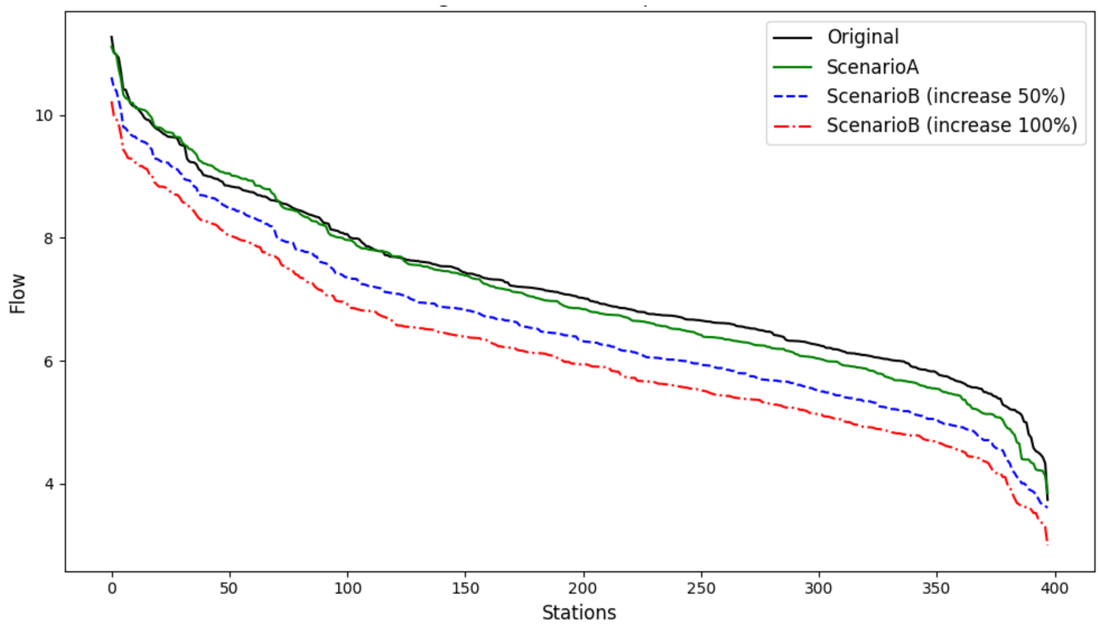


Figure 14. Comparison

In conclusion, while Scenario A's impact is more localized to Canary Wharf, Scenario B's alterations in transport costs have a network-wide impact on commuter flows. The 100% increase in Scenario B would probably result in the most significant redistribution of flows as it affects the entire network and alters the fundamental economics of commuting, pushing commuters to re-evaluate their travel and employment choices more broadly. This scenario underscores the sensitivity of commuter flows to transportation costs and the potential need for systemic adjustments in urban transport policy and infrastructure planning.

For future research, Transport for London (TfL) should examine the impact of annual ticket price increases, as well as consider adjusting the timing of off-peak traffic discounts based on varying traffic flows throughout the day to better understand of how pricing strategies influence commuter behavior and network efficiency.

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