Counterclockwise Rotation

If you want to rotate a vector r with coordinates (x, y) and angle α counterclockwise over an angle β to get vector r' with coordinates $(x$'$, y$'$)$ then the following holds:

$$x = r * cos(\alpha)$$

$$y = r * sin(\alpha)$$

$$x\text{'} = r\text{'} * cos(\alpha + \beta)$$

$$y\text{'} = r\text{'} * sin(\alpha + \beta)$$

Trigonometric addition gives us:

$$cos(\alpha + \beta) = cos(\alpha)cos(\beta) - sin(\alpha)sin(\beta)$$

$$sin(\alpha + \beta) = cos(\alpha)sin(\beta) + sin(\alpha)cos(\beta)$$

As the length of the vector stays the same,

$$x\text{'} = r * cos(\alpha)cos(\beta) - r * sin(\alpha)sin(\beta)$$

$$y\text{'} = r * cos(\alpha)sin(\beta) + r * sin(\alpha)cos(\beta)$$

This equates to:

$$x\text{'} = x * cos(\beta) - y * sin(\beta)$$

$$y\text{'} = x * sin(\beta) + y * cos(\beta)$$

Written as matrix multiplication with row vectors, this becomes,

$$[x', y'] = [x, y] \cdot \begin{bmatrix} cos(\beta) & sin(\beta) \\ -sin(\beta) & cos(\beta) \end{bmatrix}$$

with the rotation matrix equal to,

$$R = \begin{bmatrix} cos(\beta) & sin(\beta) \\ -sin(\beta) & cos(\beta) \end{bmatrix}$$

Written as matrix multiplication with column vectors, this becomes,

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} cos(\beta) & -sin(\beta) \\ sin(\beta) & cos(\beta) \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix}$$

with the rotation matrix equal to,

$$R = \begin{bmatrix} cos(\beta) & -sin(\beta) \\ sin(\beta) & cos(\beta) \end{bmatrix}$$

Note that the position of $-sin(\beta)$ in the rotation matrix has changed.

Clockwise Rotation

If rotation is clockwise, then the rotation matrix for multiplication with row vectors becomes,

$$R = \begin{bmatrix} cos(-\beta) & sin(-\beta) \\ -sin(-\beta) & cos(-\beta) \end{bmatrix}$$

As $sin(-\beta) = -sin(\beta)$ and $cos(-\beta) = cos(\beta)$

this equates to

$$R = \begin{bmatrix} cos(\beta) & -sin(\beta) \\ sin(\beta) & cos(\beta) \end{bmatrix}$$

So clockwise rotation of a vector $[x, y]$ can be expressed as,

$$[x', y'] = [x, y] \cdot \begin{bmatrix} cos(\beta) & -sin(\beta) \\ sin(\beta) & cos(\beta) \end{bmatrix}$$

So clockwise rotation of a vector $[x, y]$ can be expressed as,

$$[x', y'] = [x, y] \cdot \begin{bmatrix} cos(\beta) & -sin(\beta) \\ sin(\beta) & cos(\beta) \end{bmatrix}$$

The rotation matrix for multiplication with column vectors becomes,

$$R = \begin{bmatrix} cos(-\beta) & -sin(-\beta) \\ sin(-\beta) & cos(-\beta) \end{bmatrix}$$

which equates to,

$$R = \begin{bmatrix} cos(\beta) & sin(\beta) \\ -sin(\beta) & cos(\beta) \end{bmatrix}$$

So clockwise rotation of a vector $\begin{bmatrix} x \\ y \end{bmatrix}$ can be expressed as,

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} cos(\beta) & sin(\beta) \\ -sin(\beta) & cos(\beta) \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix}$$

Author: Reinoud Bosch