

Urban Simulation 5

a. Cellular Automata

Physical Model Representations of Urban Systems

Michael Batty

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Outline of Today's Lecture

- Some References to Begin with
- Representing City Systems: Points, Lines, Polygons, Cells, Agents: Statics v Dynamics
- A Recapitulation about Space Filling and Fractal Systems: I indicated this last week
- Early One Dimensional Automata – our first programming interlude
- The Game of Life: John Conway's Contribution
- Applications through Cellular Automata – our second programming interlude
- Different Model Applications: DUEM – Demo
- Moving to Agent-Based Models: Schelling's Model

Some References to Begin With

Cellular Automata and Urban Form: A Primer

Michael Batty

Artificial processes for locating urban activities based on simple rules pertaining to local circumstances give rise to complex global patterns that mirror the spatial organization of cities. These systems are called Cellular Automata (CA). They provide a useful means of articulating the way highly decentralized decision-making can be employed in simulating and designing robust urban forms. CA can be easily programmed in a variety of software, and as such provide a suggestive way of exploring actual as well as optimal patterns and plans. This primer provides a pedagogic guide to these ideas and to potential computer applications.

Batty is Director of the Centre for Advanced Spatial Analysis and a professor in the Bartlett School and Department of Geography, University College, London. He has been a professor and Chair of the Department of City Planning in the University of Wales at Cardiff, where he also acted as Dean (1979–1989), and was Director of the National Center for Geographic Information and Analysis at the State University of New York at Buffalo, New York from 1990 until 1995. His most recent books are *Fractal Cities* (Academic Press, 1994) and *Spatial Analysis: Modelling in a GIS Environment* (Pearson Geoinformation, 1996). He is the editor of the journal *Environment and Planning B*.

Journal of the American Planning Association, Vol. 63, No. 2, Spring 1997. ©American Planning Association, Chicago, IL.

266 APA JOURNAL • SPRING 1997

M. Batty (1997) Cellular Automata and Urban Form: A Primer, **AIP Journal**, 63, 266-274

Sketch Planning as Computational Pedagogy

Computer models of cities either attempt to simulate existing urban form or provide procedures for the design of optimal forms, but rarely both. The mechanisms used to model actual cities usually embody local behavioral descriptions without explicit optimizing,¹ whereas those that produce idealized forms seek to optimize in a more global fashion, often mirroring the viewpoint of the designer. Recently, however, a class of models has emerged that has the potential to represent both. By replacing traditional mathematical functions with rule-based procedures, functions of many kinds can be reduced to rules that mirror how actual systems work and how they might work under idealized conditions. Furthermore, as rule-based systems can be built up from the simplest modules, it is possible to strip real systems down to their fundamentals and concentrate on their essential working.

An excellent example of this discipline is based on a class of models called Cellular Automata, or CA for short. CA are models in which contiguous or adjacent *cells*, such as those that might comprise a rectangular grid, change their *states*—their attributes or characteristics—through the repetitive application of simple rules. CA models can be based on cells that are defined in more than 2 dimensions, but the 2-d form that makes them applicable to cities is the most usual. The rules for *transition* from one cell state to another can be interpreted as the generators of growth or decline, such as the change from an undeveloped to a developed cell or vice versa. This change is a function of what is going on in the *neighborhood* of the cell, the neighborhood usually being defined as immediately adjacent cells, or cells that “in some sense” are nearby. Urban growth and decline in real city neighborhoods provide excellent examples.

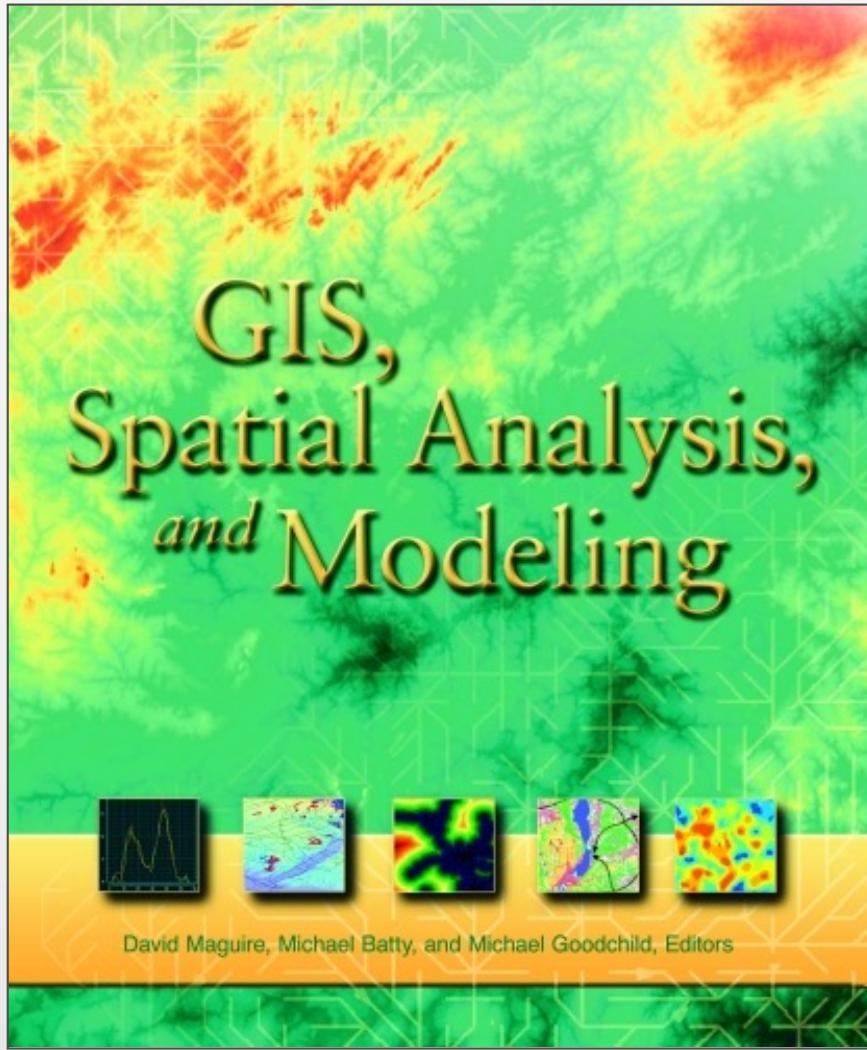
Cities and Complexity

Understanding Cities with Cellular Automata, Agent-Based Models, and Fractals

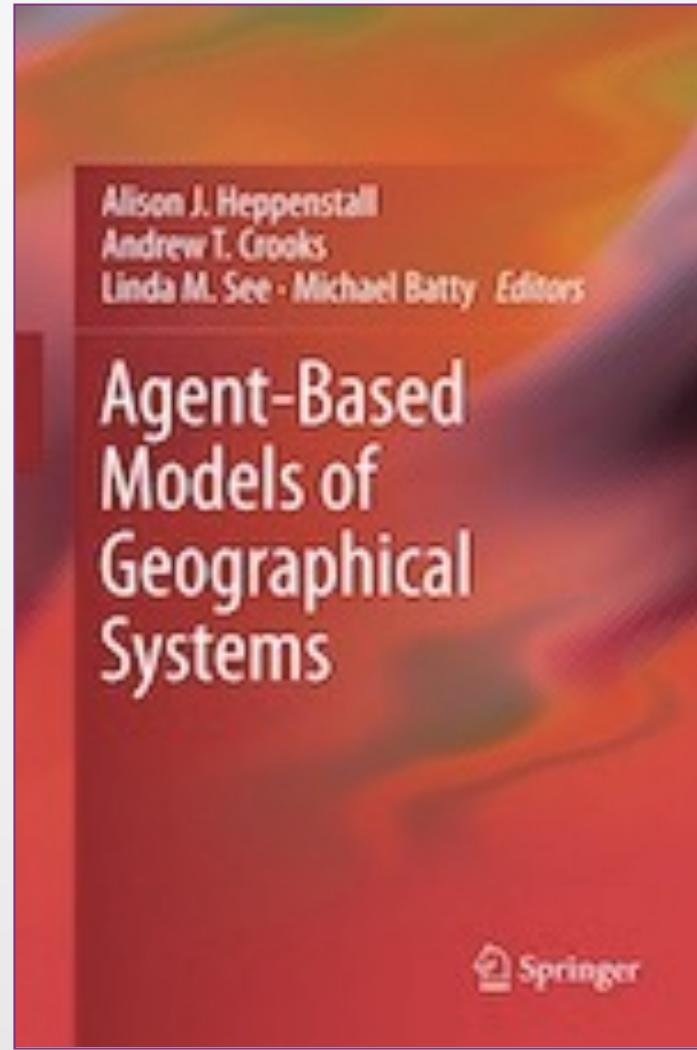


Michael Batty

M. Batty (2005) **Cities and Complexity**, The MIT Press, Cambridge, MA.



Batty, M., and Y. Xie (2005) Urban Growth using Cellular Automata Models, in D. J., Maguire, M. Batty and M. F. Goodchild (Editors) **GIS, Spatial Analysis, and Modeling**, ESRI Press, Redlands, CA, 151-172.



Springer, 2011

 Article

Modelling urban change with cellular automata: Contemporary issues and future research directions

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Michael Batty
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Siqin Wang
University of Queensland, Australia

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Abstract
The study of land use change in urban and regional systems has been dramatically transformed in the last four decades by the emergence and application of cellular automata (CA) models. CA models simulate urban land use changes which evolve from the bottom-up. Despite notable achievements in this field, there remain significant gaps between urban processes simulated in CA models and the actual dynamics of evolving urban systems. This article identifies contemporary issues faced in developing urban CA models and draws on this evidence to map out four interrelated thematic areas that require concerted attention by the wider CA urban modelling community. These are: (1) to build models that comprehensively capture the multi-dimensional processes of urban change, including urban regeneration, densification and gentrification, in-fill development, as well as urban shrinkage and vertical urban growth; (2) to establish models that incorporate individual human decision behaviours into the CA analytic framework; (3) to draw on emergent sources of 'big data' to calibrate and validate urban CA models and to capture the role of human actors and their impact on urban change dynamics; and (4) to strengthen theory-based CA models that comprehensively explain urban change mechanisms and dynamics. We conclude by advocating cellular automata that embed agent-based models and big data input as the most promising analytical framework through which we can enhance our understanding and planning of the contemporary urban change dynamics.

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Liu, Y., Batty, M., Wang, S., and Corcoran, J. (2021) Modelling urban change with cellular automata: Contemporary issues and future research directions, *Progress in Human Geography*, 45(1), 3-24

Heppenstall, A., Crooks, A., Malleson, N., Manley, E., Ge, J. & Batty, M (2021) Future Developments in Geographical Agent-Based Models: Challenges and Opportunities, *Geographical Analysis*, 53, 76-91,

geographical analysis

Geographical Analysis (2021) 53, 76–91

Special Issue

Future Developments in Geographical Agent-Based Models: Challenges and Opportunities

Alison Heppenstall^{1,2} , Andrew Crooks³, Nick Malleson^{1,2}, Ed Manley^{1,2}, Jiaqi Ge¹, Michael Batty⁴

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Despite reaching a point of acceptance as a research tool across the geographical and social sciences, there remain significant methodological challenges for agent-based models. These include recognizing and simulating emergent phenomena, agent representation, construction of behavioral rules, and calibration and validation. While advances in individual-level data and computing power have opened up new research avenues, they have also brought with them a new set of challenges. This article reviews some of the challenges that the field has faced, the opportunities available to advance the state-of-the-art, and the outlook for the field over the next decade. We argue that although agent-based models continue to have enormous promise as a means of developing dynamic spatial simulations, the field needs to fully embrace the potential offered by approaches from machine learning to allow us to fully broaden and deepen our understanding of geographical systems.

Introduction

Individual-based methods, in particular agent-based models, have seen a rapid uptake by researchers across the social and geographical sciences in the past 20 years (Macal 2016). Agent-based models were first formally proposed in the early 1990s (e.g., Epstein and Axtell 1996) but their lineage goes back much further to the development of models of individual locational decision-making in the 1950s and 1960s in the influential work of Hagerstrand (1953), Donnelly et al. (1964), and Schelling (1969) among others. They are now reaching a point of acceptance as a research tool across the geographical and social sciences, exploring such phenomena as epidemiology (Shook and Wang 2015), invasive species (Anderson and Dragičević 2020), settlement patterns (Bura et al. 1996), and segregation (Benenson and Hatna 2011) (see Polhill et al. 2019 for further discussion on the applications of ABMs and their use in policy).

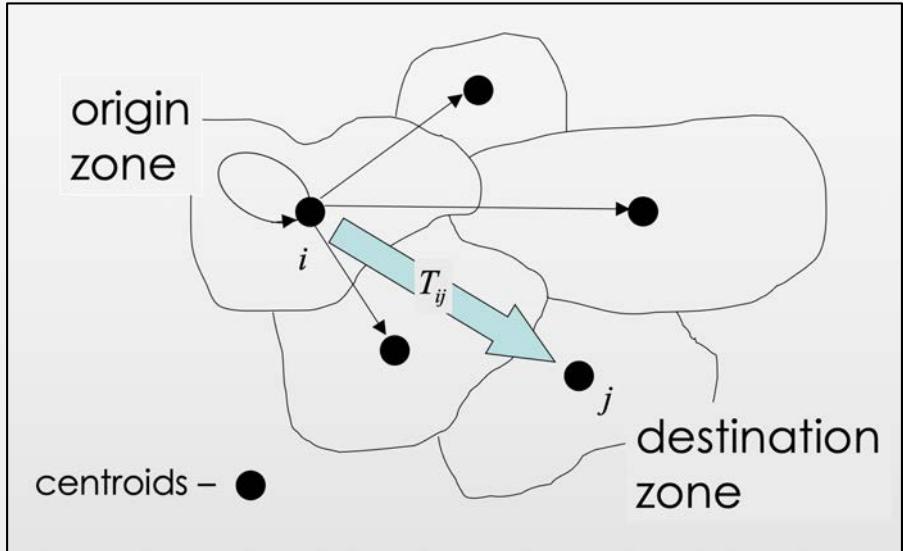
However, there remain significant methodological challenges, for example, in: recognizing and simulating emergent phenomena; agent representation; construction of behavioral rules;

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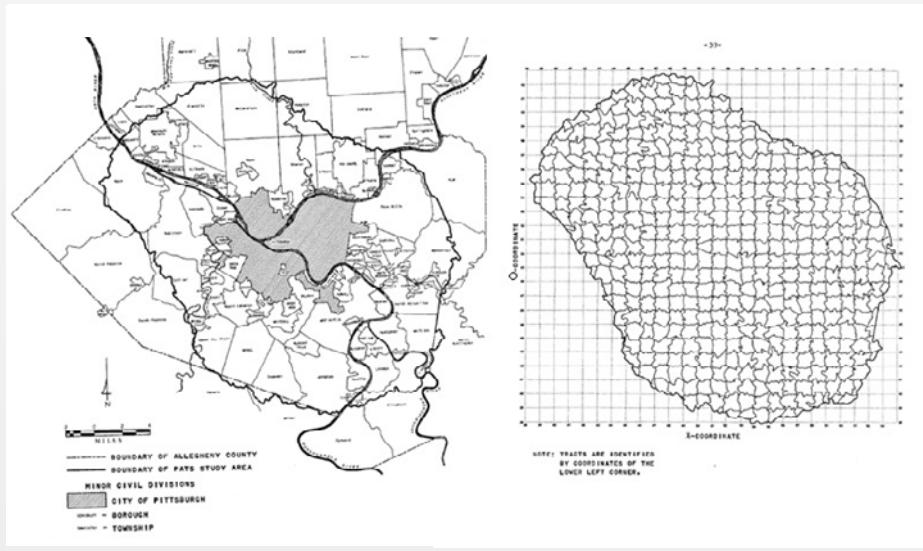
Submitted: November 13, 2019; Revised version accepted: November 16, 2020

76 doi: 10.1111/gean.12267

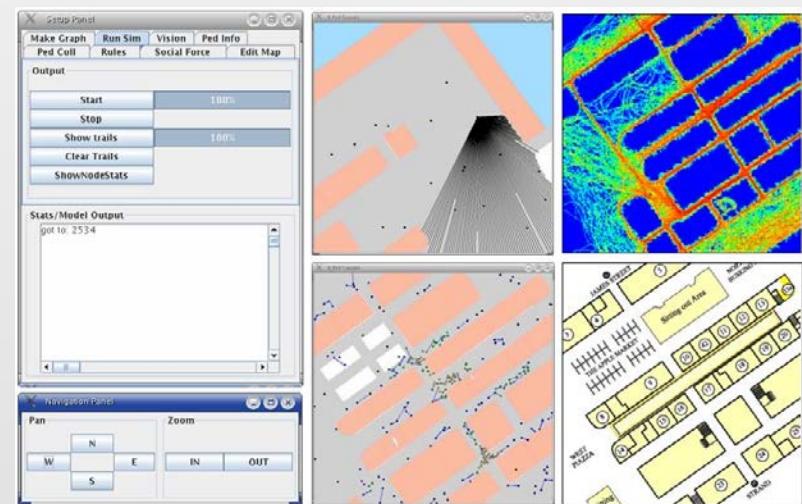
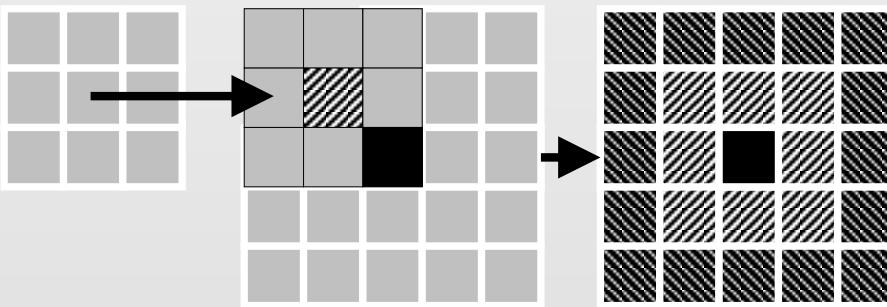
Representing City Systems: Points, Lines, Polygons, Cells, Agents: Statics v Dynamics



Movement in space – simple dynamics



Transitions in time across space



A Recapitulation about Space Filling and Fractals

Indeed most of our models are about space filling rather than time filling – time is added on as it is difficult --- I talked about fractals and how objects fill space in different ways last week and I need to give you some simple illustrations to begin with as context

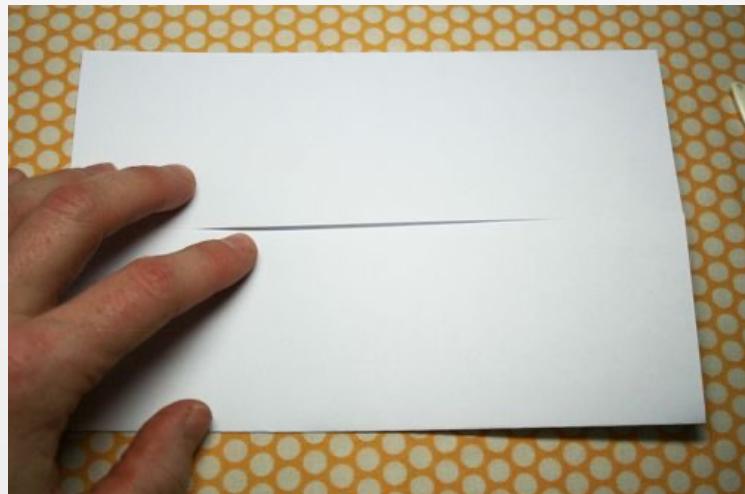
This is because cellular automata are excellent ‘machines’ or models – called automata because they operate autonomously over and over again -- for generating fractal shapes – and it will become clear from the pictures as to what is happening in terms of the way space is filled and emergent patterns formed – they fill space through time

What are Fractals? Definitions and Properties

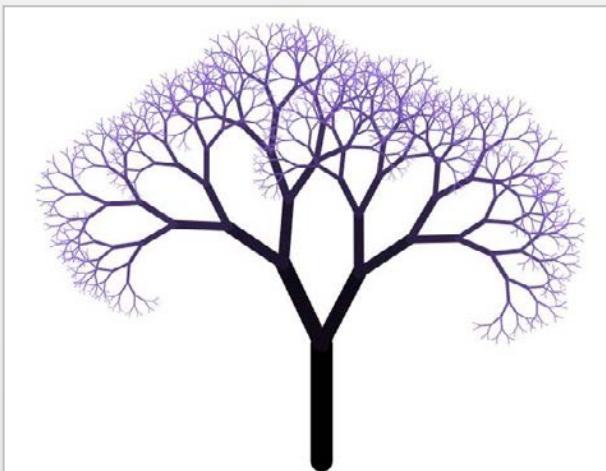
- Fractals are objects that scale – they show the same shape at different scales
- This property of scaling is sometimes called self-similarity or self-affinity
- In our world of cities, we think of this scaling as being a replication of the same shapes in 2 or 3D Euclidean space. I talked a bit about this in the first lecture and Elsa will say more.
- This suggests modularity in growth & evolution with processes uniform over many scales

- The signature of a fractal is called its dimension and usually this suggests how the fractal fills space
- If we think of 0-d as a point, 1-d as a line, 2-d as a plane and 3-d as volume, then a fractal also has *fractional dimension*.
- This means that the Euclidean world is the exception not the rule as the integral dimensions are simplifications.
- The best example of a fractal is a crumpled piece of paper

It is 2-d but when we crumple it we make it more than 2-d



Other great examples are tree structures



Cellular Automata

Here is a good example of the idea of self-similarity



Then at the idea of space-filling – how much of space gets filled – how space is scaled

- In fact in mathematics, a function is scaling if it scales under a simple transformation
- i.e. if we can scale a distance by multiplying it by 2 say and the function does not change qualitatively, then this is scaling – so power laws are scaling
- Functions like $f(y)=x^{-1}$ scale because if we multiply **x by 2**, we get $f(2y) = (2x)^{-1} = 2^{-1}x^{-1} \sim f(y)$
- So if you have a power law – this example is to a power of 1, then the scaling is proportionate; then if the power is more than 1, we have non-linear scaling where $a > 1$, if $y = x^a$ then $ky = (kx)^a = k^a x^a \sim y$ which is superlinear and more than proportionate
- They are all over the place in our world

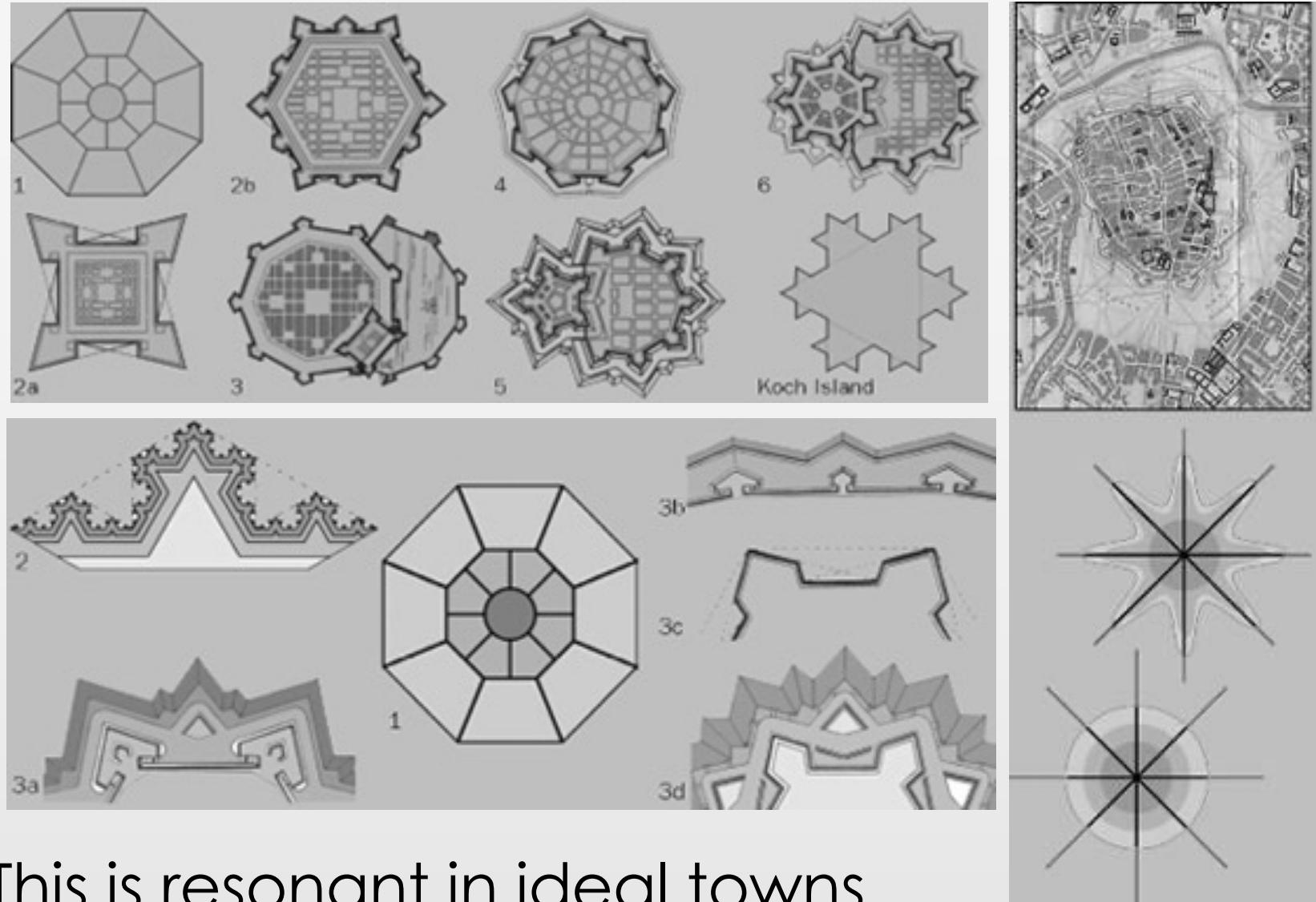
Fractal Geometries: Patterns and Processes

- There are some basic conundrums and paradoxes with fractal geometry – the clearest one is the length of a fractal line – if a line is truly fractal, it fills space more than the line and less than the plane with a fractal dimension between 1 and 2.
- As it also scales – any bit of it has the same shape as an enlarged or reduced bit but the length is infinite.

Note the famous paper in Science in 1967 by Mandelbrot –
How long is the coastline of Britain?



Cellular Automata



This is resonant in ideal towns
And in many shapes in nature as we show ...

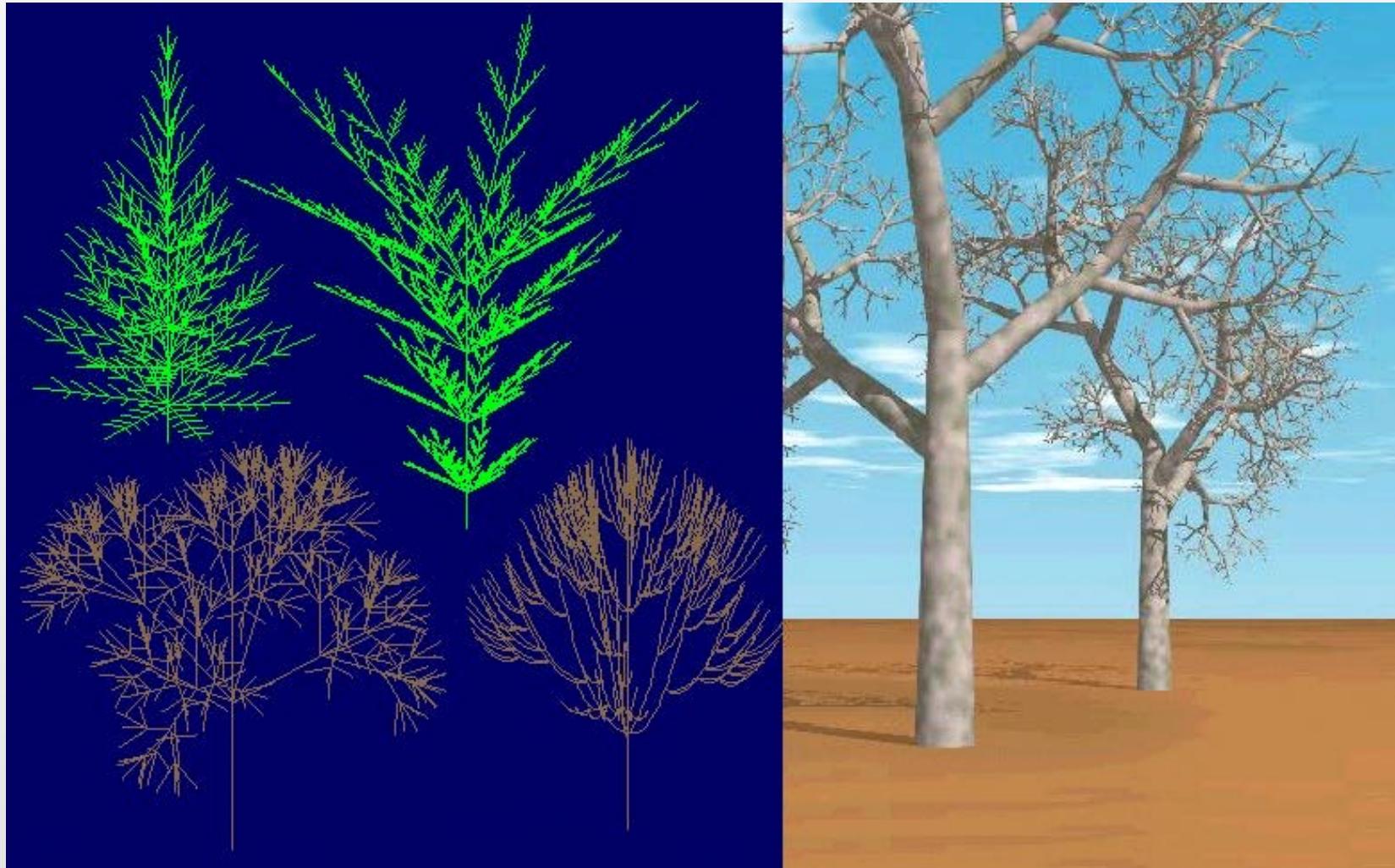


Cellular Automata

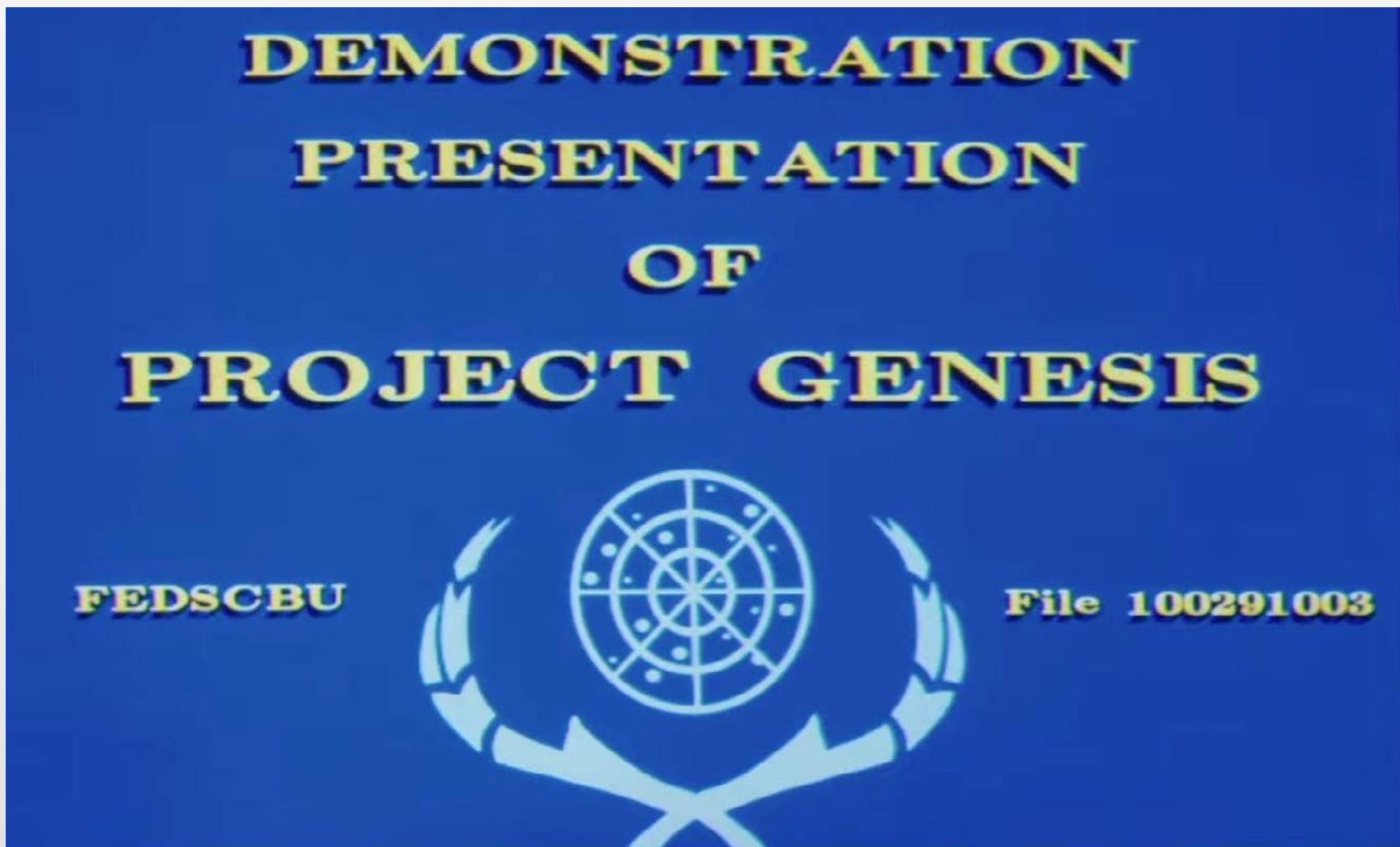


Barnsley's fern, from his book Fractals Everywhere which is generated by a rather sophisticated mathematical systems of routine and repetitive transformations called the Iterated Function System

Computer graphics depends upon fractals!
At least for natural forms such as trees



The First Computer Movie: Star Trek The Wrath of Khan

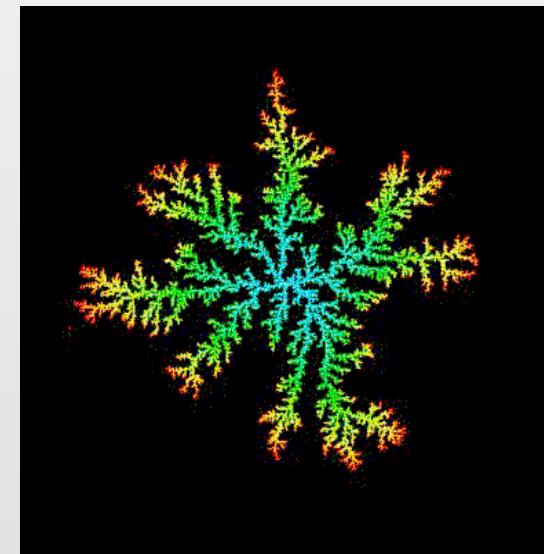
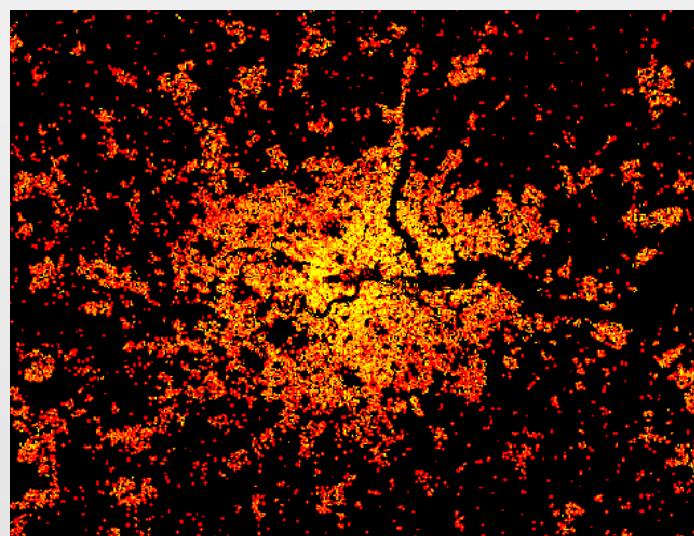
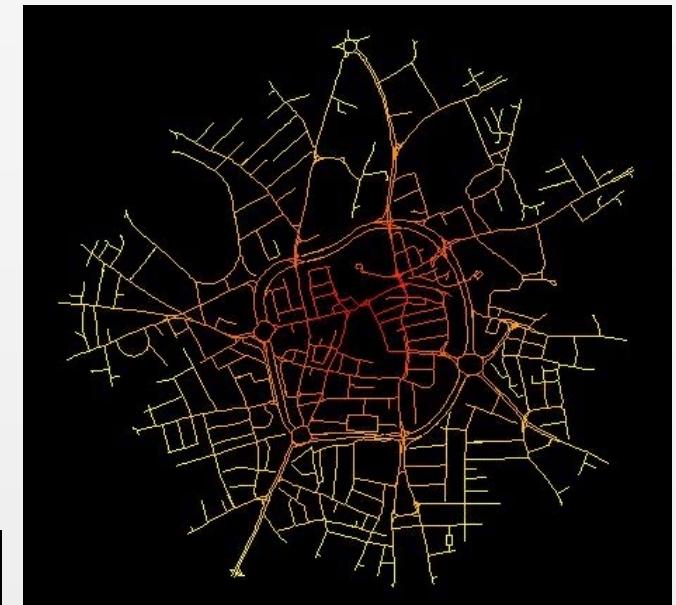
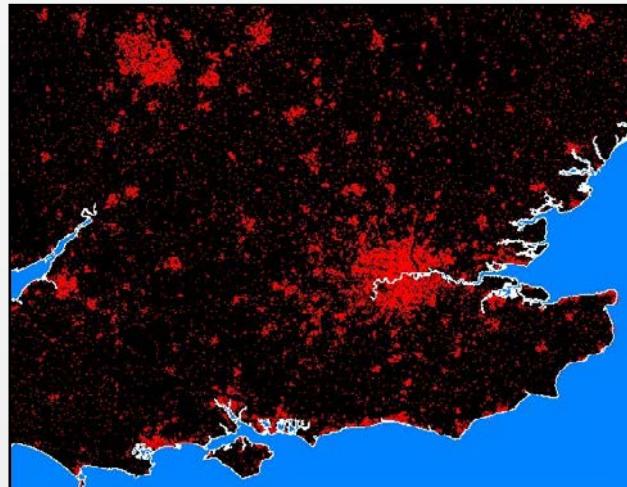
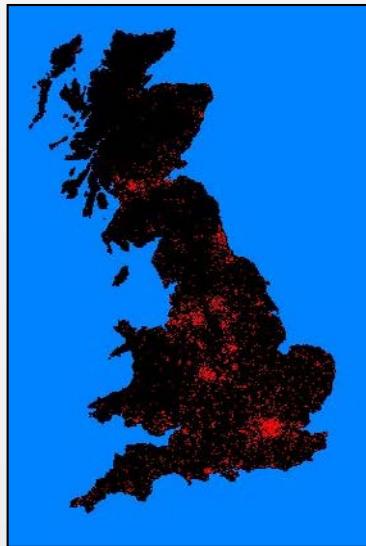


The Genesis Simulation: StarTrek: The Wrath of Kahn



https://www.youtube.com/watch?v=Tq_sSxDDE32c

City Shapes at Different Scales: Modular Growth



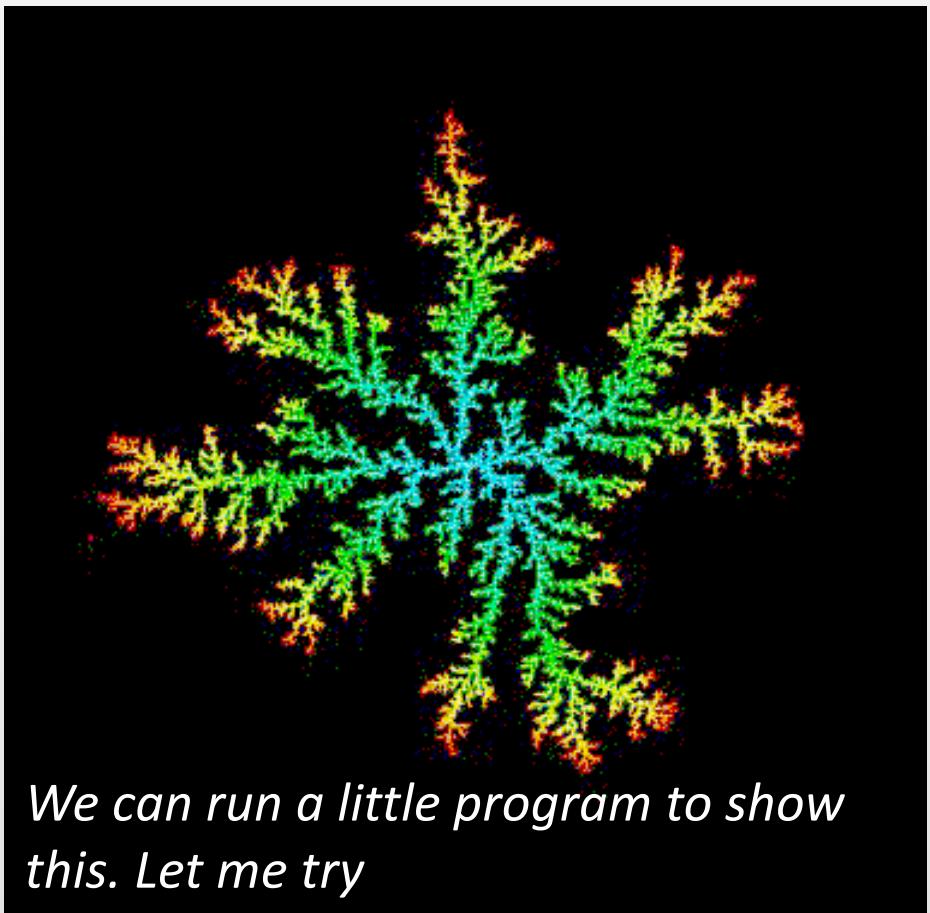
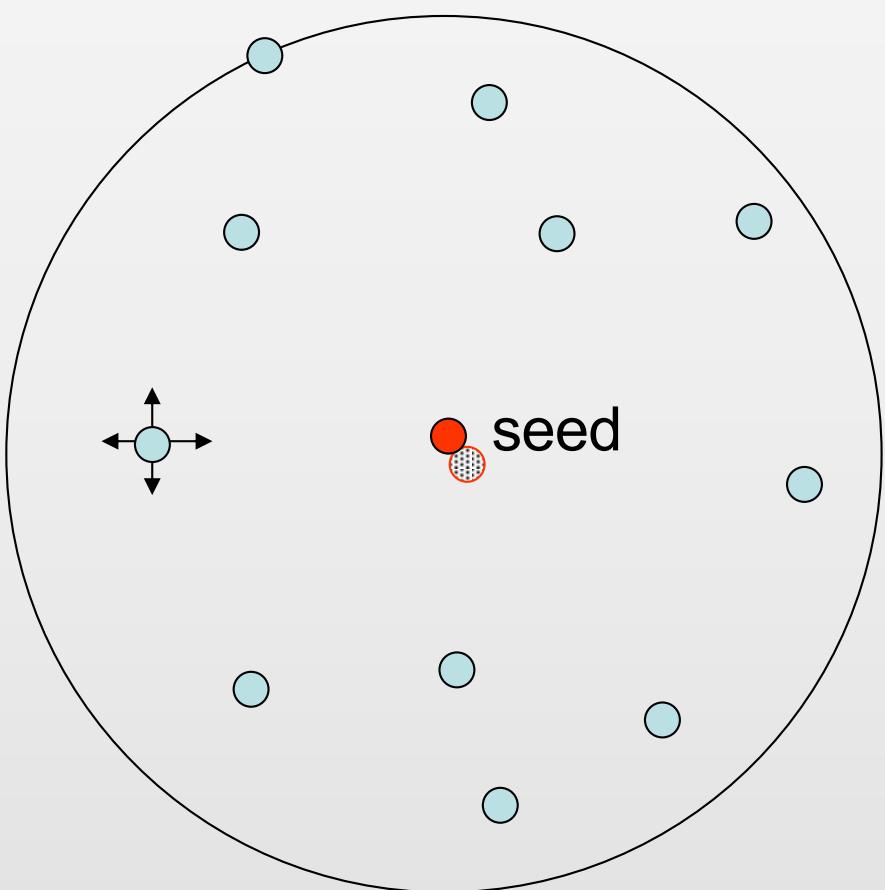
Fractal Growth Models: DLA

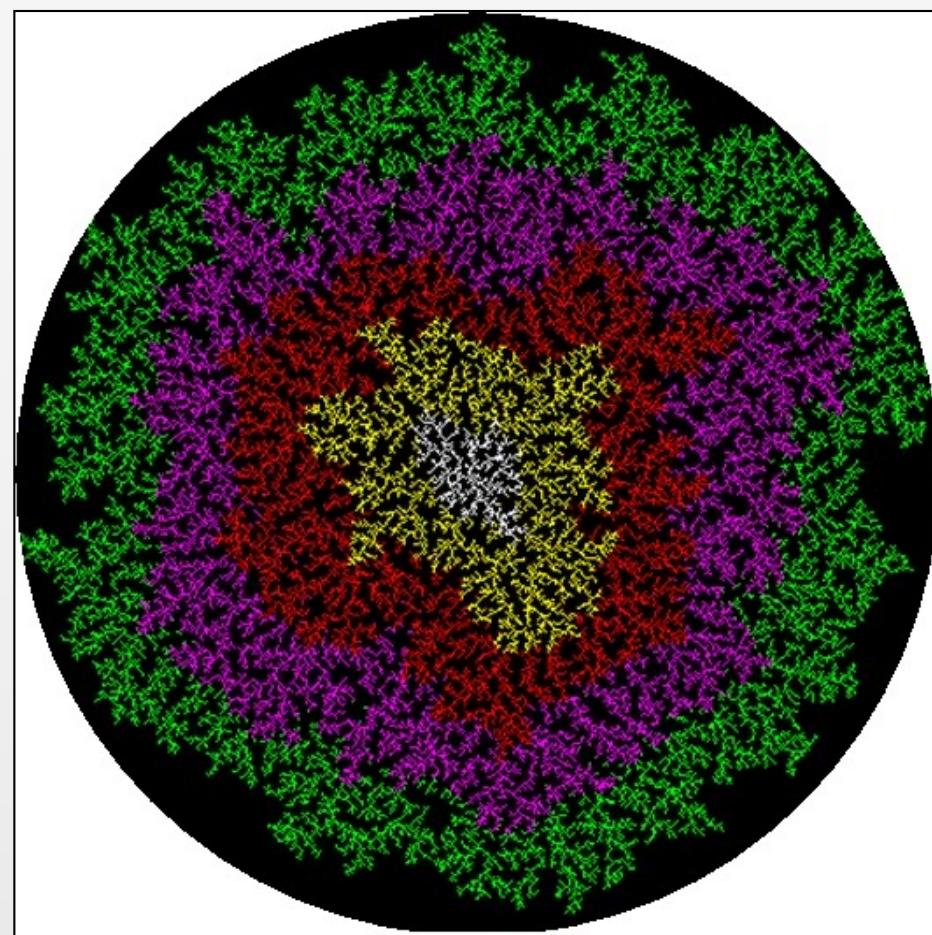
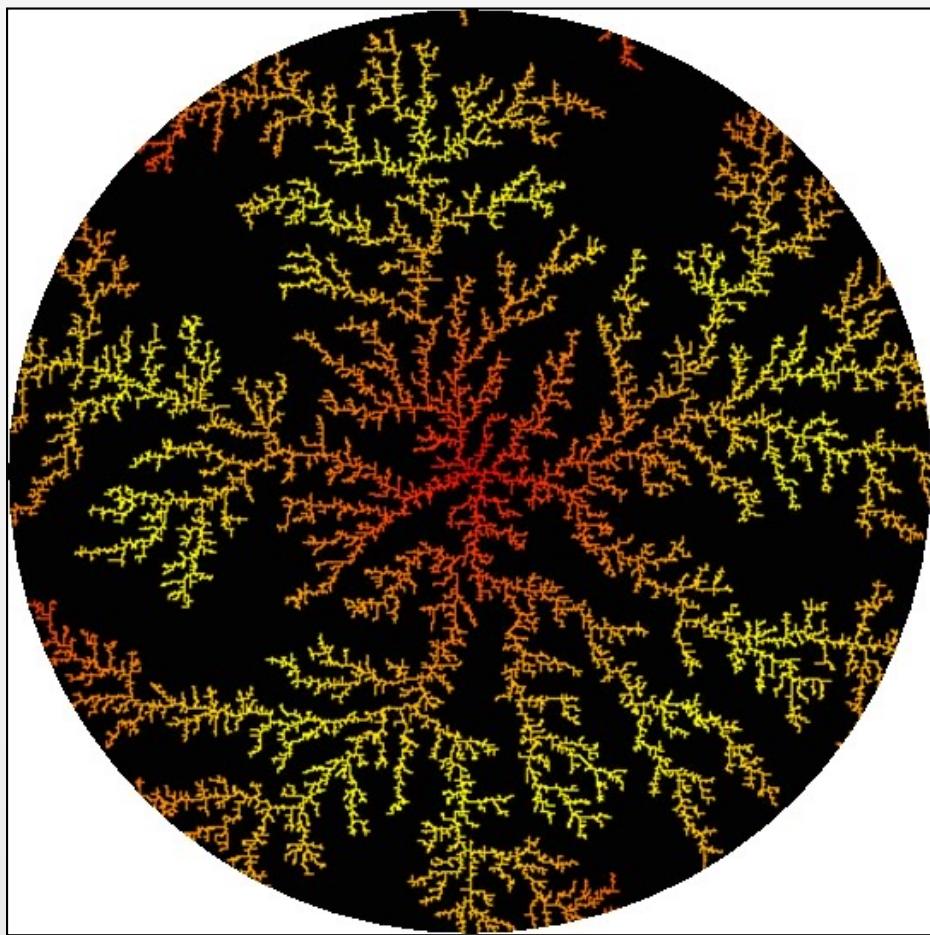
Ok, let me show you the simplest possible model of an organically growing city – based on two simple principles

- *A city is connected in that its units of development are physically adjacent*
- *Each unit of development wants as much space around it as it needs for its function.*

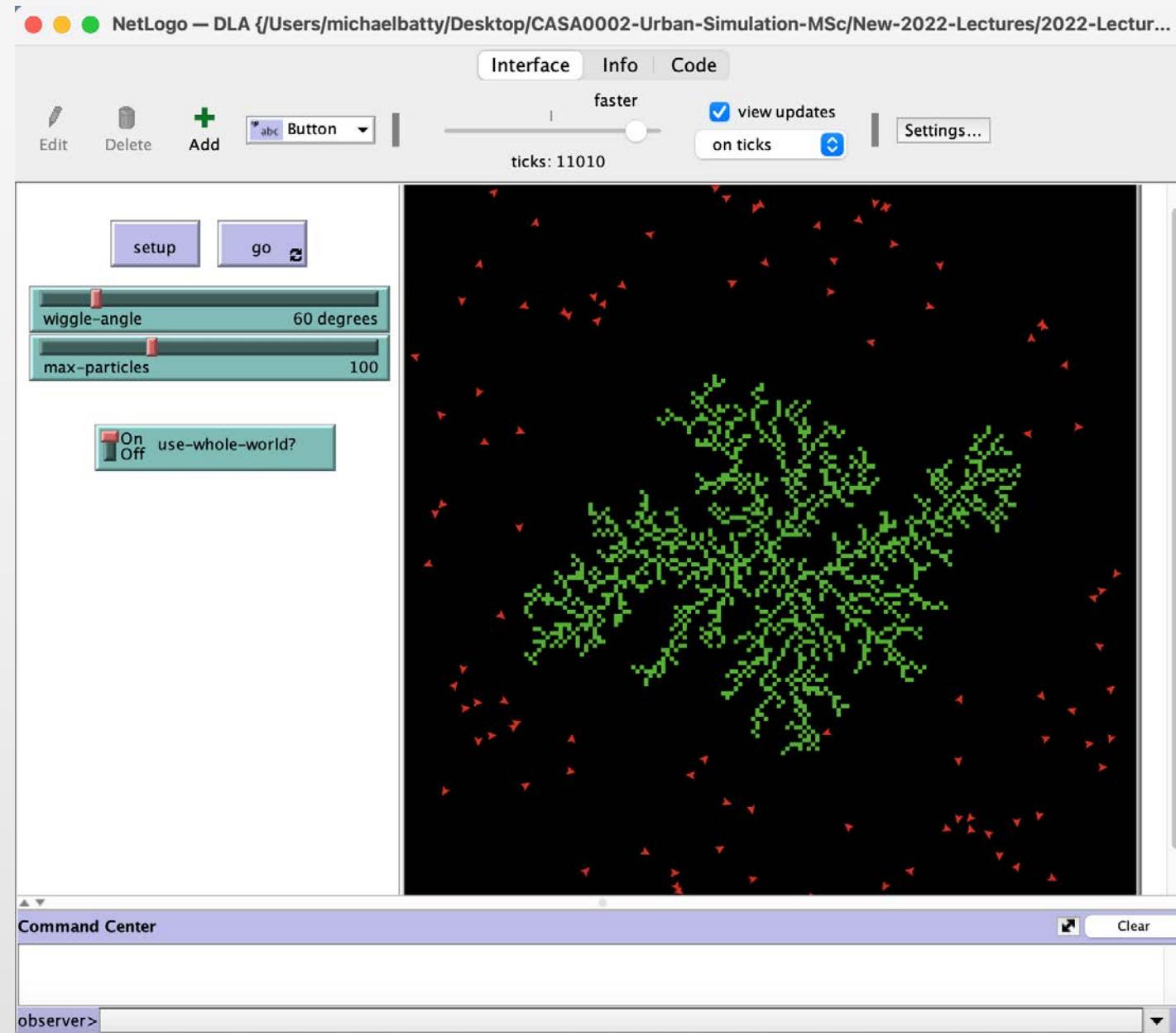
We start with a seed at the centre of a space and simply let actors or agents randomly walk in search of others who have settled. When they find someone, they stick. That is all.

In essence, this is random walk in space which is can be likened to the diffusion of particles ● around a source ● but limited to remain within the influence of the source – the city

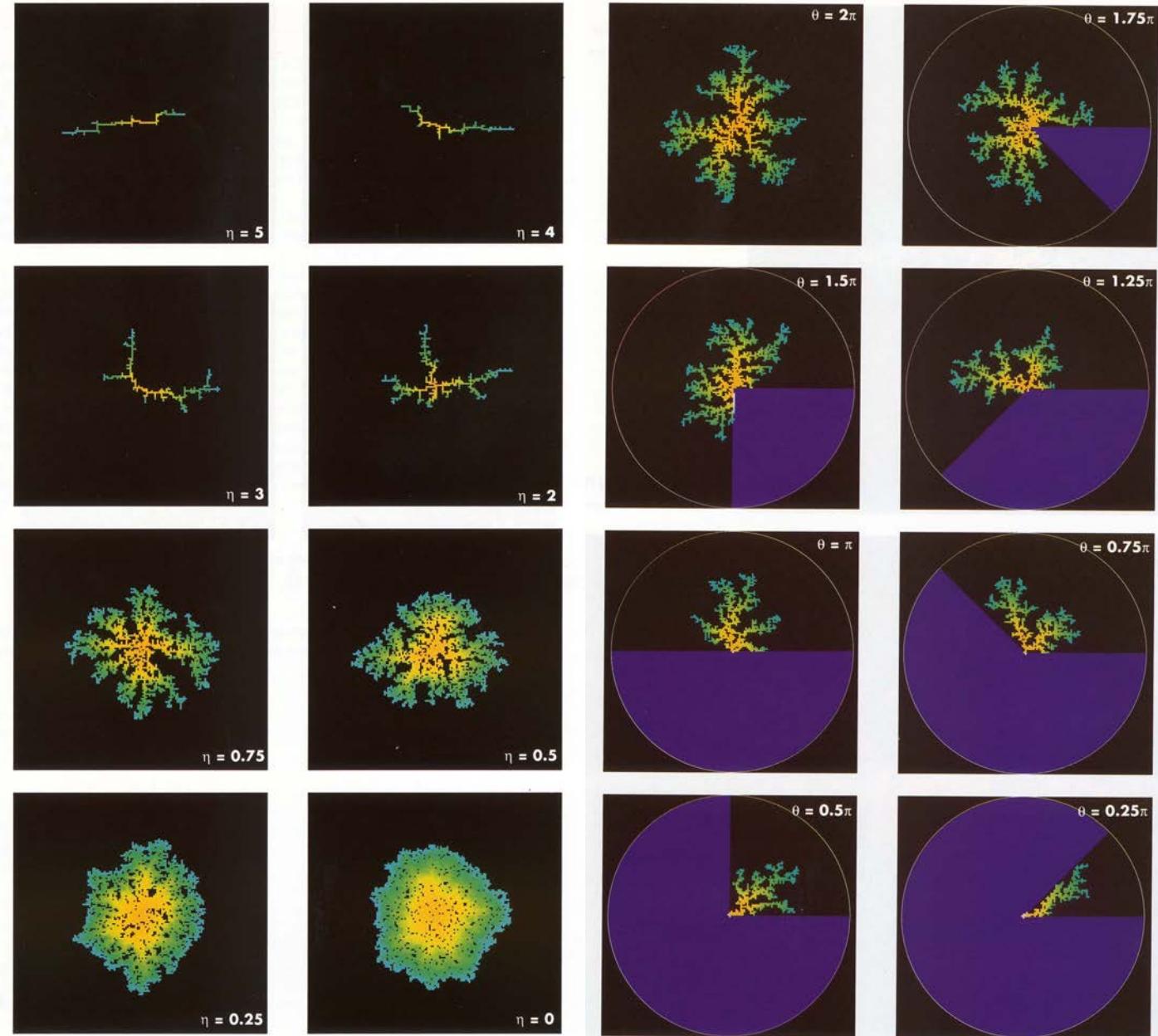




Cellular Automata



Cellular Automata



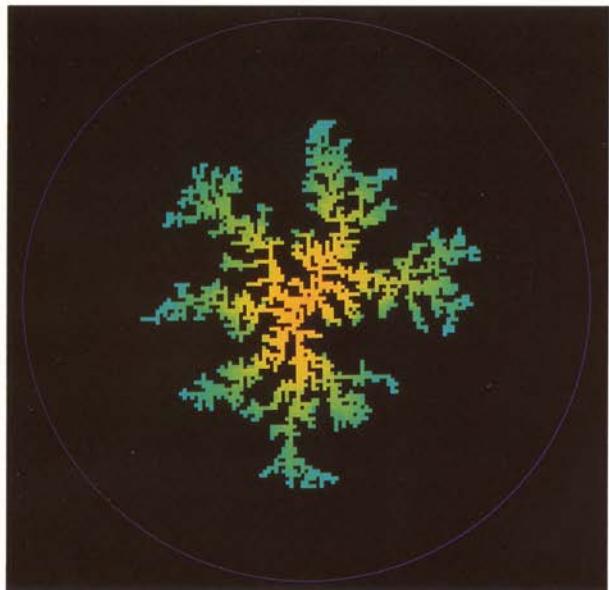


Plate 8.3 (left) The Baseline Simulation $\eta = 1$

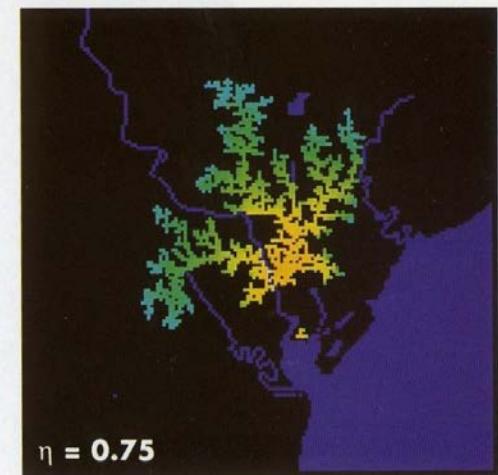
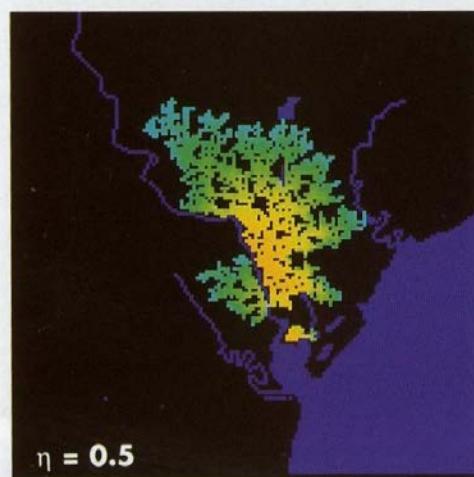
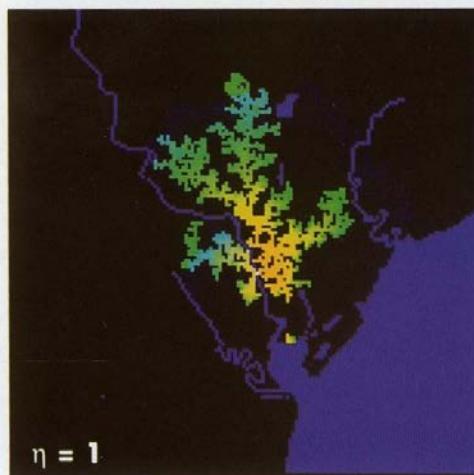
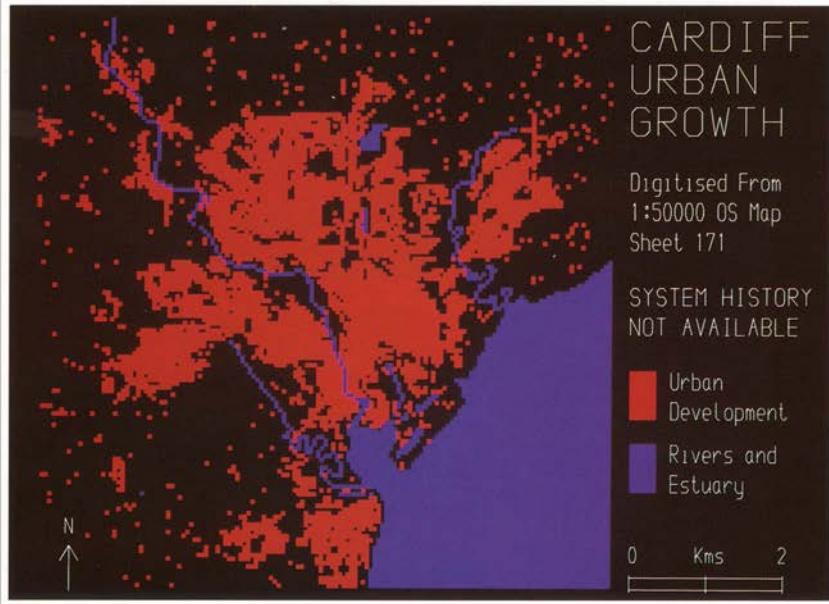
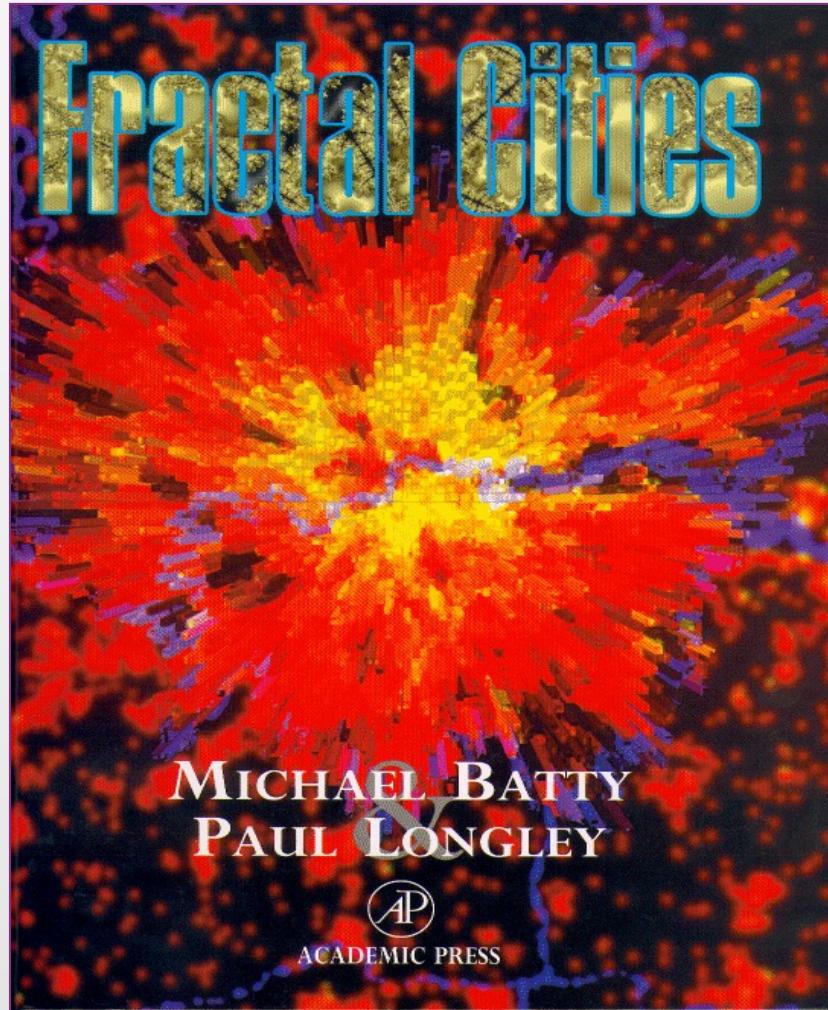


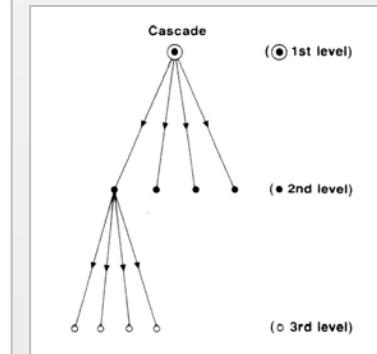
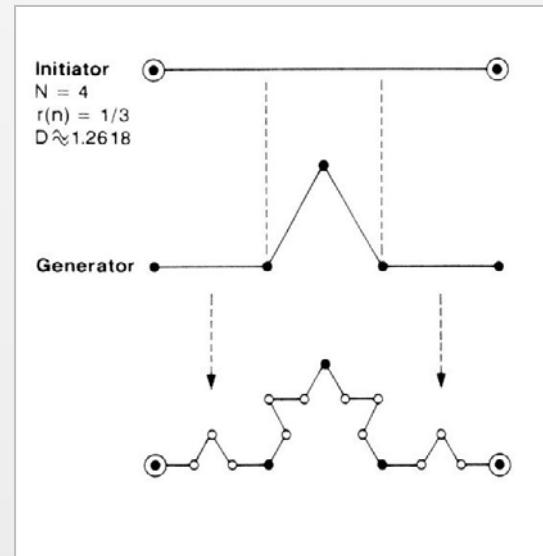
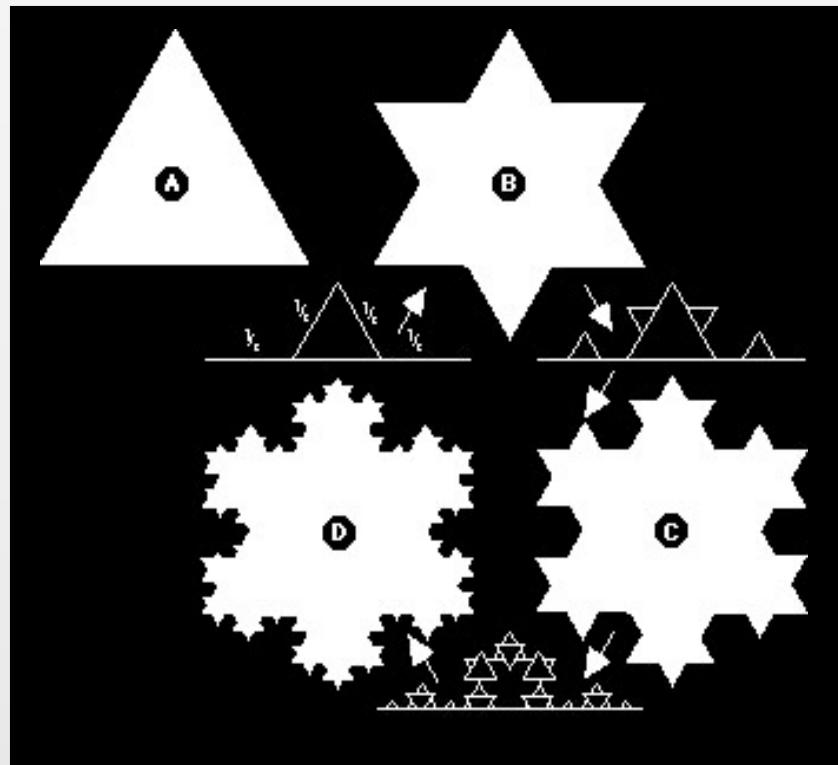
Plate 8.6 Simulating the Urban Growth of Cardiff

All this is contained in our book on
Fractal Cities. It is online



www.fractalcities.org

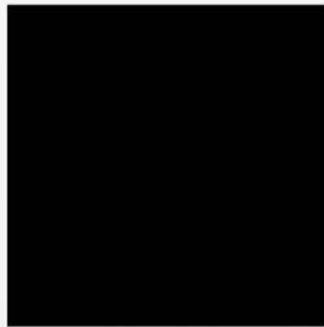
Self-similarity and space filling. Note how we construct the irregularity by adding a scaled down piece of the curve: The Koch curve



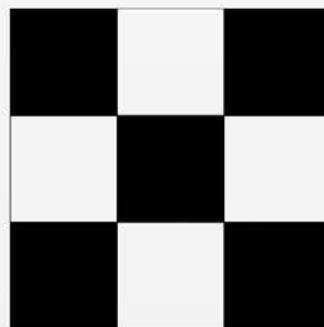
Note how hierarchy is a feature of the construction

Note how the line is infinite but the area is finite

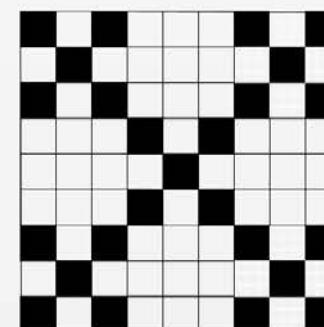
$k=0$



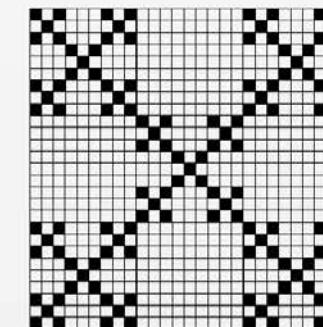
$k=1$



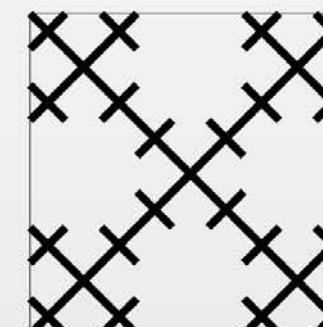
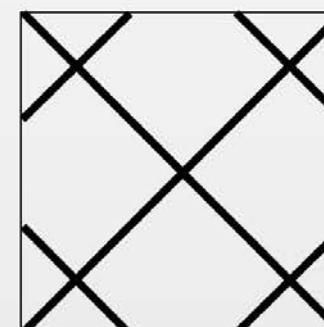
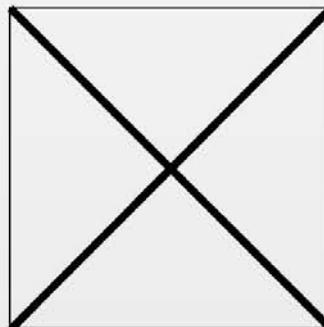
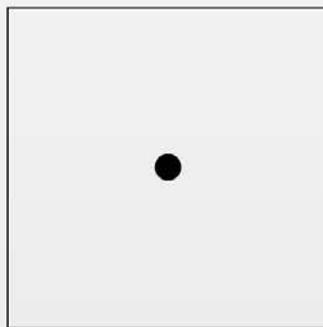
$k=2$



$k=3$

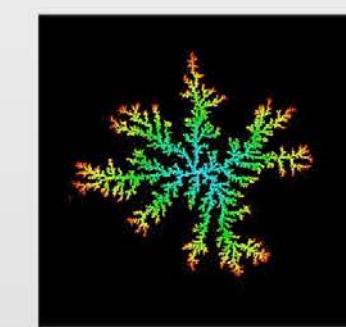
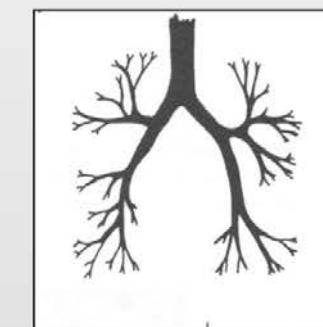
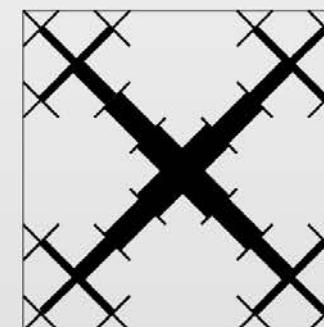
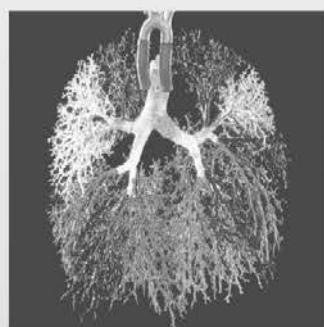


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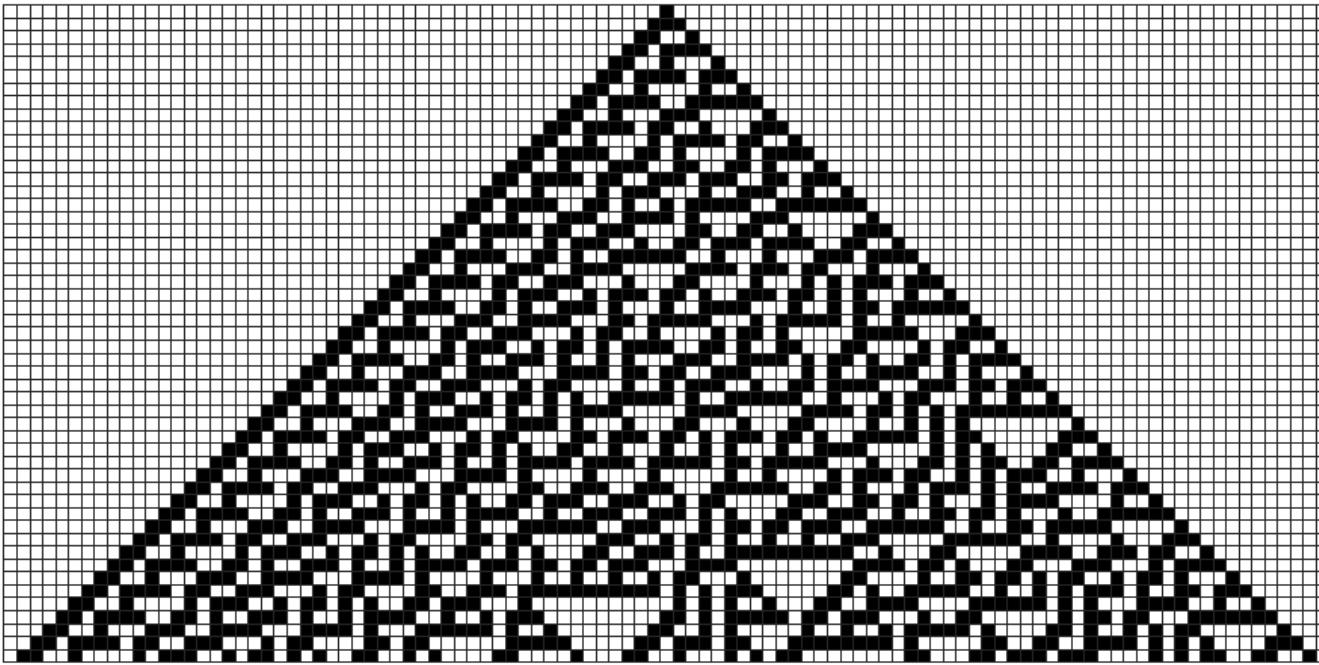
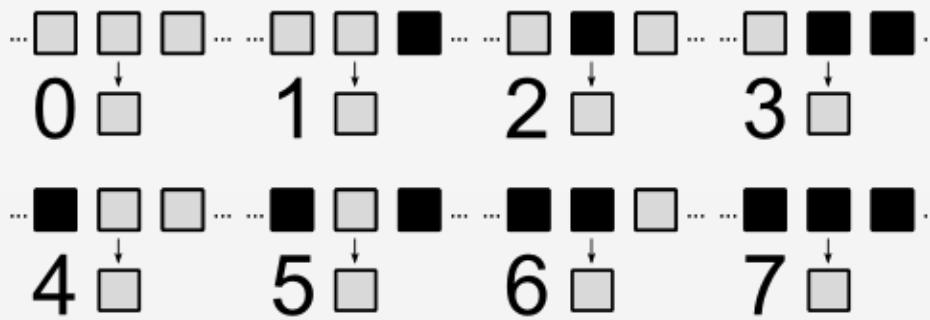
We May Break
About Here
for 5
Minutes



Early One Dimensional Automata

Essentially we can define a line of cells and we then specify a rule that basically says that the cells change from say black to white – empty to full – if a certain configuration of cells takes place in the neighbourhood

Basically this generates lots of different configurations and we can see what happens over time when we string each change in the pattern on top of each other this producing patterns that might diverge or converge or oscillate over time as we show on the next slide – ok we will use Netlogo to show these ideas



Time
↓

A vertical line with a downward-pointing arrow, labeled "Time" to its left, indicating the direction of time in the cellular automaton's evolution.

Space →

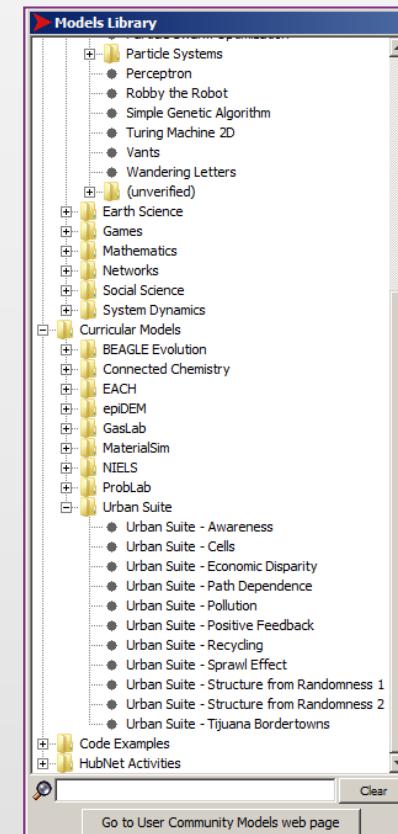
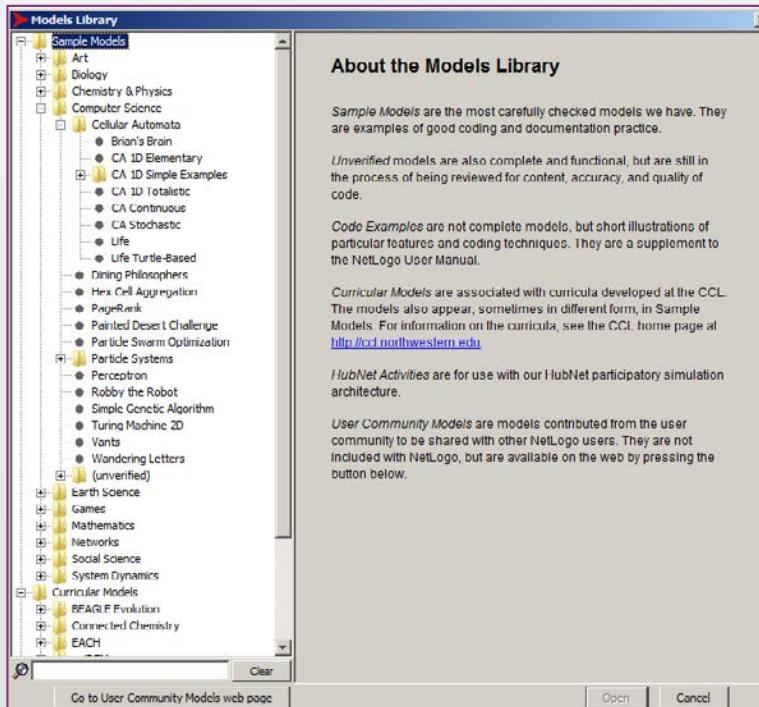
A horizontal arrow pointing to the right, labeled "Space" above it, indicating the dimension of space in the cellular automaton.

Cellular Automata

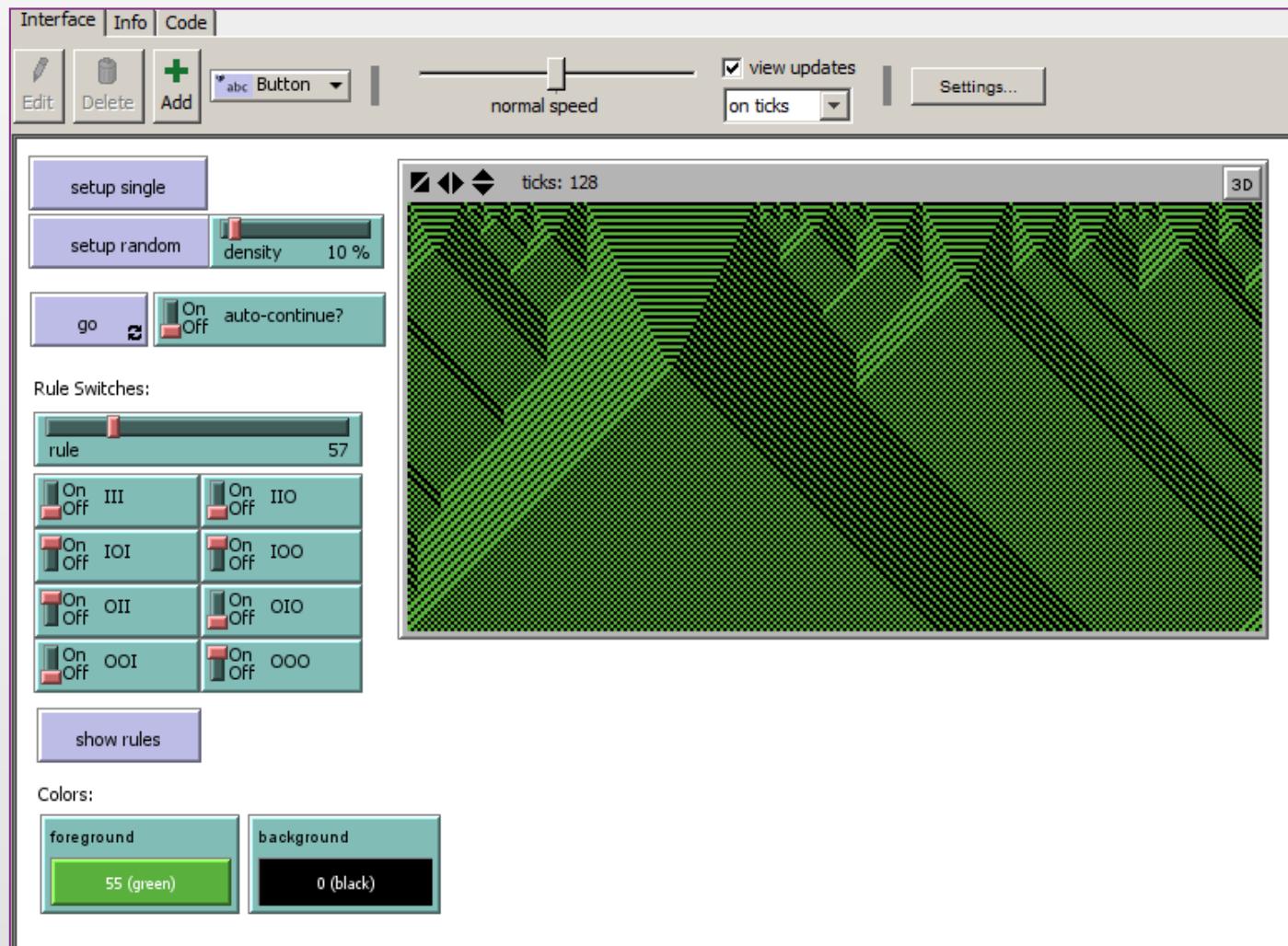
First Programming Interlude

NetLogo has an excellent library of model types – some of which are CA, some of which are close to my own. Here is an example of the library

<https://ccl.northwestern.edu/netlogo/download.shtml>



I am going to look at two basic CA models now –that we have looked at – first the 1 d model, then the 2-d Game of Life



The Game of Life

Imagine we have a world where you are located in space – randomly – then there are three rules – you spawn a new cell adjacent if there are two or three cells around you that are active – i.e. you are not active but you become active if there are two or three cells there

If you are active you die if there are more than 3 cells around you i.e. over population too much density

If you are active you die if there are 0 or 1 cells around you

I think that is it – what do we get – ok NetLogo again



John Conway invented it – look at the NetLogo version for a further explanation and look at William Poundstone's book **The Random Universe**.

<http://www.cuug.ab.ca/dewara/life/life.html>

Another Interlude: For a link to the Beatles look at this

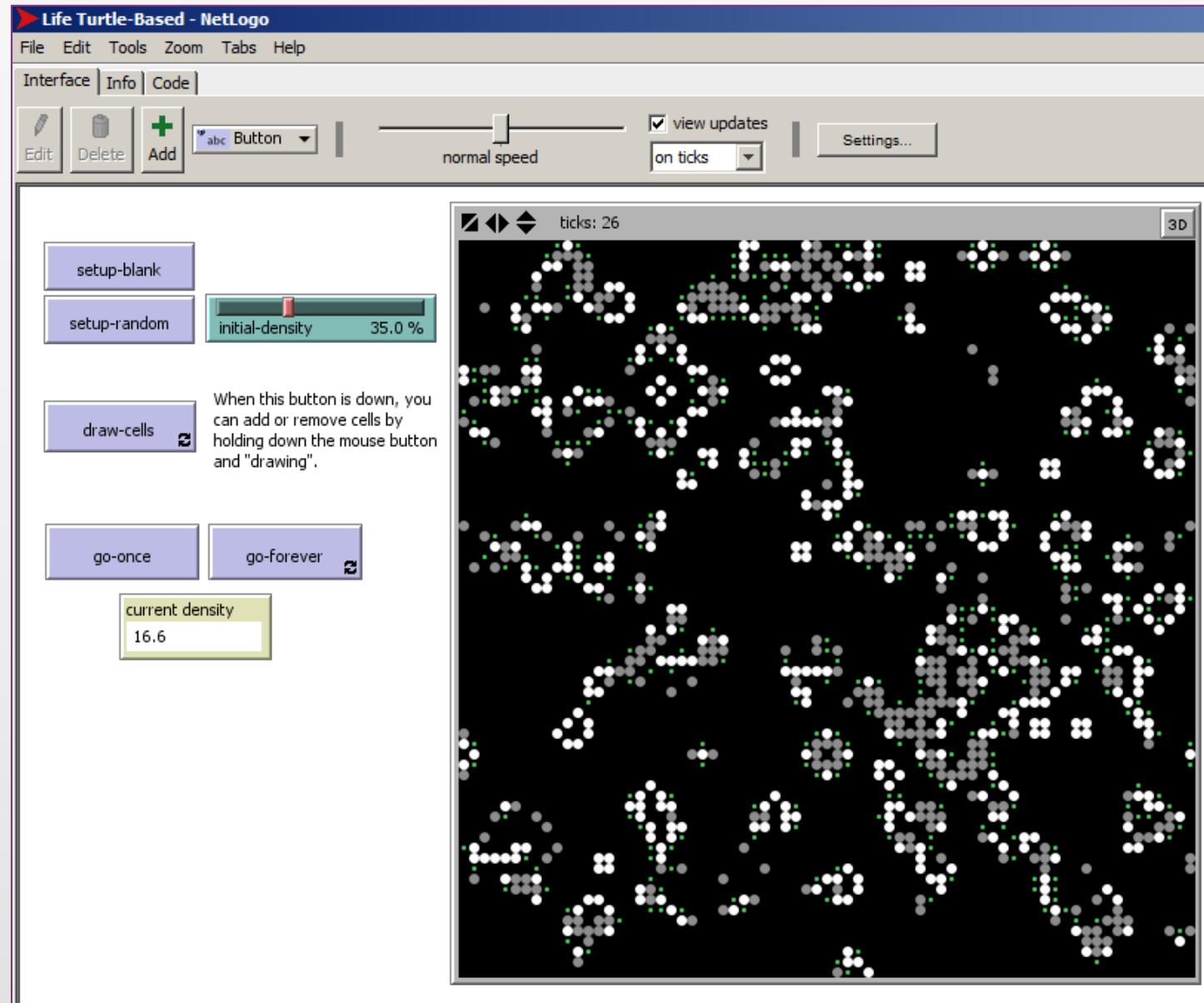
<http://www.nytimes.com/1993/10/12/science/scientist-at-work-john-h-conway-at-home-in-the-elusive-world-of-mathematics.html?pagewanted=all&src=pm>

JL didn't go the same school but ...

<http://www.complexcity.info/files/2013/08/school.jpg>



The 2-d Game of Life



Applications through Cellular Automata

To illustrate how CA works, we first define

- a grid of cells, (or it could be irregular but to simplify we will assume a square grid)
- a neighbourhood around each cell which is composed of the nearest cells,
- a set of rules as to how what happens in the neighbourhood affects the development of the cell in question
- a set of states that each cell can take on – i.e. developed or not developed
- an assumption of universality that all these features operate uniformly and universally

This defines a (cellular) automata machine that can be applied to all cells that define the system: i.e. each cell is an automata

Some things to note: cells are irregular and not necessarily spatially adjacent.

Neighbourhoods can be wider than those which are formed from nearest neighbours- they could be formed as fields – like interaction fields around a cell

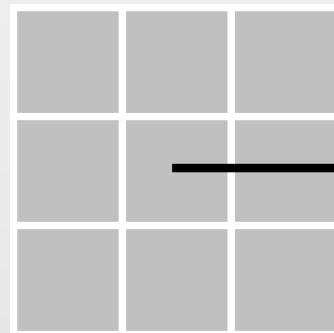
Strict CA are models whose rules work on neighbourhoods defined by nearest neighbours and exhibit emergence – i.e. their operation is local giving rise to global pattern

Cell-space models can relax some or all of these rules

This is how a CA works defined on a square grid of cells with two states – not developed & developed

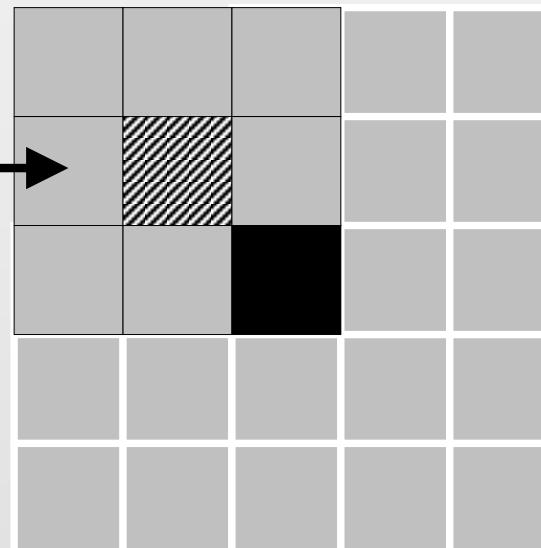
(a)

The neighbourhood is composed of 8 cells around the central cell



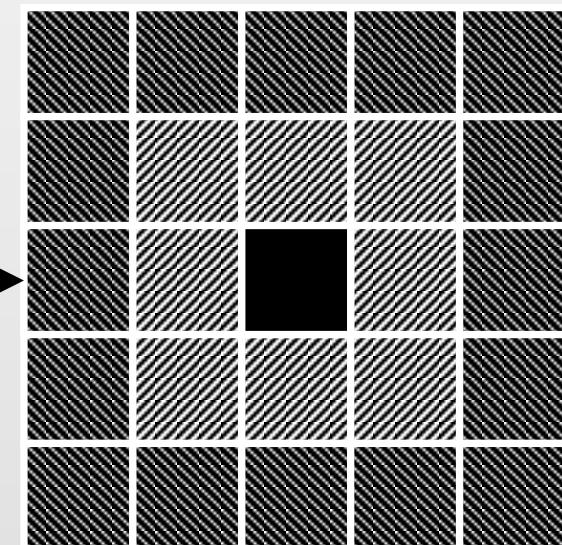
(b)

Place the neighbourhood over each cell on the grid. The **rule** says that if there is one or more cells developed (black) in the neighbourhood, then the cell is developed.



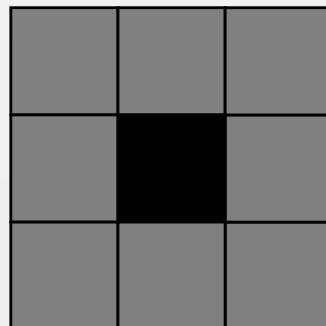
(c)

If you keep on doing this for every cell, you get the diffusion from the central cell shown below.

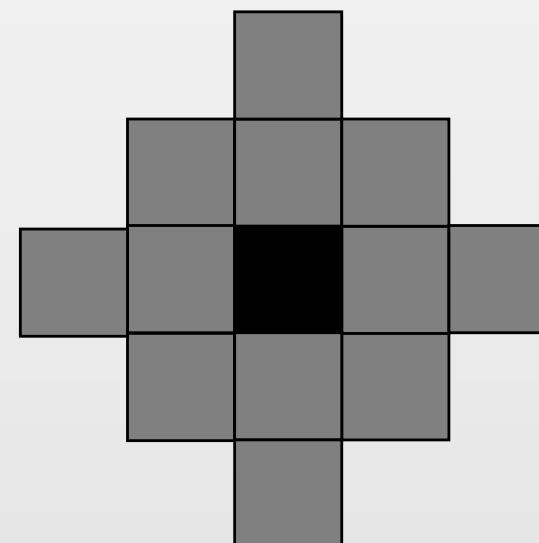


These are strictly deterministic CA models and we can have different shaped local neighbourhoods composed of different combinations of cells e.g.

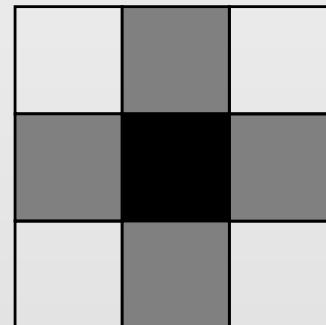
(a) Moore



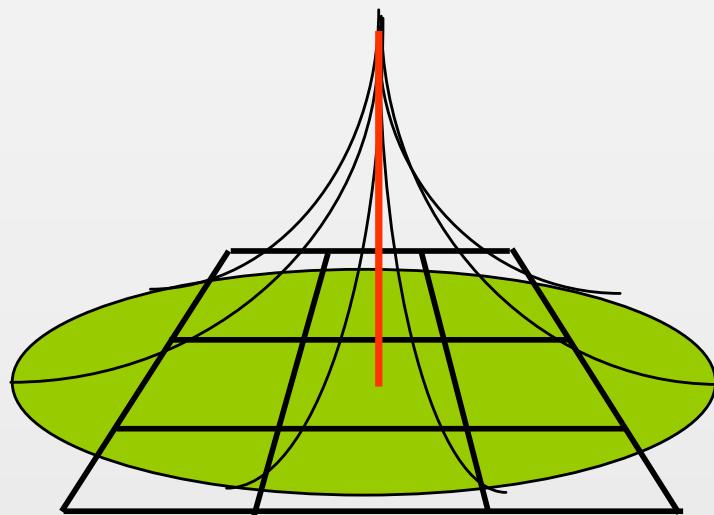
(c) Extended Moore
von Neumann



(b) von Neumann



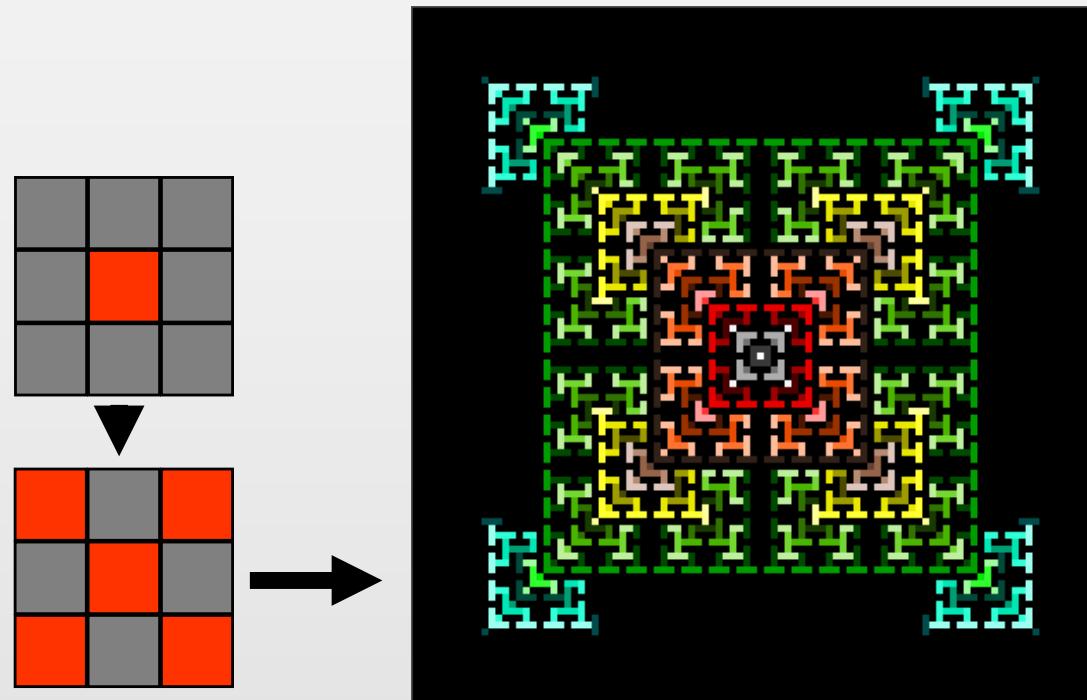
And we can have probabilistic fields defining neighbourhoods where there is a probability that a cell changes state – where the probabilities might vary regularly reflecting say action-at-a-distance principles e.g.



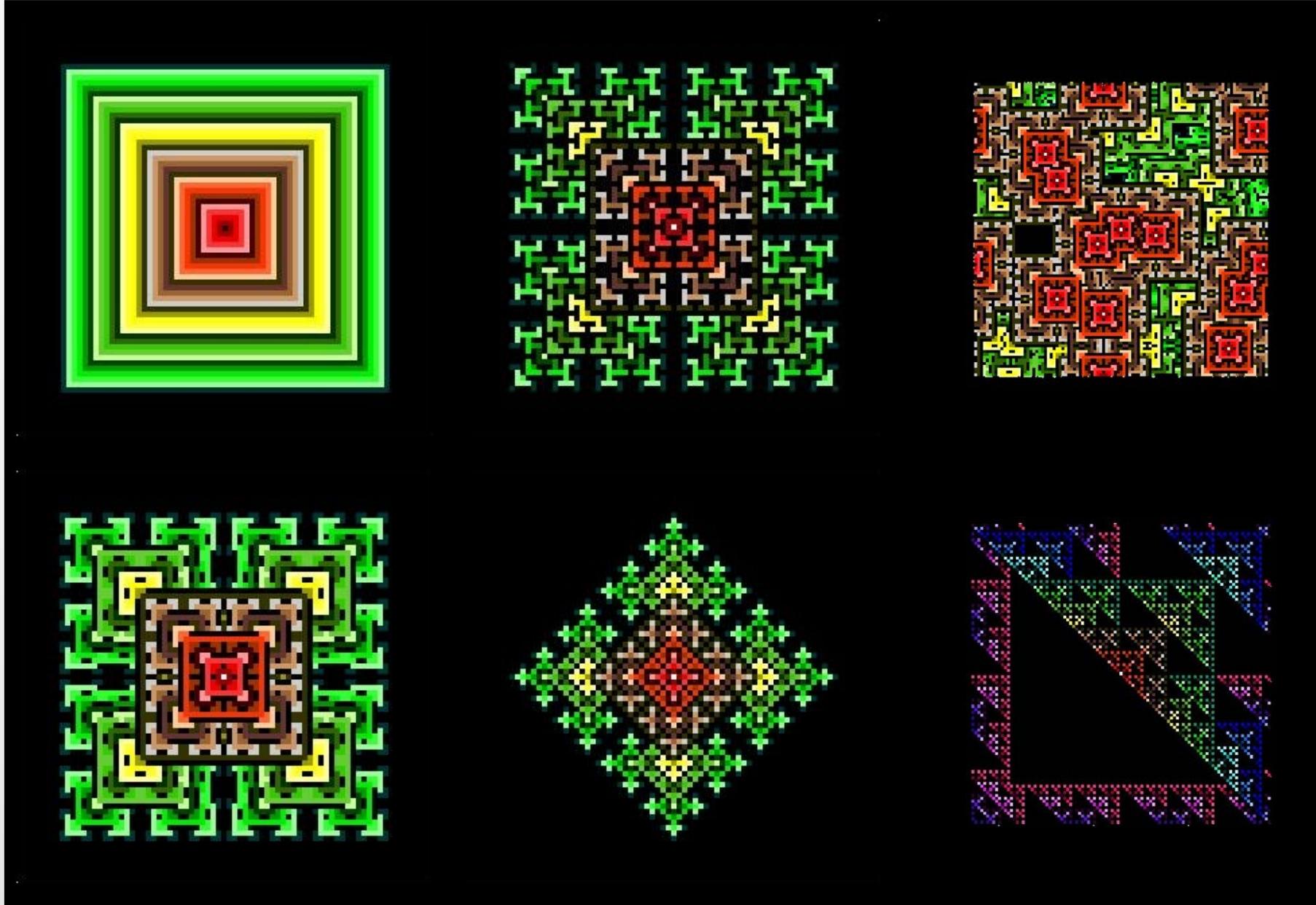
We will now show some examples of how one can generate idealised patterns that illustrate emergence

For example, for any cell $\{x,y\}$,

- **if** only one neighbourhood cell either NW, SE, NE, or SW other than $\{x,y\}$ is already developed,
- **then** cell $\{x,y\}$ is developed according to the following neighbourhood switching rule



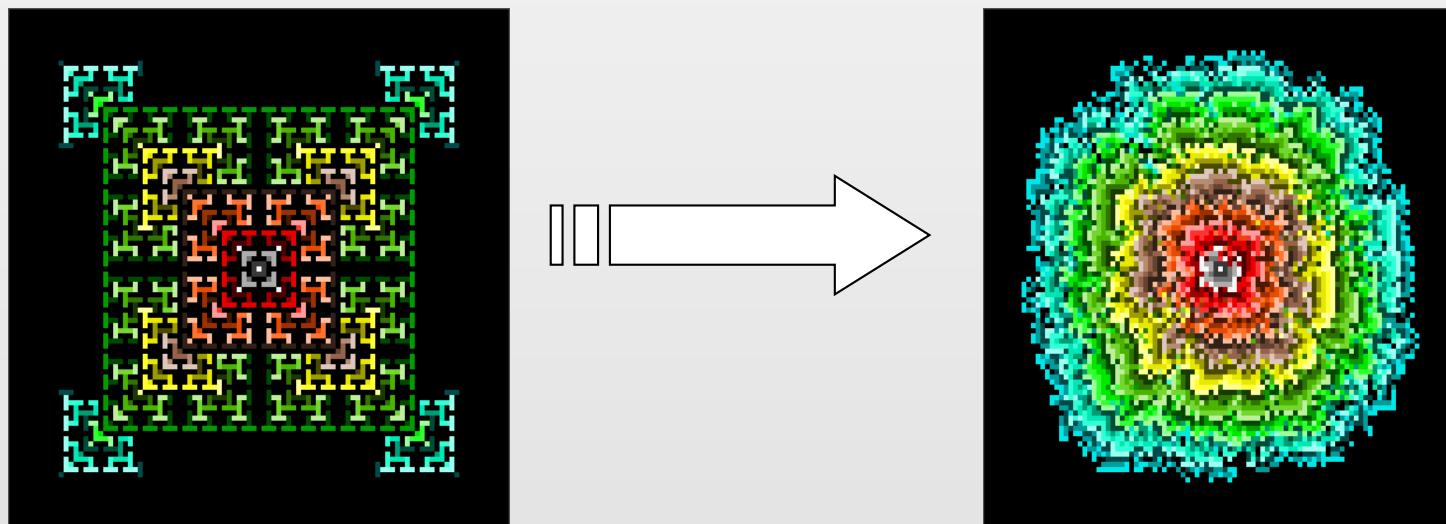
And changing
There rules in
various ways
lead to
many different
patterns



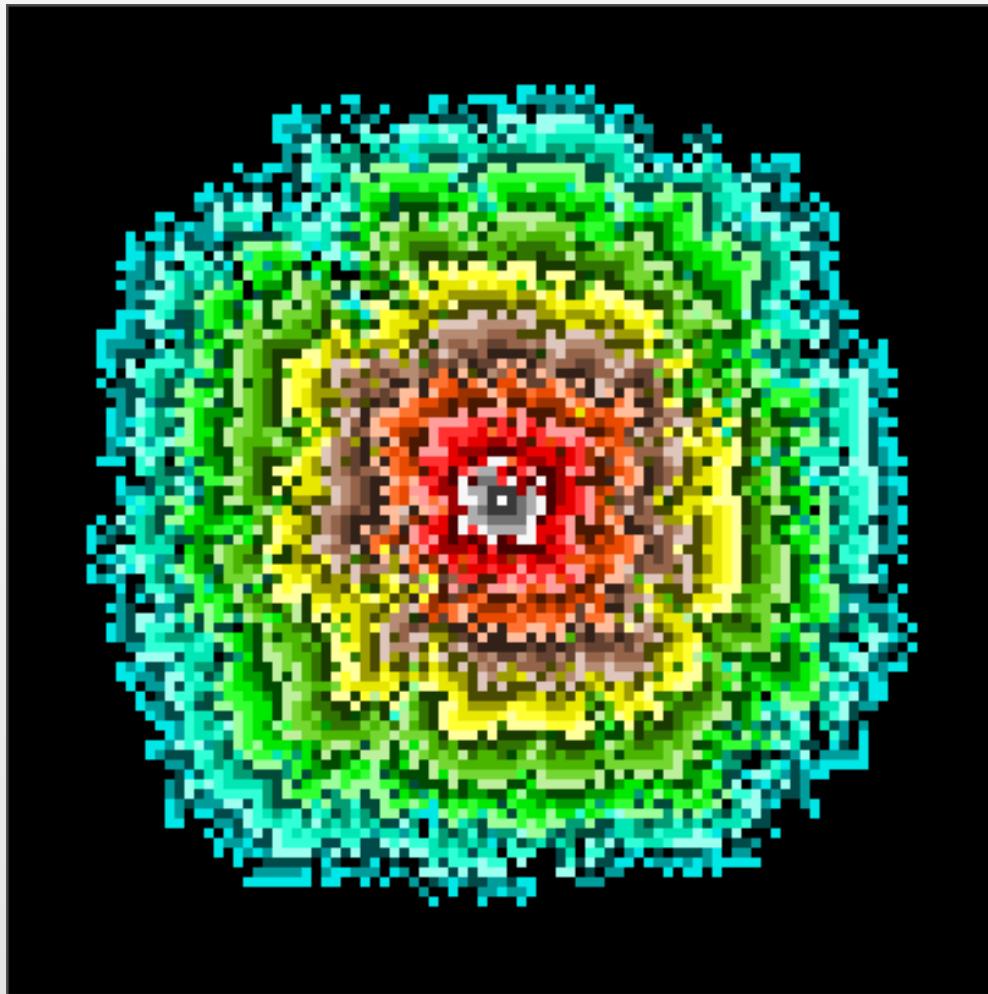
Cellular Automata

For probabilistic rules, we can generate statistically self-similar structures which look more like real city morphologies. For example,

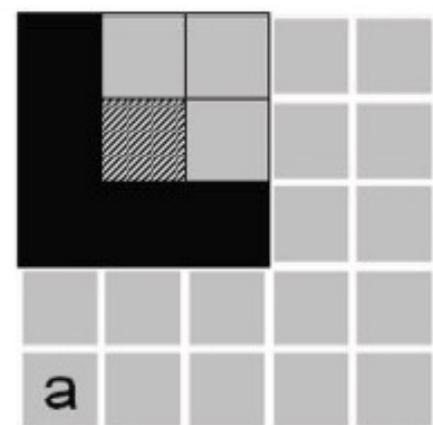
- if** any neighbourhood cell other than $\{x,y\}$ is already developed, **then** the field value $p \{x,y\}$ is set &
- if** $p \{x,y\} >$ some threshold value, **then** the cell $\{x,y\}$ is developed



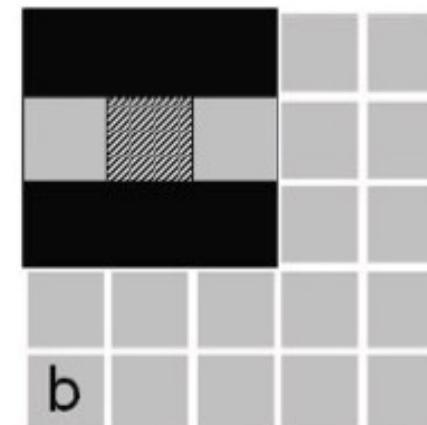
Here are the constructions we have seen overlayed so you can see how neighbourhood rules make a distinct difference



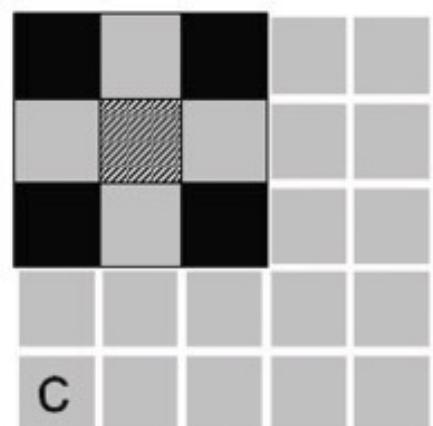
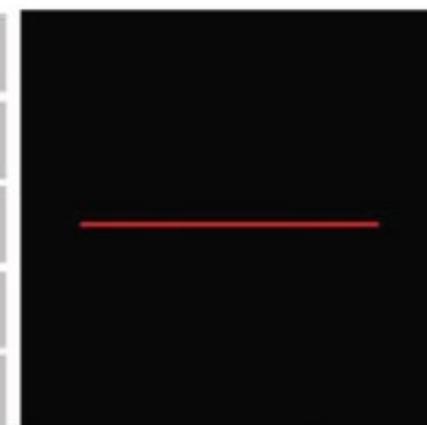
We can steer the development in different ways by constructing rules based on ‘ruling’ out or ‘admitting’ certain cells for development – embodying constraints – Look at the paper I will also put up on Moodle from Architectural Design



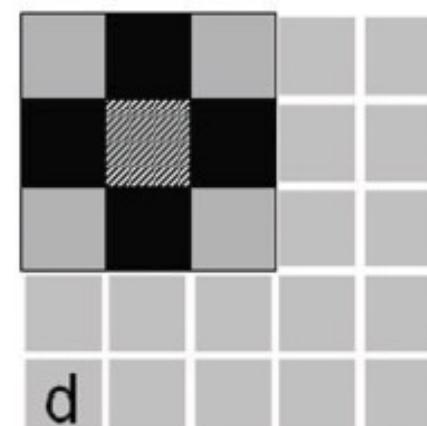
a



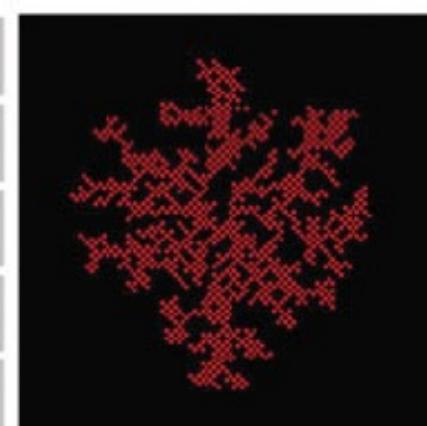
b



c

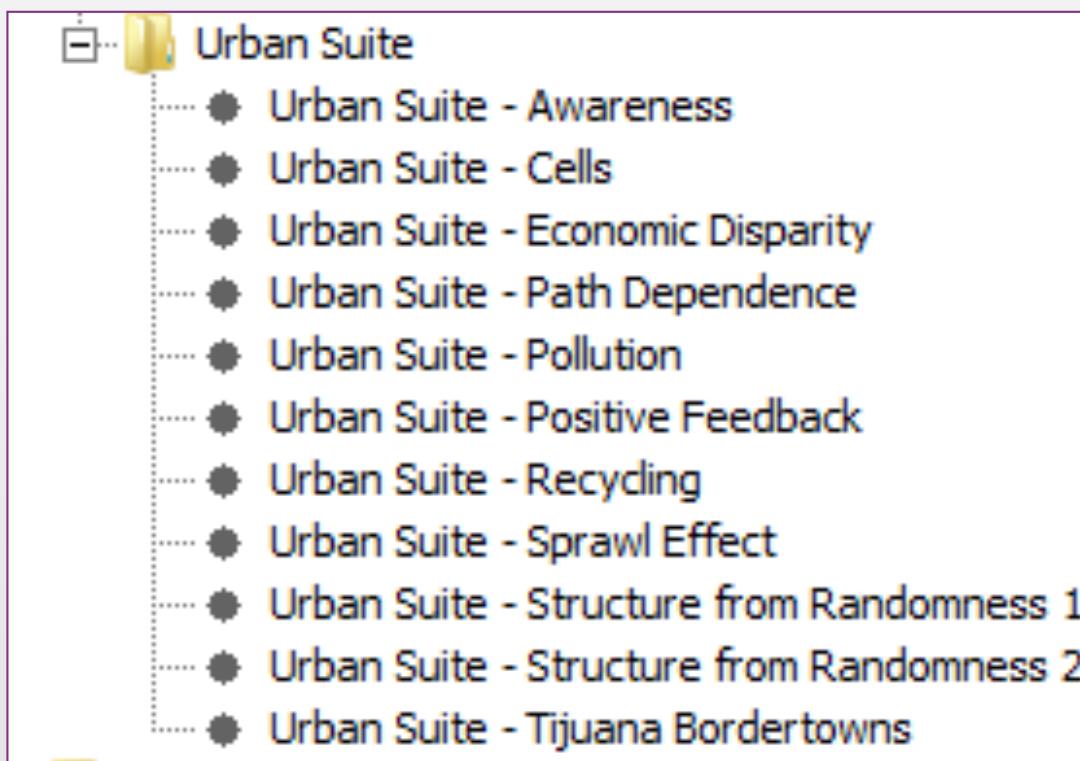


d

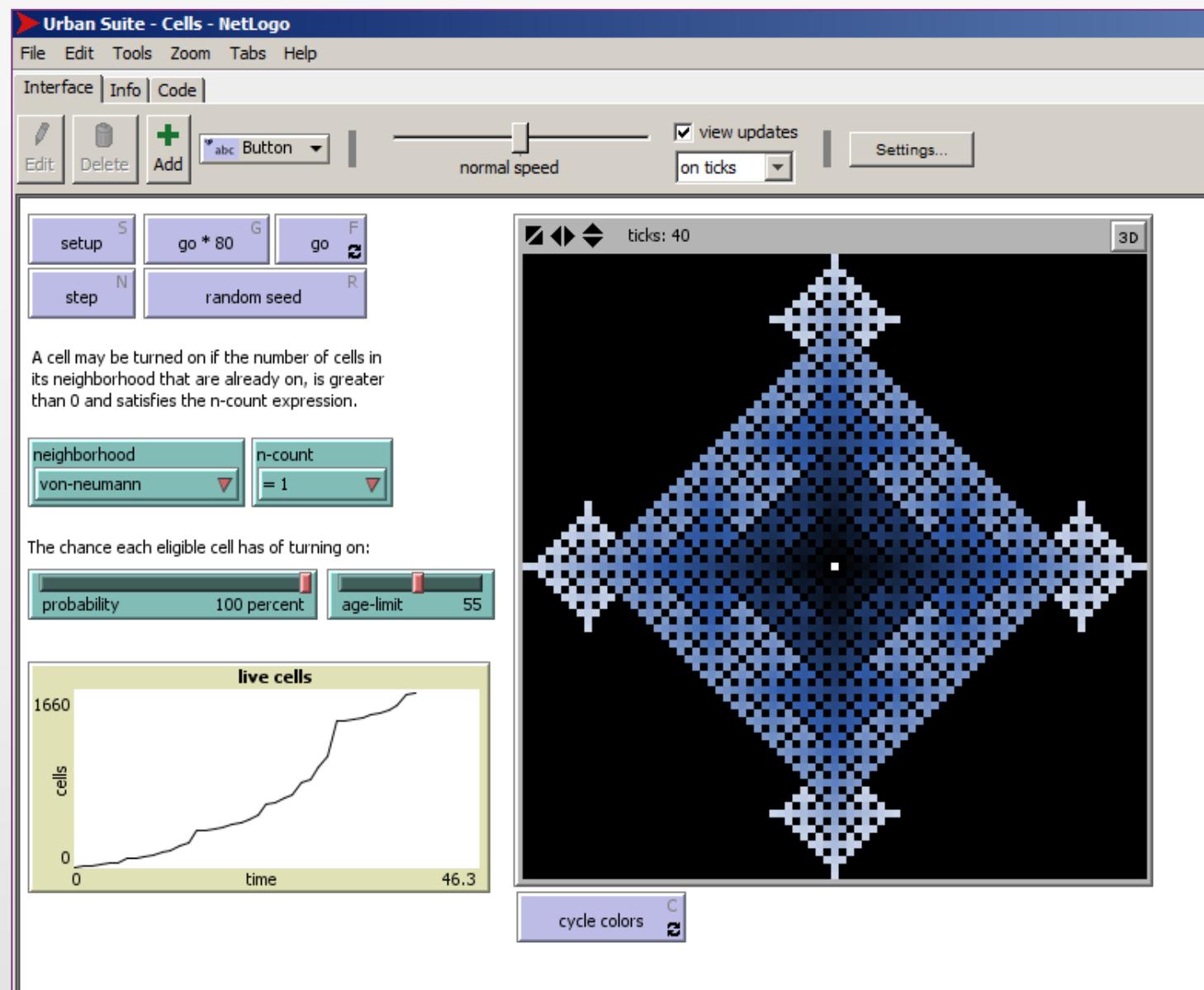


Second Programming Interlude

Netlogo has an Urban Suite – I don't know who did it
but some of the models in my **Cities and Complexity** Book are in there



I will run a few of these – let us take a look



Different Model Applications

There are many groups around the world

- White and Engelen, RIKS, Holland – **GeoDynamica, METRONamica**
- Clarke, UCSB/NCGIA, USA – **SLEUTH**
- Yeh and Li, Hong Kong – Pearl River – RS bias
- Wu/Webster – Southampton/Cardiff – urban economics
- Xie/Batty – Ypsilanti/London, US/UK – **DUEM**
- Cechinni/Viola – Venice, Italy – AUGH
- Rabino/Lombardi – Milan/Turin, Italy – NN Calibration
- Semboloni – Florence, Italy – links to traditional LU models
- Phin/Murray – Brisbane/Adelaide, Aus – visualization
- Portugali/Benenson – Tel-Aviv, Israel – **CITY** models
- Various applications in INPE (Brazil), China (Beijing), Japan, Portugal, Taiwan, Canada, Haifa (Technion), Ascona, France (Pumain's group), Louvain-la-Neuve, Netherlands (ITC), JRC (Ispra+Dublin+RIKS), even at CASA Kiril Stanilov's model



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Progress in Human Geography

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Articles



[Modelling urban change with cellular automata: Contemporary issues and future research directions](#)

Yan Liu , Michael Batty, Siqin Wang, Jonathan Corcoran

First Published December 23, 2019; pp. 3–24

Abstract

[▶ Preview](#)



[Check for updates](#)

Article

Modelling urban change with cellular automata: Contemporary issues and future research directions

Yan Liu

University of Queensland, Australia

Michael Batty

University College London, UK

Siqin Wang

University of Queensland, Australia

Jonathan Corcoran

University of Queensland, Australia

Abstract

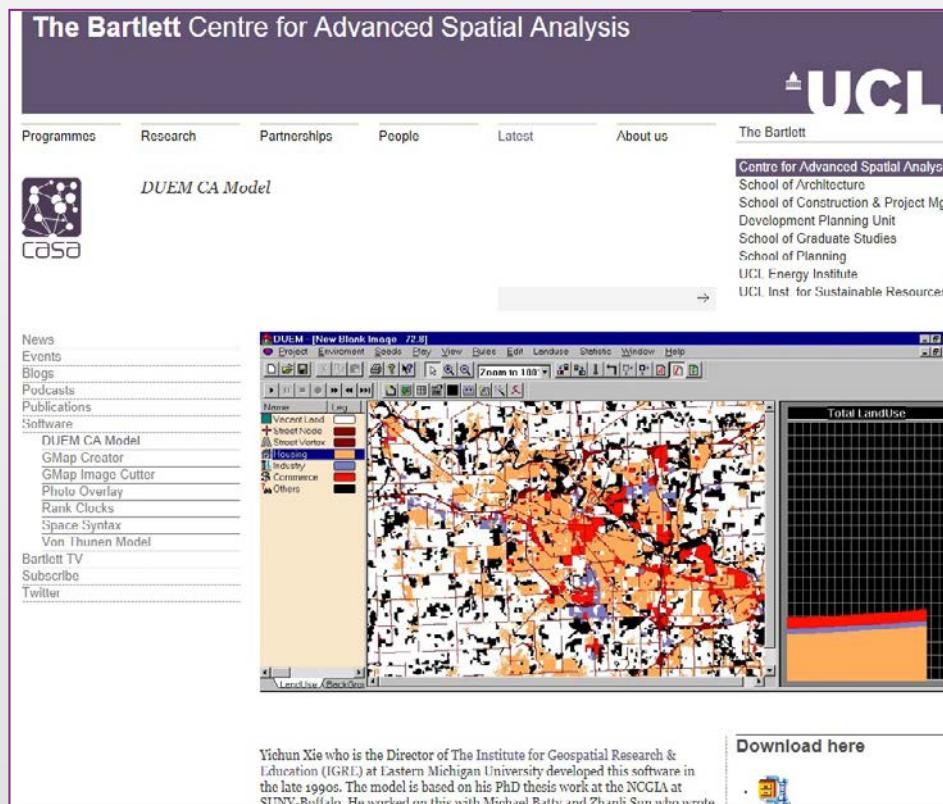
The study of land use change in urban and regional systems has been dramatically transformed in the last four decades by the emergence and application of cellular automata (CA) models. CA models simulate urban land use changes which evolve from the bottom-up. Despite notable achievements in this field, there remain significant gaps between urban processes simulated in CA models and the actual dynamics of evolving urban systems. This article identifies contemporary issues faced in developing urban CA models and draws on this evidence to map out four interrelated thematic areas that require concerted attention by the wider CA urban modelling community. These are: (1) to build models that comprehensively capture the multi-dimensional processes of urban change, including urban regeneration, densification and gentrification, in-fill development, as well as urban shrinkage and vertical urban growth; (2) to establish models that incorporate individual human decision behaviours into the CA analytic framework; (3) to draw on emergent sources of 'big data' to calibrate and validate urban CA models and to capture the role of human actors and their impact on urban change dynamics; and (4) to strengthen theory-based CA models that comprehensively explain urban change mechanisms and dynamics. We conclude by advocating cellular automata that embed agent-based models and big data input as the most promising analytical framework through which we can enhance our understanding and planning of the contemporary urban change dynamics.

Corresponding author:

Yan Liu, University of Queensland, School of Earth and Environmental Sciences, Brisbane, Queensland 4072 Australia.

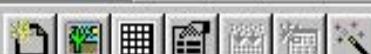
DUEM – Dynamic Urban Evolutionary Model

Let me demo it first – you can download it from
<http://www.bartlett.ucl.ac.uk/casa/latest/software/duem-ca> I haven't checked this yet – it may be long gone ---it is very very old – you can read about in my C&C book



DUEM - [New Blank Image 85.0]

Project Enviroment Seeds Play View Rules Edit Landuse Statistic Window Help

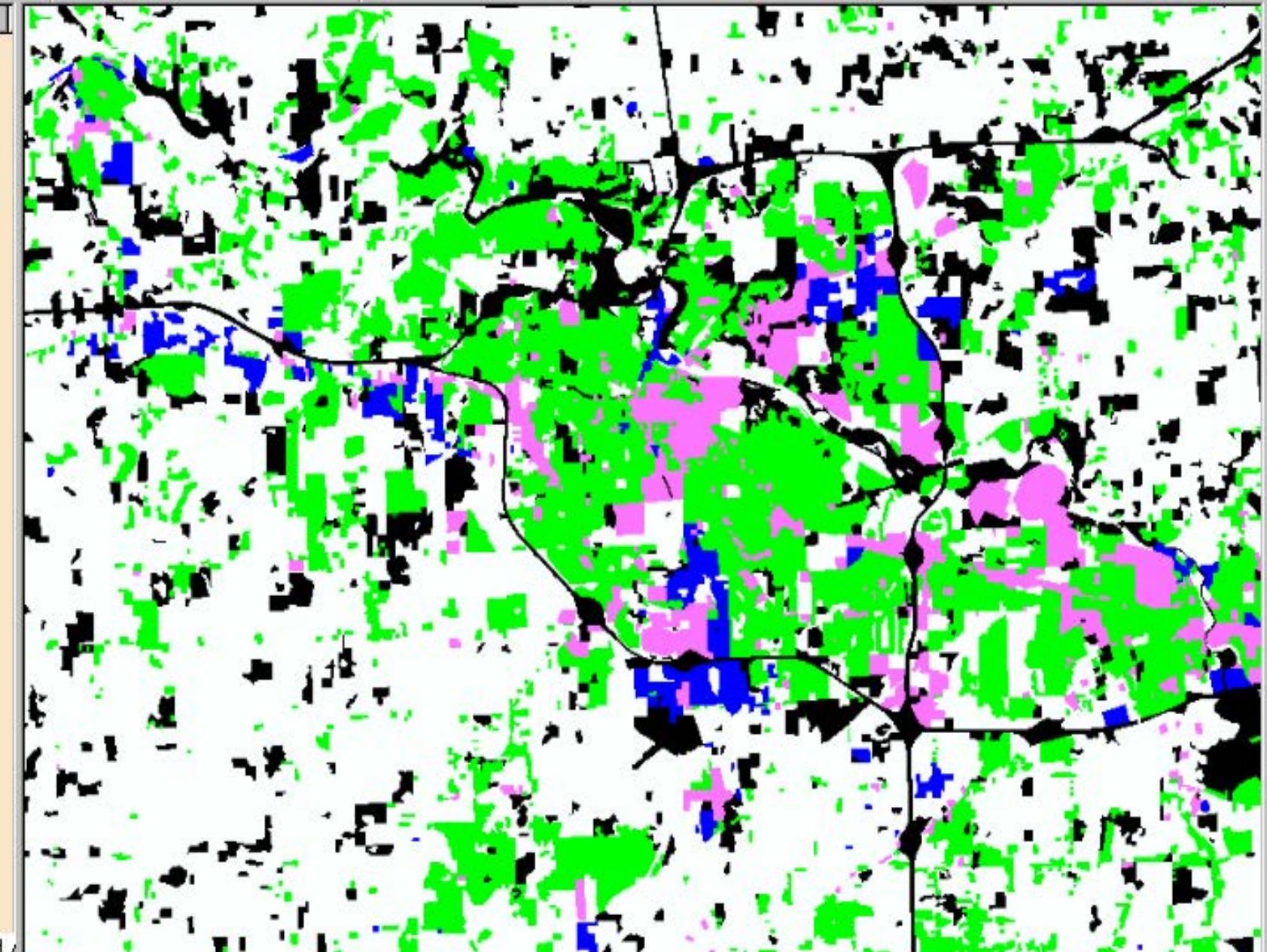


Zoom to 100%



Allow Legend

Nothing
Housing
Industry
Shopping
Hous_Indu
Hous_Shop
Shop_Indu
All



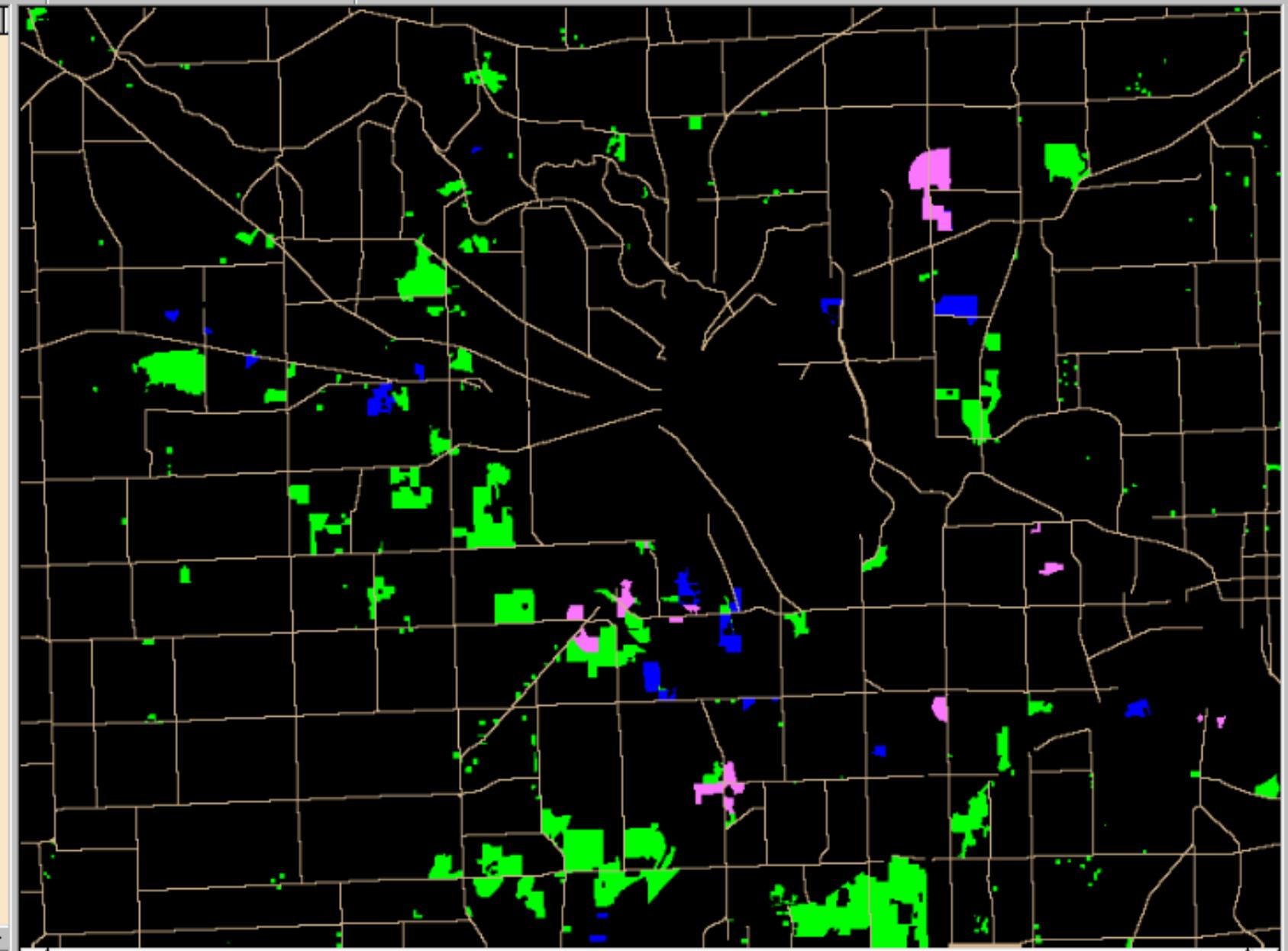
\LandUse BackGround

DUEM - [New Blank Image 87.1]

Project Enviroment Seeds Play View Rules Edit Landuse Statistic Window Help



Name	Legend
Blank Land	[Solid Black Box]
Normal Node	[Red Cross]
Normal Street	[Grey Line]
Housing	[Green House]
Industry	[Blue Factory]
Shopping	[Pink Store]
Other Type	[White Question Mark]



LandUse BackGround



Name

Legend

Blank Land



Normal Node



Normal Street



Housing



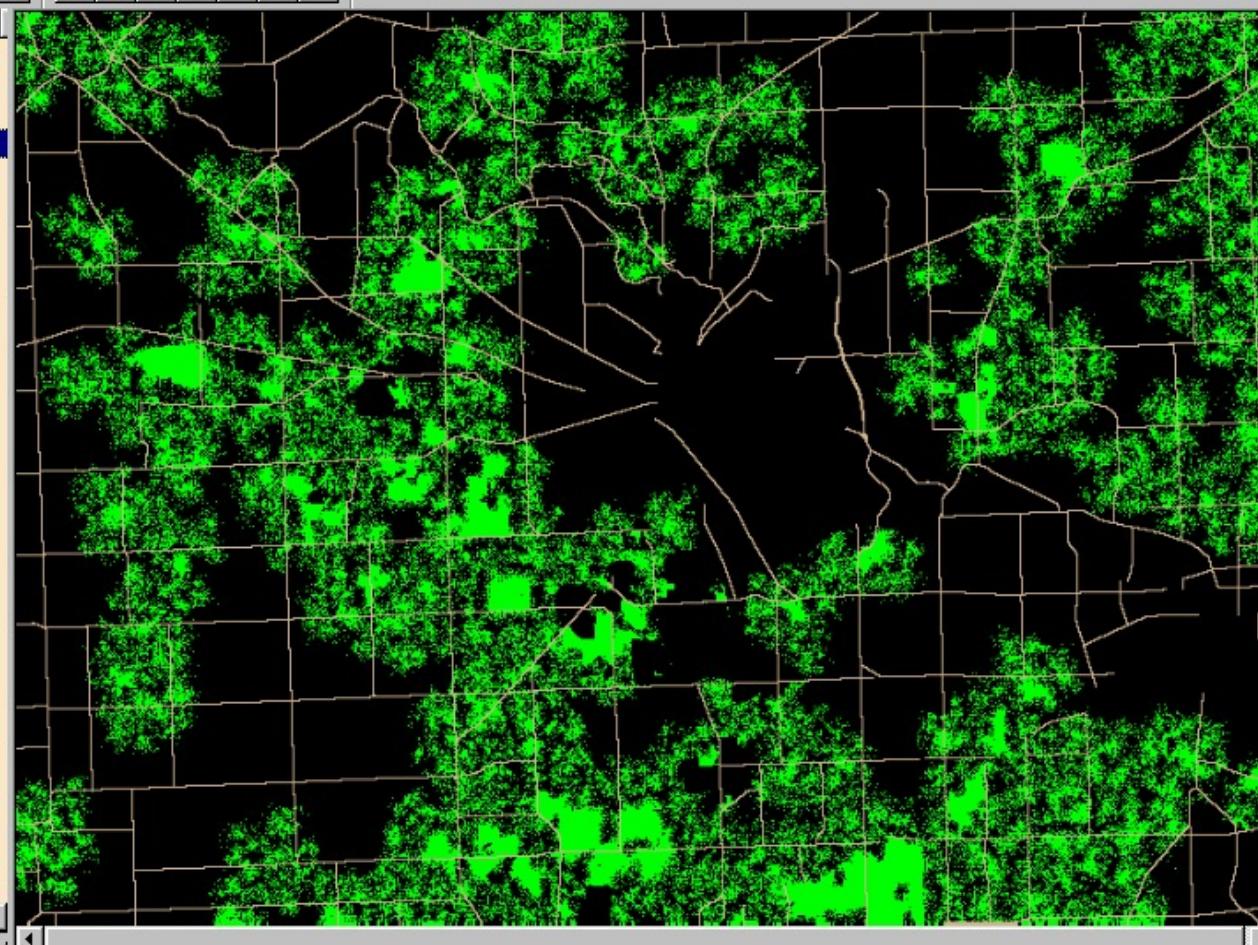
Industry



Shopping



Other Type



Total LandUse



LandUse BackGround



Housing

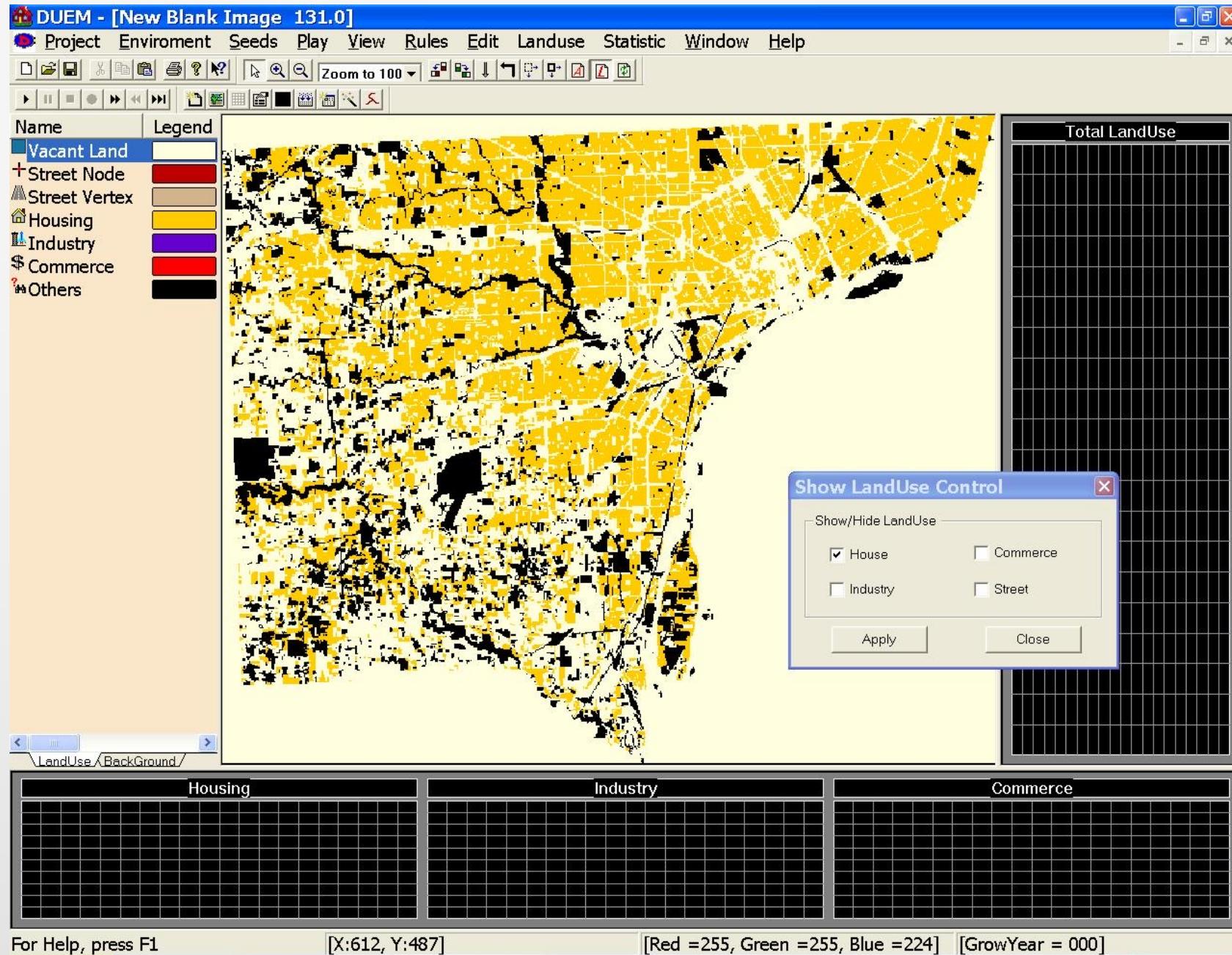
105752

Industry

3742

Shopping

31855
22931

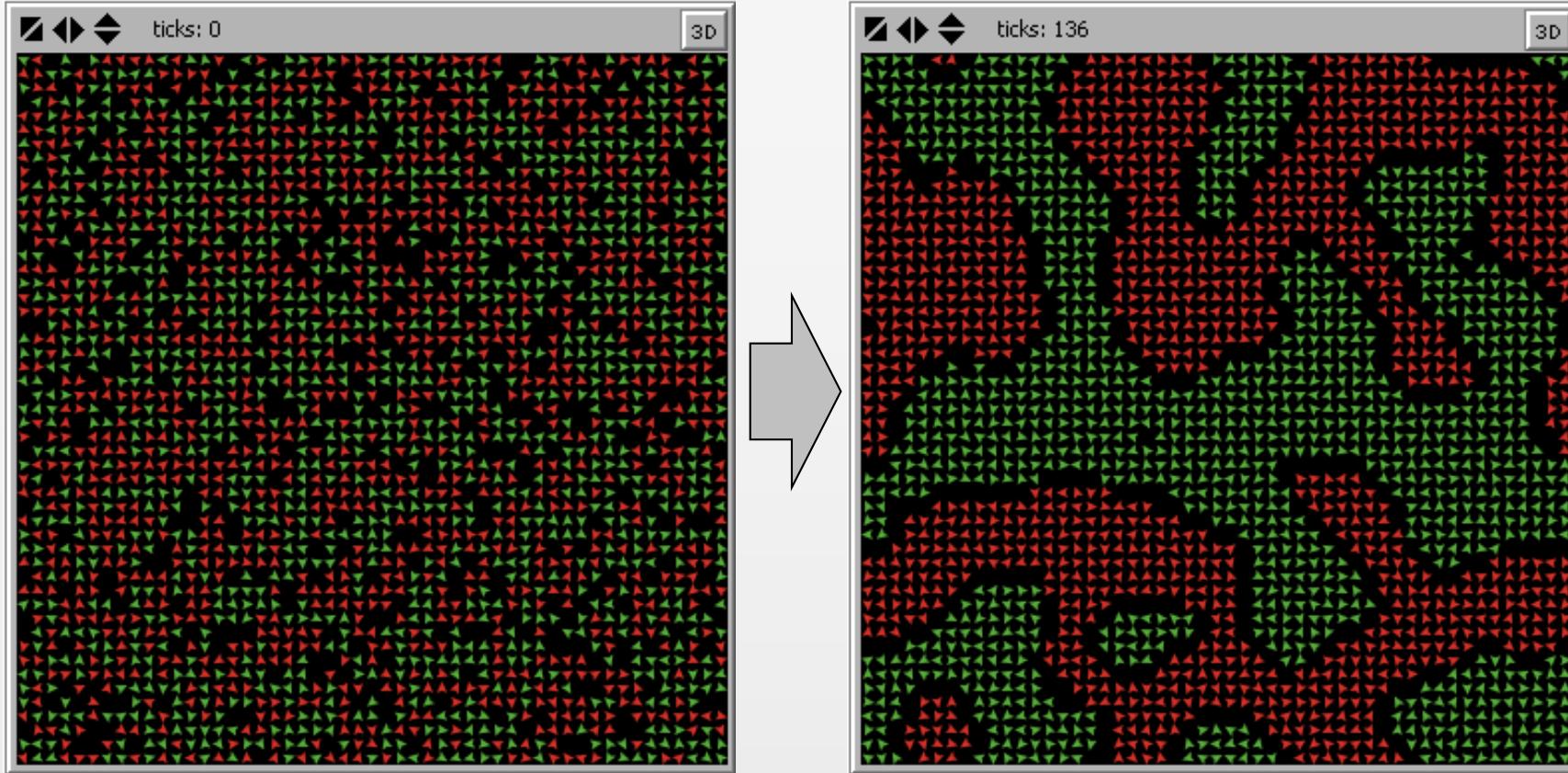


Moving to Agent-Based Models: Schelling - NetLogo

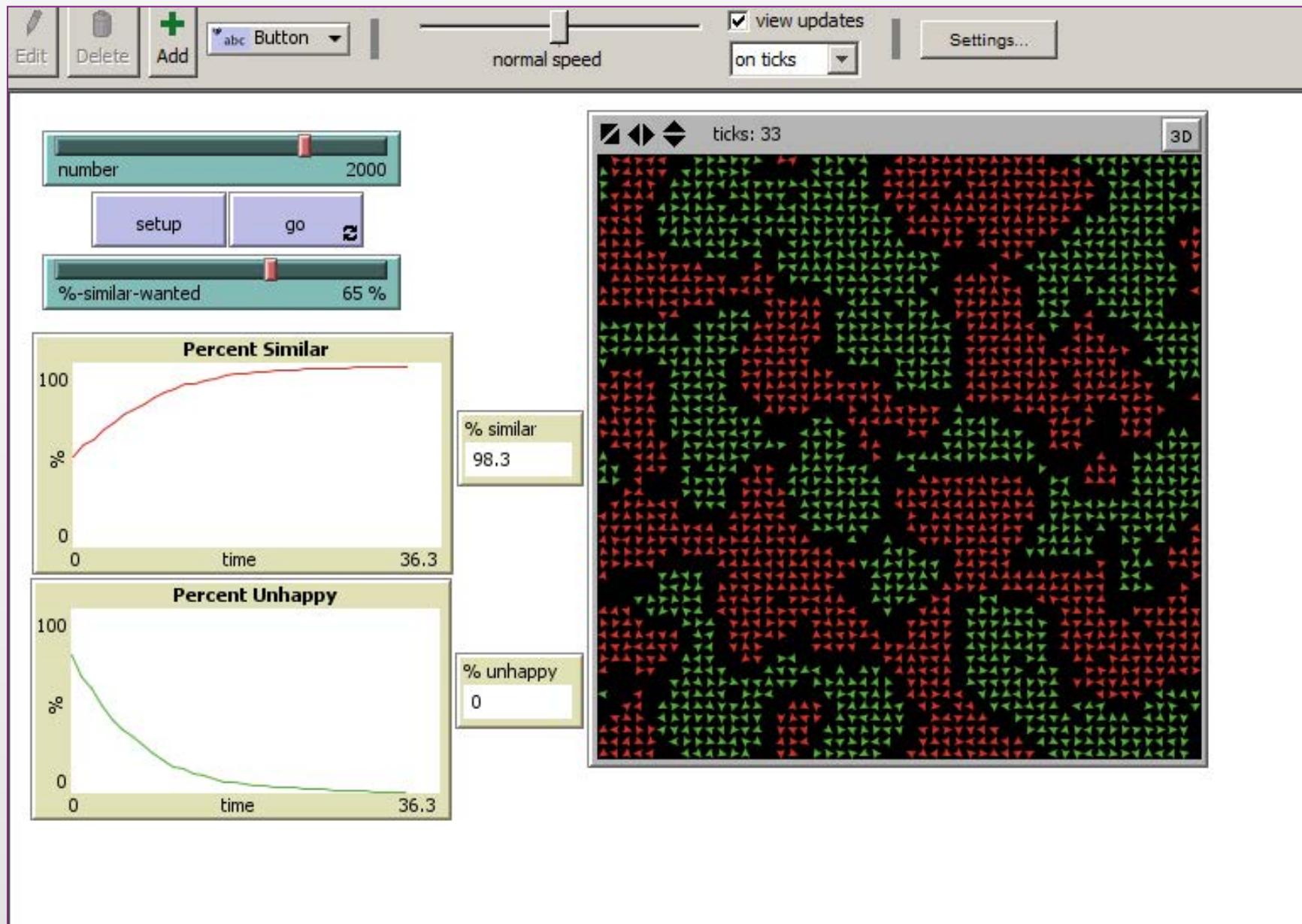
Ok this model essentially redistributes people – we divide our landscape up into two types of people and we allocate them randomly.

Now the rule is dead simple – if there are more people of another type than yourself in your Moore 8x8 cell neighbourhood, you switch your type or opinion

If there are less you do not shift – i.e. you are quite happy say with 50-50 of each type – but unhappy with a majority against you – this is not blind prejudice but mild preference



From a random distribution of two unlike groups, each with a very mild preference to live amongst their own kind, people shift if more than half are different, the picture unravels and dramatic segregation emerges.
Netlogo demo



Urban Simulation 5 continued if time

Agent Based Models

Individuals Representing the Element of Urban Systems

Michael Batty

<http://www.spatialcomplexity.info/>

5th February 2024

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