MIE 1603 / 1653 - Integer Programming
Winter 2020
Assignment #1

Due Date: January 21, Tuesday, no later than 11:59PM.

- You may work in groups of two students (from the same course code) or individually. In the former case, turn in *only one* solution set with both group members listed on it.
- Groups must work independently.
- You **must** cite any references (texts, papers, websites, etc.) you have used to help you solve these problems.
- Submit your written/typed solutions via Quercus (pdf file only). For Questions 1 and 2, additionally submit your code along with your solutions in Quercus.

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1. [20pts] UofT catering company purchases coffee from three major brands: Starbucks, Second Cup and Tim Hortons. Each brand offers packages of 100 bags with different combinations of coffee roasts. The cost and combination of the coffee bags from each brand are given in Table 1:

   Table 1: Cost and combination of orders

Brand	Cost/package	% Dark	% Medium	% Light
Starbucks	512	42	45	13
Second Cup	451	32	37	31
Tim Hortons	373	24	22	54

   Each month the company places orders with each brand. At least 330 dark, 300 medium and 100 light roast coffee bags must be purchased each month. Because of limited inventory, at most 500 coffee bags per month can be purchased from each brand. How many packages UofT catering would order from each company to minimize the total purchase cost?

   (a) Model this problem using (Mixed) Integer Linear Programming. Clearly state and explain all the components of your model (e.g., variables, constraints and objective function).

   (b) Implement your model in Gurobi and report the results, including the optimal cost and number of orders. Also provide a printout of the output of your code (i.e., what is printed to the screen when you run your code).

   (c) Modify your model in part (a) such that you cannot order from Second Cup and Tim Hortons at the same time. How does your solution change?

2. [40pts] In his famous book "Reason", Isaac Asimov described a space station located in outer space that collects energy and transmits it across the solar system. The idea of Space-Based Solar Power (SBSP) originated from this science fiction and is still alive and attractive after 50

years of research. Despite vast investments from well-known space agencies across the globe like NASA and JAXA, there has been no practical proposal for actually building it.

The main idea of an SBSP system is that a set of reflectors are collecting the solar energy. Then, a set of wireless power transmitters transmit the generated power to Earth via microwave, with high efficiency. Assume that part of a viable proposal is being able to plan and schedule these wireless power transmitters such that the associated costs are minimized.

Your job is to plan and schedule an SPBS system with 4 transmitters for 5 days. Table 2 presents the amount of power generated by the reflectors sent for transmission in each day. If on a certain day you don't have enough transmitters, the remaining power is stored in solar cells, which can be transmitted in the next day. The cost of storage of 1 kW power is given in Table 2 for each day. To prevent overheating, a transmitter cannot be used for more than two consecutive days (i.e., if it was on for two consecutive days, it has to be off for at least one day before getting turned on again). Every transmitter has a transmission rate (the maximum amount of power a transmitter can transmit in a day) and cost of transmission per 1 kW, shown in Table 3. We want to know how much every transmitter is used during a day in an optimal plan.

Time	$t=1$	$t=2$	$t=3$	$t=4$	$t=5$
Generated power (kW)	214	532	623	545	871
Storage cost (\$/1kW)	95	65	121	79	400

Table 2: Generated power over 5 days

Transmitter	a	b	c	d
Transmission rate (kW/day)	350	250	200	150
Transmission Cost (\$/kW)	200	136	95	78

Table 3: Transmission rate of the transmitters

(a) Model this problem using (Mixed) Integer Linear Programming. Clearly state and explain all the components of your model (e.g., variables, constraints and objective function).

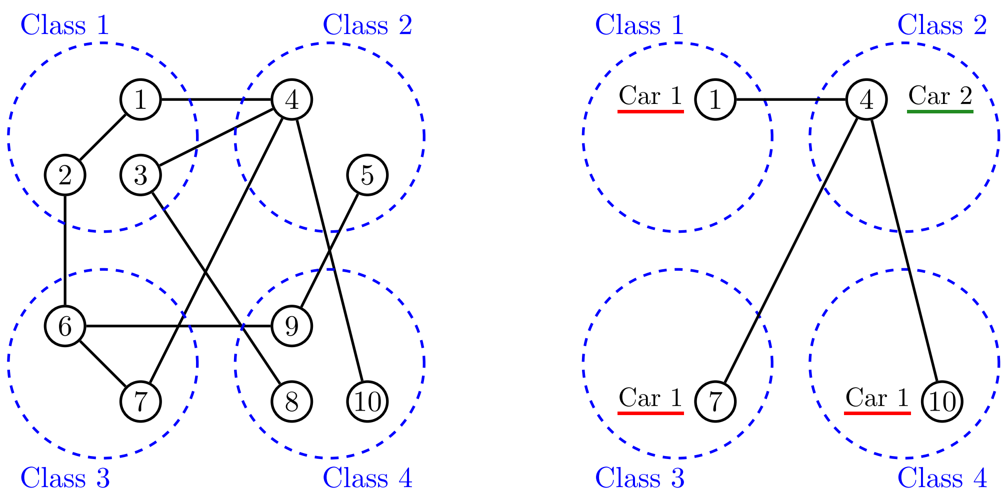
(b) Read the data from Q2_data.xls file. Implement the model in Gurobi and report the results, including the cost and the details of your optimal plan, along with a printout of the output of your code.

(c) Assume that we have two special transmitters e and f that each can only be used once during the planning period, but transmitter f can only be used after transmitter e is used to transmit power. Explain how to modify your model in part (a) accordingly.

3. [40pts] A Toronto middle school (with a set of students S) is choosing one representative student from each class $c \in \{1, \ldots, C\}$ to attend a national student conference on industrial engineering in Montréal. The set of students in class c is denoted by S_c. The school would like

to select the students so that the number of parent chaperones who have to drive the students is minimized. As is normal with teenagers, not all of the students get along, i.e., they can have a conflict. If two students have a conflict, they cannot travel together in the same vehicle.

This problem can be represented by a graph $G = (S, E)$ where S is the set of nodes (students) and E is the set of edges. There exists an edge $e = (i, j)$ between students i, j if they have a conflict. The nodes (students) are partitioned into a set of clusters (classes) C and clearly a student can only be in one class. An example is given in Figure 1. In the example, students 1,4,7 and 10 are chosen as class representatives, however students 4 and 7 have a conflict and so cannot travel in the same car. We see there is a feasible solution which uses two cars, however an optimal solution would be to select students $3, 5, 7, 10$ and take only one car.



(a) A student-conflict graph with four classes.

(b) An feasible solution which requires two parent chaperones to drive.

Figure 1: A small example of a student-conflict graph and an feasible solution to the problem.

(a) Model the above problem using (Mixed) Integer Linear Programming. Clearly state and explain all the components of your model (e.g., variables, constraints and objective function).

(b) Modify your model in part (a) to consider that each parent has a total of three seats in their car. Explain any new variables and/or constraints that you use.

(c) **(BONUS, [5pts])** Given an optimal solution to the model in part (a) which uses n chaperones, that is, a selection of students and assignment to cars, we can create another optimal solution by simply re-indexing the cars. For instance, in the example solution provided in Figure 1b, we can assign students 1,7 and 10 to Car 2, and assign student 4 to Car 1, to obtain another feasible solution.

   (i) How many possible solutions can we achieve by re-indexing the cars?

(ii) Is it possible to modify the model in part (a) to eliminate these “repeated” solutions? Explain any new variables and/or constraints that you use.