A Hierarchical Incentive Mechanism for Federated Learning

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***Abstract*—With the explosive development of mobile computing, federated learning (FL) has been considered as a promising distributed training framework for addressing the shortage of conventional cloud based centralized training. In FL, local model owners (LMOs) individually train their respective local models and then upload the trained local models to the task publisher (TP) for aggregation to obtain the global model. When the data provided by LMOs do not meet the requirements for model training, they can recruit workers to collect data. In this paper, by considering the interactions among the TP, LMOs and workers, we propose a three-layer hierarchical game framework. However, there are two challenges. First, information asymmetry between workers and LMOs may result in that the workers hide their types. Second, incentive mismatch between TP and LMOs may result in a lack of LMOs' willingness to participate in FL. Therefore, we decompose the hierarchical-based framework into two layers to address these challenges. For the lower-layer, we leverage the contract theory to ensure truthful reporting of the workers' types, based on which we simplify the feasible conditions of the contract and design the optimal contract. For the upper-layer, the Stackelberg game is adopted to model the interactions between the TP and LMOs, and we derive the Nash equilibrium and Stackelberg equilibrium solutions. Moreover, we develop an iterative Hierarchical-based Utility Maximization Algorithm (HUMA) to solve the coupling problem between upper-layer and lower-layer games. Extensive numerical experimental results verify the effectiveness of HUMA, and the comparison results illustrate the performance gain of HUMA.**

***Index Terms*—Incentive mechanism, federated learning, contract theory, Stackelberg game.**

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I. Introduction

The rapid development of mobile computing has led to an explosive growth in various types of sensing data, which facilitates various machine learning enabled services and applications. With the tremendous growth of data generated by mobile devices, it is challenging to upload all the data to the cloud servers in a traditional cloud computing paradigm due to limited network resources for sending the data as well as the issue of data privacy [1], [2], [3].

Federated learning (FL) is emerging as a privacy-preserving and cost-efficient distributed computing paradigm [4], [5]. In particular, FL enables a collaborative machine learning approach that allows a group of clients (which are called as the local model owners, i.e., LMO, in this work) to train their local models individually using their local data. Instead of uploading raw data to the FL server which is called as the task publisher (TP) in this work, the LMOs upload their locally trained models, whose sizes are usually much smaller than that of the sampling data, to the TP. Then, the TP aggregates all LMOs' local models to obtain the global one by using some weighted averaging algorithms, such as Federated Averaging (FedAvg) [6]. Compared with the traditional cloud computing paradigm, FL enhances the data privacy and reduces the network communication latency.

The advantage of FL has attracted lots of applications and research interests in recent years. By utilizing an effective decentralized training protocol, FL enables training models with diverse training resources. Meanwhile, the presence of data bias among the participating parties can lead to the generation of non-IID data, which in turn can impact the model performance [7], [8]. In particular, most existing studies assume that the LMOs have a sufficient amount of sampling data for their local model training. Nevertheless, in practice, the LMOs may suffer from insufficient local sampling data (e.g., due to the limited sensing capacity or storage) [9]. As a result, recruiting workers to collect additional sampling data provides a promising approach for addressing this issue. For example, NVIDIA Claran FL, which takes advantage of a distributed collaborative learning technique, conducts medical analysis by using patient data with multiple hospitals [10]. Although hospitals can access mandatory data such as patients' age, weight and blood group, they are not allowed to access privacy data without personal consent. In response to this situation, there are workers who can collect information or data samples by the mobile devices for LMOs to obtain reward [11].

In the scenario of Unmanned Aerial Vehicle (UAV)-enabled Internet of Vehicles [12], LMOs can deploy UAVs for time-sensitive tasks, including sensing and model training. And LMOs have the capability to recruit UAVs for sensing tasks through Internet-based sensing platforms [13]. In mobile crowd-sensing system, LMOs can announce a sensing task and interested workers collect the required data [14]. Additionally, LMOs can create a reward pool to encourage data owners to contribute their data to LMOs for FL tasks [15]. Workers can also collect data at their convenience and submit it to the nearest edge server [16]. These examples demonstrate the real-world applicability of worker recruiting strategies in FL. Furthermore, the rapid growth of crowdsourcing platforms such as Amazon Mechanical Turk (MTurk), Prolific, and Clickworker exemplify effective worker recruitment strategies.

However, there are still challenges in FL system to incentivize LMOs to join FL and to design the number of data samples collected by workers and their corresponding rewards. Most of the existing studies address the problem of designing incentive mechanisms between the TP and LMOs or LMOs and workers in FL through a game theory approach (e.g., contract theory, Stackelberg game and auction) [9], [17], [18]. With the popularity of FL [19], there are several papers that investigate the hierarchical FL. The study in [20] use a Stackelberg differential game to solve the optimal bandwidth allocation problem for the LMOs. The studies in [15], [21] leverage an evolutionary game to simulate the population selection process, and propose a deep learning-based auction model to simulate the competition between model owners. And the contract theory is adopted to motivate workers to provide more data with a linear cost function under information asymmetry [9], [14], [18]. Nevertheless, the above studies do not take into account the feature between the TP and LMOs when the data provided by LMOs do not meet the requirements for model training. And considering the process of data collection by workers, the linear cost function is not consistent with the law of diminishing marginal utility.

In this paper, we propose a hierarchical-based game framework for FL with three main roles including the TP, LMOs and workers. There are two challenging issues regarding the incentive mechanism design in our framework. The incentive mechanism of the lower-layer is designed between LMO and workers. Due to the information asymmetry, selfish workers will conceal their types, i.e., workers with a higher ability to collect data have an incentive to collect less data to reduce cost. Therefore, an effective incentive mechanism should enable workers to collect data samples according to their real types. The incentive mechanism of the upper-layer is designed between TP and LMOs. The purpose of TP is to maximize its utility, which is defined as the benefit from the model minus the total payment to the LMOs. And each LMO would like to maximize its utility, i.e., the payment from the TP minus its total cost. If the total payment of TP is too large, the utility of TP is reduced. Conversely, the data samples contributed by LMOs are reduced, which also leads to a decrease in model accuracy. The main contributions of this paper are summarized as follows.

1) We propose a three-layer framework combining contract theory and Stackelberg game to analyze the interactions among the TP, LMOs and workers in an FL scenario. Due to the incentive mismatches and information asymmetry between TP and LMOs, as well as between LMOs and workers, we design effective incentive mechanisms to incentivize workers to collect data and LMOs to participate in FL. And then, we describe the problem under this framework as a utility maximization problem.
2) We decompose the framework into upper-layer and lower-layer. For the upper-layer, we model the interactions between the TP and LMOs as the Stackelberg game. We analyze the existence of Nash equilibrium solution for non-cooperative game between LMOs. Afterwards, the Nash equilibrium and Stackelberg equilibrium are derived. For the lower-layer, we apply the contract theory to address the issue of information asymmetry between the LMOs and workers. Then, we simplify the feasible conditions of the contract, and finally design the optimal contract.
3) Due to the fact that the lower-layer and the upper-layer are coupled in the hierarchical framework, i.e., the strategies of the upper-layer and the lower-layer are influenced by each other, it is difficult to find the analytical solution directly. After analyzing the hierarchical structure of the framework, we introduce an auxiliary variable and prove the monotonicity of unit data purchase cost with the number of data samples collected by workers. Then, we propose an iterative Hierarchical-based Utility Maximization Algorithm called HUMA to solve the utility maximization problem.
4) We measure the model accuracy under different number of data samples on the EMNIST [22] and Fashion-MNIST [23] datasets and perform extensive simulations to evaluate the performance efficiency of HUMA. The results show that HUMA is consistent with the theoretical analysis. Moreover, the comparison results with the benchmark schemes, including the discriminatory pricing scheme and uniform pricing scheme, demonstrate the advantage of HUMA.

The remainder of this paper is organized as follows. Section II reviews related works. Section III introduces the hierarchical-based game framework and the utility maximization problem formulation. Section IV addresses the incentive mechanism problems of the upper-layer and lower-layer respectively. Section V proposes the HUMA algorithm to solve the coupled problem between upper-layer and lower-layer. Section VI evaluates the algorithm performance. Finally, Section VII concludes this work and discusses the future directions.

II. Related Work

The rapid expansion of the Internet of Things (IoT) has resulted in a substantial increase in internet-connected devices [24], [25]. To address the problems of user privacy leakage and constrained communication resources for data transmission, FL is proposed as a promising framework for distributed learning [4], [6], [26]. In contrast to traditional cloud-centric training methods, adopting FL for model training enhances user privacy, makes efficient use of network bandwidth, and reduces latency [27], [28], [29], [30]. With these advantages, FL has been used in various scenarios [31], such as smart city, smart transportation, smart vehicle services [32], etc. Chen et al. explored the challenge of training FL algorithms over a practical wireless network [30] and optimized the uplink RB allocation and user selection to minimize the FL loss function. Yang et al. provided an overview of FL applications for 6G wireless networks and discussed essential requirements, potential applications, associated problems and challenges in FL [31].

Although FL shows tremendous benefits in terms of cooperative learning and user privacy protection, the performance of FL degrades if the edge nodes do not have sufficient training resources [33], [34]. Therefore, incentivizing LMOs to participate in FL and ensuring that they have sufficient data are crucial. Many techniques had already been applied to the design of incentive mechanism, including contract theory [9], [12], [18], [35], [36], Stackelberg game [37], [38], [39] and auction [33], [40]. Khan et al. [37] presented design aspects and incentive mechanisms for FL in edge networks to address the resource optimization problem. And The interactions based on incentives between the global server and local model owners were modeled by the Stackelberg game to incentivize local model owners to engage in FL process. Zhan et al. [39] formulated the mobile crowdsensing problem as a Stackelberg game involving multiple leaders and followers. In their model, the task publisher assumed the role of the leader, while the mobile users acted as followers. Then, they proposed a DDIM algorithm utilizing deep reinforcement learning to address the challenge to compute the Stackelberg Equilibrium. Ding et al. [36] studied the optimal incentives for servers in the presence of multidimensional privacy information of users in FL. They considered a multidimensional contract-theoretic approach to verified the good performance of the contract algorithm under three information scenarios.

As FL gains widespread adoption, [19] introduces a novel Hierarchical Federated Edge Learning (HFEL) framework. This innovative approach involves the partial migration of model aggregation from the cloud to edge servers. Xiong et al. [41] developed a hierarchical game framework that combined contract theory and Stackelberg game, and proposed a new joint optimization approach in which only one content provider was considered that did not reflect the competitive relationship between content providers. Lim et al. [14] proposed a framework of hierarchical incentive mechanism considering hierarchical structure of participating subjects under FL. And the challenges related to incentive mismatch among workers and model owners, as well as model owners were addressed using contract theory and coalition game theory approaches, respectively. Ng et al. [15] addressed the performance limitation problem by employing a two-tiered incentive design strategy, incorporating both an evolutionary game and a deep learning-based auction model. Huang et al. [34] introduced a hybrid training FL framework. Within this framework, each user had the option to conduct either centralized training or local training. Leveraging the hierarchical structure of the framework, they devised an efficient algorithm to address the non-convex joint optimization problem.

Different from the above existing works, we consider a more complex FL framework. Specifically, the LMO can design the optimal contract to attract workers to collect data for it, so that the LMO can get more payment from the TP and thus maximize its utility. In addition, to capture the diminishing returns of workers in the data collection process, we use a nonlinear function based on the law of diminishing marginal utility to describe the negative utility of workers, which is more complex and improves the reliability of our analysis. The characteristics of leader-follower interactions and competition among LMOs motivate us to use the Stackelberg game to optimize the respective utility of the TP and each LMO. In addition, we use contract theory to account for the information asymmetry between LMOs and workers and to model their interactions. Due to the non-convex nature of the optimization problem, we propose a hierarchical-based utility maximization algorithm to solve the coupled problem in a three-level framework.

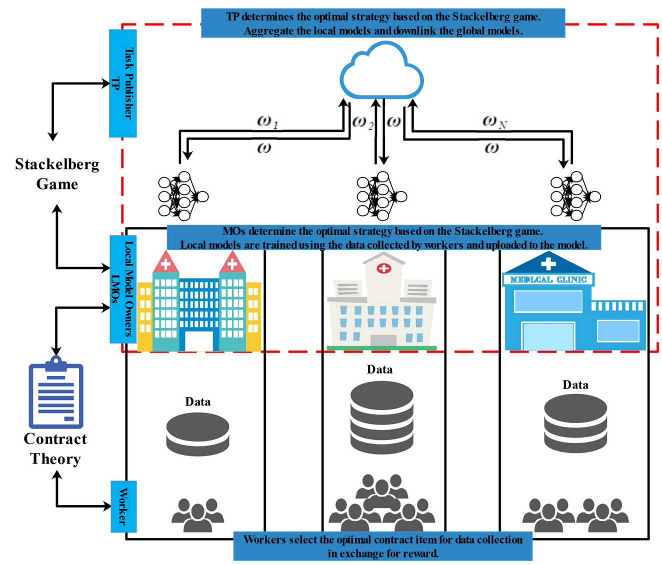


Fig. 1. Hierarchical game model in FL framework.

III. System Model and Problem Formulation

In this section, we propose the distributed FL framework. Then, the utility functions of different parts of the system are presented, and the utility maximization problem is introduced.

A. Proposed FL Framework With Two Layered Games

As shown in Fig. 1, we consider an FL framework consisting of a TP, multiple LMOs, and multiple workers. The TP publishes FL tasks and total payment to attract interested LMOs to participate to gain the reward. According to the FL tasks, each LMO makes a strategy regarding its level of participation, i.e., the data samples x_n contributed by LMO n to maximize its utility. In some cases, LMOs are short of training data. To address this issue, they may publish data collection tasks that workers can process in exchange for rewards. Our framework is structured into two distinct layers to enhance privacy while leveraging FL. In the upper layer, the interaction is between the TP and LMOs. In this layer, FL plays a key role in enabling a collaborative machine learning approach. This design allows LMOs to train their local models independently using local data. This arrangement significantly improves data privacy and reduces network communication latency, since data does not need to be pooled or shared centrally [6]. In the lower layer, the interaction is between LMOs and workers. In this layer, the focus is on the data collected from workers. Workers' data forms a private dataset that is processed locally in LMOs for FL. LMOs act as trusted third-party platforms, ensuring that data is processed and analysed locally in combination with other sensing data from connected devices. This structure eliminates the need for direct data submission to a global platform, thereby preserving local privacy at the LMO level [16].

The FL process contains three key steps in each global iteration: local computation at each LMO, involving several local iterations; transmission of local model parameters instead of raw data from each LMO; and aggregation of results and broadcast of the global model at the TP. The number of local iterations and the number of global iterations are denoted as I_l and I_g, respectively. Following FL algorithm in [28], we use Gradient Descent (GD) for local training at each LMO, since it achieves higher accuracy and requires fewer global iterations compared to Stochastic Gradient Descent (SGD) and mini-batch method [29].

Denote $\mathcal{N} = \{1, 2, \ldots, N\}$ as the set of LMOs in the system. Since LMOs are rational, they will only participate in FL training if they can benefit from it. Denote $\mathcal{M} = \{1, 2, \ldots, M\}$ as the set of all participating LMOs in the system. LMO $m \in \mathcal{M}$ owns the dataset $\mathcal{X}_m$ with X_m data samples from the workers. For the dataset $\mathcal{X}_m = \{\boldsymbol{I}_{mi}, O_{mi}\}_{i=1}^{X_m}$, $\boldsymbol{I}_{mi}$ represents the input vector, while O_{mi} represents the corresponding output. In order to facilitate the explanation, the FL model trained by each LMO is termed as local model, while the FL model aggregated by TP is termed as global model. In the process of FL, $\boldsymbol{w}_m$ denotes the vector parameter of the local model trained by LMO m, the loss function $l_m(\boldsymbol{w}_m, \boldsymbol{I}_{mi}, O_{mi})$ represents the performance over $\boldsymbol{I}_{mi}$ and O_{mi}. For LMO m with the dataset $\mathcal{X}_m$, the total loss function is denoted as

$$L_m\left(\boldsymbol{w}_m\right) := \frac{1}{X_m} \sum_{i=1}^{X_m} l_i\left(\boldsymbol{w}_m, \boldsymbol{I}_{mi}, O_{mi}\right). \tag{1}$$

Then, the objective of FL is to minimize the subsequent global loss function

$$\min_{\boldsymbol{w}} L\left(\boldsymbol{w}\right) := \sum_{m=1}^{M} \frac{X_m}{X} L_m\left(\boldsymbol{w}_m\right), \tag{2}$$

where $X = \sum_{m=1}^{M} X_m$ is the total number of data samples, $\boldsymbol{w}$ denotes the vector parameter related to the global model. According to [28], the problem (2) can be solved by an iterative approach. We consider the function $L_m(\boldsymbol{w}_m)$ is $\pi - Lipschitz$ and $\gamma - strongly$ convex, i.e.,

$$\gamma \boldsymbol{I} \preceq \nabla^2 L_m\left(\boldsymbol{w}_m\right) \preceq \pi \boldsymbol{I}, \tag{3}$$

where π and γ represent parameters associated with the loss function. A number of widely used loss functions such as $f_i(\boldsymbol{w}, \boldsymbol{I}_{mi}, O_{mi}) = \frac{1}{2}(\boldsymbol{I}_{mi}^T \boldsymbol{w} - O_{mi})^2$ for linear regression and $f_i(\boldsymbol{w}, \boldsymbol{I}_{mi}, O_{mi}) = -\log(1 + \exp(-O_{mi}\boldsymbol{I}_{mi}^T \boldsymbol{w}))$ for logistic regression satisfy condition (3). Each LMO m solves the following optimization problem in local training

$$\min_{\phi_m} \mathcal{G}_m\left(\omega^k, \boldsymbol{\phi_m}\right) = L_m\left(\omega^k + \boldsymbol{\phi_m}\right) - \left(\nabla L_m\left(\omega^k\right) - \xi \nabla L\left(\omega^k\right)\right)^T \boldsymbol{\phi_m}, \tag{4}$$

where ω^k is the global parameter at global iteration k, $\boldsymbol{\phi_m}$ represents the difference between the global model and local model for LMO n, $\nabla L_m(\omega^k)$ is the gradient of function $L_m(\omega^k)$, ξ is a constant value. The lower bound on the number of local iterations I_l in one global iteration is derived as $C_1 \log_2(1/\eta)$, where $C_1 = \frac{2}{(2-L\delta)\delta\gamma}$ and the step size $\delta < 2/L$ of local training in LMOs. The lower bound on the number of global iterations I_g is derived as $I_g = C_2/(1-\eta)$, where $C_2 = \frac{2L^2}{\gamma^2 \xi} \ln(1/\varepsilon)$ and $0 \leq \xi \leq \frac{\gamma}{L}$ [42]. Similar to [28], we can replace the assumption of γ-strong convexity with the γ-bounded nonconvexity condition for the nonconvex loss function. It has been proven that the regularized versions of $L_m(\boldsymbol{w}_m)$ and $L(\boldsymbol{w})$ satisfy γ-strong convexity and $(\pi + 2\gamma)$-Lipschitz properties. Therefore, the convergence analysis of convex loss function can be directly applied. We can observe that a higher local accuracy has the potential to decrease the count of global iterations required to achieve a predetermined global accuracy. However, a high local accuracy requires more rounds of each LMO's local iterations as well as the corresponding sources.

During the process of local training, each LMO allocates computing resources to train the local model. To facilitate model aggregation, all LMOs upload the same size of local model parameters to the TP. Following prior research [12], [14], [17], we consider that the local computational cost of LMO n is proportional to the amount of data, and is independent of the model parameters. The unit computation cost of LMO n in one local iteration is denoted as c_n^{cmp}. Furthermore, we consider that the communication cost e_n^{com} of LMO n in each round of the global iteration is a constant, regardless of the number of data samples.

B. Problem Formulation

1) Utility of Task Publisher: The TP generates an FL task and announces the total payment $\tau > 0$, while each LMO determines the level of participation based on τ and the decisions made by other LMOs. The utility of the TP is expressed as

$$\mathcal{U}\left(\tau\right) = \lambda g(X) - \tau, \tag{5}$$

where $\lambda > 0$ is a system parameter of the conversion coefficient between model accuracy and revenue, $g(X)$ is a function of the model accuracy with respect to the total number of data samples X. Similar to related works in [12], [14], [17], the model performance experiences diminishing returns with an increasing number of data samples. The utility of the model can be aptly characterised using a concave function. And we adopt a similar concave function of $g(X) = \alpha \ln(1 + \beta X)$. The logarithmic function, a classic representation of the law of diminishing marginal utility, effectively captures this concept. It indicates that the incremental benefit from each additional data unit decreases as the model training of TP. In addition, the logarithmic function helps us derive closed-form solutions.

2) Utility of Local Model Owners: For FL tasks, LMOs train the local models for profit with local data samples, while incurring some training cost such as computation, communication, and data purchase cost. We denote the unit data purchase cost as C_n^{pur}. Therefore, the utility function of LMO n in FL-based system is defined as

$$\mathcal{U}_n\left(x_n, \boldsymbol{x_{-n}}\right) = \frac{x_n}{X}\tau - I_g\left(I_l c_n^{cmp} x_n + e_n^{com}\right) - C_n^{pur} x_n, \tag{6}$$

where x_n represents the number of data samples contributed by LMO n, $\boldsymbol{x_{-n}} = (x_1, x_2, \ldots, x_{n-1}, x_{n+1}, \ldots, x_N)$ is the strategy set of other LMOs exclude LMO n. For an FL task, the total computation cost of LMO n is $I_g I_l c_n^{cmp} x_n$, the total communication cost is $I_g e_n^{com}$, and the total data purchase cost is $C_n^{pur} x_n$. To facilitate the expression of symbols, we define the unit computation cost as $C_n^{cmp} = I_g I_l c_n^{cmp}$ and the communication cost as $E_n^{com} = I_g e_n^{com}$. Thus, the utility function of LMO n in (6) is redefined as

$$\mathcal{U}_n\left(x_n, \boldsymbol{x_{-n}}\right) = \frac{x_n}{X}\tau - C_n^{cmp} x_n - E_n^{com} - C_n^{pur} x_n. \tag{7}$$

3) Utility of Workers: There are many different types of workers, which differ in the ability to collect data in the system. In the hierarchical incentive mechanism framework, we simplify the model by assuming a uniform quality of data across all worker types [9], [12], [39]. Denote M_n as the number of worker types included in the n-th LMO. The willingness of type-m worker belonging to LMO n to participate is denoted as δ_n^m, where $m \in \{1, \ldots, M_n\}$. Similarly, the data quantity collection is denoted as d_n^m, where $m \in \{1, \ldots, M_n\}$ [14]. Without loss of generality, we consider that $0 < \delta_n^1 < \delta_n^2 < \cdots < \delta_n^{M_n}$ and the higher δ_n^m indicates the worker can incur lower costs in data collection. LMOs can utilize statistical information to classify workers into different types based on their data collection behaviors [12]. This classification is achieved using well-established data mining methods, such as k-means clustering. Once the workers are classified into types, the self-revealing property of the contract-theoretic mechanism design comes into play. This property ensures that workers are appropriately rewarded according to their types. Workers will naturally reveal their true types, allowing LMOs to allocate rewards effectively based on these self-revealed types [43]. Regarding the function h, LMOs can also estimate it through historical data and past experiences.

From the perspective of economics, a nonlinear function called fatigue loss function based on the law of diminishing marginal utility is used to describe the negative benefit of workers when collecting data, like in [38], [39], [41]. The fatigue loss function $h(x)$ is

$$h(x) = (1-\theta)\, x^{\frac{1}{1-\theta}}, \tag{8}$$

where $0 < \theta < 1$ is the given coefficient. Note that the fatigue loss function is non-decreasing and convex with increasing marginal loss, i.e., $(\mathrm{d}h(x))/(\mathrm{d}x) \geq 0$ and $(\mathrm{d}^2 h(x))/(\mathrm{d}x^2) \geq 0$, which can be used to model the increasing marginal loss for every additional unit of data. Thus, the the utility function of type-m worker belonging to LMO n is formulated as

$$\mathcal{U}_n^m\left(R_n^m, d_n^m\right) = R_n^m - \frac{h\left(d_n^m\right)}{\delta_n^m} - c d_n^m, \tag{9}$$

where c is the cost associated with collecting each unit of data, R_n^m is the reward that the worker received by collecting d_n^m data samples. The expression of $h(d_n^m)/\delta_n^m$ indicates that for workers with higher data collection abilities, the same amount of data collection results in lower associated costs.

4) Utility Maximization: In our proposed hierarchical-based game framework, the interactions between the parties involved in FL are highly complex. The TP needs to decide on an appropriate payment to attract LMOs to participate to obtain a high-quality model (10a). LMOs need to decide on their level of participation and design optimal contract items (10b). Meanwhile, workers need to choose the contract item to maximize their utility (10c). To sum up, since all participants in the system have the desire to maximize their utilities. The utility maximization problem of the hierarchical game is formulated as:

$$TP: \text{ Maximize } \mathcal{U}(\tau) \tag{10a}$$

$$LMO: \text{ Maximize } \mathcal{U}_n\left(x_n, \boldsymbol{x_{-n}}\right) \tag{10b}$$

$$Worker: \text{ Maximize } \mathcal{U}_n^m\left(R_n^m, d_n^m\right) \tag{10c}$$

IV. Incentive Mechanism Design for FL-Based Sub-Layers

In this section, we decompose the three-layer FL framework into two parts. The first part includes the TP and the LMOs, and the second part includes the LMOs and the workers.

A. Optimal Pricing Design for TP and LMOs

In the FL network, the TP attracts LMOs to participate in model training by paying a total payment τ, while each LMO gets part of τ by training the local model with local data samples. In fact, there are trade-offs to be made in the determination of payment τ. On the one hand, if τ is set higher, the LMOs will have a higher level of participation, resulting in a better global model, but it will also result in a higher cost for the TP. On the other hand, if the TP sets a lower τ, the TP will have lower cost, but the LMOs will have a lower level of participation and make the global model worse. Therefore, the determination of payment τ is important.

We formulate the interactions between TP and LMOs as a two stage Stackelberg game [44]. In the stage I, the TP announces a total payment τ. Then, in the stage II, each LMO determines its level of participation to maximize its own utility.

1) LMO Strategy Determination: LMO n's strategy is the number of contributed data samples x_n. Since each LMO is selfish and rational, each LMO n aims to maximize its own utility. In this case, the competition between LMOs can be formulated as a non-cooperative game and their optimal strategies form a Nash equilibrium. The detailed definition and proof in this non-cooperative game are as follows:

Definition 1 (Nash Equilibrium): The strategy of each LMO $\boldsymbol{x}^* = (x_1^*, x_2^*, \ldots, x_N^*)$ forms a Nash equilibrium [44] if for any

LMO n,

$$\mathcal{U}_n\left(x_n^*, \boldsymbol{x}_{-\boldsymbol{n}}^*\right) \geq \mathcal{U}_n\left(x_n, \boldsymbol{x}_{-\boldsymbol{n}}^*\right).$$

Theorem 1: There exists a Nash Equilibrium $\boldsymbol{x}^* = (x_1^*, x_2^*, \ldots, x_N^*)$ in stage II of the Stackelberg game among the LMOs.

Proof: The non-cooperative game between LMOs is composed as follows:

- *Players:* LMO $n \in \mathcal{N}$.
- *Strategy:* The number of contributed data samples x_n of any LMO n.
- *Utility:* The utility function of any LMO n in (7).

First, it is clear that the number of players N is a constant.

Second, based on (7) and the fact that $\mathcal{U}_n(x_n, \boldsymbol{x}_{-\boldsymbol{n}})$ must be positive, we obtain that

$$0 \leq x_n \leq \frac{\tau - E_n^{com}}{C_n^{cmp} + C_n^{pur}}. \tag{11}$$

Third, the first and second-order derivative of $\mathcal{U}_n(x_n, \boldsymbol{x}_{-\boldsymbol{n}})$ with respect to x_n can be derived as

$$\frac{\partial \mathcal{U}_n\left(x_n, \boldsymbol{x}_{-\boldsymbol{n}}\right)}{\partial x_n} = \frac{-\tau x_n}{X^2} + \frac{\tau}{X} - C_n^{cmp} - C_n^{pur}. \tag{12}$$

$$\frac{\partial^2 \mathcal{U}_n\left(x_n, \boldsymbol{x}_{-\boldsymbol{n}}\right)}{\partial x_n^2} = -\frac{2\tau \Sigma_{m\neq n} x_m}{X^3} < 0. \tag{13}$$

To sum up, there exists a Nash equilibrium of the Stackelberg game [44].

Then, the optimal strategy of each LMO can be derived as

$$\frac{-\tau x_n}{X^2} + \frac{\tau}{X} - C_n^{cmp} - C_n^{pur} = 0. \tag{14}$$

Solving (14), we obtain

$$x_n = \sqrt{\frac{\tau \sum_{m\neq n} x_m}{C_n^{cmp} + C_n^{pur}}} - \sum_{m\neq n} x_m. \tag{15}$$

It is noted that in actual situations, x_n does not always achieve the optimal solution. We define the maximum number of data samples that LMO n can collect as $q_n^{\max}$. Therefore, when the calculation result of (15) is greater than $q_n^{\max}$, the optimal strategy of LMO n is $q_n^{\max}$. When the result of (15) is negative, LMO n does not participate, i.e., the optimal strategy of LMO n is 0. Therefore, we have

$$x_n^* = \begin{cases} 0, x_n \leq 0, \\ \sqrt{\frac{\tau \sum_{m\neq n} x_m}{C_n^{cmp} + C_n^{pur}}} - \sum_{m\neq n} x_m, 0 \leq x_n \leq q_n^{\max}, \\ q_n^{\max}, \text{ otherwise}. \end{cases} \tag{16}$$

Since the calculation of x_n^* in (16) requires the information of other LMOs, which are complicated to carry out, we give the analytical expression of the optimal strategy for each LMO in the closed form in Theorem 2.

Theorem 2: For any $m \in \mathcal{M} \subseteq \mathcal{N}$ participating in the game, its optimal strategy is

$$x_m^* = \frac{(M-1)\,\tau}{\sum_{n\in\mathcal{M}} C_n}\left(1 - \frac{(M-1)\,C_m}{\sum_{n\in\mathcal{M}} C_n}\right), \tag{17}$$

where $M = |\mathcal{M}|$ and $C_n = C_n^{cmp} + C_n^{pur}$ is the unit cost of LMO n, $\forall n = 1, 2, \ldots, M$.

Proof: According to (16), for any $m \in \mathcal{M}$, we have

$$\sum_{n=1}^{M} x_n^* = \sqrt{\frac{\tau \sum_{m\neq n} x_n^*}{C_m^{cmp} + C_m^{pur}}}. \tag{18}$$

Transform the (18), we obtain

$$\sum_{m\neq n} x_n^* = \frac{\left(C_m^{cmp} + C_m^{pur}\right)\left(\sum_{n=1}^{M} x_n^*\right)^2}{\tau}. \tag{19}$$

By setting $\psi_{-m} = \sum_{m\neq n} x_n^*$, we can derive that

$$\begin{cases} \psi_{-1} = \frac{\left(C_1^{cmp}+C_1^{pur}\right)\left(\sum_{n=1}^{M} x_n^*\right)^2}{\tau} \\ \psi_{-2} = \frac{\left(C_2^{cmp}+C_2^{pur}\right)\left(\sum_{n=1}^{M} x_n^*\right)^2}{\tau} \\ \quad\vdots \\ \psi_{-M} = \frac{\left(C_M^{cmp}+C_M^{pur}\right)\left(\sum_{n=1}^{M} x_n^*\right)^2}{\tau}. \end{cases} \tag{20}$$

It can be observed that $\sum_{n=1}^{M} \psi_{-n} = (M-1)\sum_{n=1}^{M} x_n^*$, we have

$$\sum_{n=1}^{M} x_n^* = \frac{\tau(M-1)}{\sum_{n=1}^{M}\left(C_n^{cmp} + C_n^{pur}\right)}. \tag{21}$$

Substituting (21) into (18), we get

$$\sum_{n=1}^{M} x_n^* = \sqrt{\frac{\tau\left(\left(\sum_{n=1}^{M} x_n^*\right) - x_m^*\right)}{C_m^{cmp} + C_m^{pur}}}. \tag{22}$$

Therefore, we have

$$x_m^* = \frac{(M-1)\,\tau}{\sum_{n\in\mathcal{M}} C_n^{cmp} + C_n^{pur}}\left(1 - \frac{(M-1)\left(C_m^{cmp} + C_m^{pur}\right)}{\sum_{n\in\mathcal{M}} C_n^{cmp} + C_n^{pur}}\right). \tag{23}$$

By setting $C_n = C_n^{cmp} + C_n^{pur}, \forall n = 1, 2, \ldots, M$, (17) can be derived. □

2) TP's Payment Determination: The TP's strategy involves setting the payment τ with the aim of maximizing its utility. According to (17) and (5), we have

$$\mathcal{U}(\tau) = \lambda g\left(\sum_{n=1}^{M} x_n^*\right) - \tau. \tag{24}$$

The optimal strategy τ is determined by the TP after observing the optimal strategy of each LMO. Therefore, the TP can maximize its utility by determining the payment τ. And the optimal strategy of TP together with the Nash equilibrium forms the Stackelberg equilibrium. The existence of Stackelberg equilibrium between TP and LMOs is illustrated in Theorem 3.

Theorem 3: There exists a unique Stackelberg equilibrium $(\tau^*, \boldsymbol{x}^*)$ between TP and LMOs.

Proof: The first and the second-order derivative of $\mathcal{U}(\tau)$ are as follows

$$\frac{\partial \mathcal{U}(\tau)}{\partial \tau} = \lambda g'(X)\left(\frac{\partial x_1^*}{\partial \tau} + \frac{\partial x_2^*}{\partial \tau} + \cdots + \frac{\partial x_M^*}{\partial \tau}\right) - 1. \tag{25}$$

$$\frac{\partial^2 \mathcal{U}(\tau)}{\partial \tau^2} = \lambda g''(X)\left(\frac{\partial x_1^*}{\partial \tau} + \frac{\partial x_2^*}{\partial \tau} + \cdots + \frac{\partial x_M^*}{\partial \tau}\right)^2 + \lambda g'(X)\left(\frac{\partial^2 x_1^*}{\partial \tau^2} + \frac{\partial^2 x_2^*}{\partial \tau^2} + \cdots + \frac{\partial^2 x_M^*}{\partial \tau^2}\right) = \lambda g''(X)\left[\sum_{n=1}^{M} \frac{(M-1)}{\Sigma_{m\in\mathcal{M}} C_m} \times \left(1 - \frac{(M-1)\,C_n}{\Sigma_{m\in\mathcal{M}} C_m}\right)\right]^2. \tag{26}$$

Given that $g(X)$ represents a concave function of X, $(\partial^2\mathcal{U}(\tau))/(\partial\tau^2) < 0$. Thus, $\mathcal{U}(\tau)$ is a strictly concave function of τ and there exists a unique Stackelberg equilibrium. □

B. Optimal Contract Design for LMOs and Workers

Since LMOs are not aware of the private attributes of workers, workers will conceal their types to get more rewards. Therefore, we apply contract theory to address the information asymmetry between LMOs and workers. In this part, we present the feasible conditions of contract theory, optimize its feasible conditions, and finally give the analytical solution of the optimal contract.

1) Conditions for Contract Feasibility: To attract the workers to participate, the individual rationality (IR) and incentive compatibility (IC) constraints should be satisfied.

Definition 2 (IR): Workers will choose a contract only if their utility is non-negative,

$$\mathcal{U}_n^i\left(R_n^i, d_n^i\right) = R_n^i - \frac{h\left(d_n^i\right)}{\delta_n^i} - c d_n^i > 0, \quad \forall i \in \{1,\ldots,M_n\}, \forall n \in \{1,\ldots,N\}. \tag{27}$$

Definition 3 (IC): Each worker can maximize utility only if it chooses the contract item designed for its type, i.e., $\mathcal{U}_n^i(R_n^i, d_n^i) \geq \mathcal{U}_n^i(R_n^j, d_n^j), \forall i,j \in \{1,\ldots,M_n\}, i \neq j$,

$$R_n^i - \frac{h\left(d_n^i\right)}{\delta_n^i} - c d_n^i \geq R_n^j - \frac{h\left(d_n^j\right)}{\delta_n^i} - c d_n^j, \quad \forall i,j \in \{1,\ldots,M_n\}, i \neq j, \forall n \in \{1,\ldots,N\}. \tag{28}$$

The IR constraints are to ensure that workers receive a return that is greater than the cost when collecting data. If $\mathcal{U}_n^i(R_n^i, d_n^i) < 0$, workers will not participate. The IC constraints are to ensure that each worker's type is exposed to the LMO. A contract is deemed feasible if it complies with IR and IC constraints.

Based on contract theory, each LMO aims to maximize its utility under these constraints. Therefore, the optimal contract problem is formulated as:

$$\begin{aligned}
\max_{R_n^m, d_n^m}\ & \mathcal{U}_n\left(x_n, \boldsymbol{x_{-n}}\right) = \frac{x_n}{X}\tau - C_n^{cmp} x_n - E_n^{com} - C_n^{pur} x_n \\
\text{s.t.}\ & C_n^{pur} = \frac{\sum_{m=1}^{M_n} \rho_n^m R_n^m}{\sum_{m=1}^{M_n} \rho_n^m d_n^m}, \\
& x_n - p_n \sum_{m=1}^{M_n} \rho_n^m d_n^m = 0, \\
& \mathcal{U}_n^i\left(R_n^i, d_n^i\right) \geq 0, \forall i \in \{1,\ldots,M_n\}, \\
& \mathcal{U}_n^i\left(R_n^i, d_n^i\right) \geq \mathcal{U}_n^i\left(R_n^j, d_n^j\right), \forall i,j \in \{1,\ldots,M_n\}, \\
& d_n^i \geq 0, R_n^i \geq 0, \delta_n^i \geq 0, \forall i \in \{1,\ldots,M_n\},
\end{aligned} \tag{29}$$

where ρ_n^m denotes the proportion of workers with type m in LMO n, $\sum_{m=1}^{M_n} \rho_n^m = 1$. P_n refers to the number of participating workers in LMO n, and R_n^m refers to the reward received by type m workers.

2) Design Optimal Contract: As there exist M_n IR constraints and $M_n(M_n-1)$ IC constraints, which are non-convex and coupled, solving the problem in (29) directly proves challenging. According to the feasibility conditions of contract, Lemma 1 can be derived.

Lemma 1: For any feasible contract, we have $d_n^i \geq d_n^j$ if and only $\delta_n^i \geq \delta_n^j$.

Proof: From (28), we have

$$\begin{cases} \mathcal{U}_n^i\left(R_n^i, d_n^i\right) \geq \mathcal{U}_n^i\left(R_n^j, d_n^j\right) \\ \mathcal{U}_n^j\left(R_n^j, d_n^j\right) \geq \mathcal{U}_n^j\left(R_n^i, d_n^i\right). \end{cases} \tag{30}$$

The above (30) is equivalent to

$$\begin{cases} R_n^i - \frac{h\left(d_n^i\right)}{\delta_n^i} - c d_n^i \geq R_n^j - \frac{h\left(d_n^j\right)}{\delta_n^i} - c d_n^j \\ R_n^j - \frac{h\left(d_n^j\right)}{\delta_n^j} - c d_n^j \geq R_n^i - \frac{h\left(d_n^i\right)}{\delta_n^j} - c d_n^i. \end{cases} \tag{31}$$

Then, we add up the two inequalities to obtain

$$h\left(d_n^i\right)\frac{\left(\delta_n^i - \delta_n^j\right)}{\delta_n^i \delta_n^j} \geq h\left(d_n^j\right)\frac{\left(\delta_n^i - \delta_n^j\right)}{\delta_n^i \delta_n^j}. \tag{32}$$

$$\frac{h\left(d_n^i\right) - h\left(d_n^j\right)}{\delta_n^j} \geq \frac{h\left(d_n^i\right) - h\left(d_n^j\right)}{\delta_n^i}. \tag{33}$$

Since $\delta_n^i - \delta_n^j \geq 0$ and $\mathrm{d}h(x)/\mathrm{d}x \geq 0$, it follows that $d_n^i \geq d_n^j$ and $\delta_n^i \geq \delta_n^j$. Thus, we prove that $d_n^i \geq d_n^j$ if and only $\delta_n^i \geq \delta_n^j$. □

Lemma 1 indicates that workers with higher willingness (i.e., δ) to participate will provide more data samples. And according to (28), we obtain that

$$R_n^i \geq R_n^j + \frac{h\left(d_n^i\right)}{\delta_n^i} - \frac{h(d_n^j)}{\delta_n^i} + c d_n^i - c d_n^j. \tag{34}$$

Thus, $R_n^i \geq R_n^j, \forall i,j \in \{1,\ldots,M_n\}, i \geq j$. The contract items should satisfy the property such that the more data samples contributed, the more reward for workers. And the necessary conditions for the contract design can be derived in Theorem 4.

Theorem 4: A feasible contract must meet the monotonicity conditions:

$$\begin{cases} 0 \leq R_n^1 \leq \cdots \leq R_n^m \leq \cdots \leq R_n^{M_n} \\ 0 \leq d_n^1 \leq \cdots \leq d_n^m \leq \cdots \leq d_n^{M_n}. \end{cases} \tag{35}$$

Next, the IR constraints can be reduced by Lemma 2.

Lemma 2: If the IR constraints are satisfied for workers of type-1, i.e.,

$$R_n^1 - \frac{h\left(d_n^1\right)}{\delta_n^1} - c d_n^1 \geq 0, \tag{36}$$

the IR constraints for all types of workers are satisfied as well.

Proof: Following (28) and $\delta_n^1 < \cdots < \delta_n^m < \cdots < \delta_n^{M_n}$, we have

$$R_n^i - \frac{h\left(d_n^i\right)}{\delta_n^i} - cd_n^i \geq R_n^1 - \frac{h\left(d_n^1\right)}{\delta_n^i} - cd_n^1$$
$$\geq R_n^1 - \frac{h\left(d_n^1\right)}{\delta_n^1} - cd_n^1.$$

As such, the IC constraints can be reduced. And it is noted that type-1 workers are the most reluctant to collect data samples. □

After reducing the IR constraints, we proceed to reduce the IC constraints and introduce the relevant definitions in the following.

Definition 4 (Local Upward Incentive Constraint, LUIC): LUIC($\delta_n^i, \delta_n^{i+1}$): $\mathcal{U}_n^i(R_n^i, d_n^i) \geq \mathcal{U}_n^i(R_n^{i+1}, d_n^{i+1}), \forall i \in \{1, \ldots, M_n - 1\}, \forall n \in \{1, \ldots, N\}$. Specifically,

$$R_n^i - \frac{h\left(d_n^i\right)}{\delta_n^i} - cd_n^i \geq R_n^{i+1} - \frac{h\left(d_n^{i+1}\right)}{\delta_n^i} - cd_n^{i+1}. \tag{37}$$

Definition 5 (Local Downward Incentive Constraint, LDIC): LDIC($\delta_n^i, \delta_n^{i-1}$): $\mathcal{U}_n^i(R_n^i, d_n^i) \geq \mathcal{U}_n^i(R_n^{i-1}, d_n^{i-1}), \forall i \in \{2, \ldots, M_n\}, \forall n \in \{1, \ldots, N\}$. Specifically

$$R_n^i - \frac{h\left(d_n^i\right)}{\delta_n^i} - cd_n^i \geq R_n^{i-1} - \frac{h\left(d_n^{i-1}\right)}{\delta_n^i} - cd_n^{i-1}. \tag{38}$$

In Lemma 3, we reduce the IC constraints.

Lemma 3: The IC constraints can be reduced to

$$\begin{aligned}&\left\{\text{LUIC}\left(\delta_n^i, \delta_n^{i+1}\right) : \forall i \in \{1, \ldots, M_n - 1\}\right\} \cap \\ &\left\{\text{LDIC}\left(\delta_n^i, \delta_n^{i-1}\right) : \forall i \in \{2, \ldots, M_n\}\right\}.\end{aligned} \tag{39}$$

Proof: For $\delta_n^{i-1} < \delta_n^i < \delta_n^{i+1}, \forall i \in \{2, \ldots, M_n - 1\}$, we have

$$R_n^{i+1} - \frac{h\left(d_n^{i+1}\right)}{\delta_n^{i+1}} - cd_n^{i+1} \geq R_n^i - \frac{h\left(d_n^i\right)}{\delta_n^{i+1}} - cd_n^i. \tag{40}$$

$$R_n^i - \frac{h\left(d_n^i\right)}{\delta_n^i} - cd_n^i \geq R_n^{i-1} - \frac{h\left(d_n^{i-1}\right)}{\delta_n^i} - cd_n^{i-1}. \tag{41}$$

By applying the monotonicity of Theorem 4 in (41), we have

$$\left(R_n^i - cd_n^i\right) - \left(R_n^{i-1} - cd_n^{i-1}\right) \geq \frac{h\left(d_n^i\right)}{\delta_n^{i+1}} - \frac{h\left(d_n^{i-1}\right)}{\delta_n^{i+1}}. \tag{42}$$

Then, combining (40) and (42), we obtain

$$R_n^{i+1} - \frac{h\left(d_n^{i+1}\right)}{\delta_n^{i+1}} - cd_n^{i+1} \geq R_n^{i-1} - \frac{h\left(d_n^{i-1}\right)}{\delta_n^{i+1}} - cd_n^{i-1}. \tag{43}$$

It is proved that type-$(i+1)$ workers will choose contract item (R_n^{i+1}, d_n^{i+1}) instead of (R_n^{i-1}, d_n^{i-1}) to maximize its utility, i.e., $\mathcal{U}_n^{i+1}(R_n^{i+1}, d_n^{i+1}) \geq \mathcal{U}_n^{i+1}(R_n^{i-1}, d_n^{i-1})$. By using (43), it can be extended to type-1 workers, and all DIC holds.

$$\begin{aligned}R_n^{i+1} - \frac{h\left(d_n^{i+1}\right)}{\delta_n^{i+1}} - cd_n^{i+1} &\geq R_n^{i-1} - \frac{h\left(d_n^{i-1}\right)}{\delta_n^{i+1}} - cd_n^{i-1} \\ &\geq \cdots \\ &\geq R_n^1 - \frac{h\left(d_n^1\right)}{\delta_n^{i+1}} - cd_n^1.\end{aligned} \tag{44}$$

Thus, we have proved that with the LDIC and the monotonicity, all DIC will hold. Similarly, we can prove that with the LUIC and the monotonicity, the UIC will hold. □

Thus, the IR and IC constraints in feasible contract can be reduced, and the optimal contract problem can be reduced to

$$\begin{aligned}\max_{R_n^m, d_n^m} \ &\mathcal{U}_n\left(x_n, \boldsymbol{x_{-n}}\right) = \frac{x_n}{X}\tau - C_n^{cmp}x_n - E_n^{com} - C_n^{pur}x_n \\ \text{s.t.} \ \ &C_n^{pur} = \frac{\sum_{m=1}^{M_n} \rho_n^m R_n^m}{\sum_{m=1}^{M_n} \rho_n^m d_n^m}, \\ &x_n - P_n \sum_{m=1}^{M_n} \rho_n^m d_n^m = 0, \\ &\mathcal{U}_n^1\left(R_n^1, d_n^1\right) \geq 0, \\ &\mathcal{U}_n^i\left(R_n^i, d_n^i\right) \geq \mathcal{U}_n^i\left(R_n^{i-1}, d_n^{i-1}\right), \forall i \in \{2, \ldots, M_n\}, \\ &\mathcal{U}_n^i\left(R_n^i, d_n^i\right) \geq \mathcal{U}_n^i\left(R_n^{i+1}, d_n^{i+1}\right), \\ &\forall i \in \{1, \ldots, M_n - 1\}, \\ &d_n^i \geq 0, R_n^i \geq 0, \delta_n^i \geq 0, \forall i \in \{1, \ldots, M_n\}.\end{aligned} \tag{45}$$

Furthermore, the LUIC can be reduced.

Lemma 4: The optimal contract design can be achieved without considering LUIC constraints.

Proof: It is clear that the utility of LMO n is increasing with d_n^i and decreasing with $R_n^i, \forall i \in \{1, \ldots, M_n\}$. According to Lemma 2, LMO n will lower R_n^1 as much as possible to maximize its utility, until the utility of type-1 workers is zero. To maximize the utility, LMO n will lower all R_n as much as possible until the utility of type-i and type-$(i-1)$ workers are the same. Thus, the LDIC can be reduced to

$$\begin{aligned}R_n^i - \frac{h\left(d_n^i\right)}{\delta_n^i} - cd_n^i = R_n^{i-1} - \frac{h\left(d_n^{i-1}\right)}{\delta_n^i} - cd_n^{i-1}, \\ \forall i \in \{2, \ldots, M_n\},\end{aligned} \tag{46}$$

and equally it becomes

$$\delta_n^i\left(R_n^i - cd_n^i - R_n^{i-1} + cd_n^{i-1}\right) = h\left(d_n^i\right) - h\left(d_n^{i-1}\right). \tag{47}$$

Because of monotonicity, we have

$$h\left(d_n^i\right) - h\left(d_n^{i-1}\right) \geq \delta_n^{i-1}\left(R_n^i - cd_n^i - R_n^{i-1} + cd_n^{i-1}\right). \tag{48}$$

The above (48) is equivalent to

$$R_n^{i-1} - \frac{h\left(d_n^{i-1}\right)}{\delta_n^{i-1}} - cd_n^{i-1} \geq R_n^i - \frac{h\left(d_n^i\right)}{\delta_n^{i-1}} - cd_n^i. \tag{49}$$

Thus, the LUIC can be removed. □

According to Lemma 4, the optimal contract problem can be reduced to

$$\begin{aligned}\max_{R_n^m, d_n^m} \ &\mathcal{U}_n\left(x_n, \boldsymbol{x_{-n}}\right) = \frac{x_n}{X}\tau - C_n^{cmp}x_n - E_n^{com} - C_n^{pur}x_n \\ \text{s.t.} \ \ &C_n^{pur} = \frac{\sum_{m=1}^{M_n} \rho_n^m R_n^m}{\sum_{m=1}^{M_n} \rho_n^m d_n^m},\end{aligned}$$

$$
\begin{aligned}
& x_n - P_n \sum_{m=1}^{M_n} \rho_n^m d_n^m = 0, \\
& \mathcal{U}_n^1 \left(R_n^1, d_n^1\right) = 0, \\
& \mathcal{U}_n^i \left(R_n^i, d_n^i\right) = \mathcal{U}_n^i \left(R_n^{i-1}, d_n^{i-1}\right), \forall i \in \{2, \ldots, M_n\}, \\
& d_n^i \geq 0, R_n^i \geq 0, \delta_n^i \geq 0, \forall i \in \{1, \ldots, M_n\}.
\end{aligned} \tag{50}
$$

3) Optimal Contract Solution: In this part, we establish the relationship between the workers' reward $\boldsymbol{R_n}$ and $\boldsymbol{d_n}$, and then solve the optimal contract problem (50) that contains only $\boldsymbol{d_n}$, which can be summarized in the Theorem 5.

Theorem 5: The optimal reward for each type of worker is

$$
R_n^{m*} = \begin{cases} \frac{h(d_n^m)}{\delta_n^m} + c d_n^m, & \text{if } m = 1, \\ R_n^{(m-1)*} - \frac{h\left(d_n^{m-1}\right)}{\delta_n^m} & + \frac{h(d_n^m)}{\delta_n^m} - c d_n^{m-1} + c d_n^m, \\ & \text{if } m = 2, 3, \ldots, M_n, \end{cases} \tag{51}
$$

where the set of number of data samples $\boldsymbol{d_n}$ satisfies $0 \leq d_n^1 \leq \cdots \leq d_n^i \leq \cdots \leq d_n^{M_n}$.

Proof: Specifically, by iterating the third and forth constraints in (50), we have

$$
\begin{aligned}
R_n^m &= \frac{h\left(d_n^1\right)}{\delta_n^1} + c d_n^m + \sum_{k=2}^{m} \frac{h\left(d_n^k\right) - h\left(d_n^{k-1}\right)}{\delta_n^k} \\
&= \frac{h\left(d_n^m\right)}{\delta_n^m} + c d_n^m + \sum_{k=2}^{m} \left(\frac{1}{\delta_n^{k-1}} - \frac{1}{\delta_n^k}\right) h\left(d_n^{k-1}\right), \\
& \forall k \in \{2, \ldots, M_n\}.
\end{aligned} \tag{52}
$$

□

According to the first and second constraints of optimal contract problem in (50), we obtain

$$
\begin{aligned}
C_n^{pur} x_n = P_n \sum_{m=1}^{M_n} \Bigg(& \rho_n^m \frac{h\left(d_n^m\right)}{\delta_n^m} + c d_n^m \rho_n^m \\
& + h\left(d_n^m\right) \Lambda_n^m \sum_{t=m+1}^{M_n} \rho_n^t \Bigg),
\end{aligned} \tag{53}
$$

where $\Lambda_n^m = (\frac{1}{\delta_n^m} - \frac{1}{\delta_n^{m+1}})$ and $\Lambda_n^{M_n} = 0$. Substitute (53) into (50), all $R_n^m, \forall m \in \{1, \ldots, M_n\}$ are removed. Thereafter, this yields the following optimal problem:

$$
\begin{aligned}
\max_{R_n^m, d_n^m} \ & \mathcal{U}_n\left(x_n, \boldsymbol{x_{-n}}\right) = \frac{P_n \sum_{m=1}^{M_n} \rho_n^m d_n^m}{X_{-n} + P_n \sum_{m=1}^{M_n} \rho_n^m d_n^m} \tau \\
& - C_n^{cmp} P_n \sum_{m=1}^{M_n} \rho_n^m d_n^m - E_n^{com} \\
& - P_n \sum_{m=1}^{M_n} \left(\rho_n^m \frac{h\left(d_n^m\right)}{\delta_n^m} + c d_n^m \rho_n^m + \Lambda_n^m \sum_{t=m+1}^{M_n} \rho_n^t \right) \\
\text{s.t.} \ & d_n^i \geq 0, \forall i \in \{1, \ldots, M_n\}.
\end{aligned} \tag{54}
$$

In the traditional optimal contract design, we need to divide problem (54) into M_n subproblems, and the kth subproblem only contains d_n^k of the type-k workers. Then, the M_n subproblems can be solved by convex optimization and "Bunching and Ironing". Although the second and third terms of problem (54) enable the problem to be simplified, the first term is complex and difficult to split up. Thus, this optimization problem cannot be solved directly by the traditional approach.

V. Unified Game Framework Design for FL

We have solved the problem of incentive mechanism design for both the upper and lower layers. However, the strategies of the upper-layer and the lower-layer are coupled. In this section, we introduce an auxiliary variable and propose the unified algorithm to address the utility maximization problem.

A. Auxiliary Variable Design

We introduce the actual fatigue level of workers as an auxiliary variable.

Definition 6: The cost of collecting d_n^i data samples for type-i workers in LMO n is called the actual fatigue level, which is denoted as:

$$
\mu_n^i = \frac{h(d_n^i)}{\delta_n^i}, 0 < \mu_n^i < \mu_n^{\max}, \tag{55}
$$

where $\mu_n^{\max}$ is the maximum value of the workers' actual fatigue level.

For fairness, workers in the same LMO should have the same actual fatigue level μ_n. We can obtain d_n^i based on (55) as

$$
d_n^i = \left(\frac{\mu_n \delta_n^i}{1-\theta}\right)^{1-\theta} \tag{56}
$$

Due to $0 \leq \delta_n^1 \leq \delta_n^2 \leq \cdots \leq \delta_n^{M_n}$, we have $0 \leq d_n^1 \leq d_n^2 \leq \cdots \leq d_n^{M_n}$, which satisfies monotonicity in Theorem 4. Therefore, the total number of data samples collected by the workers in LMO n is $q_n = P_n \sum_{m=1}^{M_n} \rho_n^m d_n^m$ and the maximum number of data samples is $q_n^{\max}$.

B. Property of the Game Model

With the introduction of auxiliary variable, the monotonicity property can be obtained to solve the utility maximization problem, which is shown in Theorem 6.

Theorem 6: As the actual level of worker fatigue μ_n increases, the unit data purchase cost C_n^{pur} also increases.

Proof: According to the optimal contract problem in (50), we have (57) shown at the bottom of the next page. By setting f and g as the numerator and the denominator of (57) respectively, we can obtain

$$
f = \sum_{m=1}^{M_n} \left(\rho_n^m \mu_n + \delta_n^m \mu_n \Lambda_n^m \sum_{t=m+1}^{M_n} \rho_n^t + c \rho_n^m \left(\frac{\mu_n \delta_n^m}{1-\theta}\right)^{1-\theta} \right). \tag{58}
$$

$$
g = \sum_{m=1}^{M_n} \rho_n^m \left(\frac{\mu_n \delta_n^m}{1-\theta}\right)^{1-\theta}. \tag{59}
$$

Then, taking the first-order partial derivative of C_n^{pur} with respect to μ_n, we obtain

$$\frac{\partial C_n^{pur}}{\partial \mu_n} = \frac{\frac{\partial f}{\partial \mu_n} g - f \frac{\partial g}{\partial \mu_n}}{g^2}, \tag{60}$$

where $\frac{\partial f}{\partial \mu_n} g$ and $f \frac{\partial g}{\partial \mu_n}$ are given by (61) and (62) shown at the bottom of the this page, respectively. And then, we obtain (63) shown at the bottom of the this page. Due to $0 < \theta < 1$ and $g^2 > 0$, (60) is greater than 0. Therefore, C_n^{pur} increases with μ_n. □

In particular, our goal is to solve system of (64). Due to the complexity of calculation, the analytical solution cannot be calculated directly. Thus, we propose the unified algorithm to address the utility maximization problem.

Algorithm 1: CONDA: Contract and Data Contribution Determination Function.

input : LMO n; The actual fatigue level μ_n.
output: The number of data samples x_n contributed by LMO n.

1 **for** *each* $i = \{1, 2, \ldots, M_n\}$ **do**
2   Design the contract bundle $\{R_n^i, d_n^i\}$ based on (56) and (51);
3 $C_n^{pur} = (\sum_{m=1}^{M_n} \rho_n^m R_n^m)/(\sum_{m=1}^{M_n} \rho_n^m d_n^m)$;
4 $q_n = P_n \sum_{m=1}^{M_n} \rho_n^m d_n^m$;
5 Obtain the optimal x_n based on Eq. (17);
6 **return** $\{q_n, x_n\}$;

C. Algorithm Design in the Three-Layer FL Framework

We put forward the Contract and Data Contribution Determination Algorithm named CONDA, which is implemented as Algorithm 1. After LMO n determines the actual fatigue level of workers, d_n^i and the reward R_n^i of each type of workers can be determined according to the contract theory (Line 1–2), requiring the computational complexity of $\mathcal{O}(M_n)$. After each contract bundle $\{R_n^i, d_n^i\}$ is designed, the unit data purchase cost C_n^{pur} and the total number of data samples q_n collected by workers can be calculated (Line 3–4). Then, we can obtain the strategy of LMO n in Stackelberg game (Line 5), with the $\mathcal{O}(1)$ computational complexity. Therefore, the computational complexity of CONDA is $\mathcal{O}(M_n)$. Note that the result of the CONDA is a non-feasible solution that only gives the solution of the upper-layer and lower-layer game, respectively.

Then, we propose the Feasible Data Contribution Determination Algorithm called FEDA to solve the non-feasible problem above. Note that, the feasible data contribution quantity x_n can be calculated by the binary search method in Algorithm 2. The upper bound of the binary search is calculated by CONDA (Line 1). If the number of data samples contributed by LMO n is greater than the maximum number of data that the workers can provide, the strategy of LMO n is q_n (Line 2–3). After $[0, \mu_n^{\max}]$ is specified as the range of binary search for the actual fatigue level (Line 4), the feasible data contribution of LMO n is calculated in an iterative method (Line 5–12), with the $\mathcal{O}(\log(\mu_n^{\max}/\varepsilon))$ computational complexity. And for each μ_n, it takes the computational complexity of $\mathcal{O}(M_n)$ with CONDA algorithm. Thus, the computational complexity of FEDA is $\mathcal{O}(M_n \log(\mu_n^{\max}/\varepsilon))$ and the result of FEDA is a feasible solution for (64).

Finally, inspired by the structure of our hierarchical-based game framework for FL, we propose an iterative Hierarchical-based Utility Maximization Algorithm called HUMA as shown in Algorithm 3. In Algorithm 3, the TP, LMOs and workers can maximize their own utilities. We initialize the set of remaining LMOs as $\mathcal{M}$ and the unit purchase data of each LMO as 0 (Line 1). Then, each LMO broadcasts its C_n^{cmp} and C_n^{pur} (Line 3–4). The TP determines the optimal strategy τ according to (25) (Line 5). The TP would give up the opportunity to publish the FL task when $\tau < 0$, and would broadcast the payment to all LMOs when $\tau > 0$ (Line 6–9). For each LMO, the FEDA algorithm calculates the feasible solution x_n according to the strategies of TP and other LMOs. If x_n is negative or the utility of LMO n is negative, LMO n quits from the game. Since each LMO is independent and uncorrelated, this part can be computed in parallel, where the computational complexity is

$$C_n^{pur} = \frac{\sum_{m=1}^{M_n} \left(\rho_n^m \mu_n + \delta_n^m \mu_n \Lambda_n^m \sum_{t=m+1}^{M_n} \rho_n^t \right) + \sum_{m=1}^{M_n} \left(c \rho_n^m \left(\frac{\mu_n \delta_n^m}{1-\theta} \right)^{1-\theta} \right)}{\sum_{m=1}^{M_n} \rho_n^m \left(\frac{\mu_n \delta_n^m}{1-\theta} \right)^{1-\theta}}. \tag{57}$$

$$\frac{\partial f}{\partial \mu_n} g = \left(\frac{\mu_n}{1-\theta} \right) \sum_{m=1}^{M_n} \left(\rho_n^m + c \rho_n^m \delta_n^m \left(\frac{\mu_n \delta_n^m}{1-\theta} \right)^{-\theta} + \delta_n^m \Lambda_n^m \sum_{t=m+1}^{M_n} \rho_n^t \right) \sum_{m=1}^{M_n} \rho_n^m \delta_n^m \left(\frac{\mu_n \delta_n^m}{1-\theta} \right)^{-\theta}. \tag{61}$$

$$f \frac{\partial g}{\partial \mu_n} = \mu_n \sum_{m=1}^{M_n} \left(\rho_n^m + \frac{c \rho_n^m \delta_n^m}{1-\theta} \left(\frac{\mu_n \delta_n^m}{1-\theta} \right)^{-\theta} + \delta_n^m \Lambda_n^m \sum_{t=m+1}^{M_n} \rho_n^t \right) \sum_{m=1}^{M_n} \rho_n^m \delta_n^m \left(\frac{\mu_n \delta_n^m}{1-\theta} \right)^{-\theta}. \tag{62}$$

$$\frac{\partial f}{\partial \mu_n} g - f \frac{\partial g}{\partial \mu_n} = \mu_n \frac{\theta}{1-\theta} \sum_{m=1}^{M_n} \left(\rho_n^m + \delta_n^m \mu_n \Lambda_n^m \sum_{t=m+1}^{M_n} \rho_n^t \right) \sum_{m=1}^{M_n} \rho_n^m \delta_n^m \left(\frac{\mu_n \delta_n^m}{1-\theta} \right)^{-\theta}. \tag{63}$$

Algorithm 2: FEDA: Feasible Data Contribution Determination Function.

input : LMO n.
output: The number of data samples x_n contributed by LMO n.

```
1  q_n, x_n = CONDA(μ_n^max);
2  if x_n > q_n then
3      return q_n;
4  μ^l = 0, μ^r = μ_n^max ;
5  repeat
6      μ_n = (μ^l + μ^r)/2;
7      {q_n, x_n} = CONDA(μ_n);
8      if x_n < q_n then
9          μ^r = μ_n;
10     else
11         μ^l = μ_n;
12 until abs(x_n − q_n) < ε;
13 return x_n;
```

Algorithm 3: HUMA: Hierarchical-Based Utility Maximization Algorithm for FL Framework.

```
1  Initialize the number of remaining LMOs |M| = N,
   the data purchase cost C_n^pur = 0 for all LMOs.
2  repeat
3      for each LMO m in M do
4          Broadcast C_m to other LMOs and the TP.
5      Determine the optimal τ based on Eq. (25);
6      if τ < 0 then
7          End this game;
8      else
9          The TP broadcasts τ to all LMOs.
10     for each LMO m in M parallel do
11         x_m = FEDA(m);
12         if x_m ≤ 0 or LMO m's utility < 0 then
13             LMO m quits from this game;
14             M = M\{m};
15 until No LMOs change their decisions;
16 All LMOs in M train the FL model and submit
   results;
```

$\mathcal{O}(maxM_n \log(\mu_n^{\max}/\varepsilon))$ (Line 10–14). The process of the game is completed until no players change their strategies (Line 2–14). The remaining participants, i.e., the LMOs in $\mathcal{M}$, perform the FL task (Line 16). During each iteration, each LMO designs the optimal contract items using the CONDA algorithm. Rational workers will select the contract item that corresponds to their types to maximize their own utilities. By using the FEDA algorithm, the optimal data contribution of each LMO can be calculated, thus achieving the Nash equilibrium proven in Theorem 1. In addition, by using the HUMA algorithm, the TP can compute its optimal payment, achieving Stackelberg game equilibrium proven in Theorem 3, thus ensuring the optimality of HUMA. In the worst case, the game iterates N times until there are no LMOs remaining. Therefore, the computational complexity of HUMA is $\mathcal{O}(maxNM_n \log(\mu_n^{\max}/\varepsilon)), n \in \mathcal{N}$.

VI. Performance Evaluation

In this section, we conduct extensive simulation experiments to evaluate the performance of HUMA for the hierarchical-based game model in FL.

A. Setup

We have conducted experiments on the EMNIST [22] and Fashion-MNIST [23] datasets. The EMNIST dataset is a collection of handwritten character images that extends the original MNIST dataset, while the Fashion-MNIST dataset is a collection

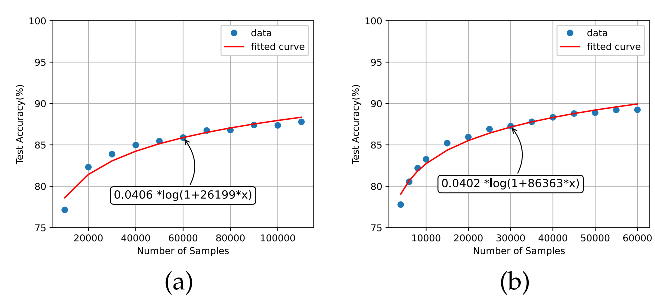


Fig. 2. The model accuracy by changing the number of samples on different dataset. (a) EMNIST. (b)Fashion-MNIST.

of grayscale images of clothing. As shown in Fig. 2, the model accuracy is measured under different numbers of training data samples on the EMNIST dataset and Fashion-MNIST dataset. For the simulation setup, we consider 5 LMOs in the system, i.e., $N = 5$. For each LMO, the number of workers' types is uniformly distributed between [6,10], and the workers' willingness to contribute is selected from the set $\{1, 2, \ldots, 10\}$, which satisfy the monotonicity constraint in Theorem 4. Similar as [17], the parameters are configured as: $c = 0.1, \rho_n^m = \frac{1}{M_n}, E_n^{com} = 4, \theta = 0.4$. We use two distinct neural networks, named CNNFashionMNIST and CNNEMNIST, to handle the

$$\begin{cases} \mu_n = \frac{(1-\theta)d_n^{i\,\frac{1}{1-\theta}}}{\delta_n^i}, \\ q_n = P_n \sum_{m=1}^{M_n} \rho_n^m d_n^m, \\ x_m^* = \frac{(M-1)\tau}{\sum_{n\in\mathcal{M}} C_n^{cmp}+C_n^{pur}} \left(1 - \frac{(M-1)C_n^{cmp}+C_n^{pur}}{\sum_{n\in\mathcal{M}} C_n^{cmp}+C_n^{pur}}\right), \\ C_n^{pur} = \frac{\sum_{m=1}^{M_n}\left(\rho_n^m \mu_n + c\rho_n^m \left(\frac{\mu_n \delta_n^m}{1-\theta}\right)^{1-\theta} + \delta_n^m \mu_n \Lambda_n^m \sum_{t=m+1}^{M_n} \rho_n^t\right)}{\sum_{m=1}^{M_n} \rho_n^m \left(\frac{\mu_n \delta_n^m}{1-\theta}\right)^{1-\theta}}. \end{cases} \tag{64}$$

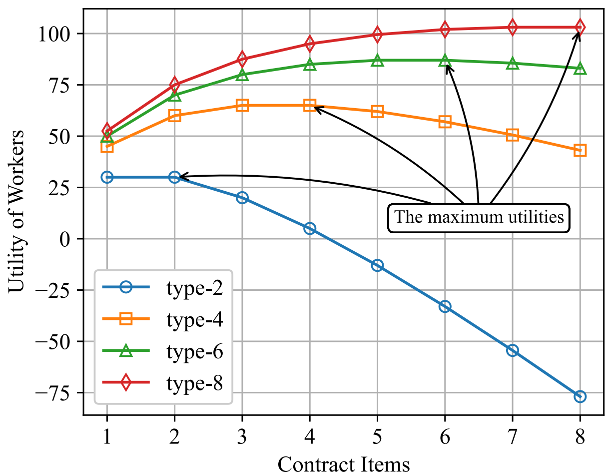


Fig. 3. Utilities of different types of Workers when opting for various contract items.

Fashion-MNIST dataset and EMNIST dataset, respectively. Both networks consist of two convolutional layers and two fully connected layers. The first convolutional layer contains 32 filters with a kernel size of 5 and a padding of 2, while the second convolutional layer consists of 64 filters with a kernel size of 5 and a padding of 2. The two fully connected layers have 512 and 62 output units for CNNEMNIST, and 10 output units for CNNFashionMNIST.

B. Parameter Analysis

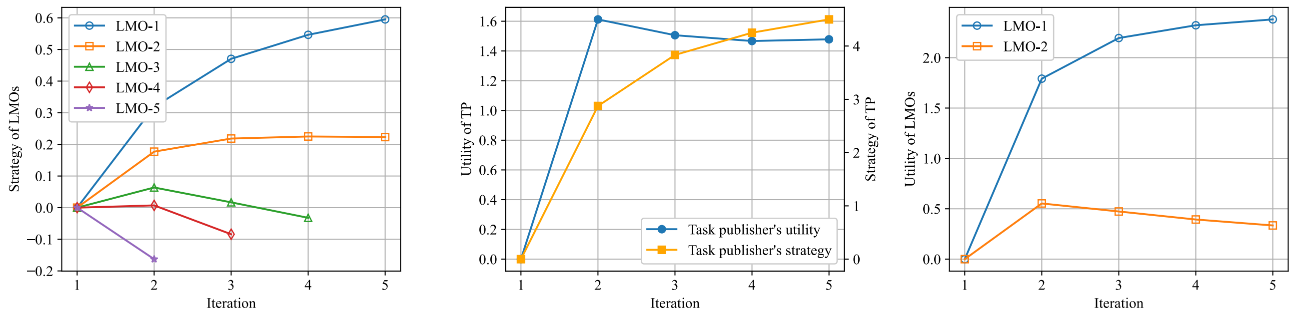
1) Contract Optimality: Fig. 3 illustrates the utilities experienced by different types of workers when opting for various contract items. From Fig. 3, we can observe that workers maximize their utility only by choosing the contract item designed for the corresponding type. Specifically, the type-8 workers will get the maximum utility when choosing the 8th contract item, otherwise they will get lower utility if they report wrong types. In addition, we observe that each worker's utility is non-negative. We can conclude that the contract theory designed in the lower-layer ensures the IC and IR constraints and allows workers to reveal their own type.

2) Pricing Optimality: Next, we evaluate the Stackelberg game process between the TP and LMOs. Fig. 4 shows the change in system performance with the number of iterations. In this experiment, we focus on the interactions between TP and LMOs, ignoring the contract design between each LMO and workers. We consider the unit data purchase cost for LMOs to be constant. As shown in Fig. 4(a), in the process of the game, each LMO continuously adapts its strategy as the system changes. If an LMO's utility turns negative, it quits the game. The whole game process continues until the strategies of all LMOs and TP remain unchanged. Specifically, since the strategy of LMO-5 is negative at the second iteration, we can realize that it is at a disadvantage in the game and quits the game. Similarly, LMO-4 and LMO-3 quit the game in the third and fourth iterations, respectively. After the fifth iteration, the strategies of LMO-1, LMO-2, and TP remain unchanged, and the FL task begins to be executed. In Fig. 4(b), with an increase in the number of iterations, the TP increase the payment to incentive more LMOs to engage, consequently leading to a decrease in the TP's utility. In Fig. 4(c), it is observed that the utility of LMO-1 is increasing and the utility of LMO-2 is decreasing from the second iteration onwards.

Fig. 5 shows the impact of performance when varying the strategy of LMOs and TP. In Fig. 5(a), for each LMO, its utility function is concave with a unique extreme value when the strategies of other LMOs reach the optimal solution and remain unchanged. For example, after the utility of LMO-2 reaches the optimal solution, the utility of LMO-1 is increasing from 0 to 1.06 and starts decreasing after reaching the maximum value at 1.06. The same is true for LMO-2, which verifies the existence of Nash equilibrium in Theorem 1. In Fig. 5(b), when the payment τ is determined by the TP, all LMOs will determine their optimal strategy, and we can obtain the utility of the TP under the current strategy. It is observed that the TP is increasing from 0 to 4.5 and then starts to decrease. The point at which it reaches a maximum is the Stackelberg equilibrium in Theorem 3. Therefore, the TP can obtain the optimal solution by determining its payment through the Stackelberg equilibrium.

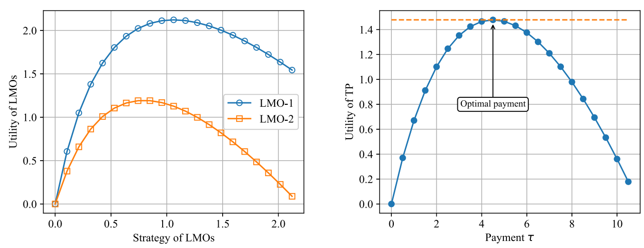
Then, we illustrate the effect of unit training cost of LMO on the system. Specifically, we study it by changing the unit computation cost C_2^{cmp} of LMO-2 from 1.8 to 3.4. In Fig. 6(a), it is observed that the participation level of all LMOs decreases as the training cost of LMO-2 increases. For example, when the unit computation cost of LMO-2 is 1.8, the number of data samples contributed by LMO-1 and LMO-2 are 1.1 and 0.92, respectively. But as LMO-2's unit computation cost is increased to 3.4, the number of data samples contributed by LMO-1 and LMO-2 is 0.72 and 0.32. Through the analysis of Stackelberg game, we know that the increase in the unit computation cost of LMO-2 will lead to the disadvantage of LMO-2 in stage II of the Stackelberg game. In Fig. 6(b), is is observed that as LMO-2's unit computation cost increases, the utility of LMO-2 is decreasing, while the utility of LMO-1 is increasing. For example, when the unit computation cost of LMO-2 is 1.8, the utilities of LMO-2 and LMO-1 are 1.38 and 2.0, respectively. As LMO-2's unit computation cost is increased to 3.4, the utility of LMO-2 decreases to 0.49, while the utility of LMO-1 increases to 2.49. As in the analysis of Fig. 6(a), the competitiveness of LMO-2 in the non-cooperative game decreases. And we can notice that as the computation cost of LMO-2 increases, the average utility of LMOs decreases. Considering the LMOs as a whole, there is more resource consumption due to the increase in computation cost, which in turn leads to a decrease in the utility. Fig. 6(c) demonstrates a decrease in both the strategy and utility of TP with increasing unit computation cost of LMO-2 increases. To sum up, the total number of data samples provided by LMOs decreases, thus, the strategy and utility of TP also decreases.

3) Algorithm Performance: Fig. 7 shows how the iterative process of Algorithm 3 HUMA influences the outcomes of LMO-1, LMO-3, LMO-5 and LMO-7. As shown in Fig. 7(a), the actual fatigue level μ of workers in each LMO can be solved by the Algorithm 2 FEDA. For example, the actual fatigue level sequence of workers in LMO-1 is (50, 25, 12.5,



(a) (b) (c)

Fig. 4. The impact of performance when varying the number of iterations. (a) The strategy of LMOs. (b) The utility and strategy of TP. (c) The utility of LMOs.



(a) (b)

Fig. 5. The impact of performance when varying the strategy. (a) The strategy of LMOs. (b) The strategy of TP.

18.75, 15.625, 14.0625, 13.28125, 12.890625, 13.0859375, 12.98828125, 12.98828125). After the 11th iteration, each LMO designs contract items for the workers according to μ. From Fig. 7(b), we can observe that the strategy of each LMO is continuously adjusted during the iterative process and finally stabilized. Similarly, in Fig. 7(c), the TP's strategy is also dynamically adjusted to maximize its utility as the LMOs' strategies changes. From Fig. 7, it is evident that the proposed algorithm reaches a stable solution after only a few iterations.

C. Comparison Results

This subsection focuses on employing benchmark schemes to assess the efficacy of the proposed three-layer FL framework.

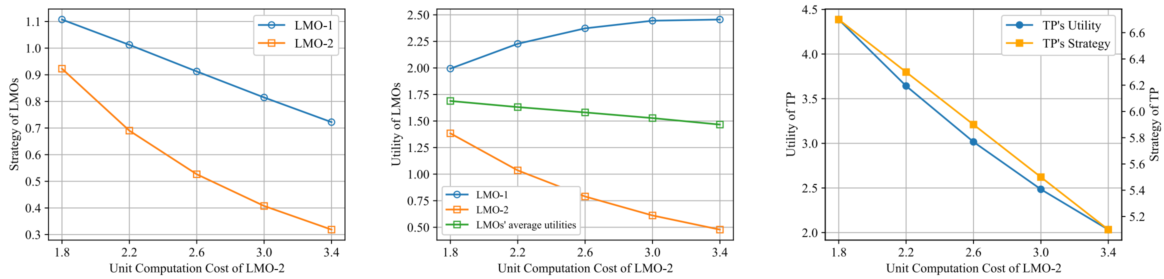
1) The Lower-Layer: Inspired by [9], [41], we discuss the following two benchmark schemes, aimed at assessing the dynamics and performance of interactions among the TP, LMOs, and workers.

- *Uniform pricing scheme:* This scheme implies that the LMO needs to pay the same price per unit of data to all workers. Obviously, LMO prefers high-quality workers to work for it, but since the attributes of each type of workers are private, LMO cannot specify which workers work for it. In addition, due to the IR constraints, the LMO must to ensure the utility for type-1 workers and to formulate the scheme accordingly.
- *Discriminatory pricing scheme:* This scheme means that the LMO needs to pay different prices per unit of data to different types of workers. Similar to the uniform pricing scheme, this scheme cannot specify workers to work for it. To obtain higher utility and satisfy the IR constraints, the LMO will reduce the unit price as much as possible.

Fig. 8 illustrates how various pricing schemes affect the performance of different worker types. In Fig. 8(a), we can observe that in the discriminatory pricing scheme, each type of workers' utility is 0. Moreover, the utility of each type of workers is higher under the uniform pricing scheme compared to our proposed scheme. In Fig. 8(b), the reward for workers increased with their types. Fig. 8(c) shows the unit price paid by LMO for different worker types to collect data.

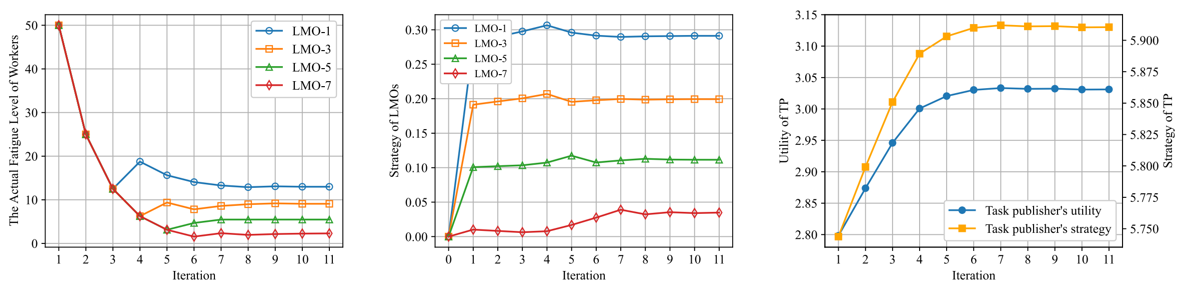
Fig. 9 illustrates the utility of choosing different contract items for different worker types under the discriminatory pricing scheme. We can observe that for different worker types, the unit price set for them by the LMO makes their utilities zeros, corresponding to the limited utilities points in the figure. However, due to the information asymmetry, workers have an incentive to hide their types to choose another unit price that maximizes their utility, i.e., the maximize utilities point in the figure. Thus, the discriminatory pricing scheme does not have the desired effect in a scenario involving information asymmetry.

To sum up, in the uniform pricing scheme, because each worker has to satisfy the IR constraints in Definition 2, the LMO is priced according to the utility of type-1 workers, resulting in high pricing and the increased utility for worker but decreased utility for the LMO. Each worker collects data for the same unit price, and they have no need to hide their types. In our proposed hierarchy-based game scheme, the quantity of collected data samples and the reward received by each type of workers are designed by contract theory. And the self-revealing nature guarantees that workers do not conceal their types. From Fig. 8, we know that the utility, reward and unit price for each type of worker based on the uniform pricing scheme surpass those in our proposed scheme. Thus, the LMO and TP can obtain higher utility in our proposed scheme. In the discriminatory scheme, the utility, reward and unit price of workers are the lowest. If the scheme works as expected, the utility of LMO and TP will be the highest among these schemes. As shown at the maximum utilities points in Fig. 9, the workers have an incentive to conceal their types to maximize their utility, the scheme will not work in practice. Specifically, the unit price



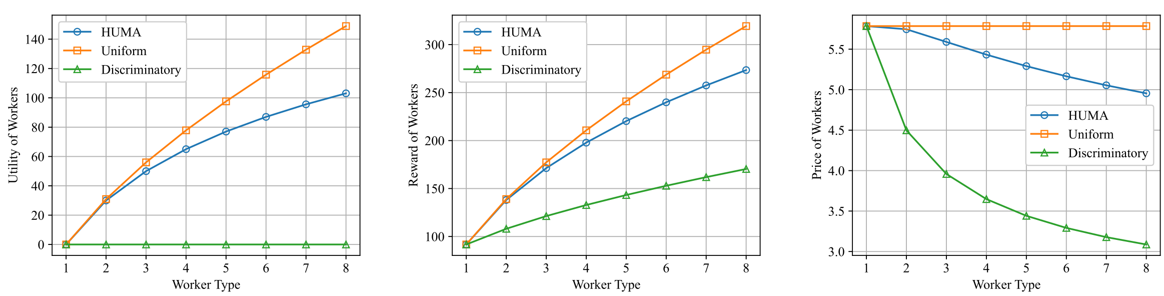
(a) (b) (c)

Fig. 6. The impact of performance when varying the unit computation cost of LMO-2. (a) The strategy of LMOs. (b) The utility of LMOs. (c) The utility and strategy of TP.



(a) (b) (c)

Fig. 7. The impact of performance in hierarchy-based game when varying the number of iterations. (a) The actual fatigue level μ of workers. (b) The strategy of LMOs. (c) The utility and strategy of TP.



(a) (b) (c)

Fig. 8. The impact of system performance for various types of workers under different schemes. (a) The utility of workers. (b) The reward of workers. (c) The price of workers.

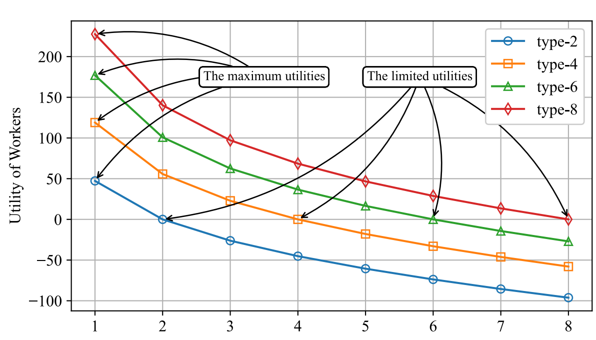


Fig. 9. The utilities of different types of workers when opting for various prices within discriminatory pricing scheme items offered by the LMO.

for type-1 workers in the scheme is the highest at 5.78, and all other types of workers pretend to be type-1 workers. Then, the unit price for all workers is 5.78, which is the uniform pricing scheme.

2) The Upper-Layer: The following two benchmark schemes are discussed to evaluate the performance of the interactions between TP and LMOs based on Stackelberg game.

- *Random pricing scheme:* At the beginning of each iteration, the TP randomly sets the payment τ from 0 to τ^*, where τ^* is the optimal strategy of TP.
- *Fix pricing scheme:* The TP sets a fixed payment τ and the strategies of all LMOs form a Nash equilibrium under τ. We consider that the fixed τ is 10 in the following.

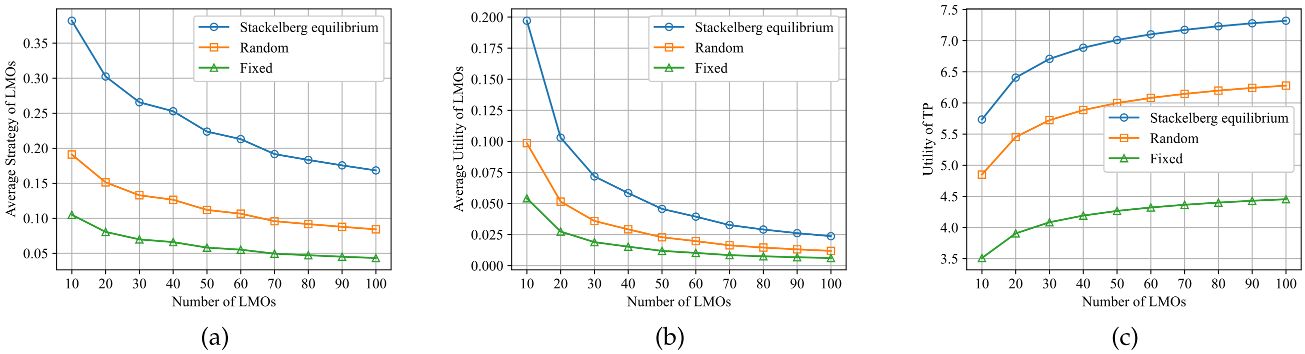


Fig. 10. The performance impact when adjusting the number of LMOs. (a) The strategy of LMOs. (b) The utility of LMOs. (c) The utility and strategy of TP.

Fig. 10 shows how varying the number of LMOs affects system performance across different pricing schemes. In this experiment, we randomly select a range of 1 to 1.5 unit computation cost and unit data purchase cost for each LMO. From Fig. 10(a) and (b), we observe that as more and more LMOs participate in the game, the competition among LMOs increases, leading to a decrease in the average strategy and average utility of LMOs. From Fig. 10(c), it is observed that the utility of TP increases with the number of LMOs. As shown above, it is evident that our proposed scheme outperforms for the others.

VII. Conclusion

In this paper, we have proposed a three-layer hierarchical game framework in FL, including a TP, multiple LMOs, and multiple types of workers. Because of the incentive mismatch problem in the framework, we decomposed the framework into two sub-layer games. For the upper-layer game, an incentive mechanism based on the Stackelberg game was adopted to incentivize LMOs to increase their level of participation. For the lower-layer game, contract theory was applied to address the problem of information asymmetry between LMOs and workers. Then, the uniform algorithm HUMA was developed to solve the utility maximization problem for the hierarchical game framework. In the hierarchical framework, we have captured the interactions among TP, LMOs and workers when the data provided by LMOs do not meet the requirements for model training, and designed the incentive mechanism accordingly to motivate them to join FL. Finally, the effectiveness and advantages of our scheme have been demonstrated through numerical experiments. In our future work, we will further consider the dynamic arrival of tasks and the environment (e.g., the time-varying channel) and study the adaptive framework of hierarchical incentive mechanism for FL. Moreover, we will consider the more complex privacy concerns and investigate that different types of worker are associated with different quality-levels of training samples in the hierarchical incentive mechanism framework.

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